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On a run-based δ-shock model with two critical levels

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Abstract: In reliability engineering, the δ-shock model is used to study shock-exposed systems that are sensitive to the length of the time distance between consecutive shocks. When the system failure depends on a certain number of consecutive shocks with an inter-arrival time within a critical range, we are dealing with a run-based δ-shock model. In this paper, a new run-based δ-shock model is introduced, under which the system fails when an inter-arrival time is less than a critical threshold δ_1 for the first time or k consecutive inter-arrival times fall in the interval (δ_1, δ_2), for $0 \le \delta_1 < \delta_2$. We study the probability behavior of the system's stopping time as well as the survival of the system under the proposed model. As an illustrative example, we examine the survival of the system when the arrival of shocks follows a Poisson process. Furthermore, an example of applications is provided to illustrate possible application aspects.

Keywords: Shock model; Inter-arrival time; Runs; Survival model.

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1 Introduction

Shock models are useful tools for analyzing systems that are subjected to random shocks from external sources. It is important to know how a system will behave when it gets shocks. Therefore, shock models have attracted much attention in applied probability, reliability theory, and engineering. Many shock models have been developed in recent years, among which the so-called *δ-shock model* is one of the most important that has a potential applications in various fields. According to the classical δ-shock model, a system subjected to shocks fails when the inter-arrival time between two consecu-

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tive shocks is smaller than a critical threshold δ for the first time. In fact, when the inter-arrival time is larger, a system has enough time to recover from the aftermath of a shock. Many extensions and generalizations have been proposed for the δ-shock model, for some of them, see e.g., Eryilmaz and Bayramoglu (2014), Parvardeh and Balakrishnan (2015), Tuncel and Eryilmaz (2018), Lorvand et al. (2020), Bohlooli-Zefreh et al. (2021), Poursaeed (2021), Goyal et al. (2022), Chadjiconstantinidis and Eryilmaz (2024), Finkelstein and Cha (2024), Lorvand and Eryilmaz (2024), and Farhadian and Jafari (2025).

The classical δ-shock model defines a criterion for system failure based on the time distance between consecutive shocks (that should be less than a critical level). However, there can be situations when a system can tolerate a number of shocks in a row with these distances larger than δ, which defines the second threshold for the model. Exceeding this number results in failure. Thus, the consecutive run of such events can still lead to a failure, as a system is not fully recovered from such shocks, and the corresponding damage accumulates, eventually leading to a failure. A similar model (but only with a single threshold δ) was first introduced and studied by Eryilmaz (2012). See also Balakrishnan and Koutras (2002) for some general results on the run models. The setting resembles the one with external shocks when a failure of a system results following either one shock with a relatively large magnitude or the run of $k > 1$ shocks with a smaller magnitude (see Eryilmaz and Unlu (2023)). See also Finkelstein and Cha (2013) and Goyal et al. (2025) for general analysis of systems with time redundancy.

In this paper, motivated by the work by Eryilmaz (2012), we introduce a new generalized δ-shock model using the concept of runs. According to the new model, the system fails whenever an intershock time is less than δ_1 for the first time or k consecutive intershock times fall between (δ_1, δ_2) for $k \geq 1$. Clearly, for $\delta_1 = 0$, the new model reduces to the δ-shock model proposed by Eryilmaz (2012). Meanwhile, Eryilmaz (2012) has considered only the exponential case, but we are dealing with arbitrary lifetime distributions. Furthermore, our approach to obtaining the survival probabilities of interest is different from that in his paper and is mostly based on the corresponding integral equations. We will also consider a new setting with the random number of runs for part of this study.

The remainder of the paper is organized as follows. Section 2 introduces some of the notation an acronyms used. The new model is described in Section 3. The probability behavior of the survival of the system under the new model is derived in Section 4. Section 5 provides an illustration of the survival model. An example of applications to illustrate possible application aspects is presented in Section 6. Section 7 concludes the paper.

2 Preliminaries: notation and acronyms

In this section, we present some of the notations and acronyms used throughout the paper. For any random variable X, we denote by $\Pr(X = x)$ the probability mass function (pmf) of X, $f(x)$ the probability density function (pdf) of X, $F(x)$ the cumulative distribution function (cdf) of X, $\bar{F}(x)$ the survival (or reliability) function of X, $G(x)$ the probability generating function (pgf) of X, and $E(X)$ the expected value of X. We denote the Laplace transform (LT) by $\mathcal{L}(s)$ and for any real function $h(t)$, we denote its LT as $\mathcal{L}(h) = h^*(s)$. Further, let $\{N(t), t \geq 0\}$ denote a point process for a sequence of shocks, representing the number of shocks arriving at time t. We also denote the set of natural numbers by $\mathbb{N}$. The acronym 'iff' stands for 'if and only if', and the acronym 'i.i.d.' means that a number of random variables are 'independent and identically distributed'.

3 Model description

Consider a system that starts operating at time $t = 0$ and is subjected to a sequence of random external shocks that are the only cause of the system's failure. Let $\{N(t), t \geq 0\}$ be the renewal process of shocks and X_i (for $i = 1, 2, \dots$) be the ith inter-arrival time, i.e., the time lag between the ith and $(i + 1)$th shocks. The inter-arrival times $X_1, X_2, \dots$ are i.i.d. random variables with a common cdf $F(x)$ and the pdf $f(x)$. We assume that the performance of the system depends on the inter-arrival times and two critical thresholds, δ_1 and δ_2, with $0 \leq \delta_1 < \delta_2$, such that the system fails whenever an intershock time is less than δ_1 for the first time or k consecutive intershock times fall between (δ_1, δ_2) for $k \geq 1$. If any inter-arrival time is larger than δ_2, this is considered a 'total renewal'. The lifetime of the system is defined as follows:

$$T = \sum_{i=1}^{N_{1,k}} X_i,$$

where $N_{1,k}$ is the stopping time for the first inter-arrival time is less than δ_1 or the first k consecutive inter-arrival times are between (δ_1, δ_2). That is,

$$\{N_{1,k} = n\} \text{ iff } \min\{n : (X_n \leq \delta_1) \text{ } or \text{ } (\delta_1 < X_{n-k+1} < \delta_2, \dots, \delta_1 < X_n < \delta_2)\}, \quad n \in \mathbb{N}.$$

According to Philippou et al. (1983) and Balakrishnan and Koutras (2002), the pmf of the event $\{N_{1,k} = n\}$ for $n > k$ is as follows:

$$\begin{aligned}\Pr(N_{1,k} = n) = &\sum_{i=1}^{n-k} C(n-k-i, k, n-k-1) p_2^{n-i} p_3^i \\ &+ \sum_{j=0}^{k-1} \sum_{i=1}^{n-j-1} C(n-i-j-1, j, n-j-2) p_1 p_2^{n-i-1} p_3^i,\end{aligned}$$

where

$$C(l, k, n) = \sum_{s=0}^{\min\left(\lfloor \frac{l}{k} \rfloor, n-l+1\right)} (-1)^s \binom{n-l+1}{s} \binom{n-sk}{n-l},$$

and $p_1 = \Pr(X \leq \delta_1) = F(\delta_1)$, $p_2 = \Pr(\delta_1 < X < \delta_2) = F(\delta_2) - F(\delta_1)$, and $p_3 = \Pr(X > \delta_2) = 1 - F(\delta_2)$.

Note that in the probability $\Pr(N_{1,k} = n)$ above, the first summation is the probability that k consecutive inter-arrival times occur in the interval (δ_1, δ_2), and the second double summation is the probability that an inter-arrival time smaller than δ_1 occurs (in this case, at most $k-1$ consecutive inter-arrival times have occurred in the interval (δ_1, δ_2) before one occurs less than δ_1).

Thus, for $N_{1,k} = 1$ and $N_{1,k} = k$, we have, respectively:

$$\begin{aligned}&\Pr(N_{1,k} = 1) = p_1, \\ &\Pr(N_{1,k} = k) = p_2^k + \sum_{j=0}^{k-1} \sum_{i=1}^{k-j-1} C(k-i-j-1, j, k-j-2) p_1 p_2^{k-i-1} p_3^i.\end{aligned}$$

Remark 3.1. *In the particular case when $\delta_1 = 0$ and external shocks arrive according to a Poisson process $N(t)$, the proposed shock model reduces to the shock model introduced by Eryilmaz (2012). Namely, the inter-arrival times are i.i.d. exponential random variables with mean $\frac{1}{\lambda}$ $(\lambda > 0)$, and the system fails when k consecutive intershock times are less than a fixed threshold, which is δ_2 here. Accordingly, we have $p_1 = \Pr(X \leq 0) = 0$, $p_2 = \Pr(0 < X < \delta_2) = \Pr(X < \delta_2) = 1 - e^{-\lambda \delta_2}$, and $p_3 = \Pr(X > \delta_2) = 1 - p_2 = e^{-\lambda \delta_2}$. Consequently,*

$$\Pr(N_{1,k} = n) = \sum_{i=1}^{n-k} C(n-k-i, k, n-k-1) p_2^{n-i} (1-p_2)^i, \qquad n = k+1, k+2, \ldots,$$

that is, the pmf of the stopping time of the shock model proposed by Eryilmaz (2012) (the stopping time is denoted there by $W_k^{(1)}$).

In subsequent sections, we will consider the system's survival model.

4 Analysis of the system's survival model

4.1 Survival model with fixed runs

In accordance with the description of the lifetime in Section 3, denote the corresponding survival probability for our system as

$$S(t) \equiv S_0(t) = \Pr(T > t). \tag{1}$$

Let r be the current length of a run with $\delta_1 < X_i < \delta_2$. We define the probability

$$S_r(t) = \Pr(T > t | run\ of\ r), \qquad r = 0, 1, 2, \ldots, k-1. \tag{2}$$

with $S_k(s) \equiv 0$.

Remark 4.1. *We assume in (1) and (2), and in what follows, for convenience and in accordance with conventional models in the literature, that the first shock had occurred at origin, i.e., $t = 0$. The model can be easily adjusted to the case when there is no shock at $t = 0$ (the shock process itself starts at $t = 0$ but not with the event) by integrating the conditional probability (1) with respect to density $f(t)$, i.e., the corresponding survival function is*

$$\hat{S}(t) = \bar{F}(t) + \int_0^t f(x) S_0(t-x) dx. \tag{3}$$

In this case, in accordance with our definition, the first shock cannot result in a failure. As our further discussion relies on the corresponding Laplace transforms (LT), and (3) can also be nicely processed in this way, the adjustment of what follows to this case is straightforward.

We will derive recursive integral equations of the Volterra type (based on relevant convolutions that are well treated then with the Laplace transform) equations with 'absorption' at $r = k$. Let an operating system at $t = 0$ already have the run of length $r \leq k-1$, and the last shock of this run had occurred at $t = 0$. It can be easily seen that the survival probability in $(0, t)$ is

$$S_r(t) = \bar{F}(t) + I_{\{r+1<k\}} \int_{\delta_1}^{\min(\delta_2, t)} f(x) S_{r+1}(t-x) dx + \int_{\delta_2}^{t} f(x) S_0(t-x) dx, \tag{4}$$

where:

- The first term is the probability that there were no shocks in $(0, t)$;
- The second term is the probability that the first shock had occurred in $(\delta_1, \min(\delta_2, t))$ and therefore, did not result in failure of a system. Then the system should operate in the rest of the interval, i.e., $[x, t)$. The corresponding probability is

$S_{r+1}(t-x)$. The indicator means that we are considering only values $r < k-1$, as under the integral there is a transition to the state with $r+1$ and the run with $r = k$ means a failure;

- The last term defines the probability that the first shock occurs at $x > \delta_2$ and, therefore, renews the setting with the initial run 0. The system is operable in $[x, t)$ with this initial condition. The corresponding probability is $S_0(t-x)$. Thus, we must obtain $S_0(t)$ from this system of integral equations.

The LT of a function $H(t)$ is defined as follows:

$$\mathcal{L}(H(t)) = \int_0^\infty e^{-st} H(t) dt.$$

Accordingly, the LT of the first integral in Eq. (4) is

$$\mathcal{L}\left(\int_{\delta_1}^{\min(\delta_2,t)} f(x) S_{r+1}(t-x) dx\right) = \int_0^\infty e^{-st}\left(\int_{\delta_1}^{\min(\delta_2,t)} f(x) S_{r+1}(t-x) dx\right) dt.$$

Changing the order of integration and then $u = t - x$ results in

$$\begin{aligned}\int_{\delta_1}^{\delta_2} f(x)\left(\int_{t=x}^{\infty} e^{-st} S_{r+1}(t-x) dt\right) dx &= \int_{\delta_1}^{\delta_2} f(x) e^{-sx}\left(\int_0^\infty e^{-su} S_{r+1}(u) du\right) dx \\ &= S^*_{r+1}(s)\int_{\delta_1}^{\delta_2} e^{-sx} f(x) dx.\end{aligned}$$

The LT of the second integral in Eq. (4) is

$$\begin{aligned}&\mathcal{L}\left(\int_{\delta_2}^{t} f(x) S_0(t-x) dx\right) = \int_0^\infty e^{-st}\left(\int_{\delta_2}^{t} f(x) S_0(t-x) dx\right) dt \\ =&\int_{\delta_2}^{\infty} f(x)\left(\int_{t=x}^{\infty} e^{-st} S_0(t-x) dt\right) dx = \int_{\delta_2}^{\infty} f(x)\left(\int_0^\infty e^{-s(u+x)} S_0(u) du\right) dx \\ =&S^*_0(s)\int_{\delta_2}^{\infty} e^{-sx} f(x) dx.\end{aligned}$$

Now, let's define

$$\begin{aligned}A(s) &= \int_{\delta_1}^{\delta_2} e^{-sx} f(x) dx, \\ B(s) &= \int_{\delta_2}^{\infty} e^{-sx} f(x) dx, \\ C(s) &= \int_0^\infty e^{-st}\bar{F}(t) dt = \frac{1 - f^*(s)}{s}.\end{aligned}$$

Following these, the LT of both sides in Eq. (4) becomes

$$S^*_r(t) = C(s) + I_{\{r+1<k\}} A(s) S^*_{r+1}(s) + B(s) S^*_0(s), \qquad r = 0, 1, \ldots, k-1.$$

This system of equations can be solved recursively. Thus, we start with

$$S_{k-1}^*(s) = B(s)C(s)S_0^*(s),$$

and back-substitute on every step to see the geometric pattern

$$S_r^*(s) = C(s)\left(1 + A(s) + A^2(s) + \cdots + A^{k-1-r}(s)\right) \\ + B(s)S_0^*(s)\left(1 + A(s) + A^2(s) + \cdots + A^{k-1-r}(s)\right). \tag{5}$$

Specifically, as we are interested in the failure-free performance, then

$$S_0^*(s) = \frac{C(s)\sum_{i=0}^{k-1} A^i(s)}{1 - B(s)\sum_{i=0}^{k-1} A^i(s)}, \tag{6}$$

in which $\sum_{i=0}^{k-1} A^i(s) = \frac{1-A^k(s)}{1-A(s)}$.

An example is given below.

Example 4.2. Specifically, for the Poisson process of shocks with $f(t) = \lambda e^{-\lambda t}$, we have

$$A(s) = \frac{\lambda e^{-(\lambda+s)\delta_1} - \lambda e^{-(\lambda+s)\delta_2}}{\lambda + s},$$
$$B(s) = \frac{\lambda e^{-(\lambda+s)\delta_2}}{\lambda + s},$$
$$C(s) = \frac{1}{\lambda + s}.$$

Accordingly, we get the following relation for the LT of the probability of survival:

$$S_0^*(s) = \frac{\frac{1}{\lambda+s}\sum_{i=0}^{k-1} A^i(s)}{1 - \frac{\lambda}{\lambda+s}e^{-(\lambda+s)\delta_2}\sum_{i=0}^{k-1} A^i(s)}. \tag{7}$$

When there are no runs, it is a classical delta shock model. It means $k \to \infty$ and the sum tends to $(1 - A(s))^{-1}$. Consequently,

$$S_0^*(s) = \frac{1}{\lambda + s}\left(\frac{1}{1 - \frac{\lambda e^{-(\lambda+s)\delta_1}}{\lambda+s}}\right),$$

whose LT is a well-known solution for the classical model, that is

$$S(t) = e^{-\lambda t}\sum_{n=0}^{\lfloor \frac{t}{\delta_1} \rfloor} \frac{\lambda(t - n\delta_1)^n}{n!}.$$

When $\delta_1 = 0$, $A(s)$ becomes simpler:

$$A(s) = \frac{\lambda - \lambda e^{-(\lambda+s)\delta_2}}{\lambda + s},$$

and from the general formula, we get only the probability of a failure due to runs.

On the other hand, if we are in the frame of (3) in the model described in Remark 3.1 (when the shock process starts at $t = 0$ and the first shock occurs only on completion of the first cycle of the renewal process), than the corresponding 'unconditional' survival probability $S_0^{*u}(s)$ (its LT) can be defined via the LT of (3) as

$$S_0^{*u}(s) = \frac{1 - f^*(s)}{s} + f^*(s)\left(\frac{C(s)\sum_{i=0}^{k-1} A^i(s)}{1 - B(s)\sum_{i=0}^{k-1} A^i(s)}\right),$$

whereas (7) becomes

$$S_0^{*u}(s) = \frac{1}{\lambda + s} + \frac{\lambda}{\lambda + s}\left(\frac{\frac{1}{\lambda+s}\sum_{i=0}^{k-1} A^i(s)}{1 - \frac{\lambda}{\lambda+s} e^{-(\lambda+s)\delta_2}\sum_{i=0}^{k-1} A^i(s)}\right).$$

4.2 Survival model with random number of runs

Due to the general stochastic nature of the model itself and its 'ingredients', the number of runs can be a discrete random variable K. Thus, let

$$\Pr(K = k) = p_k, \qquad k = 1, 2, \ldots,$$

with the corresponding pgf

$$G(x) = E(z^K) = \sum_{k=1}^{\infty} z^k p_k. \tag{8}$$

There are two ways to implement this randomness:
(**I**) First, let it be set a 'priory' at $t = 0$, and its realization is kept for all further renewal cycles. Thus, it is a kind of device-specific random K. Let us index this model by "1". Then, using Eq. (6), we can write now

$$S_{01}^*(s) = \sum_{k=1}^{\infty} p_k \frac{C(s)\sum_{i=0}^{k-1} A^i(s)}{1 - B(s)\sum_{i=0}^{k-1} A^i(s)} = E\left(\frac{C(s)\sum_{i=0}^{k-1} A^i(s)}{1 - B(s)\sum_{i=0}^{k-1} A^i(s)}\right). \tag{9}$$

(**II**) However, a more practically sound option is when K is drawn independently after each cycle, starting also at $t = 0$. Besides, it is much more appealing computationally, as will be seen in what follows. In this case, when K is redrawn at each reset/cycle, the system's life is modeled by the renewal process over cycles (not like in the previous setting, where although the shock process is renewal, the 'overall' process that describes the operation of a system is not, at least, the corresponding cycles are not independent due to the same k) . This means, in essence, that expectation with respect to K should be applied not as in Eq. (9), but to both sides of the renewal equation (5) for $r = 0$, which is

$$S_0^*(s) = C(s)\sum_{i=0}^{k-1} A^i(s) + B(s)S_0^*(s)\sum_{i=0}^{k-1} A^i(s).$$

Applying expectation t and solving with respect to $S_{02}^*(s) = E(S_0^*(s))$, results in

$$S_{02}^*(s) = \frac{C(s)E\left(\sum_{i=0}^{k-1} A^i(s)\right)}{1 - B(s)E\left(\sum_{i=0}^{k-1} A^i(s)\right)}. \tag{10}$$

Taking into account Eq. (8), we can write

$$E\left(\sum_{i=0}^{k-1} A^i(s)\right) = E\left(\frac{1 - A^k(s)}{1 - A(s)}\right) = \frac{1 - G(A(s))}{1 - A(s)}. \tag{11}$$

Thus the LT of survival probability in this case is given by Eq. (10), where the expectation of the sum is defined by Eq. (11). It is easy to see that for the fixed $K = k$, Eq. (10) reduces to Eq. (6) as $G(z) = z^k$ in this case.

As an example, consider the geometric distribution for K. Then

$$G(z) = \frac{pz}{1 - (1 - p)z},$$

and

$$E\left(\sum_{i=0}^{k-1} A^i(s)\right) = \frac{1}{1 - (1 - p)A(s)}.$$

Hence, Eq. (10) simplifies to

$$S_{02}^*(s) = \frac{1}{1 - B(s) - (1 - p)A(s)},$$

which is a remarkably simple expression.

5 Some illustrations

Assuming that the arrival of shocks follows a Poisson process (see Example 4.2), we investigate the trend of changes in the survival function curve. On this basis, Figure 1 illustrates the ordering of survival curves with respect to δ_2 and k for our main model (Eqs. (1) and (2)) with the first shock at origin. Let $\lambda = 0.05$, $\delta_1 = 1$. They show the influence of the increasing δ_2 and changing k on the corresponding survival probabilities as functions of time. We see, as expected, that the lifetimes are ordered (increasing) in the sense of the usual stochastic ordering when k is increasing. On the other hand, the initial 'drop' in survival probability is attributes to the probability of failure in $(0, 1]$ and, due to the scaling, looks rather 'abrupt'. In addition, Figure 2 depicts the survival curve with respect to the random K, where K follows a geometric distribution with mean $\frac{1}{p}$ $(p > 0)$. The corresponding lifetimes are stochastically increasing in the sense of the usual stochastic ordering when p is decreasing (the mean of K is increasing).

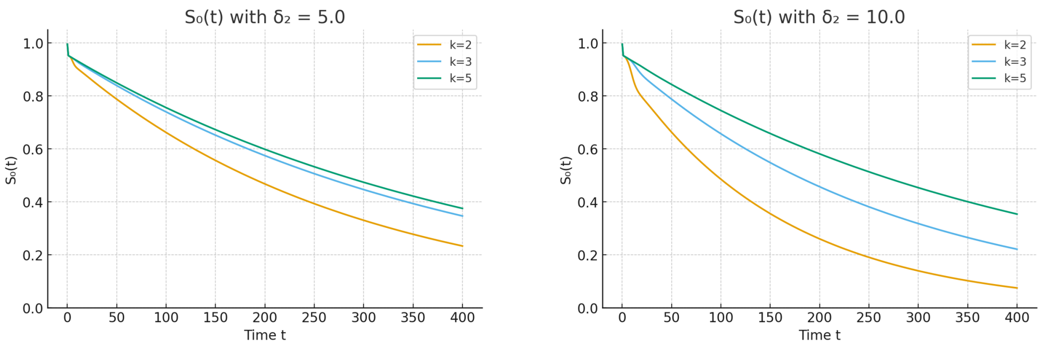

Figure 1: Survival function curve with fixed runs.

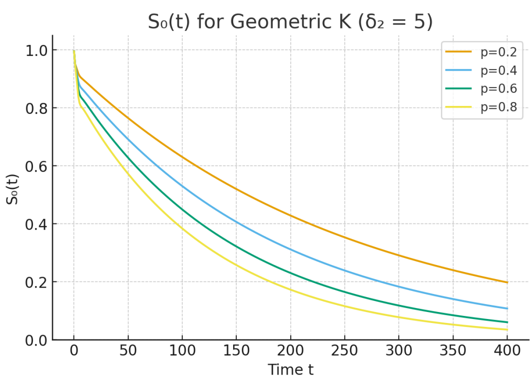

Figure 2: Survival function curve with random runs.

6 An example of application

Transistors are basic electronic components used in the manufacture of many modern electronic devices, and their main task is to control the flow of current in an electronic circuit and thereby amplify electrical signals and power in electrical devices. The structure of a transistor consists of a die placed on a base plate made of a conductive metal, and the die is encapsulated inside epoxy resin. Transistors are prone to overheating and burning due to electrical shocks such as high voltage. In fact, the permissible internal heating limit for transistors made of different materials is different, for example, for germanium transistors it is 75℃ and for silicon transistors it is 130℃ (see, e.g., Capter 10 of Kaden (1965)). Therefore, if the temperature of a transistor exceeds the permissible limit, it may burn due to overheating. However, if a transistor is not continuously heated, it will gradually lose its temperature and cool down by itself. Now consider a transistor as a system that is exposed to sequences of random electrical shocks over time. Assume its normal operating temperature is 40℃ and its permissible internal heating limit is 75℃. If each shock increases the transistor temperature by 20℃ and the transistor takes 3 minutes to cool down by itself after each shock, it is clear that there is a critical threshold of $\delta_1 (< 3)$ such that if the inter-arrival time between two consecutive shocks is less than δ_1, then the temperature of the transistor exceeds 75℃ and it burns. It is also possible to consider, for another threshold, $\delta_2 > \delta_1$, a minimum number k of consecutive shocks with inter-arrival times in the range (δ_1, δ_2) such that, with the accumulation of the temperature increase produced by them, the transistor temperature exceeds 75℃. Therefore, when faced with such scenarios, the model introduced in this paper can be applied.

7 Conclusions

We have introduced a new run-based δ-shock time shock model with two critical thresholds, δ_1 and δ_2, with $0 \leq \delta_1 < \delta_2$. According to the new model, the system fails whenever an inter-arrival time is less than δ_1 for the first time or k consecutive inter-arrival times fall between δ_1 and δ_2. We have showed that this model reduces to the δ-shock model proposed by Eryilmaz (2012) when $\delta_1 = 0$ and the arrival of shocks follows a Poisson process. We have considered the pmf of the system's stopping time as well as the system's survival probability under the new model. Furthermore, as an illustration of our findings, for the Poisson process of shocks, we have plotted the corresponding survival probabilities for the fixed and random numbers of runs, respectively. Finally, an application example was presented to illustrate possible application aspects.

Disclosure statement

No potential conflict of interest was reported by the authors.

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