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Uniform Planar Array Broadband Angle of Arrival Estimation From Polynomial Steering Matrix

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***Abstract*—In this paper, we propose a scalable broadband angle-of-arrival (AoA) estimation approach based on an estimated uniform planar array (UPA) polynomial matrix. The proposed method employs least-squares (LS) Khatri–Rao factorization in discrete Fourier transform (DFT) frequency bins to compute samples of the space-time covariance matrix in both azimuth and elevation directions. This approach eliminates the need for computing a polynomial singular value decomposition (PSVD), thereby enhancing scalability with respect to the number of sources and array elements. The proposed method's performance is showcased in a simulation where a polynomial steering matrix for three sources is deliberately perturbed with varying degrees of estimation error to simulate different signal to noise ratio (SNR) scenarios. With azimuth and elevation angles determined independently, these are paired using an economical angle pairing strategy specifically designed for broadband scenarios.**

I. Introduction

2-dimensional array geometries have received more focus over the last decade for communication [1], angle of arrival (AoA) estimation and tracking [2], and other applications [3]. We are interested in the broadband AoA estimation for a uniform planar arrays (UPA), an illustration is shown in 1. The direction of a particular source is characterised by the delays with which a wavefront arrives at different sensor. This is captured by a steering matrix which for a narrowband sources can be modelled using an ordinary matrix [4, 5]. However, for broadband sources, the delays between different sensor signals cannot be adequately represented by phase shifts alone. Instead, explicit delays must be implemented, necessitating the use of fractional delay filters for constructing broadband steering vectors [6, 7]. Consequently, a broadband steering matrix for a scenario involving P broadband sources illuminating an $M \times N$ element UPA can be effectively modeled as a polynomial matrix:

$$\boldsymbol{A}(z) = \begin{bmatrix} a_{1,1}(z) & \dots & a_{1,P}(z) \\ \vdots & \ddots & \vdots \\ a_{MN,1}(z) & \dots & a_{MN,P}(z) \end{bmatrix} \in \mathbb{C}^{MN \times P} \quad (1)$$

where each matrix element is a fractional delay filter [6]. The narrowband MUSIC algorithm has been extended to polynomial MUSIC to facilitate broadband processing [6]. However, for a UPA, polynomial MUSIC will necessitate a two-dimensional grid search, akin to the MUSIC algorithm, which is computationally expensive.

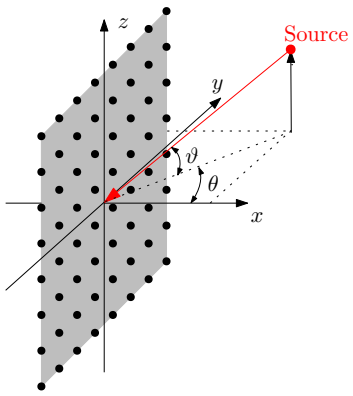

Fig. 1. Uniform planar array with azimuth or horizontal measured within the xy-plane against the x-axis and the vertical or elevation angle measured against the z-axis.

In this study, we assume that an approximate steering matrix is has been estimated from sensor measurements statistics. This assumption is supported by the availability of tensor decomposition techniques such as Parallel Factor Analysis (PARAFAC) and other tensor-based methods [1, 8, 9]. For such an approximate polynomial steering matrix, we propose an AoA estimation method based on the principle of least-squares (LS) Khatri-Rao factorization [10] to decouple the azimuth and elevation directions for a UPA.

The use of Khatri-Rao factorization methods has been considered in several signal processing related works, in problems such as channel estimation and joint channel/symbol estimation for multi-antenna systems [11–14], array processing [15], as well as in individual channel estimation for reconfigurable surfaces [16]. Unlike existing the only approach, that fully decompose a polynomial matrix into LS Khatri-Rao factors [17], we compute the LS Khatri-Rao factors only within discrete Fourier transform (DFT) bins. These bin-wise Khatri-Rao factors are then used to compute sample points of the space-time covariance matrix in both azimuth and elevation directions, thereby eliminating phase ambiguity, akin to [18], and circumventing the need for phase smoothing [19] and polynomial SVD (PSVD) [20, 21] or EVD (PEVD) [22] [23]. The DFT size is iteratively increased until the time-domain aliasing incurred in the inverse DFT (IDFT) of these space-time covariance samples falls below a predefined threshold.

Once the space-time covariance matrices are estimated, polynomial MUSIC or polynomial root algorithms [6, 23] are applied to compute the corresponding angles in each direction. Subsequently, the estimated azimuth angles are paired with their correct elevation angle counterparts via an economical approach, which avoids computing a PSVD and instead relies on an ordinary SVD with a few function evaluations. The proposed approach is validated in MATLAB by perturbing a ground-truth steering matrix with a complex Gaussian distribution under various signal-to-noise ratio (SNR) conditions.

The reminder paper is structured as follows: Section II provides an overview of UPA AoA estimation through the decoupling of azimuth and elevation directions, given the estimated steering matrix for a narrowband scenario. Section III extends the narrowband approach to the broadband scenario, while Section IV presents MATLAB simulation results to validate the proposed approach.

II. Planar Array Steering Matrix and AoA Estimation with LS Khatri-Rao Factorization

Let us assume $\mathbf{A} \in \mathbb{C}^{MN\times P}$ to be the steering matrix of an $M \times N$ plannar array capturing P source signals. This plannar steering matrix is actually the Kathri-Rao product between the vertical and horizontal steering matrices i.e. $\mathbf{A}^{\mathrm{h}} \in \mathbb{C}^{M\times P}$ and $\mathbf{A}^{\mathrm{v}} \in \mathbb{C}^{N\times P}$, defined as

$$\mathbf{A} = \mathbf{A}^{\mathrm{h}} \diamond \mathbf{A}^{\mathrm{v}} = [\mathbf{a}_1^{\mathrm{h}} \otimes \mathbf{a}_1^{\mathrm{v}}, \ldots, \mathbf{a}_P^{\mathrm{h}} \otimes \mathbf{a}_P^{\mathrm{v}}] \tag{2}$$

where $\diamond$ denotes the Khatri-Rao product and $\otimes$ denotes the Kronecker product. With $\mathbf{A}$ estimated, for example via [8], a LS Khatri-Rao factorization [10] can be applied to determine the individual steering matrices. Since Khatri-Rao factors are based on rank one estimate obtained via an SVD of $\mathbf{A}_p = \mathrm{unvec}\{\mathbf{a}_p\}, p = 1, \ldots, M$, more specifically

$$\mathbf{A}_p^{(c)} = \mathrm{unvec}\{\mathbf{a}_p\} \approx \mathbf{u}_{p,1}\sigma_{p,1}\mathbf{v}_{p,1}^{\mathrm{H}}\ , \tag{3}$$

where $\mathbf{u}_{p,1}$ and $\mathbf{v}_{p,1}$ are the dominant left and right singular vectors, respectively, and $\sigma_{p,1}$ is the corresponding dominant singular value of $\mathbf{A}_p^{(c)}$. This decomposition allows estimating $\mathbf{a}_p^{\mathrm{az}}$ and $\mathbf{a}_p^{\mathrm{el}}$ but these factors will be affected by complex scaling ambiguities. Mathematically, it can be expressed as

$$\hat{\mathbf{a}}_p^{\mathrm{az}} = \mathbf{u}_1 \mathrm{e}^{\mathrm{j}\theta_p}\alpha_p, \quad \hat{\mathbf{a}}_p^{\mathrm{el}} = \mathbf{v}_1^* \mathrm{e}^{-\mathrm{j}\theta_p}\beta_p, \tag{4}$$

where α_p and β_p are real scalar values with constraint $\alpha_p\beta_p = \sigma_{p,1}$. The phase factor in this scaling ambiguity stems from the phase ambiguity of singular vectors [24]. Among the infinite cases for α_p, β_p, we can restrict ourselves to

$$\hat{\mathbf{a}}_p^{\mathrm{az}} = \mathbf{u}_1 \mathrm{e}^{\mathrm{j}\theta_p}\sqrt{\sigma_1}, \quad \hat{\mathbf{a}}_p^{\mathrm{el}} = \mathbf{v}_1^* \mathrm{e}^{\mathrm{j}\theta_p}\sqrt{\sigma_1} \tag{5}$$

as it does not affect AoA estimation. By repeating this process for each column of $\mathbf{A}$, both individual matrices can be determined. It is reminded that both $\hat{\mathbf{A}}_{\mathrm{h}}, \hat{\mathbf{A}}_{\mathrm{v}}$ will be phase ambiguous i.e $\hat{\mathbf{A}}_{\mathrm{h}}\mathrm{diag}\{\mathrm{e}^{\mathrm{j}\theta_1}, \ldots, \mathrm{e}^{\mathrm{j}\theta_P}\}$ and $\hat{\mathbf{A}}_{\mathrm{v}}\mathrm{diag}\{\mathrm{e}^{\mathrm{j}\theta_1}, \ldots, \mathrm{e}^{\mathrm{j}\theta_P}\}$ will be a valid steering matrix in azimuth direction. The phase ambiguity is coupled between the columns of these two matrices. With horizontal and vertical spatial frequencies decoupled, AoAs in both directions i.e. $\{\theta_p, \vartheta_p\}, p = 1, \ldots, P$ can be determined either by applying MUSIC or RMUSIC separately

$$\{\theta_1, \ldots, \theta_P\} = \text{RMUSIC/MUSIC}(\hat{\mathbf{A}}^{\mathrm{h}}\hat{\mathbf{A}}^{\mathrm{h,H}}) \tag{6}$$

$$\{\vartheta_1, \ldots, \vartheta_P\} = \text{RMUSIC/MUSIC}(\hat{\mathbf{A}}^{\mathrm{v}}\hat{\mathbf{A}}^{\mathrm{v,H}}), \tag{7}$$

$$\tag{8}$$

or directly from the columns of $\hat{\mathbf{A}}^{\mathrm{h}}$ and $\hat{\mathbf{A}}^{\mathrm{v}}$ as

$$\theta_p = \underset{\theta}{\mathrm{argmax}}\ \mathbf{a}^{\mathrm{h,H}}(\theta)\hat{\mathbf{a}}_p^{\mathrm{h}},\ \vartheta_p = \underset{\theta}{\mathrm{argmax}}\ \mathbf{a}^{\mathrm{v,H}}(\theta)\hat{\mathbf{a}}_p^{\mathrm{v}} \tag{9}$$

where $\mathbf{a}^{\mathrm{h}}(\theta), \mathbf{a}^{\mathrm{v}}(\theta)$ are ULAs test steering vectors.

III. Broadband AoA Estimation From Polynomial Steering

Akin to the narrowband steering matrix of a UPA as discussed in Sec. II, a broadband steering matrix for a UPA can be expressed

$$\boldsymbol{A}(z) = \boldsymbol{A}^{\mathrm{az}}(z) \diamond \boldsymbol{A}^{\mathrm{el}}(z). \tag{10}$$

In this section, we apply the idea of Khatri-Rao factorization to determine broadband AoA in both azimuth and elevation directions assuming $\boldsymbol{A}(z)$ is known from estimation. Unlike [17], here we propose a more scalable approach without relying on computationally expensive phase smoothing procedure.

A. LS Khatri-Rao Factorization in Samples Points of $\boldsymbol{A}(z)$ on the Unit Circle

From the existence of analytic SVD, we know that analytic LS Khatri-Rao factorisation in (10) exists for a UPA steering matrix. The bin-wise LS Khatri-Rao factorization applied, akin to as discussed in Sec.II, in the sample points of $\boldsymbol{A}(\mathrm{e}^{\mathrm{j}\Omega_k}), k = 0, \ldots, (K-1)$, yields

$$\boldsymbol{A}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \mathbf{A}_k^{\mathrm{h}} \diamond \mathbf{A}_k^{\mathrm{v}},\ \text{for } k = 0, \ldots, (K-1). \tag{11}$$

Due to LS Khatri-Rao factors computed independently in frequency bins leads to loss of phase coherence between adjacent bins LS Khatri-Rao factors similar to EVD in [19]. Therefore, the bin-wise LS Khatri-Rao factors (11) can be related to the sample points of analytic $\boldsymbol{A}^{\mathrm{h}}(z)$ and $\boldsymbol{A}^{\mathrm{v}}(z)$ at $z = \mathrm{e}^{\mathrm{j}\Omega_k}, k = 0, \ldots, (K-1)$ via a diagonal phase matrix $\mathbf{D}_k = \mathrm{diag}\{\mathrm{e}^{\mathrm{j}\theta_{1,k}}, \ldots, \mathrm{e}^{\mathrm{j}\theta_{P,k}}\}$ as

$$\boldsymbol{A}^{\mathrm{h}}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \mathbf{A}_k^{\mathrm{h}}\mathbf{D}_k;\ \boldsymbol{A}^{\mathrm{v}}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \mathbf{A}_k^{\mathrm{v}}\mathbf{D}_k^{\mathrm{H}}. \tag{12}$$

These bin-wise phase matrices can be determined via phase smoothing procedure [19] which in turns provide a compact or shortest polynomial order estimate of $\boldsymbol{A}^{\mathrm{h}}(z)$ and $\boldsymbol{A}^{\mathrm{v}}(z)$.

B. Samples of Space-Time Covariance in horizontal and vertical Direction

As the underline objective of this paper is to determine broadband AoA in both horizontal and vertical directions, the polynomial MUSIC or polynomial RMUSIC algorithm can be

applied to space-time covariance matrices in both directions i.e.

$$\boldsymbol{R}^{\mathrm{h}}(z) = \boldsymbol{A}^{\mathrm{h}}(z)\boldsymbol{A}^{\mathrm{h,P}}(z);\ \boldsymbol{R}^{\mathrm{v}}(z) \quad = \boldsymbol{A}^{\mathrm{v}}(z)\boldsymbol{A}^{\mathrm{v,P}}(z).$$

This shows that the sample points of these space-time covariance matrices can be obtained from the bin-wise LS Khatri-Rao factors in (12) as

$$\boldsymbol{R}^{\mathrm{h}}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \boldsymbol{A}^{\mathrm{h}}(\mathrm{e}^{\mathrm{j}\Omega_k})\boldsymbol{A}^{\mathrm{h,H}}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \mathbf{A}_k^{\mathrm{h}}\mathbf{A}_k^{\mathrm{h,H}} \tag{13}$$

$$\boldsymbol{R}^{\mathrm{v}}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \boldsymbol{A}^{\mathrm{v}}(\mathrm{e}^{\mathrm{j}\Omega_k})\boldsymbol{A}^{\mathrm{v,H}}(\mathrm{e}^{\mathrm{j}\Omega_k}) = \mathbf{A}_k^{\mathrm{v}}\mathbf{A}_k^{\mathrm{v,H}}, \tag{14}$$

which shows that the phase ambiguity cancels out and therefore, does not require phase smoothing. With sample points of these space-time covariance matrix available from the bin-wise LS Khatri-Rao factors, an inverse DFT (IDFT) can be performed to obtain its their time-domain estimate provided the DFT size is sufficiently large to incur least time-domain aliasing.

C. Sufficient DFT Size for Estimation of $\boldsymbol{R}^{\mathrm{h}}(z)$ & $\boldsymbol{R}^{\mathrm{v}}(z)$

The DFT size K should be sufficiently large such that the time-domain aliasing metric, defined as

$$\xi = \frac{\sum_\tau \|\hat{\mathbf{R}}^{\mathrm{h},(K)}[\tau] - \hat{\mathbf{R}}^{\mathrm{h},(K/2)}[\tau]\|_{\mathrm{F}}^2}{\sum_\tau \|\hat{\mathbf{R}}^{\mathrm{h},(K)}[\tau]\|_{\mathrm{F}}^2}, \tag{15}$$

where $\hat{\mathbf{R}}^{\mathrm{h},(K)}[\tau]$ and $\hat{\mathbf{R}}^{\mathrm{h},(K/2)}[\tau]$ are the IDFTs of $\hat{\boldsymbol{R}}^{\mathrm{h}}(\mathrm{e}^{\mathrm{j}\Omega_k})|_{k=0,\ldots,(K-1)}$ and $\hat{\boldsymbol{R}}^{\mathrm{h}}(\mathrm{e}^{\mathrm{j}\Omega_{2k}})|_{k=0,\ldots,(K/2-1)}$, respectively, and $\{\cdot\}_{\mathrm{F}}$ denotes the Frobenius norm, falls below a predefined threshold $\epsilon > 0$. If for a certain DFT size, $\xi > \epsilon$, the DFT size is doubled, to exploit fast Fourier transform (FFT) advantage and reduce redundancy [25], until the time-domain aliasing falls below the threshold. Similar procedure is adopted for $\boldsymbol{R}^{\mathrm{v}}(z)$.

D. Broadband AoA Estimation

Similar to a narrowband case, polynomial MUSIC or polynomial RMUSIC algorithm can be applied independently to horizontal and vertical space-time covariance matrices. With angles computed in each directions, correct pairing has to be required to match the estimated azimuth angles to its corresponding estimated elevation angles. It is reminded that in the event of estimating horizontal and vertical steering matrices via polynomial SVD in conjunction with phase smoothing procedure, this pairing between horizontal and vertical angles are not required [17]. We perform pairing assuming spatial polynomial MUSIC algorithm. For pairing, ideally we may to compute an entire PSVD of the UPA estimated steering matrix

$$\boldsymbol{A}(z) = [\boldsymbol{U}_{||}(z) \quad \boldsymbol{U}_{\perp}(z)]\boldsymbol{\Sigma}(z)\boldsymbol{V}^{\mathrm{P}}(z),$$

where $\boldsymbol{U}_{\perp}(z) \in \mathbb{C}^{MN\times(MN-P)}$ is orthogonal to signal space, and construct a polynomial projection matrix

$$\boldsymbol{P}_{\perp}(z) = \boldsymbol{U}_{\perp}(z)\boldsymbol{U}_{\perp}^{\mathrm{P}}(z).$$

This helps construct an objective function

$$\Gamma(z,\theta,\vartheta) = \boldsymbol{s}_{\theta,\vartheta}^{\mathrm{P}}(z)\boldsymbol{P}_{\perp}(z)\boldsymbol{s}_{\theta,\vartheta}(z), \tag{16}$$

Algorithm 1: Broadband AoA From $\boldsymbol{A}(z)$

```
Initialize K that exceeds the polynomial order of A(z);
ξ_az = 1; ξ_el = 1;
while ξ > ϵ do
    Perform LS Khatri-Rao of A(e^{jΩ_k}), k = 0, ..., (K − 1);
    Compute samples points of R^h(z), R^v(z) via (13) & (14);
    Determine ξ_az, ξ_el via (15);
    ξ = max{ξ_az, ξ_el};
    K ← 2K;
end
{θ_{1:P}, ϑ_{1:P}} ←poly-MUSIC{R̂^h(z), R̂^v(z)};
[U, ∼, ∼] = SVD(S(e^{jΩ_r}));
P_⊥(e^{jΩ_r}) = U(:, P + 1 : MN)U^H(:, P + 1 : MN);
Γ(e^{jΩ_r}, θ_i, ϑ_j) = s^H_{θ_i,ϑ_j}(e^{jΩ_r})P_⊥(e^{jΩ_r})s_{θ_i,ϑ_j}(e^{jΩ_r});
Pair angles as in (17);
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where $\boldsymbol{s}_{\theta,\vartheta}(z)$ is the UPA steering vector defined in (18). To pair angle θ_i with its correct counter amongst $\vartheta_1,\ldots,\vartheta_P$, we evaluate $\Gamma(z,\theta_i,\vartheta_j)$ for $\vartheta_{j=1,\ldots,P}$ and determine

$$j_i' = \underset{i,j}{\operatorname{argmax}}\ (z,\theta_i,\vartheta_j))^{-1},\ j = 1,\ldots,P. \tag{17}$$

for each value of i which leads to pairing as $\{\theta_i,\vartheta_{j_i'}\}, i = 1,\ldots,P$. Since it is not necessary to evaluate $\Gamma(z,\theta,\vartheta)\ \forall\ z$, we can restrict evaluation to a single value of $z = \mathrm{e}^{\mathrm{j}\Omega_r}$ with Ω_r away from DC i.e., $\Omega_r > 0$, to achieve proper spatial discrimination. With this a standard SVD of $\boldsymbol{A}(\mathrm{e}^{\mathrm{j}\Omega_{\mathrm{r}}})$ is sufficient instead of a PSVD of $\boldsymbol{A}(z)$.

E. Procedure

The algorithm begins by initializing K, which exceeds the polynomial order of $\boldsymbol{A}(z)$, and performed bin-wise LS Khatri-Rao factorization in each DFT bin. The scalar ambiguity is restricted to $\alpha_{p,k}\beta_{p,k} = \sigma_{1,k}, k = 0,\ldots,(K-1)$ as highlighted in Sec. II. Thereafter, the sample points of both space-time covariance matrices are computed via (13) & (14) and subsequently ξ metric is calculated for both quantities. If the time-domain aliasing metric exceeds ϵ, K is doubled otherwise the procedure is terminated. Lastly, angles are estimated and pairing is performed to associated each horizontal and vertical angles to its corresponding source.

IV. Simulation Results for Broadband AoAs Estimation

We apply the proposed DFT domain LS Khatri-Rao factorisation based approach to a broadband steering matrix of a UPA. This simulation scenario assumes three broadband sources firing from $\{\theta_1,\vartheta_1\} = \{-41^{\mathrm{o}},34^{\mathrm{o}}\}$, $\{\theta_2,\vartheta_2\} = \{-10^{\mathrm{o}},10^{\mathrm{o}}\}$, and $\{\theta_3,\vartheta_3\} = \{30^{\mathrm{o}},-37^{\mathrm{o}}\}$ on a 6×6 UPA. To address the uncertainty of whether a wavefront is approaching from the front or the rear of the array—that is, from the

positive or negative x-direction—we define the angular ranges as $\theta, \vartheta \in [-\pi, \pi]$.

The broadband steering vectors for both horizontal and vertical directions are constructed utilizing the method based on windowed sinc function [26] restricting the polynomial order of each to 50. The Kronecker product of these two polynomial steering vectors results in a steering vector of a UPA with polynomial order 100 i.e.

$$\boldsymbol{s}_{\theta,\vartheta}(z) = \boldsymbol{s}^{\mathrm{h}}_{\theta}(z) \otimes \boldsymbol{s}^{\mathrm{v}}_{\vartheta}(z). \tag{18}$$

The entire steering matrix of a UPA for the simulated scenario can therefore also be

$$\boldsymbol{S}(z) = [\boldsymbol{s}_{\theta_1,\vartheta_1}(z), \boldsymbol{s}_{\theta_2,\vartheta_2}(z), \boldsymbol{s}_{\theta_3,\vartheta_3}(z)] \in \mathbb{C}^{36\times 3}.$$

which effectively represents a UPA broadband steering matrix with three sources at mentioned horizontal and vertical angles. Instead of computing LS Khatri-Rao factorization for a polynomial matrix which relies on analytic SVD computation involving phase smoothing procedure, we instead apply Algorithm 1 with $\epsilon = 10^{-4}$. To simulate estimation errors, we introduce a perturbed steering matrix given by

$$\tilde{\boldsymbol{S}}(z) = \boldsymbol{S}(z) + \boldsymbol{E}(z), \tag{19}$$

where the elements of $\mathbf{E}[n] \bullet\!\!-\!\!\circ\, \boldsymbol{E}(z)$, $\bullet\!\!-\!\!\circ$ denotes transform-pair, are drawn from complex Gaussian distribution. The perturbation variance is adjusted to achieve a desired SNR, which is defined as

$$\mathrm{SNR(dB)} = 10\log_{10}\frac{\|\mathbf{S}[n]\|^2_{\mathrm{F}}}{\|\mathbf{E}[n]\|^2_{\mathrm{F}}}. \tag{20}$$

The SNR is varied from 0 dB to 15 dB in increments of 5 dB. The corresponding normalized spatial polynomial MUSIC spectrum is presented in Fig. 2, demonstrating that for moderately low to high SNR values, the estimation of AoAs remains accurate. However, at very low SNR values, peak resolution deteriorates, making AoA detection challenging. This observation is further supported by estimating AoAs through peak detection in the polynomial spatial MUSIC spectrum, computed in MATLAB with a spatial resolution of 0.1°. The estimated AoAs, summarized in Table I, exhibit high accuracy across low to high SNR values but for very low SNR, error is considerably large. It is important to note that perturbation effects are not even in individual steering matrices as evident from error difference in horizontal and vertical angles w.r.t their actual values.

We further pair the estimated angles at the worst case of SNR i.e. 0 dB. With $\Omega_r = \pi/8$, we observe following values of $\Gamma^{-1}(\mathrm{e}^{\mathrm{j}\pi/8}, \theta, \vartheta)$ for estimated angles arranged in ascending order

ϑ/θ	-37.3°	9.9°	33.1°
-40.7°	0.0814	0.3298	2.3703
-10.1°	0.1375	2.8143	0.4240
29.5°	2.1451	0.1599	0.1032

,

where it can be seen that largest values are obtained only for correct horizontal and vertical angle pair. This simulation experiment suggests easy and correct pairing even at very low SNR values.

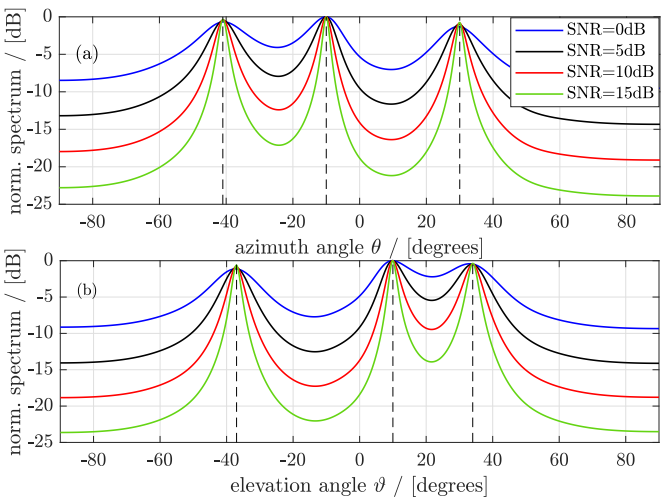

Fig. 2. Polynomial spatial MUSIC normalized spectrum for (a) horizontal, and (b) vertical directions at various SNR.

TABLE I
ESTIMATED AOAS FOR DIFFERENT SNR[dB]

AoA/SNR	θ_1	θ_2	θ_3	ϑ_1	ϑ_2	ϑ_3
0	-40.7°	-10.1°	29.5°	33.1°	9.9°	-37.3°
5	-40.8°	-10.1°	29.8°	33.8°	9.9°	-37.1°
10	-40.9°	-10.0°	29.9°	34.0°	9.9°	-37.1°
15	-41.0°	-10.0°	30.0°	34.0°	10.0°	-37.1°

V. CONCLUSION

For an estimated UPA steering matrix in broadband scenarios, a conventional approach involves performing least-squares Khatri-Rao factorization to estimate the steering matrices in each direction, followed by the estimation of AoAs independently in each direction. To circumvent the need for phase smoothing procedures or computationally expensive polynomial SVDs, we have proposed a projection-based approach that eliminates both phase smoothing and polynomial SVD computations. The proposed method demonstrates high accuracy across moderately low to high SNR values while maintaining computational efficiency. Additionally, for two-dimensional AoA estimation, we introduce an economical angle-pairing strategy tailored for broadband AoAs, which is evaluated at 0 dB SNR and shown to perform effectively.

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