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Online Damage Detection using Recursive Principal Component Analysis

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Abstract

In this paper, a novel baseline free approach for continuous online damage detection of multi degree of freedom vibrating structures using Recursive Principle Component Analysis (RPCA) in conjunction with online damage indicators is proposed. The RPCA algorithm iterates the eigenvector and eigenvalue estimates for sample covariance matrices and new data point at each successive time instants, to obtain recursive proper orthogonal modes online using the rank-one perturbation method. The proposed method when applied to streaming data, eliminates the need for offline post processing. Numerical simulations performed on 5-DOF nonlinear system under white noise excitations, with different levels of damage demonstrate the robustness and efficacy of the proposed methodology as an ideal candidate for real-time, reference free structural health monitoring.

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Keywords: Recursive Principal Component Analysis (RPCA); recursive residual error (RRE); Online Damage Detection

1. Introduction

Damage detection in structures is a topic that has received considerable attention in the literature ([1–3]). The basic idea is that the dynamic parameters are related to the physical and material properties of the structure, i.e. mass, stiffness and damping which suffer significant alteration due to damage (caused due to excessive response, fatigue, buckling, accumulation of cracks, impact of a foreign object, etc.). A robust damage detection framework should provide an early detection, estimate the severity of the damage, determine the location of the damage and predict the remaining useful life of the structure. Online damage detection entails identification of damage of a multi degree of freedom vibrating system as the recorded vibration data streams in real time. Bulk of the vibration based damage detection techniques rely heavily on post processing of data that has already been acquired. These methods can be classified as: model based methods involving a comparative analysis with respect to a detailed numerical model of the system ([1,4]), and response based methods which detect damage from only the response data of the system. This paper endeavors to address the problem of online damage detection of vibrating systems and proposes a novel

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technique utilizing the concepts of recursive principal component analysis (RPCA) and a robust online condition indicators called recursive residual error (RRE).

Classical damage detection is closely related to system identification techniques ([2,5]). In the context of adaptive system identification; recursive subspace identification, recursive least-squares ([6,7]), hybrid clustering for nonlinear systems using radial basis function networks, detection of change points in data using CUSUM ([3]) are some notable contributions. But none of the aforementioned algorithms are amenable to online damage detection. This is due to the fact that there is a close relationship between online damage detection and the time varying modal behavior of multi degree of freedom vibrating systems that evolve in real time. In recent times, a paradigm shift to data driven statistical techniques like blind source separation ([8]) and principal component analysis pitched in a semi-recursive and windowed data processing framework has paved way to opportunities in real time structural health monitoring and damage detection.

The key objective of the present work is to develop an algorithm which can process the data online and to detect damages or changes in the stiffness of the structure in real time. Since the data streams in continuously, the algorithm must work online, which further necessitates that it should be relatively parameter free and independent of baseline data. In this paper, a baseline free approach is proposed which facilitates the monitoring of structural systems directly using the acceleration data. CIs are employed in order to detect the instant of damage online. Damage in the present context is idealized as a transition from linear to nonlinear behavior (or vice-versa) of vibrating systems. The change in nonlinearity serves as an indication of damage of the monitored system which can be utilized to develop an online damage detection framework exclusive of baseline data. The cases of false detections are addressed by effective use of scatter plots and observing the change in orientation of cluster pre and post event.

The major contributions of this work are as follows: **Firstly**, a novel framework has been provided using RPCA ([9]) as a damage detection tool that has so far not been explored in the context of structural damage detection. **Secondly**, the paper proposes the use of RRE to provide a unique online damage detection technique incorporated into the RPCA framework which serves as a viable candidate for online damage detection. **Finally**, the authors have extended the utility of the proposed algorithm to assess damage detection in a 5 degree of freedom (dof) buocwen system excited by a white noise, in which the nonlinearity level is subjected to various degrees of changes to simulate different levels of damage in the structure. These numerical results were complemented with scatter plots which help in validating the exact instant of damage, thereby providing good visual aid of damage manifestation in the system.

2. RPCA and structural dynamics

Traditional PCA analyzes data in batches, offline, which cannot be utilized without the application of windowing and using a baseline value. This motivates the need of using a baseline free approach amenable to online damage detection which is addressed by the RPCA based framework. The algorithm is based on rank-one update of the eigenspace of the covariance matrix applied to the data vector and as new sampled data becomes available, the eigenstructure is updated as a whole which extracts the linear normal modes (LNMs), instead of updating the covariance matrix directly, thus providing an immediate update of the eigenvalues and the proper orthogonal matrices (POMs)(i.e., the eigen vectors) in a recursive manner ([9]). In order to understand the application of RPCA in the purview of structural dynamics, consider a linear, classically damped, and lumped parameter system with mass and stiffness matrices **M** and **K** subjected to an external force, with **x** as the displacement vector.

$$[M]\{\ddot{\mathbf{x}}(t)\} + [C]\{\dot{\mathbf{x}}(t)\} + [K]\{\mathbf{x}(t)\} = \{\mathbf{F}(t)\} \tag{1}$$

where $F(t)$ is the input excitation which is assumed to be Gaussian and broadband. The solution of the equation can be written as $\{\mathbf{x}\}_m = [\mathbf{V}]_{m\times s}\{\mathbf{q}\}_s$ From the previous discussion, it can be inferred that the POMs (**W**) can be expressed in terms of LNMs (**V**) and error terms (ε) as shown:

$$\mathbf{\Psi} = \mathbf{W}^T\mathbf{X} = (\mathbf{V}^T + \varepsilon^T)\mathbf{X} = \mathbf{Q} + \mathbf{\Gamma} \tag{2}$$

where **Q** is the modal ensemble matrix and Γ is the matrix containing the error terms. Hence, a new form of the covariance matrix **R**, is obtained:

$$\mathbf{R} = \frac{1}{N}\mathbf{X}\mathbf{X}^{\mathbf{T}} = \frac{1}{N}\mathbf{W}[\mathbf{Q} + \mathbf{\Gamma}][\mathbf{Q} + \mathbf{\Gamma}]^T\mathbf{W}^{\mathbf{T}} \tag{3}$$

The basic RPCA equation can be written as:

$$\mathbf{R}_k = \frac{k-1}{k}\mathbf{R}_{k-1} + \frac{1}{k}\mathbf{X}_k\mathbf{X}_k^T \tag{4}$$

where R_k and X_k are the covariance matrix and the matrix of the data points at the k^{th} instant, respectively; and R_{k-1} denotes the covariance matrix at the $(k-1)^{th}$ instant. The covariance estimate R_k can be expressed as an eigen decomposition as: $\mathbf{R}_k = \mathbf{W}_k\mathbf{\Omega}_k\mathbf{W}_k^T$. Thus for $(k-1)^{th}$ data point the eigenvalue decomposition of $\mathbf{R}_{k-1}$ can be expressed as, $\mathbf{R}_{k-1} = \mathbf{W}_{k-1}\mathbf{\Omega}_{k-1}\mathbf{W}_{k-1}$, and the gain depth parameter β_k is given by $\beta_k = \mathbf{W}_{k-1}^T\mathbf{X}_k$ On substituting the value of gain depth parameter and the covariance estimate in equation 4, the following expressions can be obtained ([9])

$$\mathbf{W}_k(k\mathbf{\Omega}_k)\mathbf{W}_k^T = \mathbf{W}_{k-1}\{(k-1)\mathbf{\Omega}_{k-1} + \beta_k\beta_k^T\}\mathbf{W}_{k-1}^T \tag{5}$$

For the RPCA algorithm to be stable and robust, it is important that the term $\{(k-1)\mathbf{\Omega}_{k-1} + \beta_k\beta_k^T\}$ is diagonally dominant, which can be demonstrated by expanding $\mathbf{\Omega}_{k-1}$ in terms of LNMs and error terms as follows:

$$\mathbf{\Omega}_{k-1} = \mathbf{Q}_{k-1}{}^{\mathbf{T}}\mathbf{Q}_{k-1} + \mathbf{Q}_{k-1}\mathbf{\Gamma}_{k-1}^T + \mathbf{Q}_{k-1}{}^{\mathbf{T}}\mathbf{\Gamma}_{k-1} + \mathbf{\Gamma}_{k-1}\mathbf{\Gamma}_{k-1}^T \tag{6}$$

As $N \to \infty$, $\mathbf{Q^TQ}$ is approximately diagonal ([10]) for systems having mild to moderate damping under sufficiently broadband excitations. Gershgorin's theorem can now be applied on the diagonally dominant matrix which provides recursive eigen space updates using perturbation techniques at each point in time. For a structural system, the recursive eigen space update is obtained using a first order perturbation (FOP) approach which provides a less computationally intensive algorithm in a recursive framework ([9]) for the eigen value decomposition ($\mathbf{T}_k\mathbf{\Lambda}_k\mathbf{T}_k^T$) of the term $(k-1)\mathbf{\Omega}_{k-1} + \beta_k\beta_k^T$, yielding the following iterative update equations: $\mathbf{W}_k = \mathbf{W}_{k-1}\mathrm{T}_k$ and $\mathbf{\Omega}_k = \frac{\mathbf{\Lambda}_k}{k}$

3. Damage detection using Recursive Residual Error (RRE)

In the present damage detection framework, RPCA facilitates online processing of data by tracking the eigen space by a set of damage markers which are referred to as condition indicators (CIs) or damage indicators ([11]) that can detect damage online. These indicators are based on the change in the pattern of the eigen space due to damage which is manifested through alteration of eigen vectors before and after damage. In the present work, RRE is presented as the key CI along with scatter plots which validates the instant of damage. The main motivation for this condition indicator is derived from the use of residual error as a criterion in quantification of nonlinear behavior ([12,13]) which presents the use of residual error as a measure of distortion of subspaces of a nonlinear system with increase in levels of excitation. The projection of nonlinear response at time t, denoted as $\mathbf{x}_{nl}$(t), on the matrix of proper orthogonal modes (POMs) at low level of excitation $\mathbf{U}_l$ is given by the following expression: $\mathbf{x_{nl}^*} = \mathbf{U_L^1}\mathbf{U_L^{1^T}}\mathbf{x_{nl}}(t)$ The transformed response is obtained by the transforming the nonlinear response using the linear POMs ($\mathbf{U_L^1}$) ([13]). This provides a basis for the development of a condition indicator based on RRE for damage detection. The traditional residual error works in batch mode whereas to tailor it towards online damage detection, the current modification requires it to work in an online mode where the data streams in real time. The modifications require the arrangement of eigen values in descending order of magnitude, such that the eigenvectors are correspondingly arranged as: $\mathbf{W_k}$= $[\mathbf{W_k^1W_k^2}]$, such that, $\mathbf{W_k^1}$ represents the least number of eigenvectors whose corresponding eigen values explain more than 90% of the variance. Considering a damage at the end of $(k-1)^{th}$ instant, for the initial few seconds, the updated eigenvector $\mathbf{W_k^1}$ can be approximated to be equal to the eigenvectors spanned by the previous time stamp, $\mathbf{W_{k-1}^1}$ (i.e.,$\mathbf{W_k^1} \cong \mathbf{W_{k-1}^1}$). For detecting the instant of damage, the RRE proposed in this paper (χ_{RR-1}) can be represented by the following equation:

$$\chi_{RR-1} = \|\mathbf{X}^*(k) - \mathbf{W}_k^{1^T} * \mathbf{X}(k)\|^2 \tag{7}$$

From the equation 7, χ_{RR-1} can be interpreted as the distance metric between the transformed response and its projection on the subspace at the previous time stamp.

4. Proposed Algorithm

The overall methodology followed in this paper is shown in Figure 1, which entails processing of the acceleration data by the RPCA algorithm as it streams in real-time for online temporal damage detection. Whenever a significant

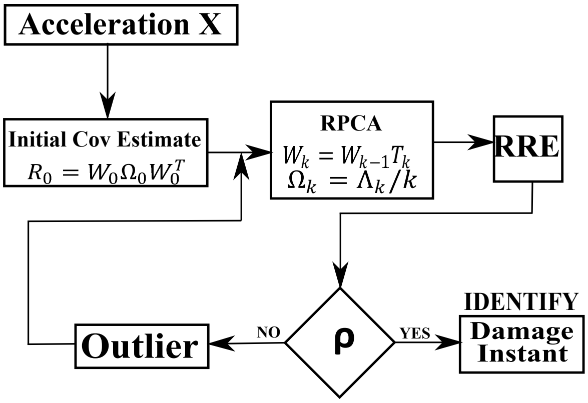

Fig. 1: Flowchart for the proposed method

alteration is observed in the behavior of RRE, the outlier detection step operates on the RRE data accumulated till the event instant (i.e. the instant of change) to determine whether the change corresponds to an outlier or a damage. The basic steps of the algorithm is shown in Figure 1.

5. Numerical Example

In order to illustrate an application of the proposed method, numerical simulations are performed on a 5-storey model with a buoc-wen oscillator at the base degree of freedom to simulate nonlinear change of state which is contextually defined as damage, by subjecting the model to white noise excitation of duration $40s$. The 5 storey structure is modeled with 4 floors and a base. The equation of motion for the system can be summarized as:

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = \mathbf{\Lambda} f - \mathbf{M}\mathbf{I}\ddot{\mathbf{u}}_{\mathbf{g}} \tag{8}$$

A simple shear building representation is assumed to arrive at the expressions for assembled mass ($\mathbf{M}$), damping ($\mathbf{C}$), and stiffness ($\mathbf{K}$) matrices respectively, which are skipped here for brevity. For each of the four floor levels above the base, the values for the respective parameters are 7461 kg, 23.71 kNs/m and 11912kN/m; while the values for the base are 6800 kg, 3.74 kNs/m and 232 kN/m respectively. In equation (8), $\ddot{\mathbf{u}}_{\mathbf{g}}$ represents the ground acceleration and $\mathbf{\Lambda}$ represents the location of the base at the point of application of the non linear force (f) due to the LRB base isolator, given by:

$$f = \kappa z Q_{pb} - k_b x_b - c_b \dot{x}_b \tag{9}$$

where, $Q_{pb} = \left(1 - \frac{k_{yield}}{k_{initial}}\right)$ and k_b and c_b are the stiffness and the viscous damping respectively, in the horizontal direction. The evolutionary variable z can be obtained by the solution of the following nonlinear differential equation:

$$z = -\gamma z \left|\dot{x}_b\right| \left|z^{n-1}\right| - \beta \dot{x}_b \left|z^n\right| + A\dot{x}_b \tag{10}$$

where γ, β, A and n are the shape parameters of the hysteresis loop. For the current model, $A = \left(\frac{k_{yield}}{k_{initial}}\right) = 6$, $\gamma = \beta =$ 39.1, $Q_y = 17800$ kg and n=1.

5.1. Results for White Noise

Temporal damage detection cases are studied by sequentially changing the κ corresponding to 30%, 40% and 50% changes in nonlinear characteristics respectively. The damage is detected at $31s$ using RRE (χ_{RR-1}) as a CI and is validated using scatter plots of the transformed responses of the system at pre and post time instants of 30s and 32s as shown in Figure 2. While the RREs show damage by a significant change at that instant and then hitting a plateau region for the rest of the duration of the excitation (Figure 2a), the scatter plot between the transformed response show a definite change in orientation which can be depicted easily from the plot shown in Figure 2b, thereby providing a robust visual CI for indicating damage. The relative change in global RREs corresponding to different levels of

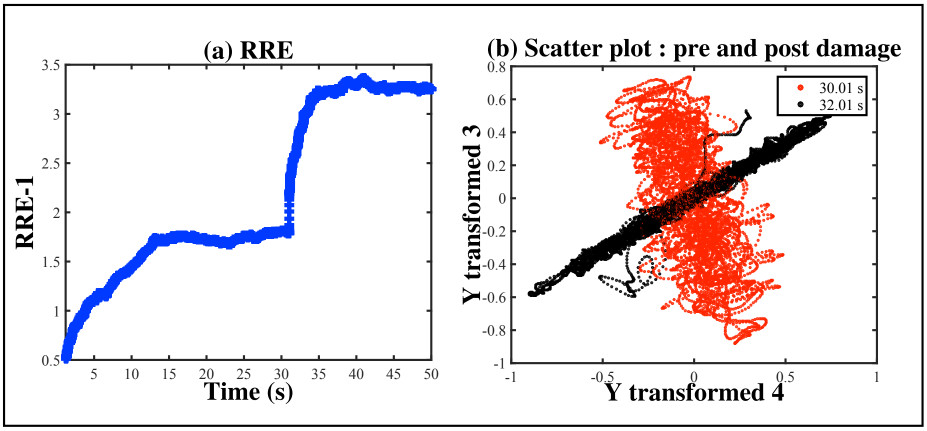

Fig. 2: Damage detection using condition indicators for 50% nonlinearity change

Table 1: Global RREs for numerical modeling (using white noise)

Damage (%)	Pre-damage RRE	Post-damage RRE	% change
30.00	0.71	0.97	36.62
40.00	0.68	1.02	50.15
50.00	0.69	1.27	84.06

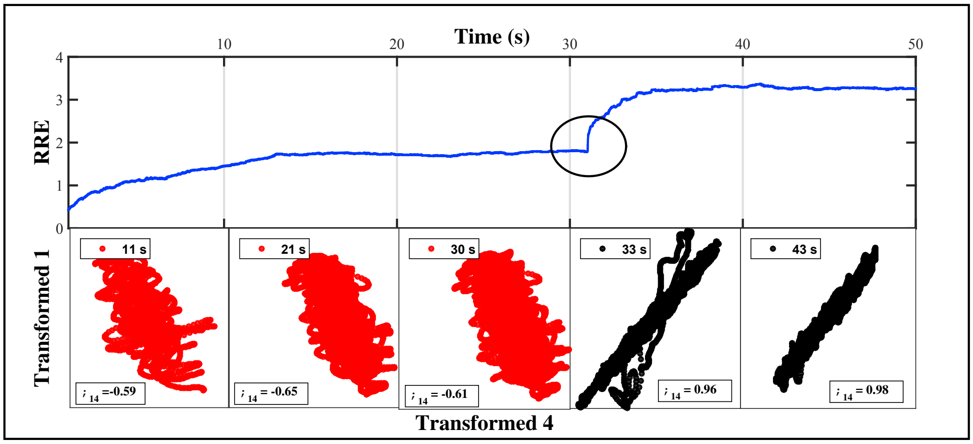

Fig. 3: Damage detection using RRE and scatter plots for 50% nonlinearity change

damage is shown in Table 1. It is clear from the results in Table 1 that the percentage change in RRE increases with the level of damage.

To demonstrate the efficacy of scatter plots, data are considered in growing windows with initial window size of $10s$ and at increments of $10s$ before damage and a slightly smaller increment in the vicinity of damage ($31s$). It can be clearly observed from Figure 3 that there is a significant change in orientation of the scatter, also reflected by the change in the signs of the correlation coefficients, between the successive windows immediately before and after damage. Considering the time of damage as 31s (as obtained from RE plot), the 4^{th} window clearly indicates the damage instant for the specified level of non linearity. Figure 4 shows the robustness of the RREs for various instants of damage for 30% and 40% damage cases, which clearly depicts that RRE shows distinct change for higher percentage change in non linearity.

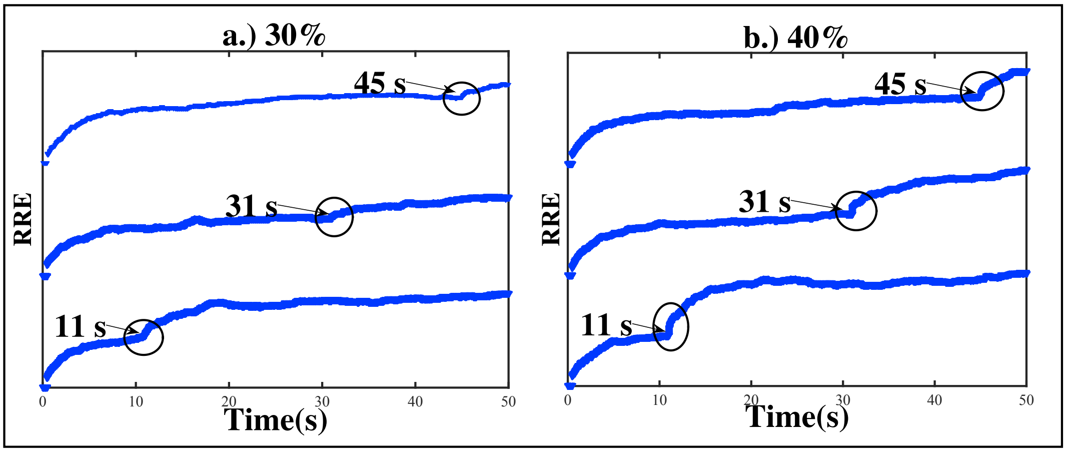

Fig. 4: Damage detection using RRE for various nonlinearity change at different time instances

6. Conclusions

A new online damage detection algorithm for vibrating structural systems using a combination of recursive principle component analysis and online condition indicators is presented. Subsequent implementation of recursive residual errors (RRE) estimated on the eigenvector updates facilitated online detection of structural damage which provided the instant of damage for the present framework. Application of cluster plots further eliminated the presence of outliers, by observing the change in correlation coefficient between the transformed responses. The proposed methodology provided successful results for numerical simulations both for white noise and earthquake excitations. Presented case studies show that the proposed approach results in successful damage detection and it can be safely concluded that the current framework is quite robust in detecting the instant of damage online.

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