Supersolid light via diffractive coupling

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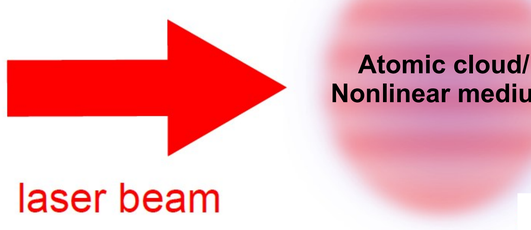
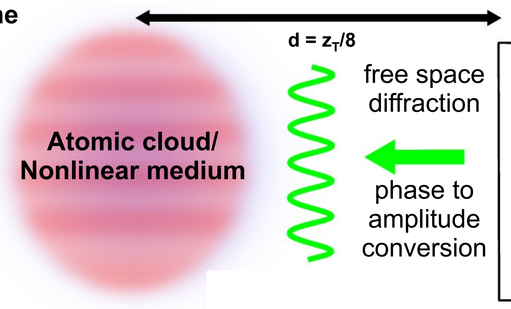
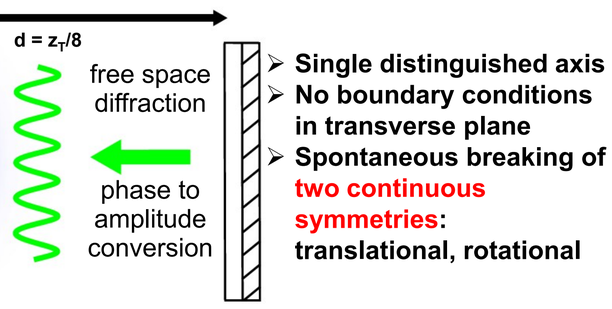
Self-organized light-matter phases via diffractive coupling:

Single mirror feedback scheme
Firth 1990 [1]







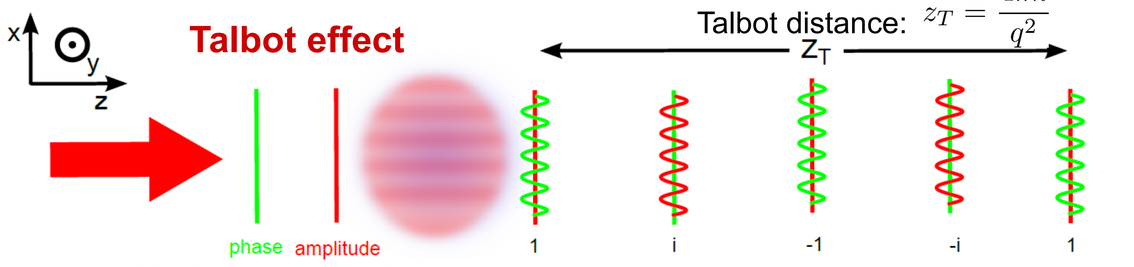
- **Single distinguished axis**
- **No boundary conditions in transverse plane**
- **Spontaneous breaking of two continuous symmetries: translational, rotational**

$$\mathbf{E}(\mathbf{r}_\perp, z+l) = \mathbf{E}(\mathbf{r}_\perp, z)e^{ikn_q l},\ n_q = n_0 + \Delta n\cos(\mathbf{q}\cdot\mathbf{r}_\perp) \rightarrow \mathbf{E}(\mathbf{r}) \approx \mathbf{E}_0(1 + i\epsilon\cos(\mathbf{q}\cdot\mathbf{r}_\perp))$$

$$\frac{\partial \mathbf{E}(\mathbf{r})}{\partial z} = -\frac{i}{2k}\Delta_\perp \mathbf{E}(\mathbf{r}) \ \rightarrow\ \mathbf{E}(\mathbf{r}) = \mathbf{E}_0(1 + i\epsilon e^{i\frac{\mathbf{q}^2}{2k}z}\cos(\mathbf{q}\cdot\mathbf{r}_\perp))$$

→ amplitude modulation from atoms
→ positive feedback on atoms

Talbot effect



Maximum amplitude modulation: $z = (n + 1/4)z_T,\ n = ..., -2, -1, 0, 1, 2, ...$

Supersolid:
- intriguing state of "matter" possessing superfluidity and spatial structure
- breaking spontaneously phase and translational symmetry

Coupled light-matter structures:
- "Trace" over light: atomic structure via light-mediated interaction
  - Matter = BEC → supersolid
- "Trace" over matter: **fluid of light [7]**
  - Unidirectional propagation of light in nonlinear medium realizes time evolution of Gross-Pitaevskii equation with contact interactions
  - No viable experimental scheme to realize long-range, oscillatory interactions in unidirectionally propagating fluid of light
  - However, **cavities or counter-propagating fluids of light** allow this and hence **supersolidity of light**
- Here: simplest system where counterpropagating fluid generated by feedback

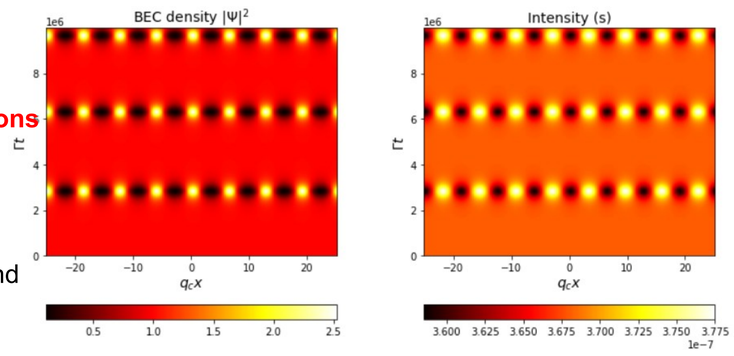
$$|E_b(\vec{r}, t + 2d/c)|^2 = R|E_0|^2\left[1 + \pi\int\int \phi(\vec{r'}, t)\cos\left(\frac{(\vec{r} - \vec{r'})^2}{4}\right)dx'\,dy'\right], \quad \phi \text{ Nonlinear phase}$$

Similar expression first derived in [8]

$$= R|E_0|^2\left[1 + \pi\int\int |E_b|^2(\vec{r'}, t)\cos\left(\frac{(\vec{r} - \vec{r'})^2}{4}\right)dx'\,dy'\right] \quad \text{For Kerr medium } \Delta\phi \sim |E|^2$$

BEC supersolid

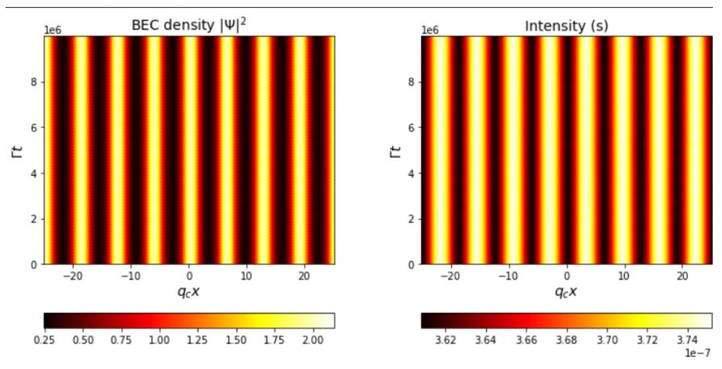
Weak seed [4]:
$|\psi(0)|^2 = 1 + 10^{-6}\cos(q_c x)$



- **Strong, persistent oscillations**
- Like in Hamiltonian Mean Field model [3,4]
- Interpretation: Strong **excitations** as total energy does not match ground state energy
- Alternative phrasing: excitation of **Higgs-mode**
- **Known to be long –lived in long-range coupled systems**

Strong seed:
$|\psi(0)|^2 = 1 + 0.48\cos(q_c x)$

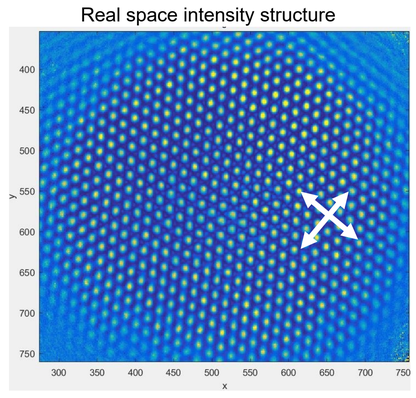
- **Essentially stationary supersolid**



Light supersolid

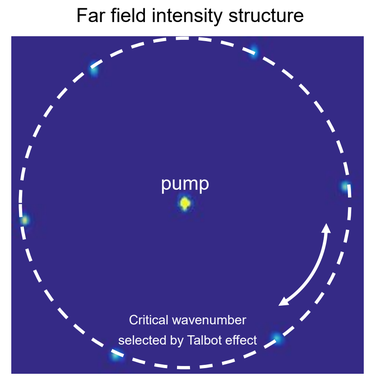
Experiment: Large thermal Rb cloud (N ≈ 10^{11}) slightly above Doppler temperature
δ = 11 Γ from F = 2 → F' = 3 line, OD 200 in line centre (similar to [2])

Real space intensity structure



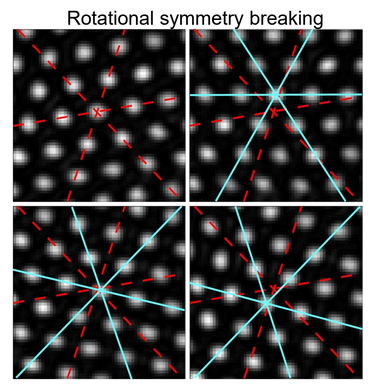
Potential **translational Goldstone modes**

Far field intensity structure



hexagon selected from all possible superpositions

Rotational symmetry breaking



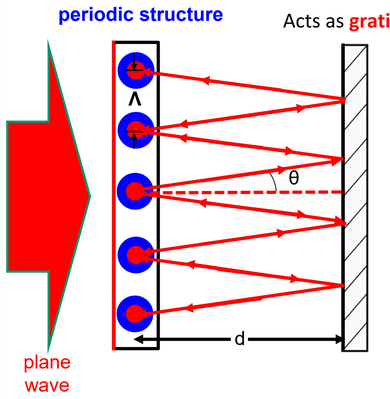
Translational symmetry breaking

✓ **Spontaneous periodic structure** ✓ **Coherence** ✓ **Spontaneous symmetry breaking**

Theory: Conservative Vlasov equation (based on [5])

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} - \frac{1}{2}\hbar\delta\frac{\nabla s(\mathbf{x})}{M}\cdot\frac{\partial f}{\partial \mathbf{v}} = 0 \qquad \rho(\mathbf{x}) = \int_{-\infty}^{+\infty} f(\mathbf{x}, \mathbf{v})d\mathbf{v}$$

periodic structure Acts as **grating**

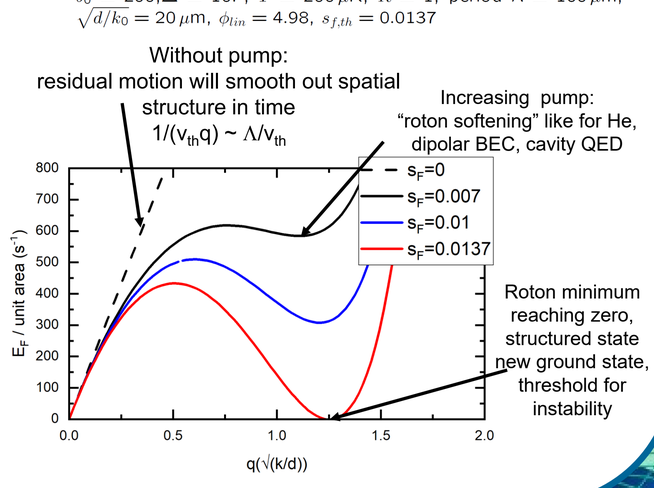


- Diffractive coupling between neighbouring spots stabilizes **defined periodicity**
- Photons **flow** between neighbouring spots

✓ Both features expected for supersolid light

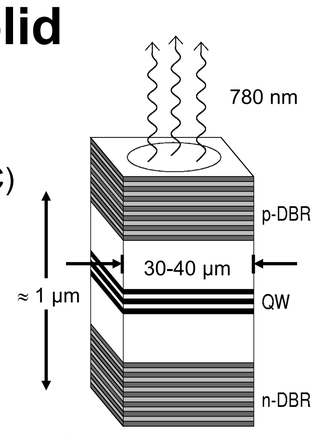
Roton spectrum

$b_0 = 200, \Delta = 10\Gamma,\ T = 200\,\mu\text{K},\ R = 1,$ period $\Lambda = 100\,\mu\text{m}$, $\sqrt{d/k_0} = 20\,\mu\text{m},\ \phi_{lin} = 4.98,\ s_{f,th} = 0.0137$



Laser light supersolid

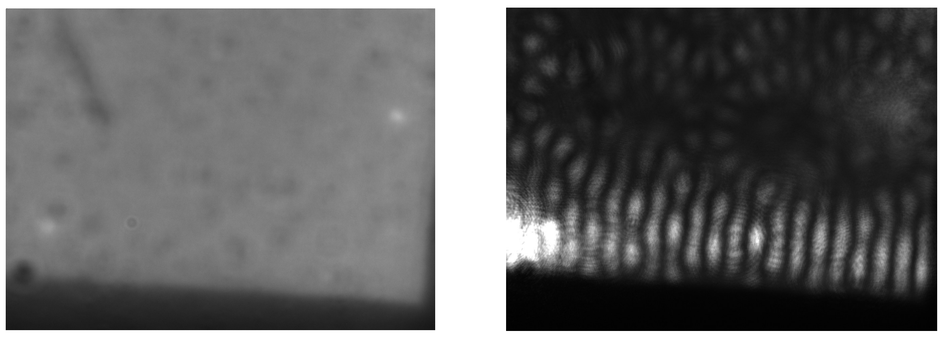
- **Nonlinear laser modes [6] as light supersolid**
  - Break phase and translational symmetry
  - Necessarily dissipative (similar to polariton BEC)
- Semiconductor microcavity in weak coupling regime (Vertical-cavity surface-emitting laser = VCSEL)
- Square aperture providing hard boundary conditions discrete supersolid (similar to cavity QED in single single-mode cavity)



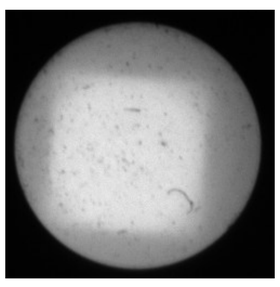
I = 5 mA thermal state

I = 13.7 mA (disordered) supersolid

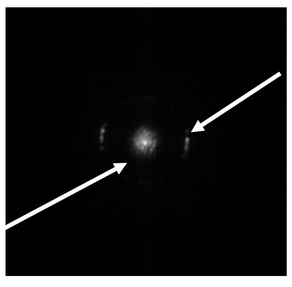
real space



Fourier space



preliminary



On-axis Background, Localized modes due to disorder

Summary and outlook

- **BEC supersolid**:
  - Strong, persistent oscillations or stationary supersolid depending on seeding
  - Connection to time crystals circumventing no-go-theorem as long-range coupled system
- **Light supersolid:** idea to "trace" over material and look only at light structure
  - Experiment demonstrates coherence and spontaneous symmetry breaking
  - Theory yields roton spectrum
- **Laser supersolid:** exciting candidate for out-of-equilibrium light supersolids
  - Connection to photon condensation

References

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