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Boosting Quality: Surveillance and Monitoring

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Abstract

Purpose: In process monitoring, manufacturers typically employ two main types of control charts: memory-less and memory-type charts. Memory-type charts, such as the *Exponentially Weighted Moving Average* (EWMA), generally outperform memory-less charts like the *Shewhart* chart. However, both types have inherent limitations that necessitate various extensions and modifications. With these considerations in mind, this study aimed to develop, present, and validate an optimal scheme for detecting non-conformities within the context of a small lamp and light bulb manufacturer based in the United Arab Emirates.

Design/methodology/approach: The developed scheme accounts for constraints related to inspection capacity and false alarm rates. Through comparative testing, we demonstrate the relative performance of the proposed optimal weighted Exponentially Weighted Moving Average (*w*EWMA) chart, which is further validated through a detection effectiveness evaluation.

Findings: Specifically, our findings indicate that, compared to other EWMA schemes with varying design specifications, the proposed optimal *w*EWMA control chart outperforms the traditional EWMA chart by 26% and the original *w*EWMA chart by 17%, based on the Average Number of Defectives (*AND*).

Practical implications: The proposed scheme relies solely on data readily available within the case organization, enabling operations managers to swiftly implement corrective actions to eliminate non-conforming items. Furthermore, the organization can integrate non-conformity detection into its broader quality initiatives, allowing the scheme to function as a strategic tool for both quality management and strategy–quality alignment.

Originality/value: The optimal *w*EWMA scheme enhances a previously modified EWMA model, which was designed to effectively detect various shifts in the fraction non-conforming (p). To achieve superior overall performance, this study optimizes the sample size (n) and sampling interval (h), factors that were not addressed in earlier research. The proposed optimal *w*EWMA scheme also holds promise for broader application across other manufacturing sectors, including household consumer goods (e.g., home appliances) and industrial products (e.g., transformers, aluminium tubes, and printed circuit boards). Future research may examine its effectiveness in monitoring multi-attribute characteristics and high-yield processes.

Keywords: Quality control; Statistical process control; Control chart; EWMA chart, Statistical monitoring.

1. Introduction

Product quality refers to an item's ability to meet user needs and expectations in terms of fitness for purpose and the absence of defects (Catenazzo and Paulssen, 2020; Liao *et al*., 2023; Haridy *et al*., 2024a). Given that quality significantly influences the profitability and competitiveness of nearly all organizations, its importance cannot be overstated (Chiarini and Kumar, 2022; Parast and Safari, 2022; Tarí *et al*., 2023; El Manzani *et al*., 2024). Quality is widely recognized as a key strategic factor in achieving business success (Antony *et al*., 2021).

However, differing perspectives on what constitutes "*quality*" can hinder its effective realization (Eriksson *et al*., 2016; Yang, 2017). Therefore, prioritizing the customer's perception and understanding of quality is essential. Factors such as reputational risk, customer complaints, product returns, defect corrections, loss of repeat business, and recall costs all highlight the need to align quality with customer expectations. Modern technology has further amplified this need, as customers now have instant access to vast amounts of comparative quality information (Anthony *et al*., 2021). In response, organizations have adopted various strategies to enhance product and service quality, including a greater emphasis on quality assurance (Wu *et al*., 2009) and the elimination of defects throughout production and operational processes (Haridy et al., 2011; Psarommatis *et al*., 2020; Caiazzo *et al*., 2022; Lv *et al*., 2022; Powell *et al*., 2022).

To minimize production defects, scholars advocate for a range of quality management practices, tools, and techniques that enhance operational efficiency and effectiveness. These include business excellence frameworks such as the Deming Prize (https://www.juse.or.jp/deming_en/; see also Gitlow, 2023), ISO 9001 (Astrini, 2021; Ikram *et al*., 2021; Camango and Cândido, 2023), the Malcolm Baldrige National Quality Award (https://asq.org/quality-resources/malcolm-baldrige-national-quality-award; see also Lapoint, 2022), and the European Foundation for Quality Management (EFQM) model (https://efqm.org/the-efqm-model/; see Fonseca, 2022). In addition, various management approaches, such as Lean, Six Sigma, Lean Six Sigma (Sindhwani *et al*., 2021; do Nascimento *et al*., 2022), Total Quality Control (Feigenbaum, 1956, 1991; Ishikawa, 1985; Takei, 1986), and Total Quality Management (TQM) (Permana *et al*., 2021; Oliveira and Gomes, 2024), have been widely adopted.

Within these quality management practices, specialized techniques focus on process control and monitoring (Xie *et al*., 2002; Qin, 2003, 2012; Ge and Chen, 2015), with control charts being a key tool (Prawatya *et al*., 2019; Jalilibal *et al*., 2024). Control charts play a crucial role in detecting shifts (i.e., changes and variations) in quality attributes. First introduced by Walter A. Shewhart (1926, 1939), these charts graphically represent sequential process data, making it easier to assess process performance in terms of quality (Haridy *et al.*, 2024a). One widely used type is the Exponentially Weighted Moving Average (EWMA) control chart, developed by Roberts (1959). EWMA charts are particularly effective in detecting minor shifts; however, various extensions and modifications have been introduced to address their limitations, especially in cases of large and sudden deviations. Some EWMA variants have improved detection speed and accuracy, enabling earlier identification of process deviations. Others have optimized objective function selection to enhance parameter efficiency.

Over time, several EWMA chart variants have been developed, including an enhanced *w*EWMA chart, which incorporates the power (w) of the difference between the actual number of defective items and its '*in-control'* value (Haridy *et al.*, 2020). While this scheme effectively detects process deviations in high-precision manufacturing, it has a key limitation: the sample size (n) is not optimized.

Given these considerations, this study aims to develop, present, and validate an improved version of the enhanced *w*EWMA chart, referred to as the optimal *w*EWMA, by optimizing the sample size (n). To achieve this goal, the study is guided by three key research objectives:

- Research Objective 1: Determine the optimal charting parameters of the optimal *w*EWMA scheme, sample size (n), sampling interval (h), power (w), weighting parameter (λ), and the upper control limit of the EWMA chart (H).
- Research Objective 2: Study the effect of sample size (n) on the performance of the optimal *w*EWMA scheme.
- Research Objective 3: Illustrate the implementation of the optimal *w*EWMA scheme through a case study and, in the process, conduct a comparative analysis between the optimal *w*EWMA and other EWMA schemes.

This study contributes to both the theory and practice of Statistical Process Control (SPC) and manufacturing monitoring. A fundamental principle of quality management is the early identification of manufacturing defects, ideally before products reach customers. In this regard, SPC charts serve as a key tool for detecting anomalies and variations in product quality attributes.

From a theoretical perspective, this study advances the use of control charts in quality control by integrating design specifications to enhance detection and predictive accuracy (see Lee *et al*., 2020). It demonstrates how incorporating specific design attributes can significantly improve the predictive performance of quality control tools.

From a practical standpoint, the improved EWMA chart offers manufacturers and decision-makers a powerful tool to ensure quality conformance under real-world constraints. It also plays a strategic role in eliminating root causes of quality issues, reducing waste, and minimizing economic losses.

To achieve these research objectives, the remainder of the paper is structured as follows: Section 2 provides contextual background on control charts and a review of the relevant literature. Section 3 presents the research methods. Section 4 examines and compares the detection effectiveness of the various schemes. Section 5 discusses the study findings, and Section 6 concludes the paper.

2. Literature

2.1 Statistical process control and monitoring

The use of Statistical Process Control (SPC) and monitoring has gained widespread adoption across various industrial sectors, including the manufacturing and production of lamps and light bulbs (Lin and Chan, 1975; Zukauskas *et al*., 2009; Ng *et al*., 2011; Semenov *et al*., 2020). SPC is particularly valuable as a tool for detecting deviations by distinguishing between special causes and common causes of variation (Suman and Prajapati, 2018; Wysocki *et al*., 2022; Haq *et al*., 2023; Joghee and Varghese, 2024; Escobar *et al*., 2025).

Statistical process control and monitoring typically involve the following tasks: (i) detection of deviations or defects; (ii) identification and analysis of these deviations or defects; (iii) estimation of the magnitude of the deviation or defect; and (iv) reconstruction, which entails evaluating stable values to ensure continuous control and monitoring regardless of the occurrence of some deviations or defects (Qin, 2003). A primary focus of process control and monitoring is on quality characteristics such as diameter and strength. Since these are continuous variables, they can theoretically assume an infinite range of values. Consequently, not all quality attributes are quantified precisely; instead, they are often classified as either *conforming* or *non-conforming* based on established quality thresholds.

One major advantage of SPC and monitoring is their dual role in both defect detection and diagnosis, as well as process improvement and learning (Rama-Maneiro *et al*., 2021; Escobar *et al*., 2025). Several recent studies have explored various methods for implementing

SPC and monitoring effectively (Tian *et al*., 2018; Prawatya *et al*., 2019; Nian *et al*., 2020; Haridy *et al*., 2024b; Baranwal *et al*., 2025; Capezza *et al*., 2025).

2.2 Application of control charts

Several tools, narrowly task-focused devices, are available for conducting SPC and monitoring, specifically for statistically reducing unwanted deviations or defects in quality attributes (Fan *et al*. 2024; Haridy *et al*. 2024). One such tool is the control chart (Prawatya *et al*. 2019; Jalilibal *et al*. 2024).

Control charts provide a structured approach to measuring and controlling quality performance within predefined limits (Chen and Chou, 2019). They facilitate variability reduction by enabling early detection of deviations or defects (Jalilibal *et al*., 2021; Jafari *et al*., 2023). These charts are defined by two control limits (Ridley and Llaugel, 2000): a Lower Control Limit (LCL) and an Upper Control Limit (UCL). When product quality falls outside these limits, it is considered '*out-of-control'* and requires appraisal and correction. Conversely, quality is deemed '*in-control'* when it lies within the LCL and UCL.

Control charts are widely utilized in industrial operations, manufacturing, and production (Ali *et al*., 2016; Haridy *et al*., 2017, 2024; Maged et al., 2019; Zwetsloot *et al*., 2024). Their implementation occurs in two distinct phases (Qiu, 2014). Phase I involves testing and validating interim quality attributes, while Phase II employs the control chart to monitor the process and ensure that quality attributes remain within acceptable limits.

There are two primary categories of control charts: (i) *Variable* control charts (Costa 2021; Chen and Wang, 2024; Slyngstad, 2021; Sałaciński *et al*., 2023) and (ii) *Attribute* control charts (Quinino *et al*., 2020; Naveed *et al*., 2024; Zheng *et al.,* 2024). The selection depends on the nature of the quality characteristic being measured. Variable control charts are appropriate for quantitative attributes (e.g., product thickness), while attribute control charts suit qualitative assessments (e.g., pass/fail or acceptance/rejection). Although the binary classification in attribute control charts simplifies quality assessments, it also introduces challenges: (i) it may oversimplify quality evaluation by overlooking minor yet significant variations; and (ii) it may exaggerate discrepancies, as small deviations could lead to outright rejection instead of nuanced quality improvements. Despite these limitations, attribute control charts remain widely used due to their simplicity, ease of implementation, and minimal reliance on precise measurement instruments. They offer a practical alternative to variable control charts, particularly where detailed quantitative measurements are unnecessary or impractical.

2.3 Types of control charts

The literature broadly categorizes control charts into two types: (i) *memory-less* charts, such as Shewhart charts (Shewhart, 1926, 1939), and (ii) *memory-based* charts, including Cumulative Sum (CUSUM) charts (Page, 1954) and Exponentially Weighted Moving Average (EWMA) charts (Roberts, 1959; Khan *et al*., 2023; Haq, 2024; Haq and Woodall, 2024). A comprehensive taxonomy of control charts has been reported in the literature (Haridy *et al*., 2024b), but for brevity, this study does not review them in detail.

Shewhart charts detect process shifts based on current test samples but are limited in identifying small variations due to their design (*Haridy et al*., 2024a). To address this limitation, the EWMA chart was developed, incorporating a weighted average of past and current measurements to enhance sensitivity to small shifts in quality (Montgomery, 2019; Haridy *et al*., 2020). This enables the EWMA chart to quickly detect minor variations in non-conformance (Naveed *et al*., 2020; Haridy *et al*., 2024a).

Several studies have compared the efficacy of control charts (Diko *et al*., 2020; Nguyen *et al*., 2020; Rosa Lakus *et al*., 2022; Chen *et al.,* 2025; Mahmood *et al*., 2025). The consensus is that Shewhart charts perform worse than both CUSUM and EWMA charts in detecting process shifts. The EWMA chart stands out not only for superior shift detection but also for its ability to provide continuous updates as sampling progresses.

While CUSUM and EWMA charts perform similarly overall (Montgomery, 2013), their applications differ. For example, CUSUM is more effective for estimating when a shift occurs, whereas EWMA is better suited for tracking gradual changes in the process mean (Hawkins and Wu, 2014). Overall, the EWMA chart is widely recognized as more effective than other control charts (DeVor *et al*., 2007; Mahmood *et al*., 2025), explaining its widespread adoption (Chen *et al*., 2019; Maged *et al*., 2019; Nazir *et al*., 2019; Shamsuzzaman *et al*., 2019; Nguyen *et al.,* 2020; Haridy *et al*., 2024a; Waqas *et al*., 2024).

2.4 Prior studies on improved control charts

Despite the superiority of EWMA charts, various extensions and modifications have been proposed to address their performance limitations, particularly in detecting large and sudden deviations. Some efforts have focused on enhancing EWMA's sensitivity to enable earlier and more accurate defect detection, aiming to optimize efficacy further. Additionally, several studies have demonstrated that through optimization techniques, control charts can be designed with objective functions that identify optimal parameters maximizing both effectiveness and efficiency. These advancements have led to multiple EWMA variants, each tailored to specific industrial and quality control needs.

Khaliq *et al*. (2016) developed an optimized EWMA chart based on a Tukey chart for skewed distributions. Mukherjee (2017) presented a nonparametric EWMA system capable of joint monitoring of the scope and location of continuous production operations. Sanusi et al. (2019) focused on simultaneous monitoring, while Afshari and Ahmadi Nadi (2023) developed EWMA schemes for repetitive sampling. Anwar et al. (2020) proposed a modified *mx*EWMA chart that used information from an auxiliary variable to enhance shift detection. Zhang *et al*. (2022) proposed an improved exponential EWMA chart for detecting decreasing time between adverse events. Additional variants have been developed by Naveed *et al*. (2020, 2021), Taboran *et al*. (2020), Nawaz *et al*. (2021), and Yeganeh *et al*. (2021), with more recent contributions by Kazmi and Noor-ul-Amin (2024), Rasheed *et al*. (2024a, b), and Xue *et al*. (2024). Recent variants have also been proposed by Baranwal *et al*. (2025), Capezza *et al*. (2025), Liu *et al*. (2025), Raza *et al*. (2025), and Waqas *et al*. (2025).

Focusing on EWMA charts with inspection and false alarm rate constraints, Haridy *et al*. (2020) developed an enhanced weighted EWMA (*w*EWMA) chart, which incorporates the power (w) of the difference between the actual number of defective items and its '*in-control'* value. This scheme enhances earlier variants by Epprecht *et al*. (2010) and Haridy *et al*. (2013, 2014). The *w*EWMA effectively detects various sizes of shifts in the fraction non-conforming (p) by optimizing λ, H, and w.

Previous studies, including Haridy *et al*. (2020), developed modified EWMA charts such as the *w*EWMA scheme, which improved detection by incorporating an exponent w to increase sensitivity to shifts in fraction non-conforming (p). However, these studies did not optimize critical parameters such as sample size (n) and sampling interval (h). The absence of this optimization may lead to suboptimal detection efficiency. Our study addresses this gap by optimizing (n, h, w, λ, and H) using a three-level design algorithm, ensuring optimal detection performance across a range of process shifts.

While previous control charts, including standard EWMA and *w*EWMA, focused on improving shift detection, they did not fully account for constraints related to inspection and false alarm rates. Our proposed optimal *w*EWMA chart explicitly incorporates an inspection rate constraint and allowable false alarm rate to balance detection efficiency with practical considerations.

Prior studies often evaluated control chart performance under fixed shift values, typically testing a single shift magnitude. In contrast, our study systematically evaluates detection performance over a wide range of shifts, demonstrating that the proposed optimal *w*EWMA outperforms both traditional EWMA and previous *w*EWMA schemes under varying conditions. Many prior studies provide theoretical frameworks for EWMA-based charts without real-world validation. Our study bridges this gap by implementing the optimal *w*EWMA scheme in a real-world case study of a lamp and light bulb manufacturing company in the UAE, demonstrating its practical applicability and cost-effectiveness.

3. Method

3.1 The case

To develop, present, and validate our proposed optimal *w*EWMA process control and monitoring scheme, we conducted a case study, a widely recognized and accepted methodology in operations research (Barratt *et al.*, 2011; Ketokivi and Choi, 2014). Case studies are particularly appropriate in operations management, serving as a robust means of generating, testing, and refining theories, while also contributing to relevant knowledge creation (Ketokivi and Choi, 2014). They may employ quantitative or qualitative approaches (Ketokivi and Choi, 2014). Given the limited literature on the application of EWMA control charts under non-optimized sample size conditions (n), we adopted a quantitative case study approach.

Our study was conducted in collaboration with a small manufacturing firm based in Sharjah, UAE, referred to here as ‘Organization X’, which specializes in the manufacturing and production of lamps and light bulbs. Notably, 96% of Sharjah’s GDP is derived from non-oil sectors, with manufacturing playing a central role (Oxford Business Group, 2023). Sharjah is recognized as the UAE’s manufacturing hub, accounting for approximately 35% of the country’s manufacturing firms (Oxford Business Group, 2021).

The lamp and lighting industry is a significant contributor to the UAE economy, driven by consumer demand for aesthetically appealing, energy-efficient, and sustainable lighting solutions. Current projections estimate the sector’s contribution to the national economy at approximately US$441.60 million (Statista, 2024). Despite its economic importance, the sector is highly competitive and mature, with numerous manufacturers producing a diverse range of products from basic lamps and bulbs to elaborate chandeliers.

Organization X currently employs an EWMA control chart for process control and monitoring. To enhance its effectiveness in detecting shifts in the fraction non-conforming (p), we sought to develop and present an improved *w*EWMA control chart through an optimization approach (Haridy *et al.*, 2024b). Our objective was to refine the EWMA control chart by creating an optimal scheme capable of responding effectively to various magnitudes of p shifts.

This was achieved by optimizing the sample size (n) and sampling interval (h), along with other key chart parameters: power (w), weighting parameter (λ), and upper control limit (H). The proposed optimization algorithm is designed to enhance overall shift detection by minimizing the average number of defectives (*AND*) as the objective function.

Data collection was conducted over an 11-week period, from June 6, 2022, to August 27, 2022. Our engagement with Organization X involved embedding an intern within the company in a quality control role. This embedded approach ensured comprehensive access to (i) relevant process control and monitoring records for data collection and analysis and (ii) extensive discussions with process control operators, providing deeper insights into operational practices (see Stapelbroek *et al.*, 2024). The other authors acted as study supervisors, providing strategic guidance and oversight throughout the research process.

3.2 Quality problems in Organization 'X's bulb manufacturing

Organization 'X''s manufacturing configuration involves five production lines (A19, A21, R20, R30, and Globe 5 bulbs) organized around three working shifts (each of eight hours). Figure 1 shows a selection of the bulb models manufactured by the company.

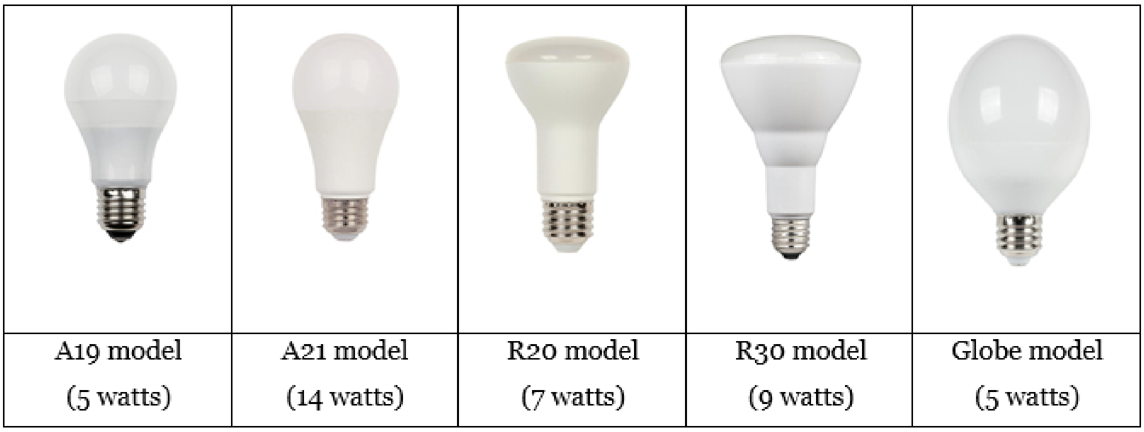

Figure 1: The manufactured bulb models

Organization X has faced significant challenges on its A21 model light bulb manufacturing line. Defects such as spatter, poor stem-to-cap adhesion, and dumet wire deformation have led to cracked glass, non-functioning bulbs, broken bases, filament failures, and light flickering. Concerned about the potential impact on customer safety and confidence, the company sought to reduce quality non-conformance by implementing a more efficient monitoring system. To achieve this, they aimed to detect increasing shifts in the 'out-of-control' fraction non-conforming (p) through an optimized EWMA scheme.

3.3 Use of control charts in Organization 'X'

For process control and monitoring, the company currently employs a traditional EWMA control chart, which we briefly review. We then introduce the layout of the optimal weighted EWMA (wEWMA) scheme, followed by a detailed exploration of its design specifications and operational mechanisms. Next, we present the implementation process, and the optimization algorithm used to refine the wEWMA scheme. Finally, we demonstrate its practical application within the company.

The optimization approach is guided by both theoretical considerations and practical constraints. Established SPC methodologies highlight the importance of parameter optimization in enhancing detection efficiency while maintaining operational feasibility (Haridy *et al*., 2022; Zago *et al.,* 2024; Li *et al*., 2025). However, practical constraints, such as resource availability, operational schedules, and cost considerations, necessitated adopting an optimal monitoring strategy tailored to the case organization.

Several assumptions were made based on input from quality engineers: (i) the number of non-conforming items (d_t) follows a binomial distribution; (ii) the '*in-control'* fraction non-conforming (p_0) is assumed known; (iii) the distribution of shifts follows a uniform distribution; and (iv) only increasing shifts in p are considered, as decreasing shifts indicate process improvement.

3.4 Notations and definitions

For brevity, we articulate the notations (abbreviations and nomenclature) employed for the model formulation as:

Abbreviations

AND	Average Number of Defectives
ATS_0	'In-control' Average Time to Signal
ATS	'Out-of-control' Average Time to Signal
RAND	Ratio of Average Number of Defectives

Nomenclature

d_t	Non-conforming/defective items at t^{th} sample

d_0	In-control value of d_t
h	Sampling interval
H	Upper control limit of the EWMA chart
w	The power of the difference between the actual number of defective items and its *'In-Control'* value
n	Sample size
τ	Allowable value of ATS_0
λ	Weighting parameter
p	Out-of-control fraction non-conforming
p_0	In-control fraction non-conforming
p_{max}	Maximum out-of-control fraction non-conforming
t	The sample number
E_o	Initial plotting statistic of EWMA chart
E_t	Plotting statistic of the EWMA chart at t^{th} sample

3.5 The design, operation, and implementation of the optimal wEWMA scheme

The traditional EWMA control chart entails real-time accumulation of all samples rather than just the most recent sample. To identify a change in fraction non-conforming p at any sample t, the EWMA scheme can be employed by plotting statistic E_t, which is updated for each t^{th} sample as follows:

$$E_0 = 0$$

$$E_t = \lambda\,(d_t - d_0) + (1 - \lambda)\,E_{t-1}, \qquad t \geq 1 \tag{1}$$

where d_t is the number of non-conforming items found in the t^{th} sample, d_0 (= $n \times p_0$) is the *'In-control'* value of d_t, λ ($0 < \lambda \leq 1$) is the weighting/smoothing parameter.

In general, research has shown that the traditional three-sigma control charts produce good chart performance, particularly with larger values of λ. The memory (functionality) of the control chart is controlled by λ, or, in other words, how much weight the chart has giving the historical data. When λ is small, the chart has long memory and, in this case, it is perfect for detecting small shifts in the process. On the other hand, if λ is equal to 1, which is the maximum value that it could be, the chart is memoryless (non-functional) and is considered as a Shewhart chart which will perform well in detecting significant process shifts (Montgomery, 2013).

To design an optimal wEWMA chart, four design specifications were set; namely, the (i) '*In-Control'* fraction non-conforming (p_0), (ii) inspection rate (r), (iii) allowable '*In-control'* ATS_0 for the chart τ, and (iv) maximum '*Out-of-control'* fraction non-conforming value p_{max}. The usual approach for choosing the appropriate p_0 tends to be based on desired quality levels which can be estimated based on historical data during Phase I of the control chart implementation. The decision on the inspection rate r is usually based on available resources such as manpower and measurement instruments. Generally, the τ value depends on the false alarm rate that users can handle. In current practice, many industries use $\tau = 500$ & 1000 (Li *et al*., 2019; Haridy *et al*., 2020). If users cannot handle additional resource requirements, large τ (e.g., 1000) might be used to reduce the false alarm rate and minimize the probability of Type I error. However, doing so is likely to reduce detection effectiveness of the control chart. The maximum fraction non-conforming value p_{max} can be chosen according to the defective rate that the customer can accept, and it can also be estimated from the historical '*Out-of-control*' data.

In terms of the operation of the optimal *w*EWMA scheme, the plotting statistic of the optimal binomial *w*EWMA scheme is given by:

$$E_0 = 0$$

$$E_t = \lambda\, v_t + (1 - \lambda)\, E_{t-1} \tag{2}$$

$$\text{where } v_t = \begin{cases} (d_t - d_0)^w, \text{if } d_t \geq d_0 \\ -(d_0 - d_t)^w, \text{if } d_t < d_0 \end{cases} \tag{3}$$

It is the same as that in Equation (1) except that $(d_t - d_0)$ is replaced by v_t. The optimal *w*EWMA scheme then works as follows: when the plotting statistic E_t falls below the control limit H, the process is '*In-control*' and no action needs to be taken. If the plotting statistic E_t exceeds the control limit H, an '*Out-of-control*' signal will be produced, and the production should be stopped as quickly as possible to preserve the quality of the products and prevent any damages and economic losses. We are attempting to identify the optimal choice of n, h, w, λ, and H to enhance the performance and the detection speed of the optimal *w*EWMA scheme versus different sizes of shifts in p.

For the implementation of the optimal *w*EWMA scheme, we took the following steps, depicted in Figure 2.

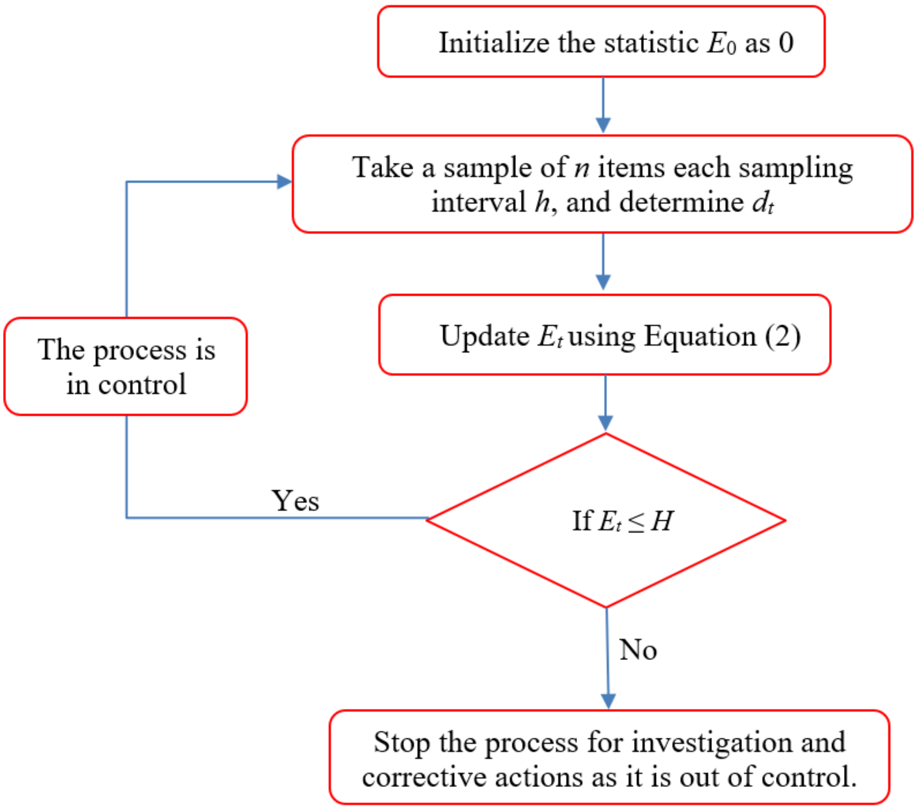

Figure 2: Implementation of the optimal *w*EWMA scheme

3.6 Measure of performance (Objective Function)

Performance metrics play a critical role in the creation stage of control charts. Metrics are defined as "*…verifiable measure[s], stated in either quantitative or qualitative terms and defined with respect to a reference point*" (Melnyk *et al*., 2004, p. 211). The importance of performance metrics is well-documented in the literature (e.g., Gunasekaran and Kobu, 2007; Ojiako *et al*., 2023), particularly in their ability to quantify quality characteristics in numerical terms. Melnyk *et al*. (2004) identified three key roles of performance metrics: (i) control measures which enable the evaluation and regulation of resource performance (ii) models of communication which facilitate clear information exchange regarding performance standards, and (iii) tools for improvement which highlight the gap between expected performance and actual outcomes, thus supporting continuous enhancement efforts.

Under the condition that the actual process is '*Out-of-control'* and a shift occurred in the process, the '*Out-of-control'* average run length (*ARL*) is the expected number of samples to run until the first signal of a control chart. It is a popular measure for charting performance evaluation. Conversely, under the condition that the process is '*In- control*', the '*In-control'* average run length (ARL_0) is the expected number of samples until a false alarm is set off by the control chart. Small *ARL* values will point to more effective control charts. It is commonly practiced in designing a control chat to fix the ARL_0 to a desired value, such as τ, and attempting to achieve an ARL_0 value that is equal to or close to it (Reynolds and Stoumbos, 1999). Another commonly used performance measure is the Average Time to Signal (*ATS*). This measure mainly depends on the average run length, and it is the average time until the control chart signals (Montgomery, 2013).

$$ATS = ARL \times h \tag{4}$$

where h is the sampling interval and when (h = 1), ATS is equal to the ARL.

In our current study, the average number of defectives (*AND*), which is the average number of non-conforming items produced in various '*Out-of-control'* instances throughout a process shift range ($p_0 < p \leq p_{max}$), are considered as a measure of performance; in other words, "*…metric[s] used to quantify the efficiency and/or effectiveness*" (Neely *et al*., 1995; p. 80). It also serves as a means of evaluating the detection speed of the control charts. This is because it also evaluates overall control chart performance across the entire shift range. However, as there is no certainty when the shift will occur during the process, the *AND* equation only evaluates overall chart performance of the chart, not only at a specific point but also across the entire shift range, hence its consideration as an objective function to be minimized. The *AND* function (see Haridy *et al*., 2020) is given as:

$$AND = \int_{p0}^{pmax} pATS\ (p) fp(p) dp \tag{5}$$

where p_0 is the in control non-conforming, p_{max} is the maximum '*Out-of-control'* fraction non-conforming; p is the '*Out-of-control'* fraction non-conforming; $ATS(p)$ is the '*Out-of-control' ATS* produced by the optimal *w*EWMA for an increased fraction non-conforming p; and $f_p(p)$ is the probability density function of p. $ATS(p)$ can be calculated via a Markov chain shown in the Appendix. If any shifts occur in the process, the quality will be affected, and the fraction non-conforming p will change to:

$$p = \delta \times p_0 \tag{6}$$

where δ is the intensity of the shift and, in the '*In-control'* situation, δ = 1 and $p = p_0$. In this research, we assume a uniform distribution for the p shift, a common approach when prior experience is lacking (Castagliola *et al*., 2011; Sparks, 2000). The corresponding density function is given by:

$$f_p(p) = \frac{1}{p_{max} - p_0} \tag{7}$$

3.7 Optimization algorithm

We now present the optimization design as a means of enhancing the detection effectiveness of the optimal *w*EWMA scheme based on the *AND* (Equation (5)) as an objective function:

Objective function:	Minimize *AND*
Constraints:	
	$ATS_0 \geq \tau$ (8)
	$r = n/h$ (9)
Design variables: n, h, w, λ and H	

The purpose of this algorithm is to identify optimal values of n, h, w, λ, and H in a manner that produces the smallest *AND* across a shift range of $p_0 < p < p_{max}$. As shown in Figure 3, a three-level design algorithm is used to allocate the optimal charting parameters of the optimal *w*EWMA scheme.

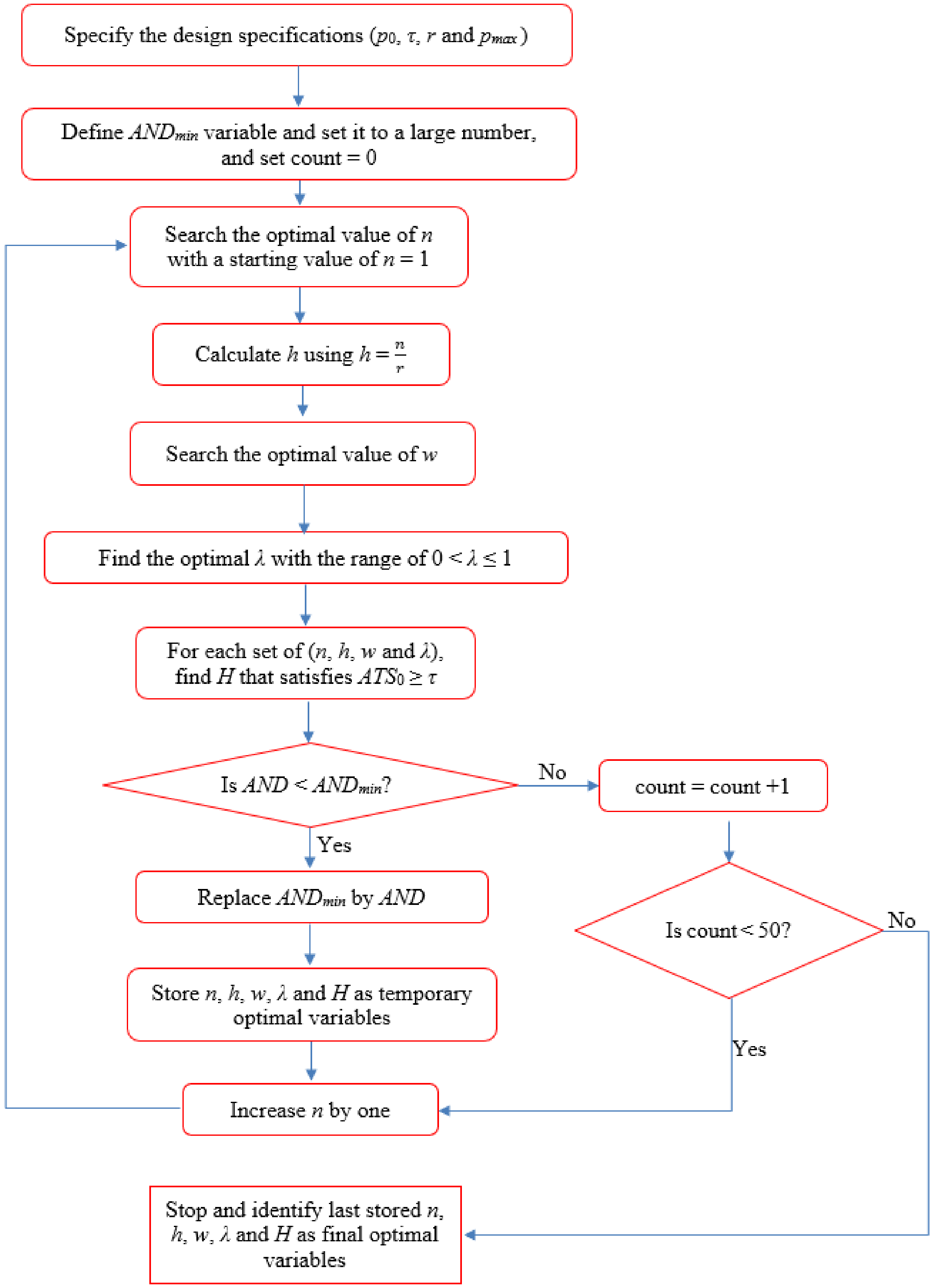

Figure 3: Optimization algorithm of the optimal *w*EWMA scheme

4. Results

In this section, we evaluate and compare the detection effectiveness of the EWMA, *w*EWMA, and optimal *w*EWMA charts across different shift sizes in the fraction non-conforming *p*, relative to a specified shift.

We collected 40 samples, each consisting of 50 bulbs per day. The manufacturing of lamps and light bulbs involves multiple sub-processes, each influencing different aspects of product quality. Ideally, a control chart should be implemented for each sub-process to ensure the overall quality of the final product. However, due to resource constraints, this approach is often impractical. Consequently, organizations must prioritize specific defects and classify a bulb as defective if any of these identified defects are present (see Goh *et al*., 1998).

Figure 4 presents a Pareto chart illustrating the most common defects encountered by Organization 'X' in the manufacturing of its best-selling product, the A21 model, based on a sample of 50 defective bulbs. This model has the highest defect rate among the company's offerings. The chart reveals that 40% of defects stem from non-functioning bulbs, while a combined 82% of defects are attributed to non-functioning bulbs, broken bases, and cracked glass. Organization 'X' classifies a bulb as defective if it exhibits any of the five defects depicted in Figure 4.

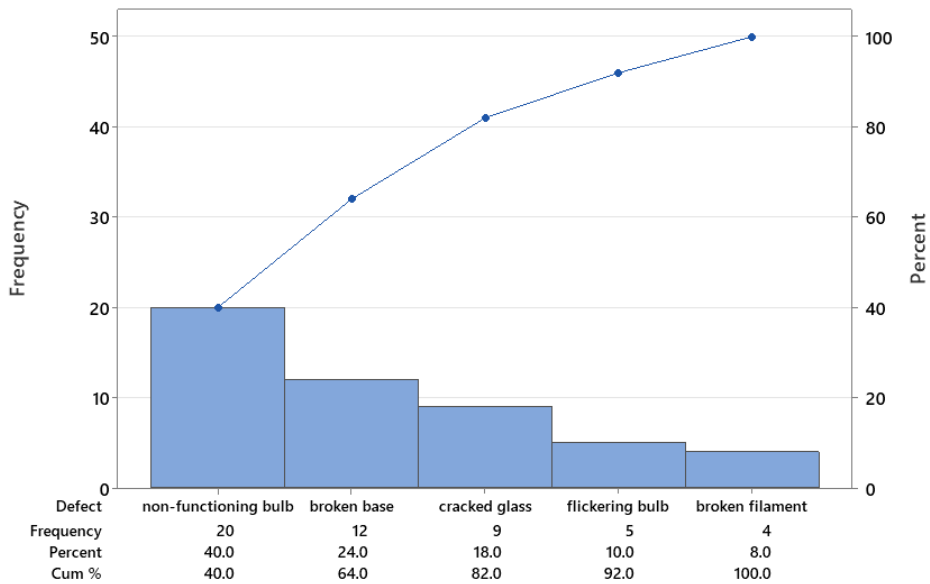

Figure 4: Defects in A21 light bulb

In Phase I of the control chart implementation, 40 samples were collected, each consisting of 50 bulbs. The number of defective bulbs in each sample was recorded and used to construct the np chart, as shown in Figure 5. We can infer that, at Phase I of the control chart implementation, all sample points fell within the control ranges, indicating that the process is under control. Consequently, the '*In-control*' p_0 can be estimated as follows:

$$p_0 = \frac{\sum_{t=1}^{m} d_t}{m \times n} = \frac{10}{40 \times 50} = 0.005 \tag{10}$$

where m is the number of samples (=40) and n is the sample size (=50).

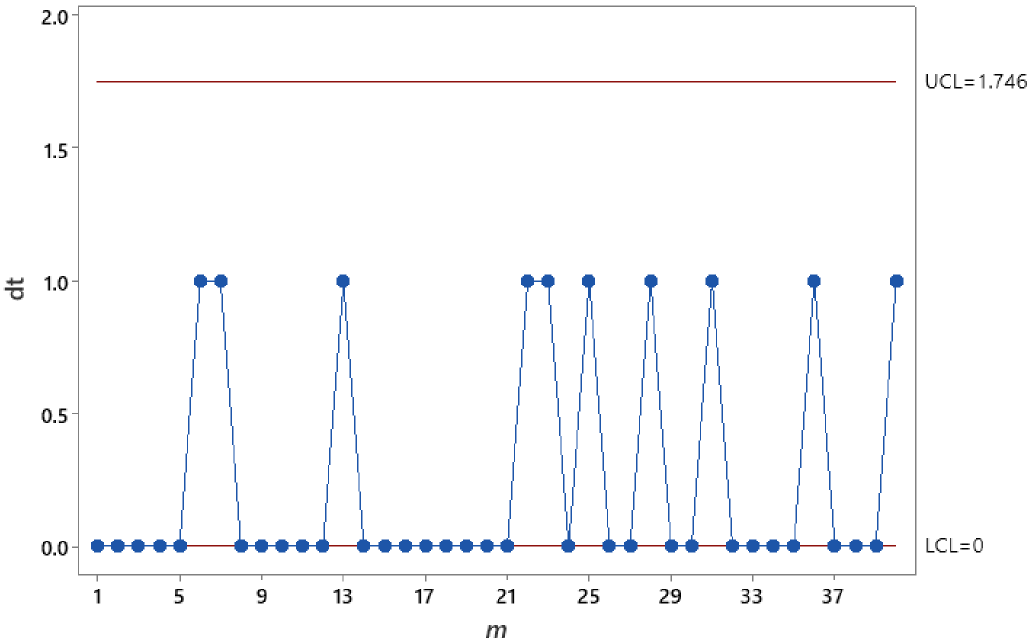

Figure 5: Phase I—*np* chart

4.1 Optimizing n and the other charting parameters

Resource constraints may have a significant adverse impact on the use of control charts (Rohleder and Silver, 2002; Celano, 2011; Haridy *et al.*, 2024a). Taking into consideration resources available to Organization 'X', and following discussions with process control and monitoring operators, an inspection rate (r) of 50 and τ of 1000 were chosen. The maximum fraction non-conforming the factor wants to detect is p_{max} = $5p_0$. To detect this shift, the EWMA, *w*EWMA, and optimal *w*EWMA schemes are used. The design specifications are summarized as follows:

$$\tau = 1000,\ p_0 = 0.005,\ p_{max} = 5p_0 \text{ and } r = 50$$

For Phase II of the implementation of control charts, the optimal *w*EWMA, *w*EWMA, and EWMA schemes were optimized. Based on the optimization algorithm introduced above (see section 3.7), the optimal design was conducted. In the EWMA chart, only λ and H are optimized, while w, λ, and H are optimized in the wEWMA chart. For both charts, n is considered equivalent to r and h is taken as the time unit (i.e., $h = 1$), to fulfil constraints (9). This is a common practice in quality control in most industries where n and h are decided based on the rational subgroup concept (i.e., $n = r$ and $h = 1$) without any optimization (Hawkins and Olwell, 1998, Wheeler, 2004). However, this approach doesn't ensure the best performance of the control charts. Contrary to other charts, all charting parameters including n and h are optimized in the design of the optimal wEWMA chart. The values of n and h are governed by the inspection rate (r) as indicated by Equation (9). As shown in Figure 3, n is treated as an independent variable, starting from an initial value of 1 and incremented iteratively with a step size of 1. For each value of n, the corresponding h is determined based on constraint (9), ensuring that available resources, such as manpower, cost and measurement tools, are utilized to enhance the control chart's detection capabilities. The n and h values that provides the smallest AND and satisfies the constrain on r are identified as the optimal ones.

The performance of the optimal wEWMA control chart was examined using both AND and ATS as performance metrics while ensuring that the constraints $ATS_0 \geq \tau$ and $r = n/h$ are satisfied. Table 1 shows the charting parameters, AND along with the Ratio of Average Number of Defectives ($RAND$) value which is the ratio of the AND value of the EWMA or wEWMA chart with the AND value of the optimal wEWMA. In other words, $AND_{EWMA} / AND_{\text{optimal } w\text{EWMA}}$ and $AND_{w\text{EWMA}} / AND_{\text{optimal } w\text{EWMA}}$.

Table 1: The *AND*, *RAND* and charting parameters of the EWMA, *w*EWMA, and optimal *w*EWMA schemes

Chart	Charting Parameters					Performance	
	n	***h***	***w***	***λ***	***H***	***AND***	***RAND***
EWMA	50	1	1	0.470	1.304	0.3664	1.53
***w*EWMA**	50	1	0.45	0.515	0.954	0.3392	1.42
Optimal *w*EWMA	140	2.8	0.60	0.515	1.235	0.2395	1

4.2 Comparison in terms of ATS

The average time to signal ATS represents the speed at which a control chart detects an '*Out-of-control'* signal. Table 2 presents the ATS values for the EWMA, wEWMA, and optimal wEWMA charts across the entire range of shifts ($1 < \delta \leq 5$). The results demonstrate that the optimal wEWMA achieves the lowest ATS values, confirming its superiority over the EWMA, *w*EWMA for detecting *p* shifts. Figure 6 visually compares the *ATS* values of the three schemes, illustrating the consistent advantage of the optimal *w*EWMA chart.

To further clarify this comparison, Figure 7 presents the normalized *ATS* values ($ATS_{\text{EWMA}}/ATS_{\text{optimal } w\text{EWMA}}$ and $ATS_{w\text{EWMA}}/ATS_{\text{optimal } w\text{EWMA}}$). The normalized results highlight that the optimal *w*EWMA consistently outperforms the other charts for all values of δ, particularly for small to moderate shifts. The use of normalized *ATS* provides a clearer visualization of relative performance and emphasizes the extent of the optimal *w*EWMA's advantage.

Table 2: *ATS* of the EWMA, *w*EWMA, and optimal *w*EWMA

		ATS	
δ	**EWMA**	***w*EWMA**	**Optimal *w*EWMA**
1.0	1010.200	1011.101	1014.872
1.5	216.254	201.316	139.798
2.0	77.648	70.259	45.984
2.5	37.687	34.053	22.689
3.0	21.841	19.792	14.024
3.5	14.382	13.504	10.029
4.0	10.205	9.719	7.721
4.5	7.719	7.576	6.274
5.0	6.053	6.122	5.364

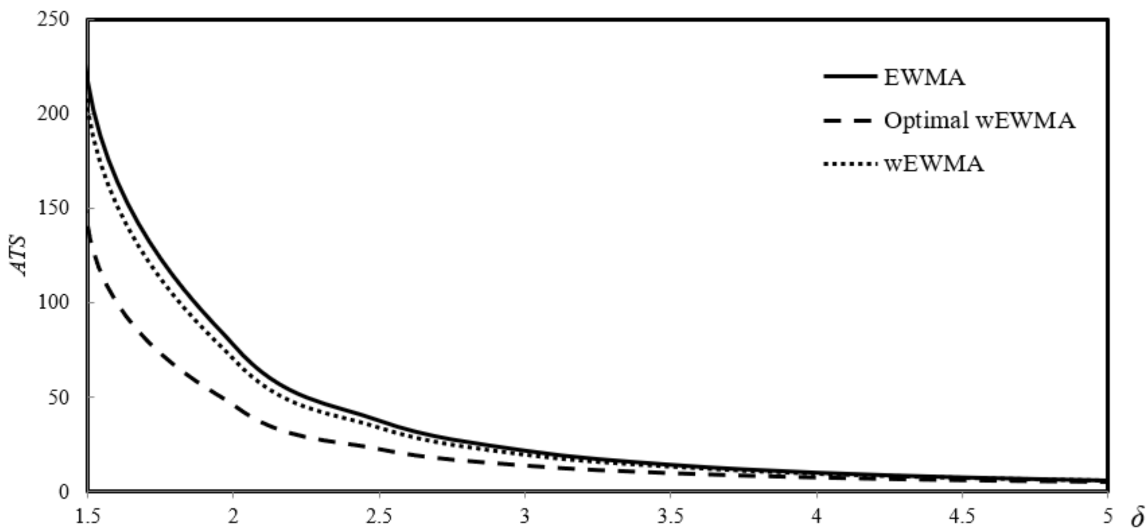

Figure 6: *ATS* of the three charts under the case (r = 50, τ= 1000, p_o=0.005, $\delta_{\max}$=5)

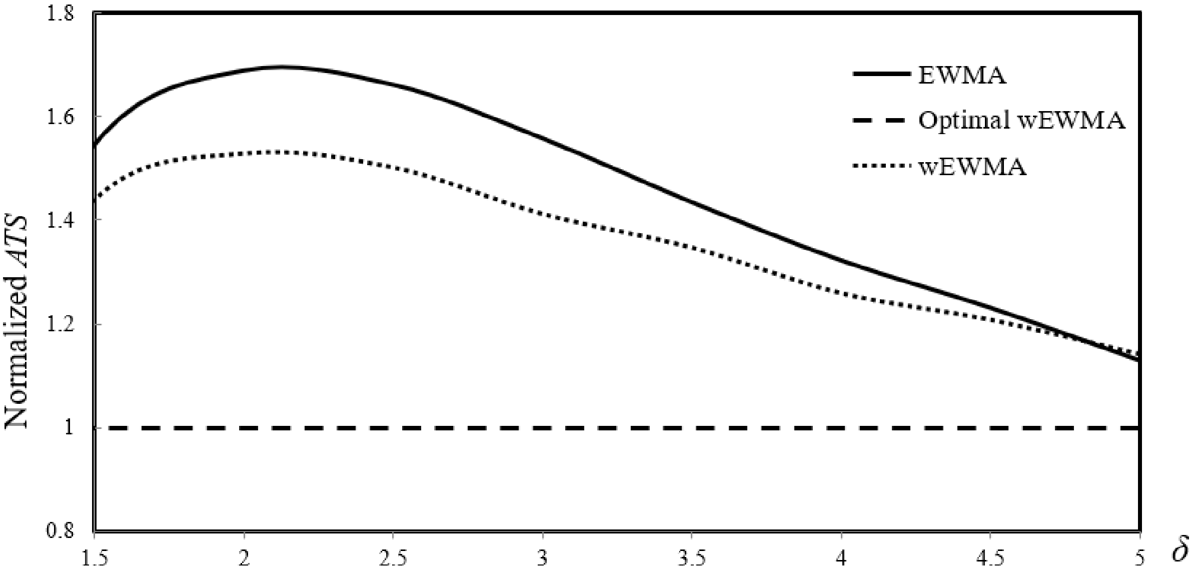

Figure 7: Normalized *ATS* of the three charts under the case ($r = 50$, $\tau = 1000$, p_o=0.005, δ_{max}=5)

4.3 Comparison in terms of AND

The values of *AND* of the EWMA and *w*EWMA charts are studied and compared with that of the optimal *w*EWMA chart based on the *RAND* and the results are found as follows:

$$RAND = AND_{\text{EWMA}} \,/\, AND_{\text{optimal } w\text{EWMA}} = 1.53$$
$$RAND = AND_{w\text{EWMA}} \,/\, AND_{\text{optimal } w\text{EWMA}} = 1.42$$

The *RAND* value displays the degree of overall improvement over the *w*EWMA and EWMA charts that the optimal *w*EWMA chart has achieved. In the examined scenario, the *AND* of the optimal *w*EWMA chart is reduced by 53% compared to the EWMA chart and by 49% compared to the *w*EWMA chart, based on the *RAND* value.

Random data for the quantity (d_t) of faulty bulbs were generated, following a binomial distribution b (50,0.005) with a shift of $\delta = 3$ occurred at the 20th sample. Using Equation (2), E_t is calculated for the EWMA, *w*EWMA, and optimal *w*EWMA schemes. Sample runs for these schemes are illustrated in Figures 8, 9, and 10, respectively. Figure 8 shows that the EWMA chart is not able to produce any '*Out-of-control*' signal. Figure 9 demonstrates that the first signal of the *w*EWMA scheme appears at sample 31 when $E_{31} = 1.042$ exceeds $H = 0.954$. Figure 10 reveals that the optimal *w*EWMA scheme is the fastest, signalling at the 22nd sample when $E_{22} = 1.55$ exceeds $H = 1.235$. This rapid detection capability of the optimal wEWMA scheme enables quality engineers to promptly identify '*Out-of-control*' scenarios and quality issues as they occur. Consequently, this allows for immediate investigation of assignable causes and quick corrective actions, thereby reducing the number of defective bulbs and mitigating potential economic losses.

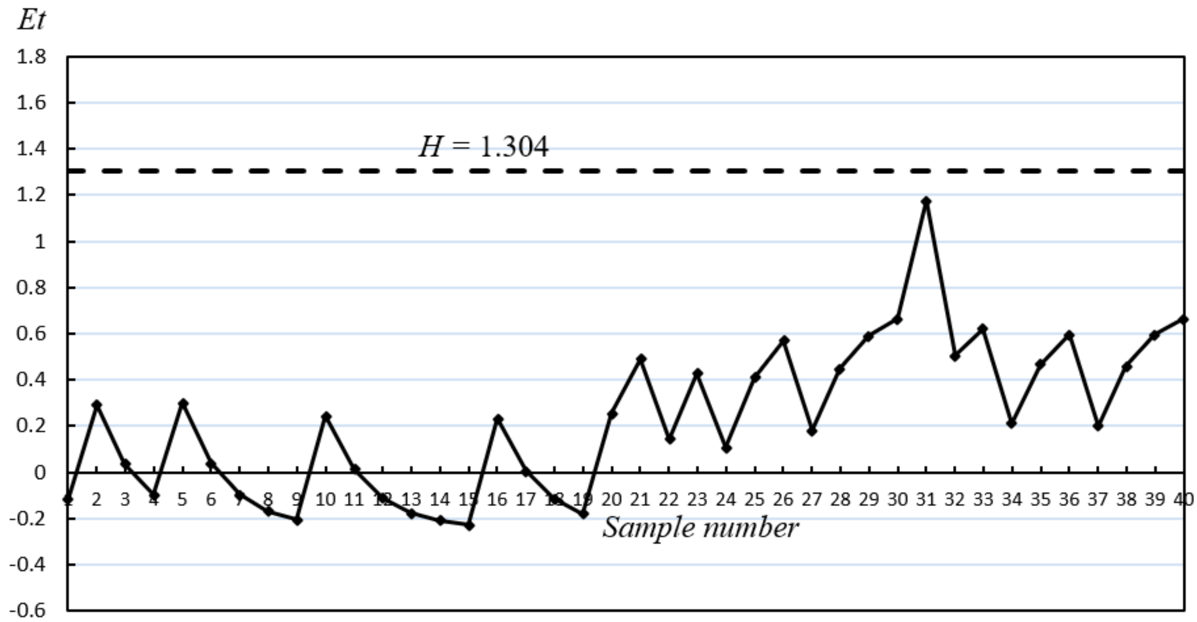

Figure 8: A sample run of the EWMA scheme

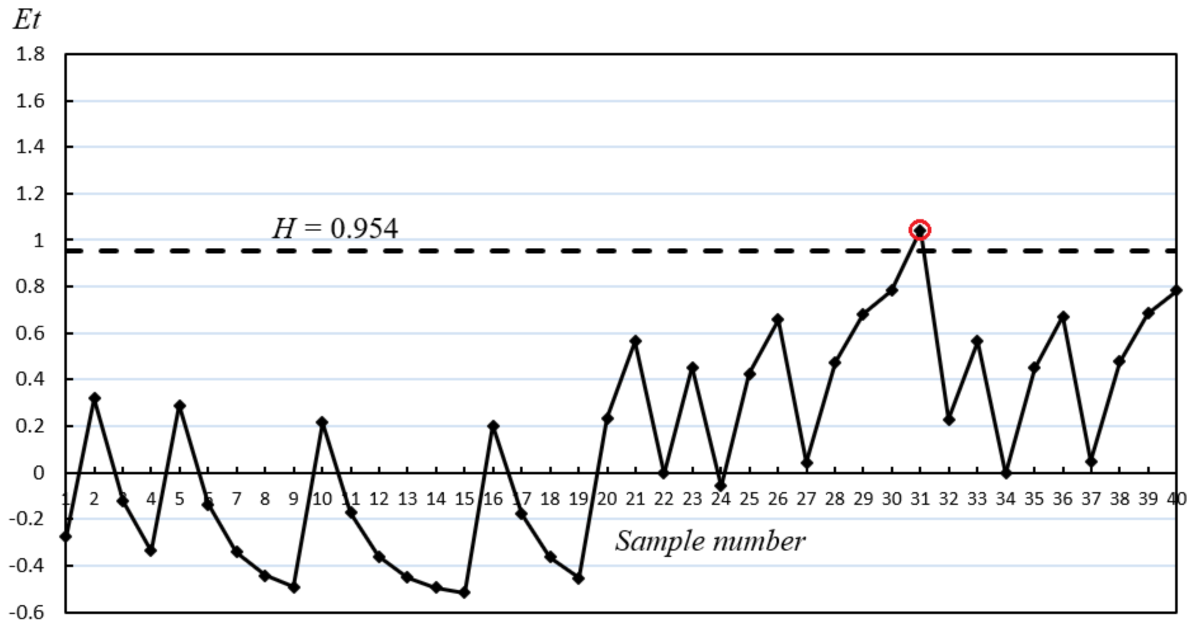

Figure 9: A sample run of the *w*EWMA scheme

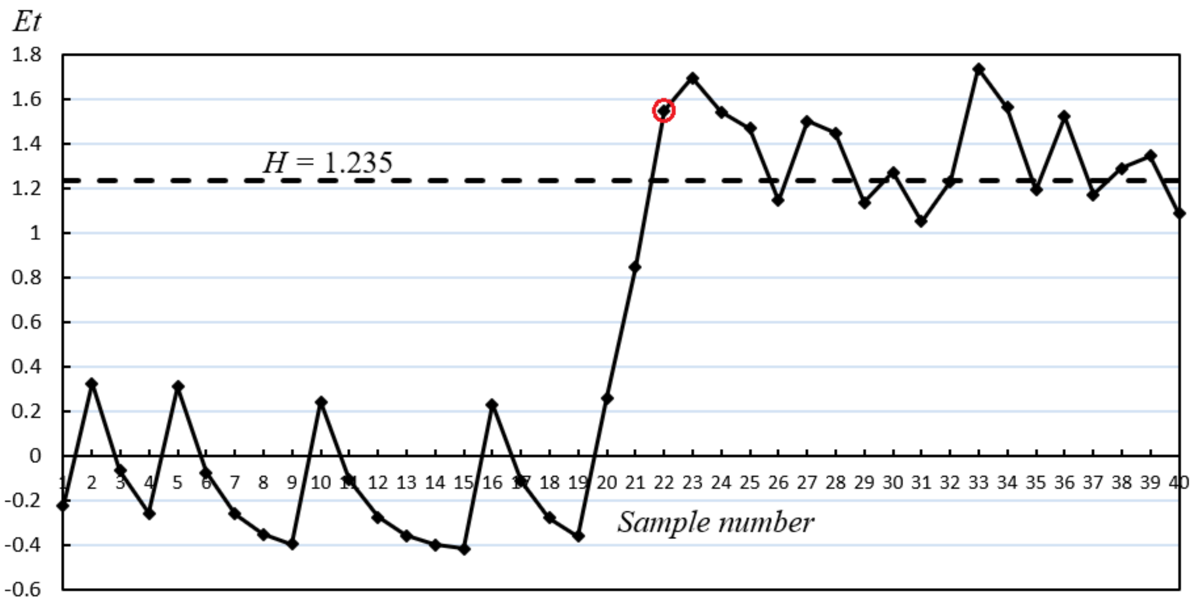

Figure 10: A sample run of the optimal *w*EWMA scheme

4.4 Validation under different design specifications

For validation, we compared the detection effectiveness of the three charts which are evaluated against 16 different cases shown in Table 3, which are derived from the literature (e.g., Haridy *et al*., 2020). Our interest being to show resultant improvement derived by optimizing *n*. The entirety of these improvements is displayed in terms of *AND*. The three schemes all result in an ATS_0 which is somewhat near to τ. The *RAND* and the charting parameters (n, h, w, λ and H) are shown in the table for each of the 16 cases. The *RAND* column reference ratio between schemes is never less than 1. As is observed, the optimal *w*EWMA scheme appears to always outperform both the EWMA and *w*EWMA control schemes.

The $\overline{AND_{\text{EWMA}}/AND_{\text{optimal } w\text{EWMA}}}$ and $\overline{AND_{w\text{EWMA}}/AND_{\text{optimal } w\text{EWMA}}}$ representing the mean of the *RAND* values for all the 16 cases are also calculated. The result of $\overline{AND_{\text{EWMA}}/AND_{\text{optimal } w\text{EWMA}}}$ is 1.26 and the result of $\overline{\text{ANDwEWMA/ANDoptimal wEWMA}}$ is 1.17. This demonstrates that the optimal *w*EWMA scheme outperforms the EWMA and *w*EWMA schemes by 26% and 17%, respectively, while considering different values of τ, n, p_0, and p_{max}.

Figure 11 presents a comparison of *AND* values across 16 cases using the three monitoring schemes. The optimal *w*EWMA consistently achieves the lowest *AND* values across all cases, indicating its superior overall detection effectiveness under diverse monitoring scenarios.

Similarly, Figure 12 compares RAND values across the same 16 cases for the three monitoring schemes. The optimal *w*EWMA scheme consistently outperforms the EWMA and *w*EWMA schemes, as reflected in its lower *RAND* values across all cases.

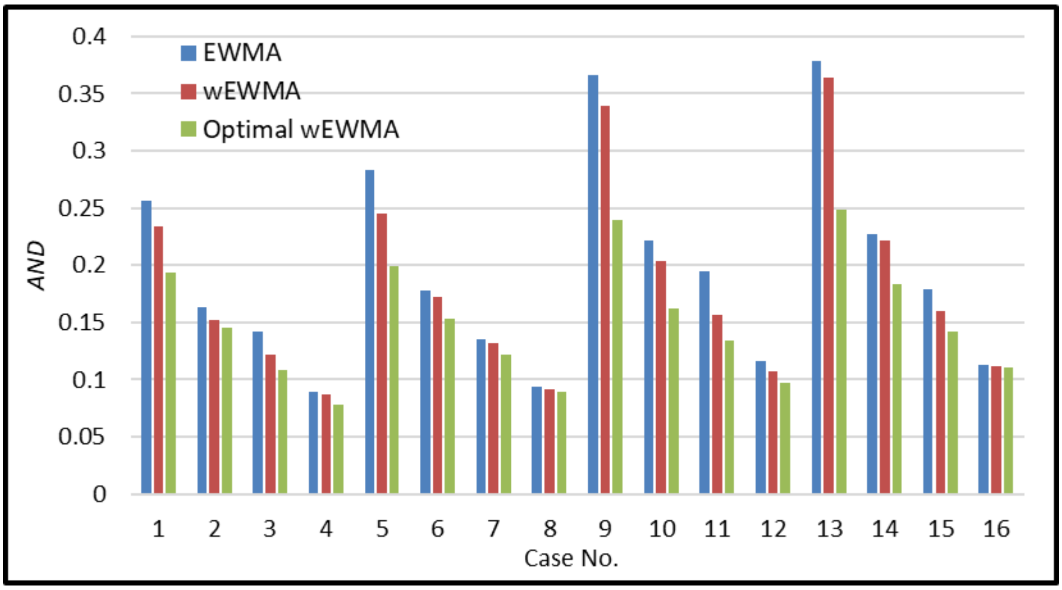

Figure 11: *AND* of the EWMA, *w*EWMA, and optimal *w*EWMA schemes under all considered cases

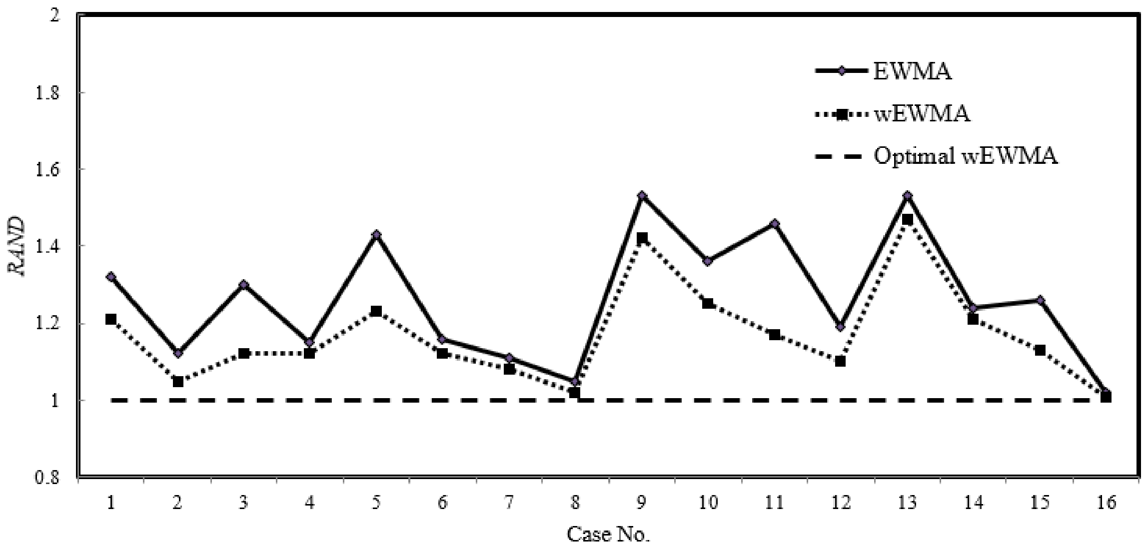

Figure 12: *RAND* of the EWMA, *w*EWMA, and optimal *w*EWMA schemes under all considered cases

Table 3: Comparison of the EWMA, *w*EWMA, and optimal *w*EWMA schemes under different conditions

Case	τ	p_0	r	δ_{max}	Chart	n	h	w	λ	H	*AND*	*RAND*
1	500	0.005	50	5	EWMA	50	1	1	0.485	1.227	0.2565	1.32
					*w*EWMA	50	1	0.75	0.470	0.985	0.2335	1.21
					Optimal *w*EWMA	94	1.88	0.75	0.470	1.099	0.1936	1.00
2	500	0.005	50	10	EWMA	50	1	1	0.485	1.225	0.1628	1.12
					wEWMA	50	1	0.75	0.440	0.935	0.1526	1.05
					Optimal wEWMA	62	1.24	0.9	0.470	1.129	0.1453	1.00
3	500	0.005	100	5	EWMA	100	1	1	0.485	1.559	0.1416	1.30
					*w*EWMA	100	1	0.9	0.395	1.213	0.1217	1.12
					Optimal *w*EWMA	186	1.86	0.75	0.500	1.432	0.1086	1.00
4	500	0.005	100	10	EWMA	100	1	1	0.485	1.555	0.0896	1.15
					wEWMA	100	1	0.9	0.485	1.411	0.0871	1.12
					Optimal wEWMA	160	1.6	1.05	0.440	1.697	0.0780	1.00
5	500	0.01	50	5	EWMA	50	1	1	0.485	1.551	0.2838	1.43
					*w*EWMA	50	1	0.6	0.470	1.098	0.2451	1.23
					Optimal *w*EWMA	158	3.16	1.2	0.410	2.276	0.1990	1.00
6	500	0.01	50	10	EWMA	50	1	1	0.470	1.509	0.1775	1.16
					*w*EWMA	50	1	0.6	0.470	1.099	0.1717	1.12
					Optimal *w*EWMA	90	1.8	1.05	0.440	1.735	0.1531	1.00
7	500	0.01	100	5	EWMA	100	1	1	0.440	1.884	0.1354	1.11
					*w*EWMA	100	1	0.75	0.470	1.506	0.1315	1.08
					Optimal *w*EWMA	149	1.49	0.9	0.470	1.947	0.1217	1.00
8	500	0.01	100	10	EWMA	100	1	1	0.485	2.039	0.0936	1.05
					*w*EWMA	100	1	0.9	0.455	1.727	0.0913	1.02
					Optimal *w*EWMA	115	1.15	1.2	0.380	2.220	0.0893	1.00
9	1000	0.005	50	5	EWMA	50	1	1	0.470	1.304	0.3664	1.53
					*w*EWMA	50	1	0.45	0.515	0.954	0.3392	1.42
					Optimal *w*EWMA	140	2.8	0.6	0.515	1.235	0.2395	1.00
10	1000	0.005	50	10	EWMA	50	1	1	0.470	1.306	0.2214	1.36
					*w*EWMA	50	1	0.75	0.485	1.105	0.2039	1.25
					Optimal *w*EWMA	146	2.92	0.75	0.485	1.348	0.1626	1.00
11	1000	0.005	100	5	EWMA	100	1	1	0.485	1.730	0.1944	1.46
					*w*EWMA	100	1	0.45	0.410	0.998	0.1566	1.17
					Optimal *w*EWMA	173	1.73	0.6	0.440	1.243	0.1336	1.00
12	1000	0.005	100	10	EWMA	100	1	1	0.485	1.730	0.1165	1.19
					*w*EWMA	100	1	0.75	0.455	1.293	0.1070	1.10
					Optimal *w*EWMA	192	1.92	1.05	0.515	2.260	0.0975	1.00
13	1000	0.01	50	5	EWMA	50	1	1	0.470	1.675	0.3788	1.53
					*w*EWMA	50	1	0.9	0.470	1.512	0.3639	1.47
					Optimal *w*EWMA	139	2.78	0.9	0.500	2.008	0.2481	1.00
14	1000	0.01	50	10	EWMA	50	1	1	0.470	1.675	0.2275	1.24
					*w*EWMA	50	1	0.75	0.455	1.287	0.2217	1.21
					Optimal *w*EWMA	128	2.56	1.05	0.500	2.375	0.1838	1.00
15	1000	0.01	100	5	EWMA	100	1	1	0.470	2.191	0.1787	1.26
					*w*EWMA	100	1	0.75	0.440	1.556	0.1603	1.13
					Optimal wEWMA	185	1.85	0.9	0.485	2.315	0.1418	1.00
16	1000	0.01	100	10	EWMA	100	1	1	0.440	2.105	0.1131	1.02
					*w*EWMA	100	1	0.75	0.485	1.663	0.1122	1.01
					Optimal *w*EWMA	100	1	0.9	0.470	1.935	0.1109	1.00

Based on the *AND* reported for the three schemes across the 16 cases in Table 3, a paired t-test is performed to determine whether the optimal *w*EWMA chart's performance is statistically different from that of the EWMA and *w*EWMA schemes. This t-test is structured as follows:

Null hypothesis (H_0): $\mu_d = 0$

Alternative hypothesis (H_1): $\mu_d < 0$

where μ_d represents the mean of the difference (d) between the *AND* values of the optimal *w*EWMA chart and those of the other EWMA schemes.

To ensure the validity of the paired t-test, the normality of the *AND* values for the three schemes was verified using the Anderson-Darling test at a significance level of 0.05. Figure 13 shows the normal probability plot of the three schemes. As their p values exceeded 0.05, the *AND* values can be considered normally distributed, thus allowing the paired t-test to determine whether there is a significant difference in the *AND* values across the three schemes. Table 4 summarizes the results of the t-test.

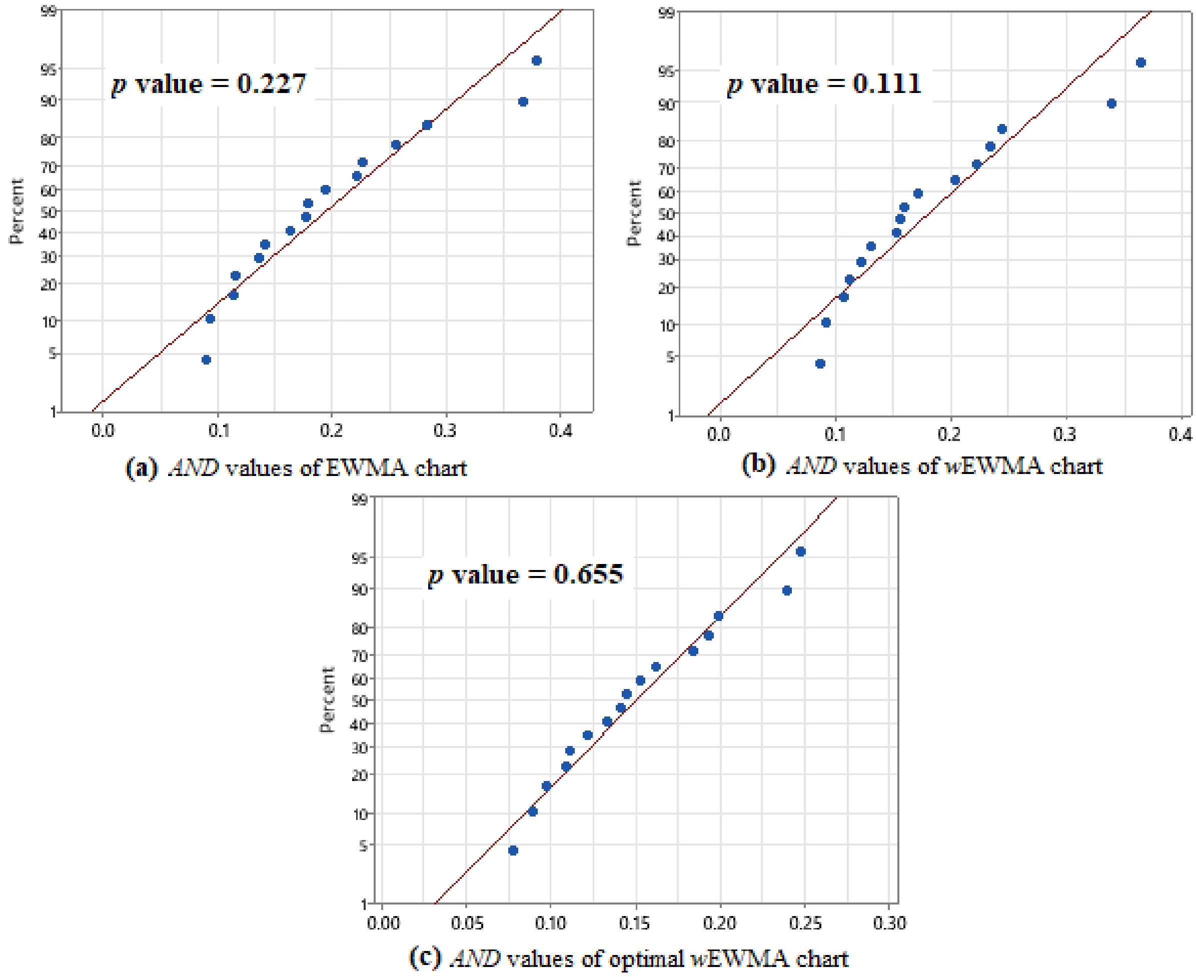

Figure 13: Normal probability plots of the *AND* values for the three schemes

Table 4: Results of the paired t-test

Compared schemes	t value	*p* value
Optimal *w*EWMA vs. EWMA	-4.57	< 0.001
Optimal *w*EWMA vs. *w*EWMA	-3.69	0.001

Since both *p* values are less than the significance level, which is considered as 0.05 in this test, then the H_0 can be rejected. Consequently, we conclude that the optimal *w*EWMA scheme significantly lowers the average number of defectives compared to both the EWMA and *w*EWMA charts.

Figures 14 and 15 illustrate the differences *d* between the *AND* values of the optimal *w*EWMA chart and those of the EWMA and *w*EWMA schemes, respectively, across the 16 cases in Table 3. Both figures include a one-tailed 95% confidence interval for the mean difference ($\overline{X}$), which represents the average of the *AND* values for the 16 cases. Clearly, $\overline{X}$ lies well below the H_0 reference line, indicating that the optimal *w*EWMA chart achieves significantly lower *AND* values than the other schemes across all 16 cases.

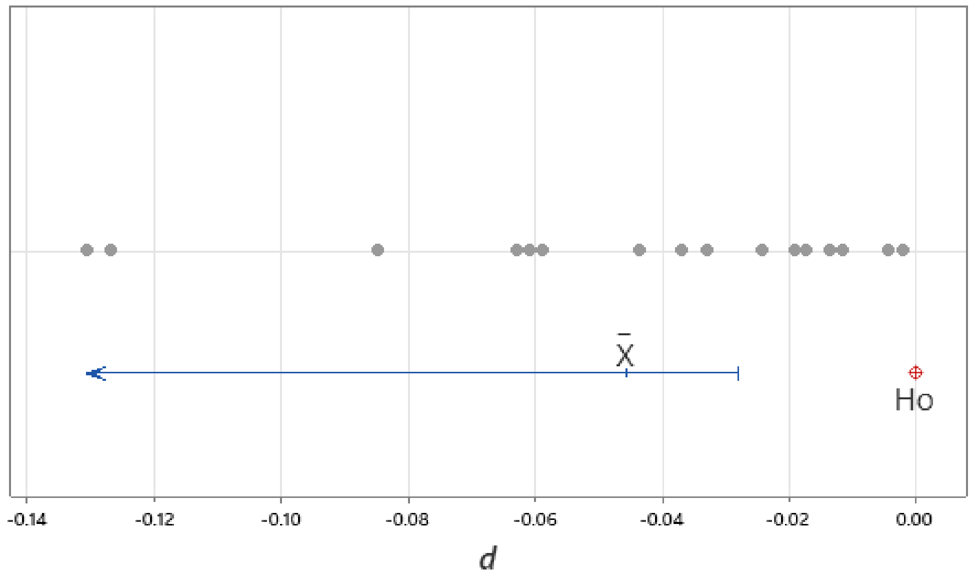

Figure 14: Individual plot of *d* values between the *AND* of the optimal *w*EWMA chart and those of the EWMA schemes, considering the 16 cases in Table 3

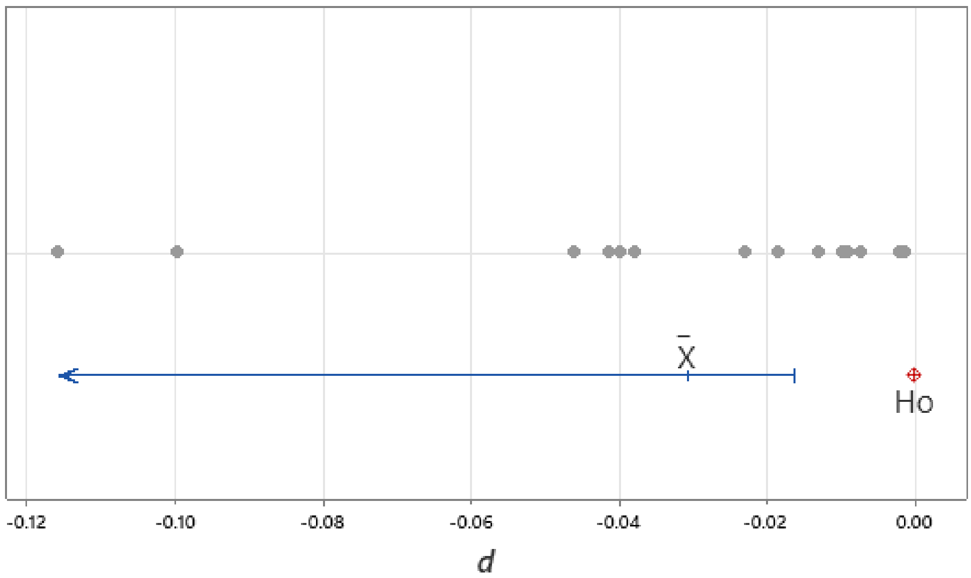

Figure 15: Individual plot of *d* values between the *AND* of the optimal *w*EWMA chart and those of the *w*EWMA schemes, considering the 16 cases in Table 3

4.5 Cost-effectiveness analysis

This section employs the 16 cases outlined in Section 4.4 to conduct a cost-effectiveness analysis of the optimal *w*EWMA chart compared to its primary competitor, the *w*EWMA chart. The objective is to compare the Expected Total Cost (*ETC*) associated with each chart, from the occurrence of an assignable cause to the generation of an '*Out-of-control'* signal. The *ETC* represents the cost of poor quality incurred during the '*Out-of-control'* phase and is calculated using the following equation:

$$ETC = (Nc_p)\int_{p0}^{pmax} pATS\,(p) fp(p)dp \qquad (11)$$

where *N* represents the production rate and c_p denotes the cost of a poor-quality item. *N* is estimated as 750 per unit time, while c_p is determined to be \$0.8.

Figure 16 illustrates the results, highlighting the reduction percentage (*Rp*) in *ETC* between *w*EWMA and optimal *w*EWMA charts. The analysis reveals considerable cost reductions across most cases. For instance, Cases 1, 5, 9, 10, 13 and 14 exhibit higher *Rp* values exceeding 15%. This demonstrates significant cost-effectiveness when using optimal *w*EWMA scheme compared to the *w*EWMA scheme. The *ETC* values for the *w*EWMA chart in these cases are notably higher, emphasizing the impact of optimization.

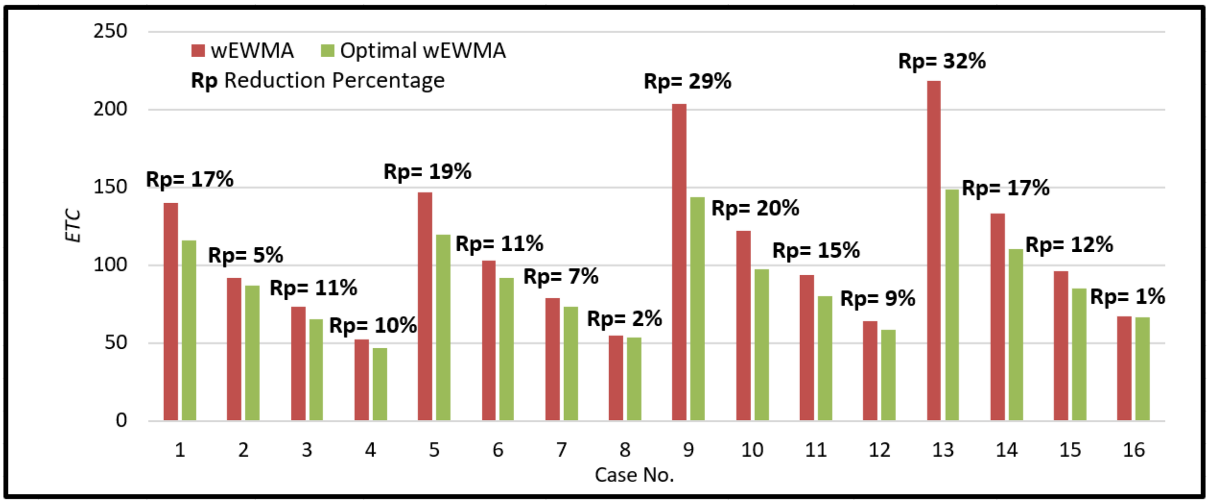

Figure 16: Cost-benefit analysis of the *w*EWMA, and optimal *w*EWMA schemes using *ETC*

5. Discussion

Defects in manufacturing and production, arising from deviations from established quality standards, represent the most fundamental type of quality failure in terms of the presence of product defects. Often, defective products fail to meet the design specifications set by the manufacturer. In other cases, they may not comply with regulatory standards mandated by government authorities. When defective products bypass the manufacturer's quality control measures, the risk of failure during use increases significantly, potentially leading to severe business consequences especially in terms of profitability and competitiveness.

Given these risks, the elimination of product defects through specialized process control and monitoring techniques and tools are critical components of manufacturing. Quality control plays a vital role in ensuring that both the production process and the final product conform to specific quality requirements, with the EWMA control charts representing one of its key tools. This study was motivated by the case organization's ongoing challenges with manufacturing defects in its popular A21 model light bulb. The company recognized that persistent defects could damage its brand reputation, ultimately undermining customer trust and loyalty. In response to these concerns, noting the effectiveness of EWMA charts in the detection of minor shifts, we set out in our study to develop a quality control EWMA chart that is both practical and theoretically robust, capable of detecting a broad range of shifts in the fraction non-conforming (p) while minimizing the average number of defectives.

Recognizing that the existing SPC control chart employed by the case organization, specifically, the traditional EWMA control chart, was limited in its effectiveness, particularly in detecting certain magnitudes of *p* shifts, we sought to address this shortcoming. To reduce the high frequency of product non-functionality, we developed, presented, and validated an optimized EWMA chart. Our approach involved designing an improved control scheme, the optimal weighted EWMA (*w*EWMA) chart, which builds on both the existing EWMA chart used by the case organization and the earlier optimized *w*EWMA scheme proposed by Haridy *et al*. (2020).

We employed *ATS*, *RAND* and *AND* comparisons in addition to a validation exercise involving detection effectiveness comparisons of the three charts against 16 cases to show performance difference between the three EWMA schemes. In terms of the *ATS* comparison, normalized *ATS* of the three charts under the case (r = 50, τ= 1000, p_o=0.005, δ_{max}=5) pointed to the optimal *w*EWMA chart being more effective than the EWMA and the *w*EWMA charts for detecting *p* shifts over the whole given range in the case of the *w*EWMA. In terms of the *AND* comparison, based on our findings, while maintaining the same constraints on the false alarm rate and inspection rate $r = n/h$, examining 140 samples every 2.8 days will yield higher performance than inspecting 50 samples every day.

Based on the comparison of the average number of defectives (*AND*) and considering the degree of improvement, 53% over the traditional EWMA and 42% over the *earlier w*EWMA, it can be inferred that inspecting 140 bulb samples every 2.8 days yields better quality control outcomes than inspecting 50 samples daily. This inspection schedule provides a balanced approach compared to other policies such as 100% inspection, sample-based inspection, or no inspection at all (Fink and Margavio, 1994; Farooq e*t al*., 2017).

However, despite the aspiration of achieving zero defects in household consumer goods (Lee and Unnikrishnan, 1998; Farooq *et al*., 2017), practical considerations must be considered. These include the costs of maintaining inspection equipment, such as scanning laser systems and measuring machines (Lee and Unnikrishnan, 1998), as well as the increased financial burden associated with higher inspection rates. Consequently, 100% inspection of the A21 model light bulb remains impractical, despite the product's relatively high defect rate (Lee and Unnikrishnan, 1998; Farooq *et al*., 2017).

The validation exercise further confirmed the superior performance of the optimal *w*EWMA scheme over both the traditional EWMA and the earlier *w*EWMA schemes, as demonstrated by improved results in the relative average number of defectives (*RAND*).

6. Conclusion

Statistical Process Control (SPC) is widely recognized as a powerful methodology for enhancing business processes. Among its various tools, the control chart remains one of the most effective and straightforward real-time graphical techniques for monitoring and maintaining process stability. The application of control charts offers several advantages, including the reduction and eventual elimination of defects, thereby contributing to overall quality improvement.

Traditionally, process anomalies and shifts have been detected using classical control charts such as the Exponentially Weighted Moving Average (EWMA) chart. However, these conventional charts possess well-documented limitations, particularly concerning performance optimization. Their detection capabilities are not always ideal, often leaving a significant gap in sensitivity to various magnitudes of process shifts. This limitation has opened avenues for optimization techniques to assume a more prominent role in quality process control. In this context, optimization refers to identifying the most efficient and effective process control strategy to achieve targeted performance objectives while adhering to real-world operational constraints.

A traditional response to the quality issues faced by the case organization would typically involve identifying assignable causes, analysing them, and implementing corrective actions. However, this approach would likely be counterproductive in the present context, leading to inefficient resource allocation, an especially critical issue for a small enterprise. Moreover, valuable time might be consumed exploring solutions that ultimately prove unfeasible. Given these limitations, the organization risked significant losses in both time and resources without any assurance of attaining an optimal outcome.

This study was conducted within a single case organization, a light bulb manufacturing facility and focused on the development and optimization of an enhanced version of the weighted exponentially weighted moving average (*w*EWMA) scheme. We introduced an optimal *w*EWMA scheme by systematically optimizing the sample size (n) and the sampling interval (h), two critical parameters that had not been addressed in prior studies. This advancement marks a significant contribution to the existing body of research on statistical process control and monitoring.

The proposed scheme is designed to enhance the overall performance of the weighted exponentially weighted moving average (*w*EWMA) chart by minimizing the average number of defectives (*AND*), which serves as the optimization objective. A comparative analysis was conducted to evaluate the effectiveness of the optimal *w*EWMA scheme relative to two benchmarks: (i) the traditional EWMA chart currently employed by the case organization and

(ii) the original *w*EWMA chart as developed by Haridy *et al*. (2020). The results demonstrate that the optimal *w*EWMA scheme exhibits superior performance across a range of design specification settings, achieving a 26% improvement over the traditional EWMA chart and a 17% improvement over the original *w*EWMA chart.

Our study is both timely and significant for several compelling reasons. Despite the widespread implementation of statistical process control (SPC) and monitoring techniques, quality deviations and manufacturing defects continue to persist across industrial production lines, representing a substantial challenge for the manufacturing sector (Kang *et al*., 2017; Catenazzo and Paulssen, 2020, 2023; Damavandi and Astvansh, 2025). These issues are particularly acute in the household lighting industry, where defects in lamps and light bulbs have, in documented cases, led to consumer injuries and fatalities (United States Consumer Product Safety Commission, 2024). Such outcomes underscore the critical need for more robust and optimized quality control and process monitoring systems.

Our study is both timely and significant for several key reasons. Although statistical process control (SPC) and monitoring techniques have been widely adopted across industrial manufacturing and production environments, defects and deviations from quality standards continue to persist, presenting ongoing challenges to the sector (Kang *et al*., 2017; Catenazzo and Paulssen, 2020, 2023). These challenges are especially critical in the context of household lamp and light bulb production, where manufacturing defects can have serious safety implications. In some cases, such defects have led to consumer injuries and fatalities (United States Consumer Product Safety Commission, 2024), highlighting the urgent need for more effective quality control and monitoring strategies.

Notably, product recalls stemming from manufacturing defects have been mandated across multiple jurisdictions, including the United Kingdom (United Kingdom Government, 2023a, 2023b), the United States (United States Consumer Product Safety Commission, 2024), and the United Arab Emirates (Kapur, 2013). These defects continue to pose a persistent and significant challenge within the manufacturing industry (Catenazzo and Paulssen, 2020). The consequences of such failures can be severe, encompassing consumer harm, such as injury or even loss of life, and substantial reputational and financial repercussions for manufacturers. For instance, high-profile cases, such as the bankruptcy of Aearo Technologies, underscore the potential for catastrophic business outcomes. Against this backdrop, the imperative to adopt advanced tools for quality control and process monitoring is both evident and urgent.

6.1 Theoretical implications

Our study makes several important theoretical contributions to the field of statistical process control (SPC). Notably, it extends existing SPC methodologies by optimizing not only traditional charting parameters such as the weighting factor (w), smoothing constant (λ), and upper control limit (H), but also the sample size (n) and sampling interval (h), dimensions often overlooked in prior research. This integrated optimization enhances the detection sensitivity of the control chart while simultaneously minimizing the occurrence of false alarms. As such, the study advances the literature by illustrating how the systematic integration of design specifications into control chart frameworks improves both detection effectiveness and predictive accuracy.

Moreover, by adopting a rigorous optimization strategy, the study offers insights into improved decision-making in SPC, particularly under resource and operational constraints. Our results show that the optimized *w*EWMA scheme delivers superior cost-effectiveness, underscoring the practical advantages of optimized SPC design models. Additionally, we acknowledge that expectations of detection performance and predictive accuracy can vary across manufacturing sectors due to differing industry-specific and regulatory requirements. Consequently, our findings reinforce the need for customized control chart designs that are responsive to contextual demands, thereby increasing their applicability and relevance across diverse industrial environments.

Beyond methodological innovation, the study deepens the understanding of the EWMA model as a critical quality management tool. By analyzing how various design parameters influence chart performance, we demonstrate the value of tailoring control schemes to enhance predictive capability (see Lee *et al*., 2020; Vega-Zambrano *et al*., 2025). However, while the optimal *w*EWMA chart outperforms traditional EWMA and prior *w*EWMA models, we also caution against the rigid application of SPC practices. Rooted in a philosophy of standardization, SPC can, if excessively emphasized, constrain the organizational flexibility needed for innovation. This concern aligns with previous studies, which suggest that overemphasis on process consistency and productivity can diminish a firm's innovative capacity (Benner and Tushman, 2003; Sony *et al*., 2020). Accordingly, our findings suggest that manufacturers must strive to balance process conformance with innovation to maintain long-term competitiveness.

6.2 Practical implications

From a practical standpoint, the enhanced EWMA chart developed in this study directly addresses several quality control challenges encountered by the case organization, offering a robust tool for reducing non-functionality and non-conformance in its flagship A21 model light bulb. As a viable alternative to the traditional EWMA control chart, the optimal *w*EWMA provides a more effective solution for real-time quality monitoring and early detection of defects. Its implementation enables timely intervention in the A21 production line, mitigating the risk of defect escalation and preventing the proliferation of nonconforming bulbs—a matter of particular concern given the potential consumer harm associated with reported product failures.

The optimal *w*EWMA scheme achieves a balanced level of detection sensitivity. It minimizes unnecessary production disruptions and inspection costs, while still maintaining a high level of defect detection accuracy. Empirical tests conducted within the case organization consistently demonstrated that the optimal *w*EWMA outperforms both the traditional EWMA and the original *w*EWMA chart. These tangible performance gains have prompted ongoing discussions regarding the chart's broader application across other bulb models produced by the organization. Its scalability indicates not only potential for internal expansion but also the possibility of its use as a strategic tool for aligning quality management with broader organizational goals, what might be termed "*strategy–quality mapping*".

Moreover, the scheme shows promise for wider adoption across the small lamp and light bulb manufacturing sector. It may also be adapted for application in other manufacturing domains, particularly those involving household consumer goods (e.g., surgical sponges) and industrial products (e.g., transformers and aluminium tubes), where early defect detection and process optimization are equally critical. The versatility and impact of the optimal *w*EWMA scheme thus extend well beyond the initial case context, suggesting its relevance to a diverse range of manufacturing environments.

6.3 Limitations

As expected, this study has several limitations, which simultaneously present opportunities for further research. These limitations arise from both the practical challenges associated with implementing the optimal *w*EWMA scheme and the foundational assumptions underpinning its development.

From a practical perspective, the successful adoption of the optimal *w*EWMA chart depends on a range of organizational factors. These include the feasibility of implementing the modelling procedures within existing operational constraints and the degree of external intervention, such as support from the original developers, that may be necessary during the deployment phase. Additionally, practical concerns relate to the number of algorithmic runs required and the computational resources needed to generate optimal solutions. Given that the optimization process involves a multidimensional design space, spanning parameters such as sample size (n), sampling interval (h), weight (w), smoothing constant (λ), and control limit (H), the algorithm necessitates many iterations to reach convergence. This increases computational complexity and necessitates a trade-off between solution accuracy and computational efficiency. One viable approach to addressing this issue is to implement the algorithm in a high-performance programming language such as *C*, which can significantly enhance execution speed and computational efficiency.

Moreover, while certain experienced operators within the case organization are equipped to apply the optimized chart, extending its use across broader production lines will require additional employee training. Although this has cost implications, we argue that such investment is justified. The long-term benefits associated with the early detection and elimination of non-conforming units, particularly in high-volume manufacturing settings such as light bulb production, are likely to outweigh the initial implementation and training costs. Consequently, the broader application of the optimal *w*EWMA scheme across the organization holds promise for driving sustained improvements in product quality and process reliability.

6.4 Future studies

There are considerable opportunities for future research, particularly in extending the applicability of the optimal *w*EWMA scheme to a broader range of industrial contexts. While previous iterations of the *w*EWMA chart have been applied in the monitoring of quality performance in consumer and industrial product manufacturing, such as surgical sponges (Haridy *et al*., 2013), electrical transformers (Haridy *et al*., 2014), and aluminium tubes (Haridy *et al*., 2020), further research could examine the effectiveness of the optimal *w*EWMA variant in these and other sectors. Such studies would benefit from assessing the scheme's utility as an alternative method for detecting nonconformities, particularly in processes characterized by discrete shifts in quality metrics.

One promising avenue for future investigation involves the integration of cost considerations into the control chart design. Specifically, studies could model appraisal costs (e.g., those related to identifying defective units through inspection or testing) alongside failure costs (e.g., warranty claims, scrapping, repairs, lost goodwill, and diminished sales). This line of inquiry would enrich the economic rationale behind control chart adoption and support the development of cost-sensitive SPC frameworks. Additionally, researchers could benchmark the performance of the optimal *w*EWMA scheme against other advanced control charts reported in the SPC literature, such as cumulative sum (CUSUM), generalized likelihood ratio (GLR), and adaptive schemes.

From a methodological perspective, one of the key assumptions in the current study, the binomial distribution of non-conforming units (d_t), presents another area for exploration. Future research could assess the robustness and sensitivity of the optimal *w*EWMA scheme under alternative statistical distributions such as the Beta, Rayleigh, or zero-inflated distributions, which may better reflect quality characteristics in specific manufacturing processes.

Finally, the model introduced in this study may be further enhanced by incorporating emerging technologies and predictive analytics. For instance, the integration of augmented reality (AR) into control chart interfaces could improve real-time visualization of process data and enhance operator interaction with digital representations of inspected units. Similarly, the incorporation of artificial intelligence (AI) could enable more adaptive, self-correcting control charts with real-time learning capabilities (see Ahmadini *et al*., 2025; Escobar *et al*., 2025; Kazmi and Noor-ul-Amin, 2025). Such developments could significantly advance the predictive power and responsiveness of SPC tools. Nevertheless, it is important to recognize that small- and medium-sized manufacturers may face practical limitations, such as limited technical expertise and financial resources, that constrain the implementation of AI- or AR-driven quality control systems. Future research could therefore explore scalable and cost-effective pathways for the adoption of such technologies in resource-constrained environments.

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Appendix: **In-control ATS_0 and out-of-control ATS of the optimal wEWMA scheme** (Haridy *et al*., 2020)**:**

Equation (A1), below, can be used to calculate the cumulative probability function $F_{v_t}(V)$ of v_t in Equation (2):

$$F_{v_t}(V) = \Pr\ (v_t \le V) = \begin{cases} \Pr\ \left(d_t \le \left[V^{1/w} + \ d_0\right]\right) & if \quad V \ge 0 \\ \\ \Pr\ \left(d_t \le \left[d_0 - (-V)^{1/w}\right]\right) & if \quad V < 0, \end{cases} \tag{A1}$$

where $[a]$ indicates the largest integer which is smaller than or equal to a. If d_t follows a binomial distribution, then

$$\Pr\ \left(d_t \le [a]\right) = \sum_{i=0}^{[a]} C_i^n p^i (1-p)^{n-i}, \qquad C_i^n = \frac{n!}{i!\ (n-i)!} \tag{A2}$$

A Markov chain procedure can describe the *w*EWMA scheme. If E_t in Equation (2) encounters M transitional states before going out-of-control state, then the in-control states can be identified as those from 0 to (M–1), while M is the out-of-control state. For each state, the width Δ can be determined in terms of the control limit H as follows:

$$\Delta = \frac{H}{M - 0.5} \tag{A3}$$

The center, O_i, of state i is given by:

$$O_i = i \times \Delta \qquad i = 0, 1, \ldots, M. \tag{A4}$$

The transition probability p_{ij} from state i to state j of the *w*EWMA scheme is determined as follows:

for j = 0,

$$p_{i0} = \Pr\ [\lambda v_t + (1-\lambda)O_i \le 0.5\Delta] = F_{v_t}\ \left[\frac{\Delta}{\lambda}(\lambda-1)i + 0.5\right] \tag{A5}$$

For $j > 0$,

$$\begin{aligned} p_{ij} &= \Pr\ [O_j - 0.5\Delta \le \lambda v_t + (1-\lambda)O_i \le O_j + 0.5\Delta] \\ &= F_{v_t}\ \left[\frac{\Delta}{\lambda}\big(j + (\lambda-1)i + 0.5\big)\right] - F_{v_t}\ \left[\frac{\Delta}{\lambda}\big(j + (\lambda-1)i - 0.5\big)\right] \end{aligned} \tag{A6}$$

Equation (A1) evaluates the cumulative probability function $F_{v_t}()$. When evaluating the in-control ARL_0, p_{ij} is calculated using $p = p_0$. Based on p_{ij}, the in-control transition probability matrix $\boldsymbol{R_0}$ is established. It is a $M \times M$ matrix excluding the elements associated with the absorbing (i.e., out-of-control) state.

$$\boldsymbol{R}_0 = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,M-1} \\ p_{10} & p_{11} & \cdots & p_{1,M-1} \\ \vdots & \vdots & \vdots & \vdots \\ p_{M-1,0} & p_{M-1,1} & \cdots & p_{M-1,M-1} \end{bmatrix} \tag{A7}$$

ARL_0 is equivalent to the first element of $\boldsymbol{U}$ calculated by the following equation:

$$\boldsymbol{U} = (\boldsymbol{I} - \boldsymbol{R_0})^{-1}\ \mathbf{1} \tag{A8}$$

where $\mathbf{1}$ is a vector with all elements of one value and $\boldsymbol{I}$ is an identity matrix. Finally, ATS_0 is determined from ARL_0.

$$ATS_0 = ARL_0 \times h \tag{A9}$$

The matrix $\boldsymbol{R_1}$ for determining ARL is similar to $\boldsymbol{R_0}$ except that p_{ij} of $\boldsymbol{R_1}$ must be calculated based on the out-of-control p. The statistic C_t is assumed to reach its stationary distribution when the shift in p occurs and that the random shift is assumed to follow a uniform distribution within the sampling interval. Consequently. The steady-state ARL can be determined as follows:

$$ARL = \boldsymbol{B}^T [(\boldsymbol{I} - \boldsymbol{R_1})^{-1}\, \mathbf{1} - \mathbf{1} / 2] \tag{A10}$$

where $\boldsymbol{B}$ is the vector representing the steady-state probability when $p = p_0$. It is determined by normalizing $\boldsymbol{R_0}$, and then solving the equation below:

$$\boldsymbol{B} = \boldsymbol{R_0^T B} \tag{A11}$$

where
$$\mathbf{1}^T \boldsymbol{B} = 1 \tag{A12}$$

Finally, ARL can be used to calculate the ATS as follows:

$$ATS = ARL \times h \tag{A13}$$