Supersolid light via diffractive coupling

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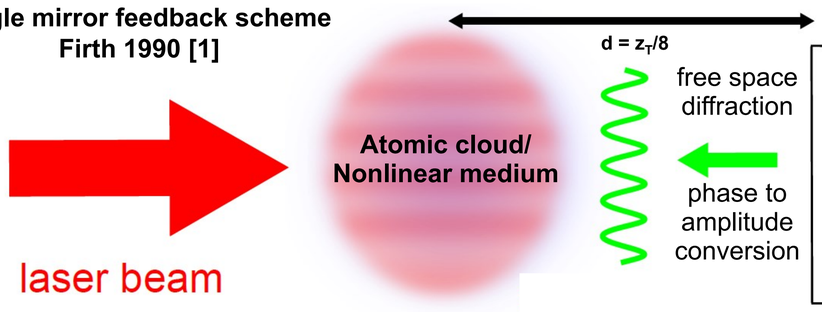
Light-mediated interactions via diffractive coupling:

Single mirror feedback scheme
Firth 1990 [1]



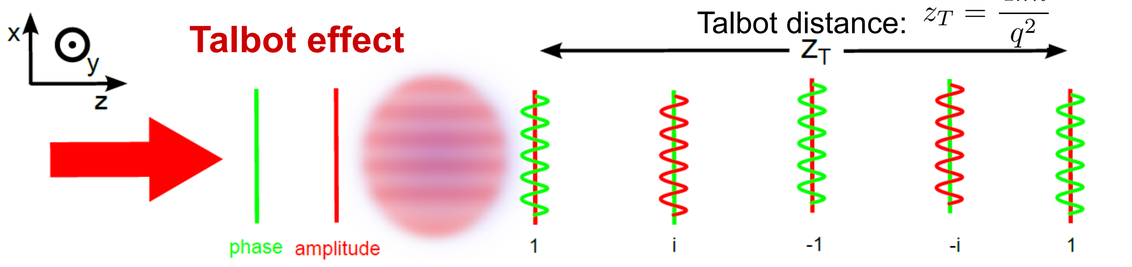
- **Single distinguished axis**
- **No boundary conditions in transverse plane**
- **Spontaneous breaking of two continuous symmetries: translational, rotational**

$$\mathbf{E}(\mathbf{r}_\perp, z+l) = \mathbf{E}(\mathbf{r}_\perp, z)e^{ikn_q l},\ n_q = n_0 + \Delta n\cos(\mathbf{q}\cdot\mathbf{r}_\perp) \rightarrow \mathbf{E}(\mathbf{r}) \approx \mathbf{E}_0(1 + i\epsilon\cos(\mathbf{q}\cdot\mathbf{r}_\perp))$$

$$\frac{\partial \mathbf{E}(\mathbf{r})}{\partial z} = -\frac{i}{2k}\Delta_\perp \mathbf{E}(\mathbf{r}) \ \rightarrow\ \mathbf{E}(\mathbf{r}) = \mathbf{E}_0(1 + i\epsilon e^{i\frac{\mathbf{q}^2}{2k}z}\cos(\mathbf{q}\cdot\mathbf{r}_\perp))$$

Talbot effect

Talbot distance: $z_T = \frac{4\pi k}{q^2}$



Maximum amplitude modulation: $z = (n + 1/4)z_T,\ n = ..., -2, -1, 0, 1, 2, ...$

Nonlinear light-matter coupling: dipole force → spontaneous crystallization
Cold, thermal atoms [2]

Nonlinear medium: quantum degenerate gas / BEC → Supersolids [3]

Dilute BEC @ T=0
Interatomic scattering a=0

BEC dynamics

$$i\frac{\partial\psi(x,t)}{\partial t} = \left[-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} + \frac{\delta}{2}s(x,t)\right]\psi(x,t)$$

Optical dipole potential

Optical field evolution in BEC (phase encoding)

$$\frac{\partial F(x,z,t)}{\partial z} = -i\frac{b_0}{2\Delta L_z}|\psi(x,t)|^2 F(x,z,t)$$

Atomic density

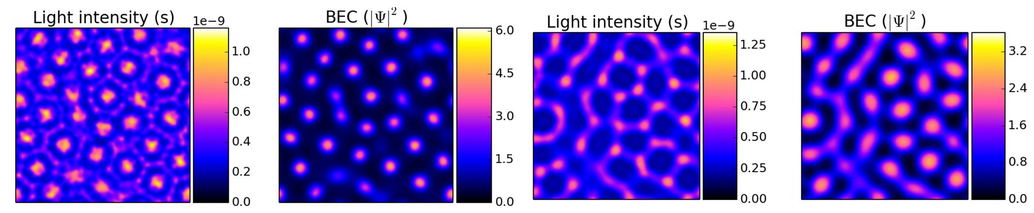
Optical field evolution in free space (diffractive dephasing)

$$B(q) = \sqrt{R}\,F_{tr}(q)\exp\left(i\frac{q^2 d}{k_0}\right)$$

Interacting intensity
s(x) = |F|²+|B(x)|²
sum of forward and backward intensities

Red detuning **Blue detuning**

Just after Saturation of instability: Quite regular



Hamiltonian Mean Field (HMF) Model

Archetypical model for long-range coupled systems
- **N particles on a ring experiencing pairwise cosine interaction**
- **Self-gravitating systems, X-Y rotors, e.g. [4]**

$$H_N = \sum_{i=1}^{N}\frac{p_i^2}{2} + \frac{J}{2N}\sum_{i,j}\left[1-\cos(\theta_i-\theta_j)\right] \qquad H_N = \sum_{i=1}^{N}\frac{p_i^2}{2} + \frac{NJ}{2}\left(1-m^2\right)$$

$$m_x = \frac{1}{N}\sum_{i=1}^{N}\cos\theta_i \qquad m_y = \frac{1}{N}\sum_{i=1}^{N}\sin\theta_i$$

- **Extension to quantum HMF: Chavanis [5], Plestid & O'Dell [6]**
- **Space-time oscillations, chevrons, solitons [6]**

Derivation of HMF [7]

Large detuning → small susceptibility

$$F_{tr} = \sqrt{p_0}\exp\left(-i\chi_0 n(x,t)\right) \approx \sqrt{p_0}(1 + i\chi_0 n(x,t)); \qquad n(x,t) = |\Psi(x,t)|^2$$

Critical Fourier component $\quad \mathrm{n}^{(q_c)} = \frac{1}{L_\perp}\int_0^{L_\perp}|\Psi(x,t)|^2 e^{-iq_c x}\,dx$

Define $\quad \theta = q_c x \qquad \mathrm{n}^{(q_c)}|\cos(q_c x - \phi) = \frac{1}{2\pi}\int_0^{2\pi}|\Psi(\theta',t)|^2\cos(\theta-\theta'))\,d\theta'$

Motivates definition of nonlocal potential $\quad \Phi(\theta,t) = \frac{1}{2\pi}\int_0^{2\pi}|\Psi(\theta',t)|^2\cos(\theta-\theta'))\,d\theta'$

Effective Gross-Pitaevskii equation $\quad i\frac{\partial\Psi'}{\partial t} = -\omega_r\frac{\partial^2\Psi'}{\partial\theta^2} - \epsilon\Phi(\theta,t)\Psi'$

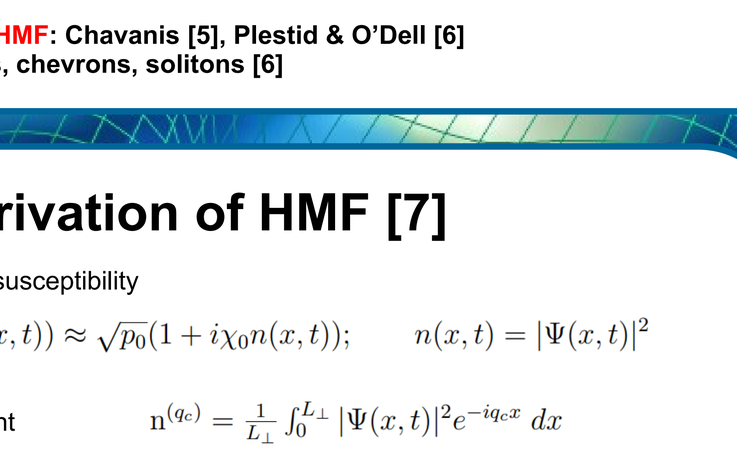
Ferromagnetic: ε > 0 $\quad \epsilon = 2\delta R p_0\chi_0 = \frac{Rp_0 b_0\Gamma}{2}$ $\quad$ Threshold: ε = 1

Order parameter: Magnetization $\quad M = \left|\frac{1}{2\pi}\int_0^{2\pi}|\Psi|^2 e^{i\theta}\,d\theta\right|$

Results: BEC supersolids

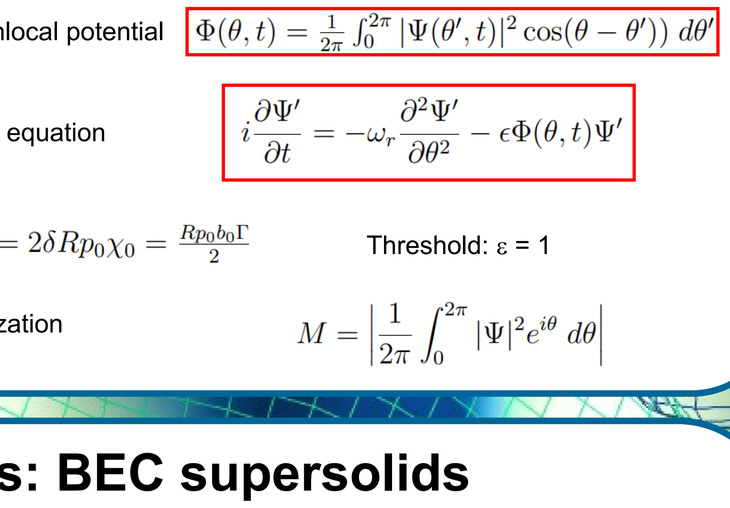
Weak seed:
$|\psi(0)|^2 = 1 + 10^{-6}\cos(q_c x)$

- **Strong, persistent oscillations**
- Like in HMF
- Interpretation: Strong **excitations** as total energy does not match ground state energy
- Alternative phrasing: excitation of **Higgs-mode**
- **Known to be long –lived in long-range coupled systems**



Strong seed:
$|\psi(0)|^2 = 1 + 0.48\cos(q_c x)$

- **Essentially stationary supersolid**

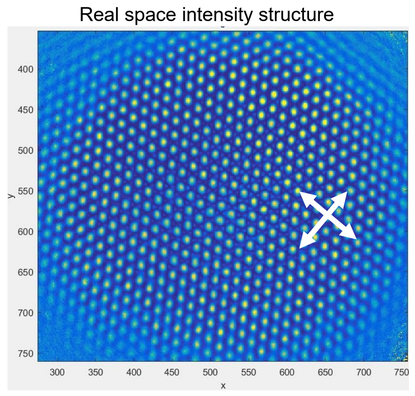


Light supersolid

Motivation/idea: Can the light part of light-matter structure alone be interpreted as supersolid ?
("tracing" over atoms/matter, even thermal matter)

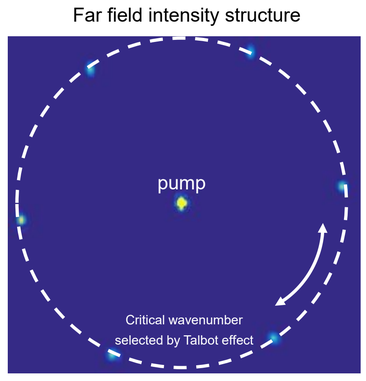
Experiment: Thermal Rb cloud slightly above Doppler temperature (similar to [2])

Real space intensity structure



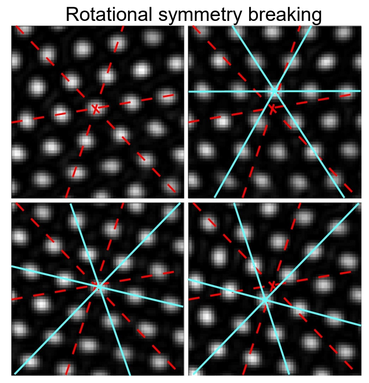
Potential **translational Goldstone modes**

Far field intensity structure



hexagon selected from all possible superpositions

Rotational symmetry breaking



Translational symmetry breaking

✓ **Spontaneous periodic structure** ✓ **Coherence** ✓ **Spontaneous symmetry breaking**

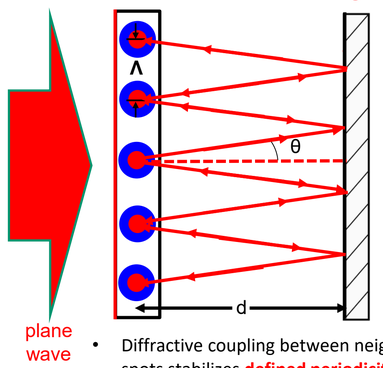
Theory: Conservative Vlasov equation (based on [8])

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} - \frac{1}{2}\hbar\delta\frac{\nabla s(\mathbf{x})}{M}\cdot\frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\rho(\mathbf{x}) = \int_{-\infty}^{+\infty} f(\mathbf{x},\mathbf{v})d\mathbf{v}$$

$b_0 = 200, \Delta = 10\Gamma,\ T = 200\,\mu\mathrm{K},\ R = 1,$ period $\Lambda = 100\,\mu\mathrm{m}$, $\sqrt{d/k_0} = 20\,\mu\mathrm{m},\ \phi_{lin} = 4.98,\ s_{f,th} = 0.0137$

periodic structure Acts as **grating**



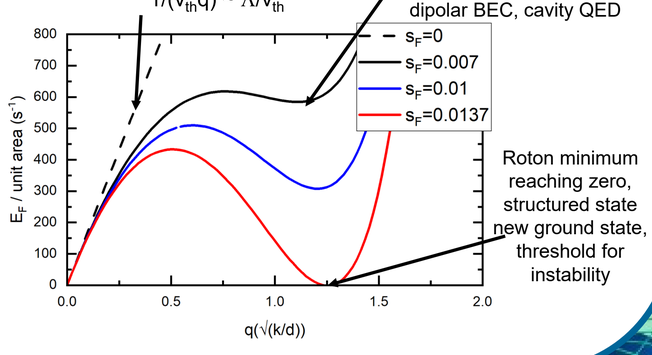
plane wave pump

- Diffractive coupling between neighbouring spots stabilizes **defined periodicity**
- Photons **flow** between neighbouring spots

✓ Both features expected for supersolid light

Without pump: residual motion will smooth out spatial structure in time $1/(v_{th}q) \sim \Lambda/v_{th}$

Increasing pump: "roton softening" like for He, dipolar BEC, cavity QED



Roton minimum reaching zero, structured state new ground state, threshold for instability

Summary and outlook

- **BEC supersolid**: Good agreement between SMF and HMF simulations
  - Prospects for experimental realization of archetypical model for long-range coupling
  - Strong, persistent oscillations or stationary supersolid depending on seedinfg
  - Connection to time crystals circumventing no-go-theorem
- **Light supersolid:** idea to "trace" over material and look only at light structure
  - Experiment demonstrates coherence and spontaneous symmetry breaking
  - Theory yields roton spectrum
- Transverse laser patterns as further exciting candidate for out-of-equilibrium light supersolids

References

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