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Addendum to Neutral Deformation Modes of Minimal Surfaces

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Addendum

Our paper [1] examines the independent deformation modes posited by the elastic theory [2] in the special setting of deformations from minimal surfaces into minimal surfaces. We find that stretching neutral deformations are also drilling neutral, and that drilling neutral deformations are also bending neutral.

Since publication, the two papers [3, 4] have appeared that study the consequences of [2] in a more general setting. Both prove that, as announced in the conclusions of [1], stretching neutral modes are drilling neutral not only for deformations of minimal surfaces but for *any* surfaces. Furthermore, the authors of [3] show by example that drilling neutral deformations of minimal surfaces do not necessarily have to be bending neutral. They point out that this implication remains valid for *conformal deformations* of minimal surfaces, which they claim is implicitly the case in the setting of [1]. This is indeed evident from the form of equations (28) and (29) in [1], see Remark 3, equation (56) in [4].

The deformations considered in [1] are actually *explicitly* conformal, a fact which we should have made clearer. Equation (12) in [1] defines the deformation $\boldsymbol{y}$ as

$$\boldsymbol{y} \circ \boldsymbol{r} = \boldsymbol{r}^* \circ h \tag{1}$$

in terms of a conformal mapping h between two domains on which $\boldsymbol{r}$ and $\boldsymbol{r}^*$ are produced by Weierstrass representations. As these latter are always conformal, also $\boldsymbol{y}$ is clearly conformal. The Weierstrass representation theorem [5, p. 112] and Riemann's mapping theorem [6, p. 283] guarantee the existence of such a deformation $\boldsymbol{y}$. Thus given two minimal surfaces, there is always a conformal deformation of one onto the other. But it remains of course possible to deform minimal surfaces in a non-conformal fashion.

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