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Relax-and-Fix and Fix-and-Optimise Algorithms to Solve an Integrated Network Design Problem for Closing a Supply Chain with Hybrid Retailers/Collection Centres

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Abstract

This paper studies a multi-echelon closed-loop supply chain network design problem that is characterised by a set of hybrid retailers/collection centres in a multi-period setting. This problem is motivated by the return-to-retail approach currently prevalent in the retail industry under the deposit return scheme. This paper proposes a mathematical programming model that integrates strategic decisions regarding the number and location of hybrid retailer/collection centre facilities, with dynamic decisions pertaining to manufacturing and remanufacturing/recycling, inventory level, and fleet size across the network. This optimisation problem is formulated as a mixed integer linear programming model to fulfil customers' demands while minimising the total network costs. To solve the problem, a matheuristic solution approach is devised, incorporating Relax-and-Fix and Fix-and-Optimise heuristics augmented by novel relaxation and fixing strategies. We introduce an integrality test which accounts for possible rounding-off errors allowing a user-defined integer feasibility tolerance. Moreover, a variable partitioning is applied to shrink the problem's dimensions and to shorten the search space. The latter is then iteratively updated to explore neighbourhoods within a given search radius size. To evaluate the validity and efficiency of the proposed model and the solution approach, 90 instances are generated using a case study within the geographical scope limited to the network of a retail chain in France. Numerical results show that the proposed solution method provides near-optimal solutions for small- and medium-size instances in a reasonable computational time and outperforms the commercial solver for large- and extra large-size problems. Managerial insights derived from the computational experiments regarding key performance indicators of the problem are presented and discussed.

Keywords:
Location, Network design, Reverse logistics, Fleet optimisation, Matheuristic

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1. Introduction

In a forward supply chain, goods/commodities are shipped from manufacturers/suppliers towards market downstream where the customer is at the end of the process. However, in a reverse

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supply chain the flow of returned items (returns) initiates from the end consumer and moves toward the upstream in the chain. The concept of "closed-loop" refers to the integration of forward and reverse supply chains, as an approach to achieving sustainability (Guide et al., 2003) in line with the principals of circular economy (MacArthur et al., 2013). In today's business world, various environmental, legal, social, and economic factors have driven numerous industries to close the loop, leading to an increase in the complexity of their processes (see for example the EPA [1] and the EEA [2]). Several retail chains worldwide have taken the initiative to adopt a Closed-Loop Supply Chain (CLSC) to collect used items for recycling purposes. These initiatives aim to promote environmental sustainability by reducing waste and encouraging responsible disposal (Üster and Hwang, 2017; Shi et al., 2011). For example, H&M has introduced garment collection boxes in their stores, where customers can not only drop off old clothing but also empty plastic bottles for recycling. IKEA, the furniture and home goods retailer, has implemented recycling programmes in some of its stores. Customers can find recycling stations where they can drop off various items, including plastic bottles. LIDL, a German international supermarket chain, has recently installed recycling machines in its stores in Glasgow, Scotland, to become the first supermarket in the UK to implement deposit return scheme (UK Government, 2023). These machines accept empty plastic bottles as well as aluminium cans and provide customers with cash vouchers in return. Research in this field shows that many industries have discovered that closing their supply chain can provide economic and environmental benefits, which serve as a key corporate asset in modern logistics (Banasik et al., 2017; Shahparvari et al., 2021; Cheng et al., 2023; Modak et al., 2023).

The problem investigated herein takes inspiration from the return-to-retail model, existing in a few European countries, US, and Canada, in which some operating retailers also act as return points so that customers are given the chance to return their used products, containers, or packaging to ensure proper recycling and increase recycling rates. In this scheme, retailers host reverse vending machines for the collection of used products or containers. Returned products or containers are then moved back from each return point to (re)manufacturing/recycling centres either directly or via intermediate facilities. However, our aim is to expand upon this concept by introducing hybrid retailers/collection centres, wherein selected retailers are designated for further expansion to accommodate the flow of returns. This suggests that hybrid retailers/collection centres fulfil the role of a collection centre within the reverse supply chain, alongside their primary functions (resulting in sales and return points). More particularly, the hybrid retailer/collection centre is expected to receive the returned items collected by other retailers/return points as well as the items collected within its own retail establishment. Aggregation of returned items at hybrid facilities is undertaken to facilitate efficient transportation to remanufacturing/recycling centres as illustrated in Figure 1. This figure shows the flow of products from a manufacturer to retail points through a distribution centre with solid arrows. The products are then sold to final consumers at retailers. However, the flow of returned items (presented with dashed arrows) initiates from the final consumers. It continues towards the selected retailers with the hybrid feature for aggregation and finally ends at the remanufacturing/recycling facilities. It is noteworthy to mention that the proposed model in this paper stands out by aiming to streamline the reverse supply chain through the elimination of an intermediate stage while enhancing capacities in storage, transportation, and production stages across available facilities. We believe that in line with achieving environmental and economic objectives, the introduction of the hybrid feature yields substantial cost savings attributed to reduced

[1] https://www.epa.gov/circulareconomy/us-recycling-system

[2] https://www.eea.europa.eu/en/analysis/indicators/waste-recycling-in-europe

investments in facilities, infrastructure, storage space, human resources, and equipment.

The research questions addressed in this paper are outlined as follows. The primary research question we seek to answer is: Which retailers within the supply chain network should be selected for expansion to serve as hybrid retailers/collection centres, and what is the optimal number of such sites required to minimise the total cost within the CLSC? It should be noted that a fixed cost for expanding such sites and an available storage capacity associated with each site have to be taken into account. The secondary research question we are interested in addressing is: What is the best plan to manage the inventory of returned items at the hybrid sites and what is the best fleet size to efficiently utilise truckload capacity for moving the products as well as the returned items? To answer the latter research question, one should consider the return rate and the periodicity over a planning horizon.

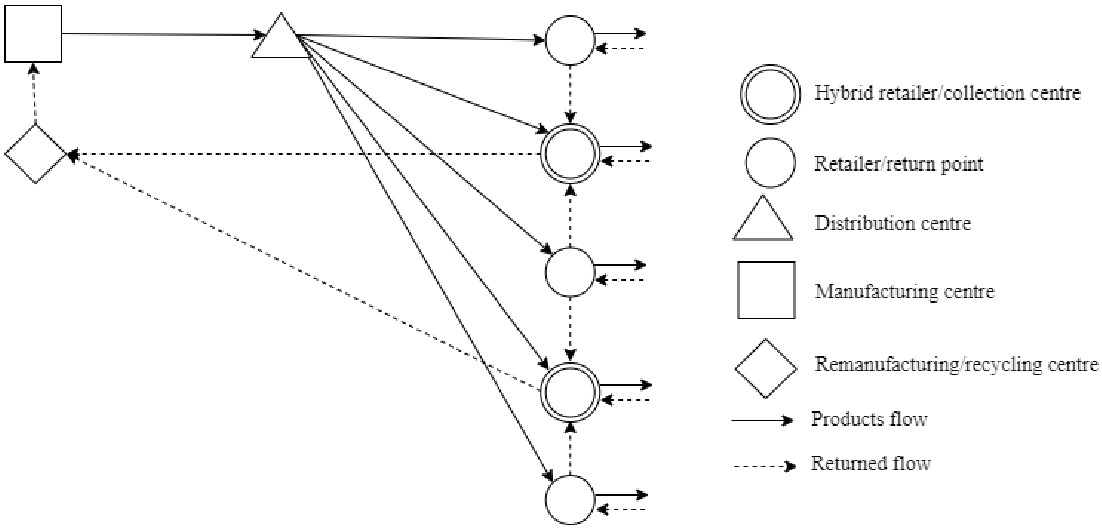


Figure 1: Closed-loop supply chain network.

In this business context, the main challenge a company may face is to optimally integrate the reverse supply chain with such hybrid sites into the forward supply chain that already exists. From a practical standpoint and based on our review of real-life cases (Beckmann et al., 2007; Wilding, 2019), it is evident that the reverse supply chain is typically integrated into an existing forward supply chain. It is uncommon to observe a real scenario where a company designs both its forward and reverse supply chains simultaneously. While the latter represents a theoretically globally optimal/optimum solution to the problem at hand, it remains distant from reality. In the realm of integration, configuration of the existing forward supply chain network remains unchanged. However, its operational and tactical decisions, including manufacturing, storage, and transportation of products, are expected to be reviewed in coordination with the insertion of the reverse supply chain. The rationale behind this approach stems from the ability to fulfil a portion of the demand through remanufacturing/recycling processes, resulting in releasing the capacity of the OEM (Original Equipment Manufacturer) for production of products. Therefore, the prior forward transportation and inventory planning at different stages of the supply chain may need to be adapted according to the new production plan. In addition to the decisions to be revisited in the forward supply chain, a set of strategic decisions need to be made for designing the reverse supply chain network (Meixell and Gargeya, 2005), such as the selection and the number of hybrid sites required in the reverse network. Moreover, multi-period operational decisions including (re)manufacturing, transportation, and inventory of returned items have to be addressed in the model. Our study, presenting a hybrid feature for retailers, constitutes a contribution to existing literature on closing supply chains. Specifically, our objective is to develop a generic mathematical model to efficiently close an open-loop supply chain by designing a reverse network, considering operational decisions across both forward and reverse supply chains as well as strategic decisions for designing the reverse network.

In numerous real-world scenarios, when product demand is subject to variability and seasonality over the planning horizon, a temporal hierarchy between strategic facility location-allocation decisions and tactical decisions needs to be considered in the model. This temporal hierarchy corresponds to operational decisions including inventory control, truck dispatching, and (re)manufacturing, to be made once after determining the number and location of new facilities. This decision-making framework leads us to a location-inventory-transportation-production problem (see e.g. Wu et al. (2020)) in which operational decisions are made periodically, subsequent to establishing the facility locations within the network. Respectively, the return would approximately follow a proportion of demand with a given time lag. Therefore it is essential to formulate a joint capacitated facility location and multi-period inventory-transportation-production model where a decision maker deals with changing parameters over a discrete-time planning horizon (Fleischmann and Klose, 2005; Arabani and Farahani, 2012). In this paper, we aim to develop an integrated location-inventory-transportation-production problem with limited capacity at different stages in a CLSC network. In our proposed mathematical model, the decisions related to the location of the collection centres (as binary variables) and those of fleet size (presented in the form of general integer variables) are made simultaneously along with decisions for production of products (manufacturing and remanufacturing/recycling) as well as flow and storage of products and returned items (as non-negative continuous variables). To the best of our knowledge, the problem with the above characteristics has not been addressed in the relevant literature. Concerning the above discussions, the proposed model in this paper is a mixed integer problem, which is complex in nature and cannot be solved optimally by commercial solvers. To overcome the non-solvability issue of the problem, we propose efficient Relax-and-Fix and Fix-and-Optimise heuristics that are integrated with novel relaxing and fixing strategies. Overall, the contribution of this paper is highlighted as follows:

- We extend the concept of return-to-retail by introducing a hybrid feature to selected retailers to function as collection centres as well.
- We propose an integrated mathematical model which collectively optimises facility location and multi-period production-transportation-inventory problems for closing the supply chain while determining optimal transportation fleet size over a planning horizon.
- We devise Relax-and-Fix and Fix-and-Optimise heuristics that empowers users to achieve near-optimal solutions for small- and medium-size instances, while surpassing a commercial optimiser for large- and extra large-size instances.
- We evaluate the proposed method within an established retail supply chain in France, gathering empty plastic bottles for recycling purposes.

The rest of the paper is organised as follows. Section 2 presents a review of the previously published relevant works. A mathematical programming model of the CLSC problem of interest is presented and discussed in section 3. In Section 4, we develop a solution algorithm based on the Relax-and-Fix and Fix-and-Optimise heuristic approaches for solving the problem. Section 5 provides experimental results for extensive realistic problem instances and a discussion on the performance of the proposed solution method. Conclusions, findings, and directions for future study are provided in Section 6.

2. Literature review

While many researchers have developed various closed-loop network design problems (Govindan et al., 2015; Jahani et al., 2023; Gunasekara et al., 2023; Simonetto et al., 2022; Zhu et al., 2008; Alegoz et al., 2020; Cheng et al., 2023; Shahparvari et al., 2021), we narrowed down our literature review to the most relevant papers, acknowledging that our review is non-exhaustive.

The concept of product recovery in supply chain was introduced in the 1990s and quickly became fashionable. This gave rise to the introduction of reverse flow in a supply chain network and incorporation of this concept into open-loop supply chain networks resulted in CLSC networks that are currently being studied extensively. Although CLSC problems have been very well received by practitioners and fairly studied by researchers over the past years, Akçalı et al. (2009) believe that reverse logistic network design problems are further studied than CLSC network design problems. This provides researchers with an outstanding opportunity to further investigate CLSC network design problems and fruitfully contribute to this field of research.

Product recovery has been regarded as a sustainable and efficient approach to reducing landfill and production costs, saving limited resources including energy resources, and reclaiming value from used and/or discarded products over the past thirty years. Recycling has been always listed by researchers and practitioners as a key activity within product recovery. Recycling is widely used in industry, see e.g. Lottermoser (2011); Geyer and Doctori Blass (2010); Sandin and Peters (2018). Kopicki et al. (1993) investigated and examined reuse and recycling activities thoroughly using interviews with seventeen firms and several trade associations active in the field of waste reduction. Oberoi (2020) provided a thorough literature review on recycling of materials as an approach for sustainable development. The author concluded that role of three main stakeholders, namely businesses, consumers, and policymakers, in the recycling process and decisions they make will highly affect the potential success of recycling. Recycling and recovery of post-consumer plastic waste in European countries is studied in Brems et al. (2012). The authors first presented the existing EU and US legislations concerning plastic waste management and then reviewed and discussed two recycling options to efficiently recycle plastic solid waste. Cui and Sošić (2019) built two supply chain models to extract a stipulation for recycling effectiveness, and then used US data to prove that recycling is effective for all common materials considered in the paper, except for glass. De Bruecker et al. (2018) took inspiration from an industry problem to optimise glass collection and developed a generic model for problem of developing shift schedules and waste collection routes while total cost is minimised.

Remanufacturing is the most advanced product recovery option that adds value to used products by bringing them to “at least an OEM functioning order with a warranty to match” (Ijomah, 2009). Remanufacturing enables companies to benefit from environmental and economic savings and consequently, remanufacturing is very well received in several industries (Matsumoto and Umeda, 2011). According to a report published by Oakdene Hollins Ltd (2020), remanufacturing is given the opportunity to play a crucial role in the future manufacturing industry with an EU market potential of €90 billion by 2030. Two types of remanufacturing problems are studied in the literature: i) remanufacturing only; ii) hybrid model (remanufacturing combined with manufacturing). Note that the latter model could be embedded into the CLSC network design problem studied in this paper. Lot-sizing in remanufacturing is also addressed in the literature extensively. One of the early studies that investigated production planning problems with remanufacturing is Golany et al. (2001) in which such problem is modelled using linear programming and then proved to be NP-complete for general concave-cost structures. Teunter et al. (2006) proposed a polynomial-time dynamic programming algorithm to efficiently solve dynamic lot-sizing problems with remanufacturing and joint set-up costs. A number of Mixed Integer Linear Programming (MILP) formulations for lot-sizing problems with remanufacturing option (separate and joint set-up costs cases) are presented and discussed in Retel Helmrich et al. (2014) and then their efficiency is evaluated using plenty of data sets. Sahling (2013) exploited column generation to solve multi-product lot-sizing problem with product returns and remanufacturing. Ali et al. (2018) used mathematical programming and polyhedral theory to derive strong valid inequalities for remanufacturing lot-sizing problem with separate setups. Brahimi et al. (2017) presented a comprehensive review of single-item lot-sizing problems, with emphasis on both manufacturing and remanufacturing contexts. Last but not least, heuristics are widely used to efficiently deal with production planning problems with remanufacturing, see e.g. Baki et al. (2014); Roshani et al. (2017); Cunha et al. (2019).

Some studies addressed the problem of fleet size optimisation as a critical equation in transportation management and highlighted the fact that it has to be integrated into strategic and operational decisions of supply chain system (Narayan et al., 2021; Ribeiro, 2024). It is clear that due to the significant contribution of transportation costs to the total logistical costs, determining the fleet size and truckload capacity utilisation should be taken into account for a cost-efficient supply chain. Beaujon and Turnquist (1991) has shown that determining the fleet size is one of the most impactful variables in optimising truckload capacity utilisation. Over-estimating the fleet size can translate into expected idleness and consequently a lower frequency of trips which can result in longer waiting time for the items to be filled in the truck and therefore longer lead time. This can end up with a higher inventory level in the warehouses. On the other hand, underestimating the fleet size will cause overstocking and cost inefficiency (Huang et al., 2014). The inclusion of these latter when dispatching trucks can improve the relevant performance indicators such as items shipped per day, empty or underutilised miles, and idling time. Moreover, it can enhance high-quality inventory control decisions at the manufacturers and distribution centres, as a trade-off between transport and inventory-related decisions plays an essential role in a cost minimisation objective. It implies that these decisions, i.e. fleet size determination and inventory level at the manufacturers and distribution centres, are strongly interdependent and have to be taken into account in supply chain network design problems. Contrary to the vast majority of studies in supply chain network design where the decision for the flow between two points is formulated as a continuous variable, we consider integer variables to determine the number of trucks carrying products/returned items. Our model proposes a discrete number of trucks with limited capacity on each link of the network, not all trucks are dispatched in full truckloads. In this paper, we cover this gap by proposing such a general integer mathematical model that considers the integration of fleet size determination and inventory level at the manufacturers and distribution centres in the network design problem.

In a supply network, integrating both the forward and reverse flows empowers companies to make more environmentally conscious decisions. As mentioned in Govindan et al. (2015), the need for reverse logistics and CLSC was first created by public awareness. Then legislations set by governments enforced manufacturers to take back their used/discarded products for recovery and this highly encouraged researchers, operations managers, and company executives to widely study CLSC problems and benefit from. Here, we present a few studies from the literature that investigate CLSC problems. The early work of Marin and Pelegrín (1998) developed a network design model for a closed-loop uncapacitated facility location problem emanating from a system of suppliers and customers in which customers are allowed to return used items to retailers. A closed-loop logistics system is modelled using MILP to determine the location of remanufacturing and distribution facilities, production, transshipment, and inventory level of remanufactured products by Jayaraman et al. (1999). Fleischmann et al. (2001) studied a closed-loop logistics network design problem with product recovery and proved that impact of product recovery on the logistics network structure is very much context dependent. Jayaraman (2006) developed a linear programming model for aggregate production planning and control in a CLSC network with product recovery, and then discussed operational and managerial implications of the model. A multi-product CLSC network design problem was studied and modelled in Üster et al. (2007). The authors developed a dual solution approach to derive efficient Benders cuts to solve instances of the network design problem of interest which outperforms branch-and-cut and traditional Benders decomposition approaches. A CLSC problem with disposal or remanufacturing of returned products is modelled in Pan et al. (2009). They used dynamic programming to solve the capacitated and uncapacitated versions of the model. Easwaran and Üster (2010) developed a MILP model for a CLSC problem with hybrid manufacturing/remanufacturing facilities and capacitated hybrid distribution/collection centres in which locations of candidate hybrid centres and hybrid sourcing facilities are all given. They used Benders' decomposition to solve the problem and gained promising results. Note that the problem studied in our paper is completely different from the problem investigated in Easwaran and Üster (2010) as we consider hybrid retailers/collections centres, in a fashion that some retailers have necessary infrastructures to serve as collection centres too, along with other novel characteristics discussed above. A multi-product multi-echelon multi-period CLSC problem is modelled in Özkır and Başlıgıl (2012). They concluded that the volume of product returns and quality conditions are key players in the design of the CLSC network.Steinke and Fischer (2015) investigated a multi-period, multi-commodity CLSC network design problem that integrates facility location, capacity, and production planning decisions for products sharing common components. The authors then used MILP to mathematically model the problem. Govindan et al. (2016) investigated a CLSC problem with product recovery option to understand how product recovery impacts sustainability in the manufacturing process. They highlighted that opening recovery facilities near resources and product recovery with less waste should be taken forward to achieve sustainability goals. Reddy et al. (2020) employed MILP to model a multi-tier, multi-period green reverse logistics network, incorporating vehicle type selection. It integrates decisions on facility locations and vehicle types while accounting for emissions generated from transportation and operational activities across various facilities. Reddy et al. (2022) proposed a MILP model for a the multi-facility green reverse logistics network design problem, aimed at minimising total costs which include expenses associated with carbon emissions from transportation and facility operations. The authors developed an enhanced solution technique, termed "Improved Benders Decomposition", which integrates various algorithmic improvements, such as a reinforced master problem, the introduction of valid inequalities, heuristic approaches, and a multi-stage framework to expedite the convergence of the decomposition process. A CLSC network design problem in smartphone manufacturing industry is modelled in Kim and Do Chung (2022) to identify whether it is more cost-effective for sourcing facilities and suppliers to return to their home country based on level of reshoring drivers.

A classification of the literature on deterministic CLSC network design problems, modelled using MILP, is depicted in Table 1 based on 5 characteristics, i.e. location, lot-sizing, transportation, periodicity, and solution approach. Table 1 also includes the characteristics of the CLSC network design problem investigated in this paper in its last line. It is noteworthy to highlight that all reference articles mentioned in Table 1 are sorted chronologically.

The model presented in this work embraces capacitated facility location problem and capacitated lot-sizing problem as well as transportation problem. To the best of our knowledge, our proposed model, which incorporates strategic decisions concerning the number and location of hybrid facilities, along with operational decisions including manufacturing and remanufacturing/recycling plans, inventory management, and fleet optimisation, has not been studied in the relevant literature. Hence, we believe this novel model adds incremental contribution to the closed-loop supply chain network design field.

3. Mathematical programming model

In this section, we begin by delineating the context of the problem at hand, followed by offering its mathematical formulation. A three-layer CLSC network with sourcing facilities, distribution centres, and demand/return locations (retailers), as shown in Figure 1, is considered in this paper. It is assumed that a subset of retailers has the hybrid feature to also serve as collection centres. This subset is called the set of candidate collection centres. On the one hand, the locations of manufacturers, remanufacturers/recycling centres, distributors and retailers are determined. On the other hand, a set of candidate collection centres is given among which we would like to pick those minimising the total cost. Furthermore, we assume that only one product is produced and transported through the network and we aim to optimise the system over a given time horizon. Hence, a multi-period multi-echelon single-product CLSC network problem is built and investigated in this study.

Each retailer has a deterministic demand of products that must be satisfied through the flow of products from sourcing facilities to distributors and then to retailers. Supplied products are used by customers by the end of their lifetime and a fraction of used products are then returned to retailers. Therefore, each retailer faces product return and consequently, a deterministic return is attached to each retailer. Returned products are then shipped to selected collection centres (hybrid retailers/collection centres) and finally from those collections centres to remanufacturers or recycling centres for remanufacturing or recycling, respectively. In the remanufacturing setting, we assume that the quality of remanufactured products is as good as manufactured products. We define *recovery rate* as a fraction of returned products that are remanufactured/recycled into new products. It is also assumed that each sourcing facility can do both manufacturing and remanufacturing/recycling. As it can be seen in Figure 1, remanufactured/recycled products are sent forward to distributors to satisfy customer demand. Furthermore, forward/reverse transportation is outsourced to a third-party logistics company with a limited fleet size that invoices the transportation cost per travelled distance per shipment.

We now introduce the notation and the decision variables required to mathematically model the problem of interest.

Sets:

$\mathcal{I}$ set of sourcing facilities (manufacturer and remanufacturer).

Table 1: Classification of relevant studies on deterministic CLSC network design

Articles	**Location**	**Lot-sizing**			**Transportation**		**Periodicity**	**Solution approach**
		Production	Set-up	Inventory	Flow	Fleet size		
Marin and Pelegrín (1998)	✓	–	–	–	✓	–	SP	Lagrangian decomposition
Jayaraman et al. (1999)	✓	✓	–	✓	✓	–	SP	–
Krikke et al. (1999)	✓	–	–	✓	✓	–	SP	–
Fleischmann et al. (2001)	✓	–	–	–	✓	–	SP	–
Beamon and Fernandes (2004)	✓	✓	–	✓	✓	–	SP	–
Sim et al. (2004)	✓	✓	–	✓	✓	–	SP	Genetic algorithm
Salema et al. (2006)	✓	–	–	–	✓	–	SP	–
Schultmann et al. (2006)	–	–	–	–	✓	–	SP	Metaheuristic
Lu and Bostel (2007)	✓	✓	–	–	✓	–	SP	Lagrangian heuristic
Sahyouni et al. (2007)	✓	–	–	–	✓	–	SP	Lagrangian relaxation
Üster et al. (2007)	✓	✓	–	–	✓	–	SP	Benders decomposition
Pishvaee et al. (2010)	✓	–	–	–	✓	–	SP	Memetic algorithm
Easwaran and Üster (2010)	✓	✓	–	–	✓	–	SP	Benders decomposition
Özkır and Başlıgıl (2012)	✓	✓	–	✓	✓	–	MP	–
Govindan et al. (2016)	✓	✓	–	✓	✓	–	MP	Multi-obj. optimisation
Pedram et al. (2017)	✓	✓	–	–	✓	–	SP	–
Jabarzadeh et al. (2020)	✓	✓	–	✓	✓	–	MP	LP-Metric
Reddy et al. (2020)	✓	✓	–	✓	✓	✓	MP	–
Pazhani et al. (2021)	✓	✓	–	✓	✓	–	MP	Integer relaxation
Reddy et al. (2022)	✓	✓	–	✓	✓	✓	MP	Improved Benders
Kim and Do Chung (2022)	✓	–	–	✓	✓	–	MP	–
This study	✓	✓	✓	✓	✓	✓	MP	Relax-&-Fix and Fix-&-Optimise

SP: Single-Period, MP: Multi-Period.

$\mathcal{J}$	set of distribution centres.
$\mathcal{K}$	set of retailers (demand/return points).
$\mathcal{L} \subseteq \mathcal{K}$	set of candidate retailers with the hybrid feature (candidate collection centres).
$\mathcal{T}$	set of discrete time periods (finite planning horizon).
$\mathcal{V}$	set of available vehicles on each arc.

We define $\mathcal{A}^p = \{\mathcal{A}_1^p \cup \mathcal{A}_2^p\}$, in which superscript p refers to the flow of **p**roducts from sourcing facilities to retailers, where $\mathcal{A}_1^p = \{(i,j)|i \in \mathcal{I}, j \in \mathcal{J}\}$ and $\mathcal{A}_2^p = \{(j,k)|j \in \mathcal{J}, k \in \mathcal{K}\}$ represent the arc from sourcing facility $i \in \mathcal{I}$ to distributor $j \in \mathcal{J}$ and the arc from distributor $j \in \mathcal{J}$ to retailer $k \in \mathcal{K}$, respectively. We also define $\mathcal{A}^r = \{\mathcal{A}_1^r \cup \mathcal{A}_2^r\}$, in which superscript r refers to the flow of **r**eturns from retailers to sourcing facilities, where $\mathcal{A}_1^r = \{(k,l)|k \in \mathcal{K}, l \in \mathcal{L}\}$ and $\mathcal{A}_2^r = \{(l,i)|l \in \mathcal{L}, i \in \mathcal{I}\}$ represent the arc from retailer $k \in \mathcal{K}$ to established collection centre $l \in \mathcal{L}$ and the arc from collection centre $l \in \mathcal{L}$ to the sourcing facility $i \in \mathcal{I}$, respectively. We also introduce $\mathcal{N} = \{\mathcal{I} \cup \mathcal{J} \cup \mathcal{K}\}$.

Parameters:

f_l	fixed cost of establishing collection centre l.
g_{it}^m	setup cost for manufacturing at sourcing facility i in period t.
g_{it}^{re}	setup cost for remanufacturing/recycling at sourcing facility i in period t.
p_i^m	unit manufacturing cost at sourcing facility i.
p_i^{re}	unit remanufacturing/recycling cost at sourcing facility i.
h_i^p	unit holding cost of products at node $i \in \mathcal{N}$.
h_i^r	unit holding cost of returns at node $i \in \{\mathcal{I} \cup \mathcal{K}\}$.
hc_l^r	unit holding cost of returns at collection centre l.
d_{kt}	amount of demand for products at retailer k in period t.
r_{kt}	amount of returns at retailer k in period t.
$e_{(i,j)}$	distance between node i and node j, for all $(i,j) \in \mathcal{A}^p \cup \mathcal{A}^r$.
c^p	unit transportation cost of products per distance per truck shipment.
c^r	unit transportation cost of returns per distance per truck shipment.
C_{it}^m	manufacturing capacity at sourcing facility i in period t.
C_{it}^{re}	remanufacturing/recycling capacity at sourcing facility i in period t.
C_l	capacity of collection centre l over the planning horizon.
α	fraction of returned items remanufactured into new products (recovery rate).
τ	number of time periods required to remanufacture returned items (recovery time).
β^p	truck capacity to transport products (forward shipment).
β^r	truck capacity to transport returns (reverse shipment).

In light of the definition of the candidate collection centres, one can readily see that $e_{(k,l)} = 0, \forall (k,l) \in \mathcal{A}_1^r \mid k = l$.

Non-negative continuous decision variables:

Q^m_{it} quantity of manufactured products at sourcing facility i at period t.

Q^{re}_{it} quantity of remanufactured/recycled products at sourcing facility i at period t.

R_{it} quantity of returned products sent forward for remanufacturing/recycling at sourcing facility i at time period t.

I^p_{it} inventory level of products held at node $i \in \mathcal{N}$ at the end of period t.

I^r_{it} inventory level of returns held at node $i \in \{\mathcal{I} \cup \mathcal{K}\}$ at the end of period t.

Ic^r_{lt} inventory level of returns held at collection centre $l \in \mathcal{L}$ at the end of period t.

F_{at} flow of products (returns) on arc $a \in \mathcal{A}^p$ $(a \in \mathcal{A}^r)$ at period t.

General integer decision variables:

B_{at} number of trucks used to transport products (returns) on arc $a \in \mathcal{A}^p$ $(a \in \mathcal{A}^r)$ at period t.

Binary decision variables:

$$Y^m_{it} \quad \begin{cases} 1, & \text{if manufacturing takes place in facility } i \text{ at time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

$$Y^{re}_{it} \quad \begin{cases} 1, & \text{if remanufacturing/recycling takes place in facility } i \text{ at time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

$$X_l \quad \begin{cases} 1, & \text{if collection centre } l \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

Having the parameters and the decision variables defined, a network representation of all flows running in a sourcing facility of a CLSC with three time periods is depicted in Figure 2 in which each time period is represented by a circle. The higher level of the figure shows the flow of returned products in sourcing facility i and the lower level demonstrates the production (manufacturing and remanufacturing/recycling) flow as well as flow of products within the sourcing facility.

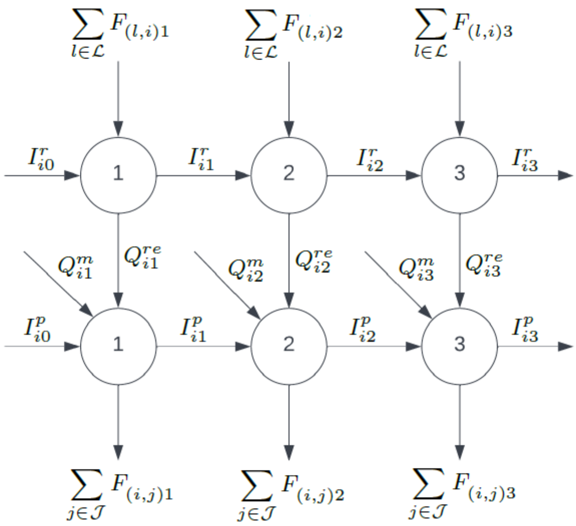


Figure 2: Network representation of flows in sourcing facility i in a CLSC problem with three time periods.

The MILP model of the CLSC problem, denoted by (P) hereafter, is shown below.

$$(\mathrm{P}) \quad \min \quad \sum_{l\in\mathcal{L}} f_l X_l + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} g^m_{it} Y^m_{it} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} g^{re}_{it} Y^{re}_{it} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} p^m_i Q^m_{it} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} p^{re}_i Q^{re}_{it} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} h^p_i I^p_{it} + \sum_{i\in\mathcal{I}\cup\mathcal{K}}\sum_{t\in\mathcal{T}} h^r_i I^r_{it} + \sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}} hc^r_l Ic^r_{lt} + \sum_{a\in\mathcal{A}^p}\sum_{t\in\mathcal{T}} e_a c^p B_{at} + \sum_{a\in\mathcal{A}^r}\sum_{t\in\mathcal{T}} e_a c^r B_{at} \quad (3.1)$$

$$s.t. \quad Q^m_{it} \le C^m_{it} Y^m_{it} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \quad (3.2)$$

$$Q^{re}_{it} \le C^{re}_{it} Y^{re}_{it} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \quad (3.3)$$

$$Q^m_{it} + Q^{re}_{it} + I^p_{i,t-1} = \sum_{j\in\mathcal{J}} F_{(i,j)t} + I^p_{it} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T} \quad (3.4)$$

$$\sum_{i\in\mathcal{I}} F_{(i,j)t} + I^p_{j,t-1} = \sum_{k\in\mathcal{K}} F_{(j,k)t} + I^p_{jt} \qquad \forall j\in\mathcal{J}, t\in\mathcal{T} \quad (3.5)$$

$$\sum_{j\in\mathcal{J}} F_{(j,k)t} + I^p_{k,t-1} = d_{kt} + I^p_{kt} \qquad \forall k\in\mathcal{K}, t\in\mathcal{T} \quad (3.6)$$

$$F_{at} \le \beta^p B_{at} \qquad \forall a\in\mathcal{A}^p, t\in\mathcal{T} \quad (3.7)$$

$$F_{at} \le \beta^r B_{at} \qquad \forall a\in\mathcal{A}^r, t\in\mathcal{T} \quad (3.8)$$

$$r_{kt} + I^r_{k,t-1} = I^r_{kt} + \sum_{l\in\mathcal{L}} F_{(k,l)t} \qquad \forall k\in\mathcal{K}, t\in\mathcal{T} \quad (3.9)$$

$$\sum_{k\in\mathcal{K}} F_{(k,l)t} + Ic^{r}_{l,t-1} = Ic^{r}_{lt} + \sum_{i\in\mathcal{I}} F_{(l,i)t} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{3.10}$$

$$\sum_{l\in\mathcal{L}} F_{(l,i)t} + I^{r}_{i,t-1} = I^{r}_{it} + R_{it}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{3.11}$$

$$Q^{re}_{i,t+\tau} = \alpha R_{it} \qquad \forall i \in \mathcal{I}, t \in \{1,\ldots,|\mathcal{T}|-\tau\} \tag{3.12}$$

$$Y^{re}_{it} = 0 \qquad \forall i \in \mathcal{I}, t \in \{1,\ldots,\tau\} \tag{3.13}$$

$$B_{at} \le |\mathcal{V}| \qquad \forall a \in \mathcal{A}^{p}, t \in \mathcal{T} \tag{3.14}$$

$$B_{(k,l)t} \le |\mathcal{V}|\, X_l \qquad \forall k \in \mathcal{K}, l \in \mathcal{L}, t \in \mathcal{T} \tag{3.15}$$

$$B_{(l,i)t} \le |\mathcal{V}|\, X_l \qquad \forall l \in \mathcal{L}, i \in \mathcal{I}, t \in \mathcal{T} \tag{3.16}$$

$$I^{r}_{lt} \le C_l\, X_l \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{3.17}$$

$$I^{p}_{it} \in \mathbb{R}^{+}_{0} \qquad \forall i \in \mathcal{N}, t \in \mathcal{T} \cup \{0\} \tag{3.18}$$

$$I^{r}_{it} \in \mathbb{R}^{+}_{0} \qquad \forall i \in \{\mathcal{I} \cup \mathcal{K}\}, t \in \mathcal{T} \cup \{0\} \tag{3.19}$$

$$Ic^{r}_{lt} \in \mathbb{R}^{+}_{0} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \cup \{0\} \tag{3.20}$$

$$F_{at} \in \mathbb{R}^{+}_{0} \qquad \forall a \in \mathcal{A}^{o}, o \in \{p,r\}, t \in \mathcal{T} \tag{3.21}$$

$$Q^{m}_{it}, Q^{re}_{it}, R_{it} \in \mathbb{R}^{+}_{0} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{3.22}$$

$$Y^{m}_{it}, Y^{re}_{it} \in \mathbb{B} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{3.23}$$

$$X_l \in \mathbb{B} \qquad \forall l \in \mathcal{L} \tag{3.24}$$

$$B_{at} \in \mathbb{Z}^{+}_{0} \qquad a \in \mathcal{A}^{o}, o \in \{p,r\}, t \in \mathcal{T} \tag{3.25}$$

where $\mathbb{R}^{+}_{0}, \mathbb{B}, \mathbb{Z}^{+}_{0}$ represent the set of non-negative real numbers, the set of binary numbers, and the set of non-negative integer numbers, respectively. Objective function (3.1) minimises the cumulative costs, including establishment cost of collection centres, setup cost for manufacturing and remanufacturing/recycling, production cost for manufacturing and remanufacturing/recycling, holding cost for products in the forward flow channel as well as returns in the reverse flow channel, and transportation cost for products and returns. Constraints (3.2) and (3.3) represent capacity restrictions for manufacturing and remanufacturing/recycling at each sourcing facility over the planning horizon and fix the binary (set-up) decisions variables to 1 whenever there is positive production. Constraints (3.4) represent the flow conservation for products at each sourcing facility over the planning horizon. Constraints (3.5) represent the flow conservation for products at each distribution centre. Constraints (3.6) represent the flow conservation for products at each retailer ensuring that customer demands are satisfied. Constraints (3.7) guarantee that at a given time period flow of products on each arc of the network (within the forward flow channel) is upper bounded by product of truck capacity and number of trucks used to transport products on that arc. Constraints (3.8) convey the same for returned products (within the reverse flow channel). Constraints (3.9) represent the flow conservation for returned products at each retailer over the planning horizon. Likewise, constraints (3.10) and (3.11) represent the flow conservation for returned products at each collection centre and each sourcing facility at time period t, respectively. Constraints (3.12) express that product of recovery rate and quantity of returns carried forward for remanufacturing/recycling at sourcing facility i and time period t results in remanufactured/recycled products at time period $t+\tau$. With respect to the definition of τ, constraints (3.13) guarantee that no product can be remanufactured/recycled from time period 1 to time period τ. Constraints (3.14) ensure that number of trucks that can be used to transport products on each arc (within the forward flow channel) over the planning horizon cannot exceed the maximum number of trucks available. Constraints (3.15) and (3.16) ensure that number of trucks used on arcs to/from each collection centre cannot exceed the maximum number of vehicles available on arcs. Constraints (3.17) guarantee that capacity of each collection centre is respected over the planning horizon, if selected. Finally, the non-negativity and integrality constraints are given by (3.18)-(3.25).

Problem (P) encompasses both the capacitated lot-sizing problem and the capacitated facility location problem, both of which are classified as NP-hard (see Pochet and Wolsey (2006) and Mirchandani and Francis (1990), respectively). Additionally, problem (P) introduces several new constraints. Consequently, it is evident that problem (P) also falls under the classification of NP-hard.

Next, we present and discuss the solution approach we designed and developed to efficiently deal with problem (P).

4. Solution approach

Since dealing with a real-world optimisation problem requires a considerable amount of time and effort, we propose a problem-specific matheuristic, based on Relax-and-Fix (R&F) and Fix-and-Optimise (F&O) approaches, to solve such problem. While the former approach sequentially solves the relaxations of a MILP model and gradually fixes variables until a feasible solution is found, the latter focuses on the improvement of feasible solutions' quality and their convergence towards optimality.

The R&F heuristic is a constructive method that iteratively solves relaxed MILP subproblems in which the integrality constraints of some integer variables are relaxed to be fractional, enforcing the rest to be an integer. However, the approaches by which the decision variables are partitioned and relaxed and the criteria used to fix the variables have a strong impact on the efficiency of the R&F heuristic (Ferreira et al., 2010). One of the widely used approaches for variable partitioning is the rolling-horizon method (De Araujo et al., 2015; Fragkos et al., 2016, 2021), which decomposes the main problem into several small subproblems according to the time horizon. Each subproblem is then solved chronologically for each time interval. See (Toledo et al., 2015; Absi and van den Heuvel, 2019; Friske et al., 2022), for example. In the relevant works, one can find a collection of variant tailor-made relaxation methods, which are problem-specific and are constructed based on the importance of critical parameters (such as demand, product, process, etc.) or the structure of the model (such as aggregation, sortation, influence in the objective function, etc.). See (Ferreira et al., 2010; Helber and Sahling, 2010; Li et al., 2017; Charles et al., 2019; Van Bulck and Goossens, 2021), for example. The F&O heuristic, however, fixes the integer variables, which have been relaxed in R&F heuristic, to solutions obtained from R&F as the initial feasible solution value and defines a set of variables to be optimised progressively until stopping criteria are met. This heuristic depends highly on the problem characteristics with user-defined parameters (De Araujo et al., 2015; Friske et al., 2022). One can refer to Pochet and Wolsey (2006) for detailed information regarding this matheuristic method. In the following, we describe the proposed solution method for solving the CLSC problem under study in this paper.

In order to apply the R&F and F&O heuristics, variable partitioning sets and a method to relax and fix relevant variable sets must clearly be defined. It should be noted that variable partitioning sets are disjunctive sets of variables, where each has a predefined type, either integer or relaxed. A time-based variable partitioning set cannot be applied to the problem studied in this paper due to the special structure of the model that incorporates time-dependent variables as well as time-independent variables. Therefore, it is necessary to introduce a more specific approach to solve this problem. Moreover, the problem under study in this paper is more complicated than the capacitated facility location problem and capacitated lot-sizing problem, due to the presence of general integer variables in the model. Below, we present our proposed solution method.

4.1. Relax-and-Fix

In our R&F heuristic, a general integer variable has one of the following three status in each iteration of the algorithm, either fixed, relaxed (i.e., non-negative continuous), or to be optimised (i.e., non-negative integer). Once a variable is fixed, it remains unchanged throughout all the consequent iterations of the algorithm. However, a relaxed one may switch to an integer or vice versa depending on certain conditions we explore in the following.

4.1.1. Relaxation strategy

The relaxation is introduced by removing the complicated constraints (3.25) which impose the integrality restriction. We consider the ensemble general integer variables of model P, denoted by $B_{at} \in \mathbb{Z}_0^+, \forall a \in \{\mathcal{A}^p \cup \mathcal{A}^r\}, t \in \mathcal{T}$. After relaxing the integrality of the latter variables to the non-negative continuous type, we denote the corresponding variables by $\tilde{B}_{at} \in \mathbb{R}_0^+, \forall a \in \{\mathcal{A}^p \cup \mathcal{A}^r\}, t \in \mathcal{T}$. While maintaining the structure of the original model, we aim to solve an easier problem that contains only binary variables, representing the facility location selection, and continuous variables, for the remaining decision variables, which can be solved in a relatively short computation time. The relaxed problem, named RP0, is represented as follows:

$$
\begin{aligned}
\text{(RP0)} \qquad \min \quad & \sum_{l \in \mathcal{L}} f_l X_l + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} g_{it}^m Y_{it}^m + \\
& \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} g_{it}^{re} Y_{it}^{re} + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} p_i^m Q_{it}^m + \\
& \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} p_i^{re} Q_{it}^{re} + \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} h_i^p I_{it}^p + \\
& \sum_{i \in \mathcal{I} \cup \mathcal{K}} \sum_{t \in \mathcal{T}} h_i^r I_{it}^r + \sum_{l \in \mathcal{L}} \sum_{t \in \mathcal{T}} hc_l^r Ic_{lt}^r + \\
& \sum_{a \in \mathcal{A}^p} \sum_{t \in \mathcal{T}} e_a c^p \tilde{B}_{at} + \sum_{a \in \mathcal{A}^r} \sum_{t \in \mathcal{T}} e_a c^r \tilde{B}_{at} \\
s.t. \quad & (3.2) - (3.6), (3.9) - (3.13) \\
& (3.17) - (3.24), \\
& F_{at} \leq \beta^p \tilde{B}_{at} && \forall a \in \mathcal{A}^p, t \in \mathcal{T} \\
& F_{at} \leq \beta^r \tilde{B}_{at} && \forall a \in \mathcal{A}^r, t \in \mathcal{T} \\
& \tilde{B}_{at} \leq |\mathcal{V}| && \forall a \in \mathcal{A}^p, t \in \mathcal{T} \\
& \tilde{B}_{at} \leq |\mathcal{V}| X_l && \forall a \in \mathcal{A}^r, t \in \mathcal{T} \\
& \tilde{B}_{at} \in \mathbb{R}_0^+ && \forall a \in \mathcal{A}^o, o \in \{p, r\}, t \in \mathcal{T}
\end{aligned}
$$

The optimal solution of RP0 provides solutions with relaxed values for the number of shipments, among them some might be infeasible to the original problem P. The following procedure is applied for fixing the relaxed variables of RP0.

4.1.2. Fixing strategy

Let $\tilde{b}_{at}$, where $a \in \mathcal{A}^o, o \in \{p, r\}, t \in \mathcal{T}$, be an obtained solution value of the corresponding relaxed variable $\tilde{B}_{at}$ after solving the problem RP0. If the obtained solution value satisfies the integrality, the corresponding variable is fixed to its integer value in the next iterations. Otherwise, the corresponding decision variable will be further improved by setting its type to either integer or relaxed depending upon some evaluations of its value. The evaluation is based on an integrality test which accounts for possible rounding-off errors allowing some feasibility tolerance. In order to do so, we carry on an integrality test as follows.

Definition 4.1. *An integrality test holds true for the solution* $\tilde{b}_{at}$ *if*

$$|\tilde{b}_{at} - \lfloor \tilde{b}_{at} + 0.5 \rfloor| \leq \theta, \forall a \in \mathcal{A}^o, o \in \{p, r\}, t \in \mathcal{T} \tag{4.1}$$

where $\lfloor . \rfloor$ *represents the floor function and* $\theta \in [0, 0.5]$ *is a user-defined integer feasibility tolerance.*

The integrality test verifies whether a fractional value is close enough to its nearest integer value with respect to a user-defined integer feasibility tolerance θ. It is important to highlight the fact that when θ is set to a relatively small (large) number, the rounding-off error is negligible (considerable). Therefore, further improvement is essential to avoid trapping in local optima due to the presence of round-off error specially when θ is relatively large. Let us introduce $\mathscr{A}^o \subset \mathcal{A}^o, o \in \{p, r\}$ a subset of arcs and $\mathscr{T} \subset \mathcal{T}$ a subset of periods on which the integrality test for the value of $\tilde{b}_{at}$ holds true, i.e., $|\tilde{b}_{at} - \lfloor \tilde{b}_{at} + 0.5 \rfloor| \leq \theta, \forall a \in \mathscr{A}^o, o \in \{p, r\}, t \in \mathscr{T}$. We then indicate its corresponding variable by $\hat{B}_{at} \in \mathbb{Z}_0^+$ to be further improved in the optimisation model. This improvement is carried out by setting the corresponding variables' type as the general integer with predefined lower and upper bounds. On the other hand, when the integrality test does not hold true for the solution $\tilde{b}_{at}$, the corresponding variable remains relaxed. For the sake of consistency, we denote such solutions by $\tilde{b}_{at}$, for $a \in \mathscr{A}^o_{\mathrm{C}} \subset \mathcal{A}^o, o \in \{p, r\}, t \in \mathscr{T}_{\mathrm{C}} \subset \mathcal{T}$, where $\mathscr{A}^o_{\mathrm{C}}$ and $\mathscr{T}_{\mathrm{C}}$ are the complementary sets of $\mathscr{A}^o$ and $\mathscr{T}$, respectively.

Definition 4.2. *When the integrality test holds true for the solution* $\tilde{b}_{at}$, $\forall a \in \mathscr{A}^o, o \in \{p, r\}, t \in \mathscr{T}$, *the corresponding variable's type becomes general integer, i.e.,* $\hat{B}_{at} \in \mathbb{Z}^+$, *and its bounds are given as follow:*

$$\hat{B}_{at} \in [lb, ub] := \Big[\max\Big\{0, \Big\lfloor \tilde{b}_{at}(1-\rho) \Big\rfloor\Big\}, \min\Big\{\Big\lceil \tilde{b}_{at}(1+\rho) \Big\rceil, |\mathcal{V}|\Big\}\Big] \tag{4.2}$$

where lb and ub represent the lower bound and upper bound, respectively, $\rho \in (0, 1)$ *is a user-defined search radius, and* $\lceil . \rceil$ *indicates the ceiling function.*

The parameter ρ allows user to introduce an interval (or a neighbourhood) within which the variable will be optimised iteratively depending on the scale of the value of $\tilde{b}_{at}$.

The variable partitioning and fixing strategy introduced above not only tightens the search space resulting in shorter computation time, also facilitates the solution procedure in order to provide integer solutions which are feasible to the original problem. Below, we present the obtained model after implementing the fixing strategy.

$$
\begin{aligned}
\text{(RP1)} \qquad \min \quad & \sum_{l\in\mathcal{L}} f_l X_l + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} g_{it}^{m} Y_{it}^{m} + \\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} g_{it}^{re} Y_{it}^{re} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} p_i^{m} Q_{it}^{m} + \\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} p_i^{re} Q_{it}^{re} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} h_i^{p} I_{it}^{p} + \\
& \sum_{i\in\mathcal{I}\cup\mathcal{K}}\sum_{t\in\mathcal{T}} h_i^{r} I_{it}^{r} + \sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}} hc_l^{r} Ic_{lt}^{r} + \\
& \sum_{a\in\mathcal{A}^p}\sum_{t\in\mathcal{T}} e_a c^p \hat{B}_{at} + \sum_{a\in\mathcal{A}^r}\sum_{t\in\mathcal{T}} e_a c^r \hat{B}_{at} + \\
& \sum_{a\in\mathcal{A}_{\mathrm{C}}^p}\sum_{t\in\mathcal{T}_{\mathrm{C}}} e_a c^p \tilde{B}_{at} + \sum_{a\in\mathcal{A}_{\mathrm{C}}^r}\sum_{t\in\mathcal{T}_{\mathrm{C}}} e_a c^r \tilde{B}_{at} \\
s.t. \quad & (3.2)-(3.6), (3.9)-(3.13) \\
& (3.17)-(3.24) \\
& F_{at} \le \beta^o \hat{B}_{at} && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& F_{at} \le \beta^o \tilde{B}_{at} && \forall a\in\mathcal{A}_{\mathrm{C}}^o, o\in\{p,r\}, t\in\mathcal{T}_{\mathrm{C}} \\
& \hat{B}_{at} \le \min\Big\{\lceil \tilde{b}_{at}(1+\rho)\rceil, |\mathcal{V}|\Big\} && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& \hat{B}_{at} \ge \max\Big\{0, \lfloor \tilde{b}_{at}(1-\rho)\rfloor\Big\} && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& \tilde{B}_{at} \le |\mathcal{V}| && \forall a\in\mathcal{A}_{\mathrm{C}}^p, t\in\mathcal{T}_{\mathrm{C}} \\
& \hat{B}_{at} \le |\mathcal{V}|\, X_l && \forall a\in\mathcal{A}^r, t\in\mathcal{T} \\
& \tilde{B}_{at} \le |\mathcal{V}|\, X_l && \forall a\in\mathcal{A}_{\mathrm{C}}^r, t\in\mathcal{T}_{\mathrm{C}} \\
& \hat{B}_{at} \in \mathbb{Z}_0^+ && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& \tilde{B}_{at} \in \mathbb{R}_0^+ && \forall a\in\mathcal{A}_{\mathrm{C}}^o, o\in\{p,r\}, t\in\mathcal{T}_{\mathrm{C}}
\end{aligned}
$$

It is important to mention that the variable partitioning shrinks the problem's dimensions noticeably allowing the model to search for optimising the corresponding integer variables within a tighter interval. This procedure continues until the stopping criteria are met that is each general integer variable is fixed to a feasible integer solution value. It is clear that once the algorithm is terminated, $\mathcal{A}_{\mathrm{C}}^o = \emptyset$ and $\mathcal{A}^o = \mathcal{A}^o$, where $o \in \{p, r\}$.

Below, we present a pseudo-code for the proposed R&F heuristic, where Θ is a list of pre-defined integer feasibility tolerance, δ is a step-size for defining the search radius, and $\rho^{max} < 1$ is the maximum user-defined search radius. Also, let z_{RP1} be the objective value of RP1 and Δz_{RP1} be the difference between the objective value of the model RP1 in two consecutive iterations (the current one and the precedent) in Algorithm 1. We introduce $RD_{R\&F} = \frac{\Delta z_{RP1}}{z_{RP1}}$ as a relative difference of z_{RP1} over iterations, representing the relative solution quality improvement, and ξ as an acceptable tolerance of the quality of objective function value obtained for the R&F heuristic.

Algorithm 1 Pseudocode of Relax-and-Fix heuristic

```
procedure R&F HEURISTIC(RP0, RP1, Θ, δ, ξ, ρ^max)
    Solve the relaxed problem (RP0)
    for all θ ∈ Θ do
        ρ ← ε                                              ▷ ε is a small number
        while ρ <= ρ^max do
            for all a ∈ A^o, o ∈ {p, r}, t ∈ T do
                if b̃_at is integer then
                    Fix the corresponding integer variable B̃_at to its value b̃_at
                else
                    Check the integrality test given in (4.1) for b̃_at with parameter θ
                    if True then
                        Change the corresponding variable's type to integer and define its bounds according to (4.2)
                        with ρ
                    else
                        Remain the corresponding variable relaxed
                    end if
                end if
            end for
            Solve the obtained Model RP1
            if RD_R&F ≤ ξ then
                Exit While
            end if
            ρ ← ρ + δ
        end while
    end for
end procedure
```

The objective value of RP1 starts from a value which contains infeasible solutions to the original problem as a consequence of relaxing the general integer variables. In Algorithm 1, the objective value progressively increases as more variables are fixed to a feasible integer value. The objective value obtained from Algorithm 1 is an admissible upper bound for the original problem P.

4.2. Fix-and-optimise

Once feasible integer solutions are constructed for all general integer variables using the R&F heuristic, we proceed with an improvement procedure allowing each general integer variable to explore its respective neighbourhood within a search radius size. If a better solution is not found, the search radius size is increased. This will allow the model to search for better solutions in a wider space. The procedure continues until no improvement in the objective value is observed.

For each general integer variable of the model P whose value is fixed by the output of the R&F algorithm, we generate respective admissible intervals over which a relaxed model, called RP2, is optimised. Let $\bar{b}_{at}$ be the value of the corresponding variable $\hat{B}_{at}$ obtained as the output of the R&F algorithm. We then propose the following bound for the variable $\hat{B}_{at}$ in the model RP2:

$$\hat{B}_{at} \in [lb, ub] := \Big[\max\Big\{0, \lfloor \bar{b}_{at}(1-\rho) \rfloor\Big\}, \min\Big\{\lceil \bar{b}_{at}(1+\rho) \rceil, |\mathcal{V}|\Big\}\Big] \tag{4.3}$$

Below, we present the model RP2 to optimise the variables toward their optimal value.

$$
\begin{aligned}
\text{(RP2)} \qquad \min \quad & \sum_{l\in\mathcal{L}} f_l X_l + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} g^m_{it} Y^m_{it} + \\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} g^{re}_{it} Y^{re}_{it} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} p^m_i Q^m_{it} + \\
& \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} p^{re}_i Q^{re}_{it} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} h^p_i I^p_{it} + \\
& \sum_{i\in\mathcal{I}\cup\mathcal{K}}\sum_{t\in\mathcal{T}} h^r_i I^r_{it} + \sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}} hc^r_l Ic^r_{lt} + \\
& \sum_{a\in\mathcal{A}^p}\sum_{t\in\mathcal{T}} e_a c^p \hat{B}_{at} + \sum_{a\in\mathcal{A}^r}\sum_{t\in\mathcal{T}} e_a c^r \hat{B}_{at} \\
s.t. \quad & (3.2)-(3.6), (3.9)-(3.13) \\
& (3.17)-(3.24) \\
& F_{at} \le \beta^o \hat{B}_{at} && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& \hat{B}_{at} \le \min\Big\{\lceil \bar{b}_{at}(1+\rho)\rceil, |\mathcal{V}|\Big\} && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& \hat{B}_{at} \ge \max\Big\{0, \lfloor \bar{b}_{at}(1-\rho)\rfloor\Big\} && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T} \\
& \hat{B}_{at} \le |\mathcal{V}|\, X_l && \forall a\in\mathcal{A}^r, t\in\mathcal{T} \\
& \hat{B}_{at} \in \mathbb{Z}^+_0 && \forall a\in\mathcal{A}^o, o\in\{p,r\}, t\in\mathcal{T}
\end{aligned}
$$

The model RP2 is iteratively solved in the proposed F&O heuristic (shown in Algorithm 2) until the termination criterion is satisfied. Let z_{RP2} be the objective value of the model RP2 and Δz_{RP2} be the difference between the objective value of the model RP2 in two consecutive iterations in Algorithm 2. Similarly, we introduce $RD_{F\&O} = \frac{\Delta z_{RP2}}{z_{RP2}}$ as a relative difference of z_{RP2} over iterations, representing the relative solution quality improvement, and ψ as an acceptable tolerance of the quality of objective function value obtained for the F&O heuristic. We also use $\rho^{max} < 0.5$ as the maximum user-defined search radius.

Algorithm 2 Pseudocode of Fix-and-Optimise heuristic

procedure F&O HEURISTIC(R&F heuristic, δ, ψ, ρ^{max})
    Read solutions found from the R&F heuristic
    $\rho \leftarrow \epsilon$ ▷ ϵ is a small number
    $z_{RP2} \leftarrow \infty$
    while $\rho <= \rho^{max}$ **do**
        for all $a \in \mathcal{A}^o, o \in \{p, r\}, t \in \mathcal{T}$ **do**
            Define bounds according to (4.3) for all general integer variables
        end for
        Solve the obtained Model RP2
        if $RD_{F\&O} \leq \psi$ **then**
            Save the best solution found so far
        else
            Save the current solution
        end if
        $\rho \leftarrow \rho + \delta$
    end while
end procedure

The solution obtained from the proposed F&O heuristic tends to approach the optimal solution of the model P. The objective value of the model RP2 in the Algorithm 2 starts with an upper bound of the model P, as earlier stated. It gradually improves (decreases) as a wider solution space is taken into account when no improvement is observed from the previous iteration.

In the next section, we present the results obtained from the proposed R&F and F&O heuristics and compare them with the solution found from the original model P using a commercial optimisation solver.

5. Computational results

In this section, we evaluate the performance of the proposed model and solution method through a set of extensive instances which are generated according to four dimensions: i. size of the network comprising sourcing facilities (SF), distribution centres (DC), retailers (RT), and potential collection centres (CC), ii. fleet size, iii. vehicle capacity, and iv. capacity of collection centres.

5.1. Experimental design and setting

The problem instances are generated within the geographical scope limited to the network of a retail chain in France, collecting empty plastic bottles at selected stores. We consider four network sizes, i.e. Small, Medium, Large, and Extra Large, varying in terms of the number of sourcing facilities, distribution centres, retailers, and potential collection centres, as presented in Table 2. The geographical location of retailers is chosen from the list of 50, 100, 200, and 300 largest communes in France in 2019 [3] for network sizes Small, Medium, Large, and Extra Large respectively. It should be noted that the network size Medium includes the 50 largest communes of the Small network size and the Large one includes the 100 largest nodes of the Medium size, and

[3] https://en.wikipedia.org/wiki/List_of_communes_in_France_with_over_20,000_inhabitants

this pattern extends to the Extra Large network size too. The locations for distribution centres and sourcing facilities are chosen randomly from the list. Candidate locations for the collection centres are selected from the largest cities. The fixed cost for establishing a CC is set to $f_l =$€$20,000$. All distances between any pairs of locations in the network are calculated based on the Haversine distance function, which returns distances between two points on a sphere from their longitudes and latitudes.

Table 2: Network sizes

Network size	Small	Medium	Large	Extra Large
Nb of sourcing facilities	2	2	3	3
Nb of distribution centres	2	4	8	8
Nb of candidate collection centres	5	10	20	40
Nb of retailers	50	100	200	300

Demand for products at each retailer is generated randomly in proportion to the inhabitants of the commune in which the retailer is located. Let us denote the population of a commune i by π_i. Therefore the periodic demand for node i is generated randomly within the following interval:

$$d_{it} \in \frac{1}{\lambda}\left[\pi_i(1-\mu), \pi_i(1+\mu)\right] \forall\, i, t, \tag{5.1}$$

where $\mu \in [0,1]$ represents the deviation from the average defined by user and λ denotes the consumption per capita. The number of returns at retailer k in period t is also a proportion to the demand at that retailer and is defined as $r_{kt} = \eta d_{kt}$, where η is the recycling rate of the sold items (demand). The parameters used to generate demand and return in this paper are as follows: $\lambda = 3.5, \mu = 0.25, \eta = 0.65$.

For each network size presented above, the following data set for transportation-related parameters after a calibration process is used (Table 3). Transportation costs are presented in € per travelled distance per truck shipment (Leslie and Murray, 2022). Truck capacity is expressed in terms of product units. Number of available trucks on each arc for each network size is given in Table 3. It should be noted that the fleet size $|\mathcal{V}| = 10$ is used only for the Large and Extra Large size instances.

Table 3: Transportation parameters

	Transportation cost	value	Truck capacity	value	Fleet size	value
Products	c_a^p	168×10^{-2}	β^p	50×10^3	$\lvert\mathcal{V}\rvert$	$\{10, 20, 50\}$
Returns	c_a^r	168×10^{-3}	β^r	$\{15, 25, 35\} \times 10^3$	$\lvert\mathcal{V}\rvert$	$\{10, 20, 50\}$

Below, we present the data used for (re)manufacturing-related operations in Table 4. The (re)manufacturing cost and the setup cost, in €, represent the variable and fixed costs, respectively, associated with (re)manufacturing operations over a one-year planning horizon ($T = 12$ months) in this study. Data related to inventory holding cost at different stages of the chain is given in Table 5. It should be noted that the recovery rate and the recovery time are set to $\alpha = 0.7$ and $\tau = 1$, respectively.

We evaluate three tiers of storage capacity of collection centres as follows: low, medium, and high, and set respectively $C_l = \{3, 6, 12\} \times 10^6$ unit items for each period in the planning horizon.

Table 4: (Re)manufacturing parameters

	(re)manufacturing cost	value	Setup cost	value	(re)manufacturing capacity	value
Products	p_i^m	0.0320	g_{it}^m	1000	C_{it}^m	2.5×10^6
Returns	p_i^{re}	0.0064	g_{it}^{re}	200	C_{it}^{re}	2.5×10^5

Table 5: Inventory holding cost parameters

	SF	Values	DC/CC	Values	RT	Values
Products	h_i^p	0.0016	h_j^p	0.0080	h_k^p	0.040
Returns	h_i^r	0.0016	hc_l^r	0.0016	h_k^r	0.016

Overall, the combination of four network sizes (Table 2), two/three fleet size values and three truck capacity quantities (table 3), and three CC capacity sizes yields 90 problem instances.

All models and algorithms are tested on a computer with 16 GB RAM and 2.40 GHz processor running on Microsoft Windows 10. All instances of the problem and the algorithms are solved with the Gurobi solver version 11.0.1 using the Python interface. When solving instances, the termination criterion of the solver was given a time limit of five hours (18,000 seconds) or reaching a MILP gap of 5×10^{-5}. If the optimal solution is not found at the end of the termination criterion, the software reports the best integer solutions found within the limit or the problem is considered non-solvable. It should be highlighted that unlike optimisation problems focused on short-term decision-making, this study addresses a network design problem involving strategic, long-term decisions. Hence, a 5-hour time limit is imposed on Gurobi to obtain (near) optimal solutions, ensuring robust solution quality. The proposed Relax-and-Fix and Fix-and-Optimise algorithms are then benchmarked against these solutions, demonstrating the exceptional performance of the matheuristic method in solving instances of the CLSC problem. In other words, had a shorter time limit been considered, the Relax-and-Fix and Fix-and-Optimise heuristics would have demonstrated superior performance compared to the results presented in this study. In the following, we present the results and provide managerial insights.

5.2. Numerical results

The detailed numerical results, including the solution value and computational time, related to all instances are presented in Tables A1 of the Appendix.

In order to measure the efficiency of the proposed modelling approach and the solution method, network design solution and the related operational decisions, we applied important metrics related to CLSC management. We present the results in the following sections.

5.2.1. Heuristics validity and efficiency assessment

In the following, we evaluate the accuracy and efficiency of the proposed heuristics by computing the associated optimality gap of each approach, and comparing it against the best solution obtained using the general-purpose solver Gurobi. The solver is either terminated upon reaching the specified time limit of 5 hours or achieving the predefined MILP optimality gap. Table 6 shows a general overview of the results obtained after solving instances for each network size. This table presents the average objective value, optimality gap, and solution time (in seconds), found by each solution method, i.e. Gurobi, R&F, and F&O, across all instances for each network size. The detailed version of this table, where such information is reported for any single generated instance, is given in Tables A1 in the Appendix section.

It can be observed from Table 6 that for all Small-size networks, Gurobi provides best integer solutions with 0.05% optimality gap (on average) within 5 hours. R&F and F&O, however, terminate overall after 1 second and 24 seconds, respectively, and, can generate best integer solutions with 1.24% and 0.32% optimality gap, respectively, with respect to the best integer solution found by Gurobi. For instances with Medium network size, the average optimality gap returned by the Gurobi and R&F increases to 0.10% within 5 hours and to 1.48% within 5 seconds, respectively, while F&O gives an average optimality gap of 0.20% w.r.t the best integer solution found by Gurobi in only 588 seconds. This shows that the proposed F&O method approaches the performance of Gurobi in a shorter computation time. For Large-size instances, the overall Gurobi's optimality gap increases to 0.42% showing a reduction in the accuracy of the solutions obtained by Gurobi within 5 hours. On the other hand, the average optimality gap of the R&F algorithm grows proportionally with the increase in the network size, with a considerably short computation time. It should be noted that the overall optimality gap of our proposed F&O algorithm is a negative number, showing that it outperforms the Gurobi solver for instances characterised by Large network size, all achieved within a reasonable solution time averaging 2293 seconds. For instances with Extra Large network size, Gurobi's optimality gap increases significantly to 4.49%, highlighting the increased complexity of these instances due to the larger network size. In contrast, the R&F algorithm achieves an average optimality gap of 0.61% in only 61 seconds (on average), benefiting from the relatively larger gaps produced by Gurobi in this scenario compared to smaller network sizes (Small, Medium, and Large). Notably, the F&O algorithm demonstrates exceptional performance, achieving an average optimality gap of -1.20% relative to Gurobi's best integer solution. This result underscores the superiority of the F&O algorithm in handling Extra Large instances, achieving these solutions within a reasonable average computational time of 3,257 seconds. The following general conclusions can be drawn from Table 6:

i. Gurobi consistently reached its 5-hour time limit, without obtaining optimal solutions given the 5×10^{-5} MILP gap, for all instances generated in this study. Furthermore, the observed Gurobi optimality gap increases as the network size expands, reflecting the growing complexity of larger networks.

ii. R&F returns feasible integer solutions within a negligible solution time for all network sizes, with on average deviation of 1.24% from the best solution found by Gurobi.

iii. F&O algorithm converges to a near-optimal solution (outperforming Gurobi) as the network size increases within acceptable solution time.

5.2.2. Network configuration and managerial insights

This subsection presents and discusses the network configuration and the essential features of the best solutions for the instances generated in subsection 5.1, achieved through the F&O algorithm. Subsequently, managerial insights derived from the case study are presented for consideration.

The number of selected CCs is depicted in Figure 3. Figures 3a, 3b, 3c, and 3d illustrate the number of established CCs in Small, Medium, Large, and Extra Large networks, respectively. In each figure, the horizontal axis represents the number of selected CCs. The light grey, beige, and dark grey circles correspond to fleet sizes of 10, 20, and 50, respectively. The numerical value

Table 6: Validity and efficiency of the proposed solution method

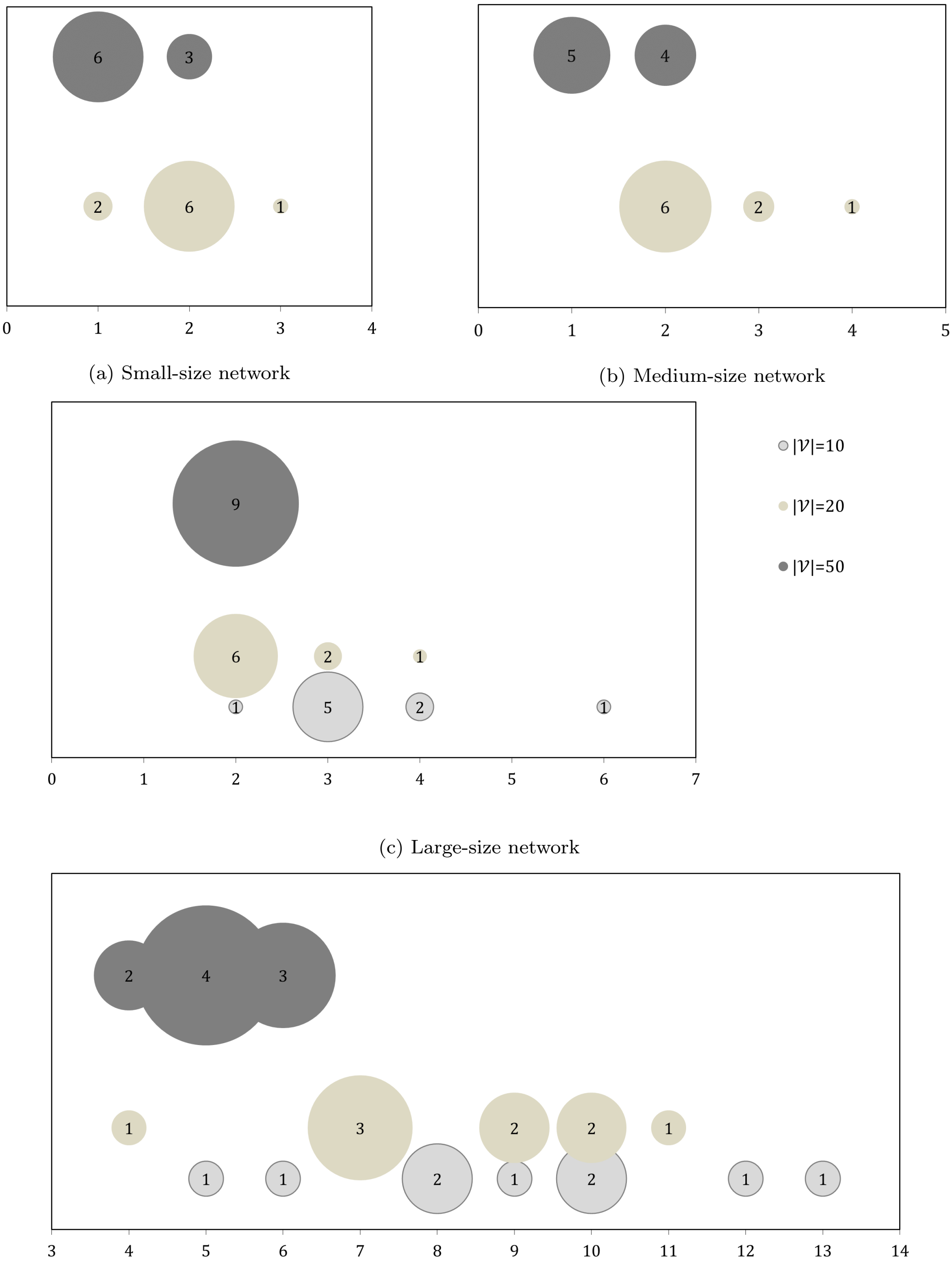
	Optimality gap			Solution time (seconds)		
Network size	Gurobi [a]	Gu-R&F [b]	Gu-F&O [c]	Gurobi	R&F	F&O
Small	0.05%	1.24%	0.32%	18,000	1	24
Medium	0.10%	1.48%	0.20%	18,000	5	588
Large	0.42%	1.61%	-0.03%	18,000	13	2,293
Extra Large	4.49%	0.61%	-1.20%	18,000	61	3,257

[a] Optimality gap of Gurobi solver within 5 hours of computation.
[b] The optimality gap of R&F w.r.t the best integer solution obtained by Gurobi.
[c] The optimality gap of F&O w.r.t the best integer solution obtained by Gurobi.

enclosed within each circle, determining its size, indicates the number of instances with the given fleet size and the number of selected CCs. Figure 3 demonstrates that for each network as fleet size decreases, there is a tendency for a greater number of instances recommending the establishment of a higher number of CCs. This trend is attributed to the diminished transportation capacity within the network resulting from lower fleet sizes, compelling the network to strategically position additional CCs spread across several regions to ensure more frequent shipments of returned items. Such strategically positioning, known as the decentralisation approach, highlights the impact of an optimised fleet size deployment in the network. This managerial insight effectively addresses the primary research question articulated in Section 1 regarding the determination of the (optimal) number of hybrid facilities to establish.

Figure 4 illustrates three key performance indicators related to transportation and inventory within the reverse flow channel. Figure 4a showcases both the quantity of transported returns and the utilisation rate of trucks, utilised to transport collected returns, across various network sizes and fleet sizes. The horizontal axis of this figure represents the network sizes and the fleet sizes (the former on the lower line and the latter on the upper line), the left-hand side vertical axis displays the average annual quantity of transported returned items (over all possible values of β_r and C_l introduced in subsection 5.1) within the CLSC network, and the right-hand side vertical axis indicates the average monthly truck utilisation rate (over all possible values of β_r and C_l) on each arc. As depicted in Figure 4a, our experimental results show that as fleet size increases, the quantity of transported returns from RTs to CCs is relatively stable, and the average quantity of transported returns from CCs to SFs rises for each network size. The former phenomenon is attributed to several factors, including the relatively low volume of collected returns at RTs, an ample number of available trucks on each transportation arc, and the comparatively higher inventory cost at RTs as opposed to CCs. The latter, however, is due to the observation derived from Figure 3, indicating that higher fleet sizes result in fewer established CCs. Consequently, with a reduced number of CCs, the model necessitates higher volumes of returns which are aggregated at each CC to be dispatched to SFs.

Concerning the average truck utilisation rates, it is notable that as the network size expands, there is a discernible decrease in the average truck utilisation rate from RTs to CCs. As explained in subsection 5.1, an increase in the network size corresponds to adding more retail points with smaller demand (returns) quantities to be moved to CCs. A higher demand granularity in a larger network size results in a reduction in the truck utilisation rate. However, for each network size, the average utilisation rate of trucks employed from RTs to CCs remains relatively constant as fleet size increases. We would like to highlight that the average utilisation rate of trucks utilised from RTs to CCs for all network sizes Small, Medium, Large, and Extra Large stands at 72%, 65%,



(a) Small-size network

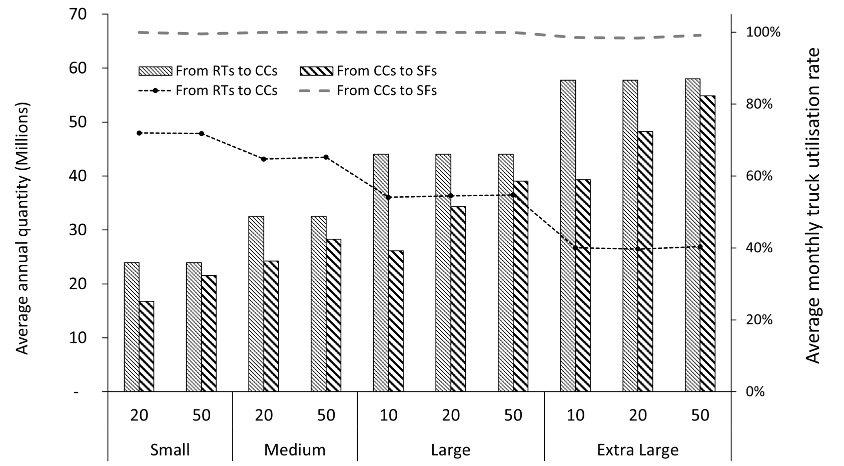
(b) Medium-size network

(c) Large-size network

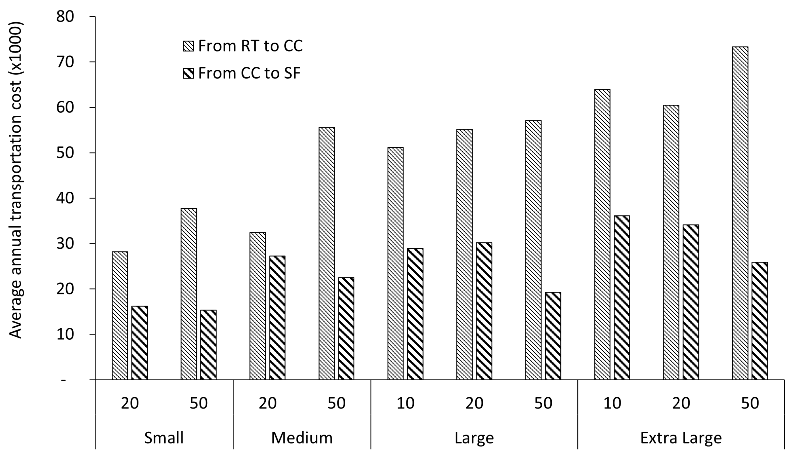
(d) Extra Large-size network

Figure 3: Number of established collection centre facilities.

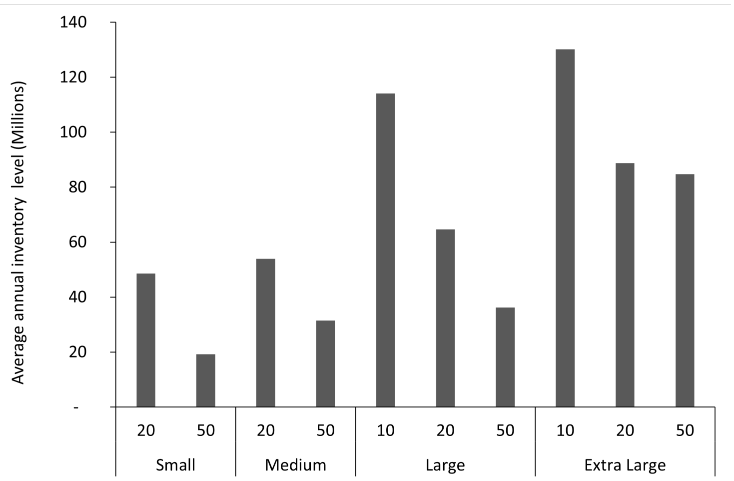
54%, and 40%, consistent with the Less-Than-Truckload (LTL) shipping policy. Hence, it is more economically advantageous to leverage LTL for transporting collected returns from RTs to CCs.



(a) Flow of returned items



(b) Transportation cost of returned items



(c) Inventory of returned items at CCs

Figure 4: Transportation- and inventory-related performance of the closed loop supply chain.

Upon consideration of trucks utilised for shipping returns from CCs to SFs, the Full-Truck-Load (FTL) policy (with an average utilisation rate of 99%) is employed due to the low number of SFs and established CCs in the network, along with the high volume of returned items to be dispatched from CCs. These findings directly contribute to answering the secondary research question, presented in Section 1, focusing on the efficient utilisation of truckload capacity for transporting returned items.

Average annual transportation cost of returns (over all possible values of β_r and C_l) within the reverse flow channel is depicted in Figure 4b, where the horizontal axis displays the network sizes and the fleet sizes (the former on the lower line and the latter on the upper line), and the vertical axis represents the average annual transportation cost of returned items. It is observed that, across various network sizes, the average transportation cost from RTs to CCs tends to increase with fleet size. This escalation arises from heightened transportation-related operations due to the greater number of available trucks. Moreover, within all network sizes, the average transportation cost from CCs to SFs typically decreases as fleet size grows. It becomes evident that, for a fixed fleet size, the average transportation cost escalates as network size expands, attributable to the changes in the network topology (network expansion). Furthermore, for any pair (network size, fleet size), the average transportation cost from RTs to CCs is consistently higher than the average transportation cost from CCs to SFs. This observation, coupled with the fact that in our study transportation cost is a function of distance, suggests that strategically positioning collection centres in close proximity to SFs is advisable. Consequently, CCs situated nearby SFs should be prioritised for further expansion to optimise costs. This observation pertains to the primary research question delineated in Section 1 regarding the selection of retailers within the supply chain network for expansion.

Figure 4c displays the average annual inventory level of returns at CCs (over all possible values of β_r and C_l). In this figure, the horizontal axis indicates the network sizes and the fleet sizes (the former on the lower line and the latter on the upper line), and the vertical axis represents the average annual inventory level of returned items at CCs. It is evident that for each network size, the average inventory of returns held at CCs declines as fleet size increases. This pattern is a direct consequence of the augmented transportation capacity within the system accompanying the increase in fleet size. Consequently, a higher volume of returned items is transferred from CCs to SFs, resulting in a diminished inventory level at CCs. It is worth noting that this pattern is aligned with increasing outgoing flow of returns from CCs when fleet size rises as discussed in the preceding paragraph. Furthermore, it is evident that for a constant fleet size, the average inventory level at CCs rises as network size expands. This trend is driven by the augmented number of RTs and the amplified volume of collected returns within the system as the network size enlarges. These insights effectively address the secondary research question regarding the optimal management of inventory of returned items at CCs.

The Box and Whisker Chart presented in Figure 5 illustrates the frequency distribution of the monthly average number of utilised trucks within the reverse flow channel across all generated instances. Specifically, it showcases the average number of used trucks from RTs to CCs and from CCs to SFs across all four network sizes. Each chart employs the five-number summary principle to portray the spread and skewness of data, delineating the minimum, first quartile, median, third quartile, and maximum number of utilised trucks (from bottom to top). Additionally, the average number of used trucks in each chart is denoted by the symbol $\times$. It is observed that the monthly average number of utilised trucks from RTs to CCs increases with the expansion of the network size. This observation is expected given the enlarged network size. Moreover, the variability of the data pertaining to transportation from RTs to CCs escalates as the network size expands. This increase is attributed to the growing number of established CCs and increasing variability in collected returns from RTs as the network size expands. The monthly average number of utilised trucks from CCs to SFs exhibits a similar trend, albeit with a reduced quantity of trucks used. This reduction is attributed to the transportation of aggregated returns from CCs to SFs, as discussed in Figure 4a.

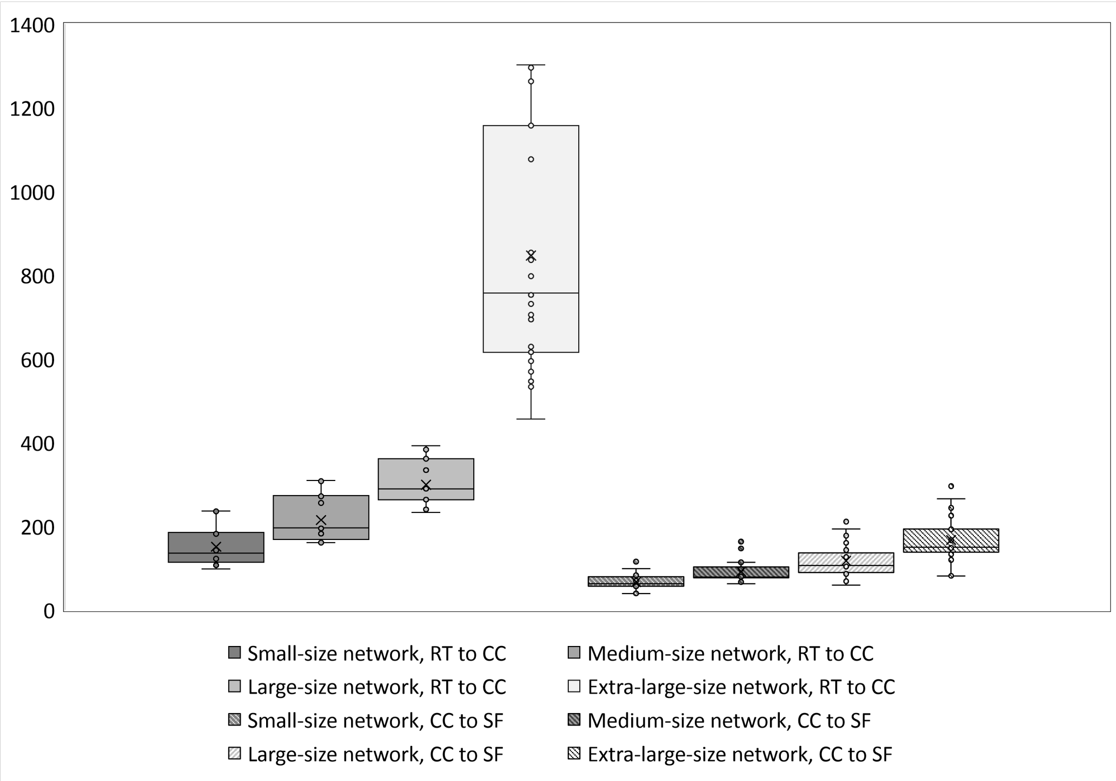
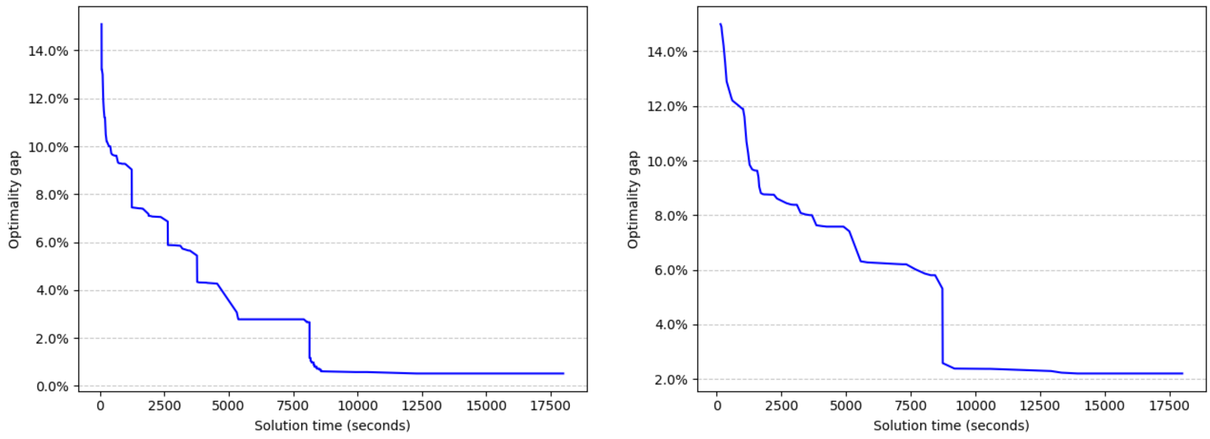


Figure 5: Number of trucks used in the reverse flow channel and its variability

Figure 6 illustrates the progression of the optimality gap reduction achieved by Gurobi within the 5-hour time limit for two representative instances: one corresponding to an instance with Large network size (Figure 6a) and another to an instance with Extra Large network size (Figure 6b). This visualisation highlights the complexity of the generated instances, Gurobi's convergence behaviour, and the increasing computational challenges associated with larger network sizes. Instances with Small and Medium network sizes exhibited similar optimality gap patterns to the Large instance presented, while other Large and Extra Large instances demonstrated trends consistent with those shown here. Consequently, these two instances were selected to provide concise yet representative insights into the solver's performance. The results emphasise the limitations of exact methods in solving large-scale instances within practical time constraints, further motivating the need for efficient heuristic approaches like the proposed Relax-and-Fix and Fix-and-Optimise algorithms developed in this study.

5.2.3. Impact of uncertain demand

In the following, we present the impact of demand uncertainty on the objective function value of the problem using different scenario realisations. To do so, we generate N individual sets of random



(a) Large-size instance

(b) Extra Large-size instance

Figure 6: Solver's optimality gap reduction.

demand values based on the expression 5.1 over the planning horizon $\mathcal{T}$, and, subsequently, solve the problem (P) for each generated demand set. It is important to note that $N \times \mathcal{T}$ demand scenarios are produced from the same probability distribution. Therefore, a relatively small number of sample sets is required to obtain good results when the demand process is stationary. We showcase $N = 4$ sets of random demand values for the Small size instances, and the problem (P) is solved using each generated demand set. Results reveal that the solution to location decisions is identical in all four demand sets for each instance. However, we observe that multi-period operational decisions and total costs vary depending on different demand sets. This result validates the qualification of solutions produced under different demand scenario sets used, given the variability associated with the operational solutions. Moreover, it is worth mentioning that the average of the N objective values represents an unbiased estimator for the lower bound of the problem (Shapiro et al., 2021).

Moreover, we assess the quality of the strategic locations of the hybrid facilities obtained by solving the deterministic problem (P) using a sampling method. To do so, the main problem is solved under a large number of random demand scenarios ($N' = 30$) while strategic (facility location) solutions are given. Let $\hat{\mathbf{X}}$ be a solution vector associated with the facility location variables in the model (P) and Ω be a set of scenarios based on which demand values are generated. Demand realisations, denoted by $d_{kt}(\omega)$, $\omega \in \Omega$, are assumed to follow a Uniform distribution: $\mathbf{U} \sim [l, u]$, which is generated using the Monte-Carlo procedure. We denote the obtained objective function value associated with the ω-th scenario, when $\hat{\mathbf{X}}$ is given, by $z(\hat{\mathbf{X}}, \omega)$.

We use the coefficient of variation indicator to assess the dispersion of the objective values across 30 scenarios for each instance. Results indicate that the coefficient of variation for each instance is less than 1%, confirming that the solution obtained by the deterministic model produces a robust enough solution to the problem under study. However, it is important to note that this indicator may vary if the deviation from the average (denoted by μ in 5.1) increases. The sampling method procedure is outlined in Algorithm 3.

Algorithm 3 Pseudocode of Mode Sampling method

procedure MONTE-CARLO METHOD(Ω, $\mathbf{U} \sim [l, u]$, $\hat{\mathbf{X}}$)
  for all $k \in \mathcal{K}, t \in \mathcal{T}$ **do**
    for all $\omega \in \Omega$ **do**
      $r \leftarrow [0, 1]$
      Generate $d_{kt}(\omega)$ using the inverse Uniform distribution $\mathbf{U}^{-1}(r)$
    end for
  end for
end procedure
procedure SAMPLING METHOD($\hat{\mathbf{X}}$)
  Recall $\hat{\mathbf{X}}$
  for all $\omega \in \Omega$ **do**
    Solve model (P) with demand scenarios $d_{kt}(\omega)$ and the given $\hat{\mathbf{X}}$
    Return $z(\hat{\mathbf{X}}, \omega)$
  end for
  Calculate the mean and standard deviation of $z(\hat{\mathbf{X}}, \omega)$
end procedure

5.2.4. Impact of integrated model

The following section illustrates the improvements achieved by the proposed integrated decision-making model compared to a sequential decision-making approach. In the sequential approach, operational decisions, such as lot-sizing, transportation, and production, are made independently of the location decisions. The former modelling approach considers long-term location decisions at the beginning of the planning horizon to address recurring future business operations. Once facility locations are determined, decisions regarding the inventory, transportation, and production in both forward and reverse flow channels are made on a periodic basis to satisfy demand and/or collect the returned items. It is clear that incorporating these operational decisions into the strategic level results in a cost-efficient outcome with quality facility location decisions (Sabri and Beamon, 2000). In contrast, the latter modelling approach focuses on a sequential decision-making process, where decisions are made independently. In this context, the process begins with the development of a facility location model. Based on the solutions derived from this model, subsequent problems related to transportation, production (lot-sizing), and inventory optimisation are addressed sequentially. It is important to highlight that each of these decisions is inherently linked to the facility location decision and ignoring this interdependence can lead to suboptimal solutions that fail to fully account for the integrated nature of the problem.

Inspired from Farahani and Hekmatfar (2009), we first develop the following location problem.

$$\text{(LOC)} \qquad \min \sum_{l \in \mathcal{L}} f_l X_l + \sum_{a \in \mathcal{A}^r} \sum_{t \in \mathcal{T}} e_a c^r r_{kt} W_a \tag{5.2}$$

$$W_a \leq X_l \qquad \forall a \in \mathcal{A}^r \tag{5.3}$$

$$W_a \geq 1 \qquad \forall a \in \mathcal{A}^r \tag{5.4}$$

$$\sum_{k \in \mathcal{K}} r_{kt} W_a \leq C_l X_l \qquad \forall a \in \mathcal{A}^r, t \in \mathcal{T} \tag{5.5}$$

where $W_a = 1$ if an allocation on arc $a \in \mathcal{A}^r$ is selected, 0 otherwise. Recall that $\mathcal{A}^r = \{\mathcal{A}_1^r \cup \mathcal{A}_2^r\}$, where $\mathcal{A}_1^r = \{(k,l)|k \in \mathcal{K}, l \in \mathcal{L}\}$ and $\mathcal{A}_2^r = \{(l,i)|l \in \mathcal{L}, i \in \mathcal{I}\}$. Once the problem (LOC) is solved, its solution is then fed into the problem (P), which evaluates the lot-sizing, transportation, and fleet optimisation problems combined. Let $\hat{X}_l^{LOC}$ be the solution to the location variable, i.e. X_l, in the problem (LOC) and $z_{\mathrm{P}}(\hat{X}_l^{LOC})$ be the Objective function value of model (P) when the solution of model (LOC) is used as input. Let z_{P}^* denote the objective value of the integrated model (P). We then can present the improvement (cost savings) obtained from the integrated model as follows: $\frac{(z_{\mathrm{P}}(\hat{X}_l^{LOC})-z_{\mathrm{P}}^*}{z_{\mathrm{P}}^*}$. We applied the sequential decision-making approach to the Extra Large-size instances and presented the resulting improvements for each instance in Figure 7, where the horizontal axis represents the fleet sizes and the truck capacities (the former on the upper line and the latter on the lower line) and the vertical axis displays the total cost. The results clearly demonstrate that the integrated model consistently outperforms the sequential model across all instances, achieving an average total cost improvement of approximately 9%. This highlights the superior efficiency and effectiveness of the integrated model.

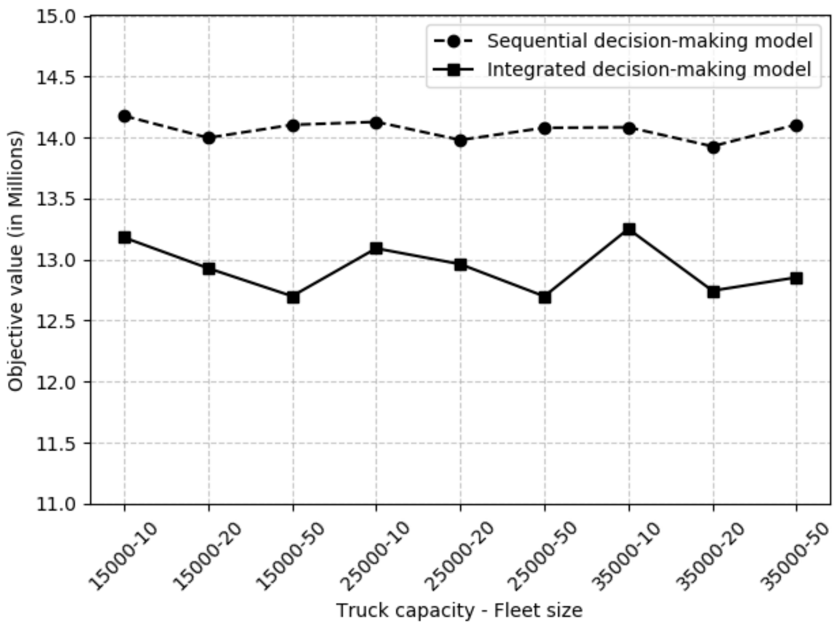


Figure 7: Cost saving of integrated decision-making vs sequential decision-making.

6. Conclusions and future work

This paper delves into the closure of an open-loop supply chain through an investigation of a multi-echelon multi-period single-product capacitated CLSC network design problem with hybrid retailers/collection centres. This problem is conceptualised as a joint capacitated facility location and production-transportation-inventory problem, formulated as a MILP model. The proposed model aims to determine optimal number and placement of CCs, timing and quantity of manufacturing and remanufacturing/recycling activities, inventory levels within the network, as well as the flow of products and returned items, alongside the fleet optimisation to fulfil customer demand while minimising the total network cost. Due to the complexity of the problem addressed in this paper and its computational challenges, we propose R&F and F&O heuristics to produce high-quality solutions in reasonable computation time. It is worth noting that tailored relaxation and fixing strategies are proposed for the R&F method and an interval search strategy is developed for the F&O method.

The mixed integer programming formulation, along with the aforementioned heuristics, are employed to solve instances of the CLSC network design problem with realistic geographical data, associated with a retail network in France, coupled with randomly generated demand and returns. Computational findings demonstrate that the R&F algorithm can efficiently solve instances within a few seconds, with an average deviation of merely 1.24% from the best feasible solution generated by Gurobi within a 5-hour timeframe. This achievement underscores the significant time-saving capability of the proposed R&F heuristic. Subsequently, the F&O algorithm is employed to refine the solution quality obtained from the R&F approach. Computational experiments reveal that the F&O heuristic produces near-optimal solutions within a reasonable time, with a negligible deviation from the best feasible solutions provided by Gurobi within the 5-hour time limit for Small- and Medium-size instances. Additionally, the F&O heuristic consistently outperforms Gurobi on average for Large- and Extra Large-size instances. It is important to note that the heuristics developed in this research can be effectively utilised to deal with MILP problems with similar structures.

From a managerial standpoint, insights gleaned from computational experiments suggest that establishing CCs in close proximity to SFs is more cost-effective. This recommendation, under the return-to-retail concept, guides the selection of retailers with the hybrid feature, yielding significant cost savings. Moreover, the case study indicates that the decentralisation approach should be deployed as fleet size decreases, offering managerial guidance on determining the optimal number of hybrid retailers/collection centres in the network. Additionally, the influence of fleet size and truck capacity utilisation on the network design solution is observed to be substantial, suggesting improved capacity utilisation with a smaller (larger) fleet size deployed towards (from) hybrid retailers/collection centres. Furthermore, employing the LTL policy for transporting returns from RTs to CCs and the FTL policy for shipping accumulated returns from CCs to SFs is economically beneficial.

Within the framework of the return-to-retail scheme, retailers possess the option to incentivise their customers, thereby motivating them to return used products to designated collection points. Consequently, it is worth investigating the incorporation of an incentive mechanism into the CLSC model outlined in this research paper. Moreover, there is a keen interest in investigating the CLSC network design problem under conditions of uncertain demand/return, as well as considering scenarios involving hybrid distribution/collection centres. These endeavours are earmarked for future exploration. Furthermore, future studies could explore extending the proposed framework by incorporating expenses associated with carbon emissions from transportation and facility operations. This would allow for a more comprehensive analysis of environmental and economic trade-offs in network design and decision-making. Future research could also explore decomposing the problem by network tiers, such as facilities, retailers, and other hierarchical levels. Such decomposition strategies could enhance computational performance, particularly for very large-scale instances, by breaking the problem into smaller, more manageable subproblems. This approach would enable tailored solution methods for each tier, potentially improving both efficiency and scalability.

CRediT authorship contribution statement

Amiri-Aref: Conceptualisation, Methodology, Investigation, Software, Validation, Writing.
Doostmohammadi: Conceptualisation, Methodology, Investigation, Writing - Original Draft.

Disclosure statement

The authors declare that they have no known competing financial interests or personal relationships regarding the publication of this paper.

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Appendix

Table A1: Results

Network size: Small											
Fleet size	Fleet capacity	CC capacity	Objective value GU	R&F	F&O	Optimality gap Gurobi	Gu-R&F	Gu-F&O	Solution time (seconds) Gu	R&F	F&O
20	15,000	Low	1,679,079	1,696,302	1,685,107	0.05%	1.03%	0.36%	18,000	2	34
		Medium	1,660,615	1,678,010	1,664,935	0.05%	1.05%	0.26%	18,000	2	43
		High	1,655,159	1,672,092	1,660,156	0.04%	1.02%	0.30%	18,000	1	18
	25,000	Low	1,632,421	1,648,350	1,637,608	0.05%	0.98%	0.32%	18,000	2	27
		Medium	1,632,296	1,648,532	1,638,051	0.05%	0.99%	0.35%	18,000	1	21
		High	1,632,078	1,652,134	1,639,855	0.05%	1.23%	0.48%	18,000	1	30
	35,000	Low	1,619,998	1,636,938	1,625,276	0.05%	1.05%	0.33%	18,000	1	22
		Medium	1,620,006	1,636,683	1,623,782	0.05%	1.03%	0.23%	18,000	2	23
		High	1,618,973	1,636,004	1,622,644	0.05%	1.05%	0.23%	18,000	1	24
50	15,000	Low	1,648,613	1,673,008	1,654,370	0.05%	1.48%	0.35%	18,000	1	22
		Medium	1,645,268	1,672,278	1,651,044	0.05%	1.64%	0.35%	18,000	1	20
		High	1,639,207	1,662,637	1,644,755	0.05%	1.43%	0.34%	18,000	1	18
	25,000	Low	1,614,527	1,636,766	1,619,648	0.06%	1.38%	0.32%	18,000	1	25
		Medium	1,614,449	1,636,752	1,619,986	0.05%	1.38%	0.34%	18,000	1	21
		High	1,614,426	1,636,752	1,618,042	0.05%	1.38%	0.22%	18,000	1	29
	35,000	Low	1,601,168	1,623,905	1,606,301	0.05%	1.42%	0.32%	18,000	1	20
		Medium	1,601,154	1,623,343	1,606,614	0.05%	1.39%	0.34%	18,000	1	14
		High	1,601,167	1,623,328	1,606,650	0.05%	1.38%	0.34%	18,000	1	18
Average			**1,629,478**	**1,649,656**	**1,634,712**	**0.05%**	**1.24%**	**0.32%**	**18,000**	**1**	**24**

Table A1. Results (*Cont.*)

Network size: Medium											
Fleet size	Fleet capacity	CC capacity	Objective value GU	R&F	F&O	Optimality gap Gurobi	Gu-R&F	Gu-F&O	Solution time (seconds) Gu	R&F	F&O
20	15,000	Low	2,395,859	2,425,218	2,400,286	0.34%	1.23%	0.18%	18,000	3	497
		Medium	2,367,487	2,395,552	2,370,278	0.10%	1.19%	0.12%	18,000	3	712
		High	2,352,555	2,381,099	2,356,492	0.08%	1.21%	0.17%	18,000	3	751
	25,000	Low	2,337,263	2,365,479	2,340,249	0.08%	1.21%	0.13%	18,000	3	297
		Medium	2,322,031	2,348,680	2,323,895	0.07%	1.15%	0.08%	18,000	4	636
		High	2,321,046	2,347,403	2,324,464	0.08%	1.14%	0.15%	18,000	4	324
	35,000	Low	2,303,914	2,330,726	2,310,512	0.08%	1.16%	0.29%	18,000	3	170
		Medium	2,303,946	2,330,726	2,306,641	0.08%	1.16%	0.12%	18,000	3	155
		High	2,304,032	2,330,529	2,310,607	0.08%	1.15%	0.29%	18,000	5	298
50	15,000	Low	2,353,396	2,393,260	2,355,949	0.09%	1.69%	0.11%	18,000	6	854
		Medium	2,349,734	2,389,266	2,352,730	0.08%	1.68%	0.13%	18,000	6	896
		High	2,343,239	2,382,806	2,346,081	0.09%	1.69%	0.12%	18,000	6	476
	25,000	Low	2,317,793	2,360,794	2,324,760	0.09%	1.86%	0.30%	18,000	7	579
		Medium	2,317,668	2,360,794	2,325,608	0.09%	1.86%	0.34%	18,000	7	1,231
		High	2,317,112	2,360,794	2,323,709	0.07%	1.89%	0.28%	18,000	6	1,464
	35,000	Low	2,302,249	2,344,387	2,307,744	0.08%	1.83%	0.24%	18,000	7	345
		Medium	2,302,459	2,343,218	2,307,777	0.09%	1.77%	0.23%	18,000	5	367
		High	2,302,506	2,343,233	2,310,895	0.09%	1.77%	0.36%	18,000	5	533
Average			**2,328,572**	**2,362,998**	**2,333,260**	**0.10%**	**1.48%**	**0.20%**	**18,000**	**5**	**588**

Table A1. Results (*Cont.*)

Network size: Large											
Fleet size	Fleet capacity	CC capacity	Objective value GU	R&F	F&O	Optimality gap Gurobi	Gu-R&F	Gu-F&O	Solution time (seconds) Gu	R&F	F&O
10	15,000	Low	3,063,710	3,090,435	3,066,299	0.25%	0.87%	0.08%	18,000	19	1,692
		Medium	3,035,765	3,064,473	3,038,442	0.27%	0.95%	0.09%	18,000	20	7,123
		High	3,019,161	3,040,325	3,016,390	0.40%	0.70%	-0.09%	18,000	23	3,219
	25,000	Low	3,014,680	3,026,044	3,000,631	0.79%	0.38%	-0.47%	18,000	22	1,495
		Medium	3,003,139	3,015,365	2,990,630	0.89%	0.41%	-0.42%	18,000	20	2,939
		High	2,988,449	3,006,811	2,984,869	0.61%	0.61%	-0.12%	18,000	16	1,900
	35,000	Low	2,988,516	2,994,139	2,968,982	1.14%	0.19%	-0.65%	18,000	21	1,380
		Medium	2,965,353	2,990,940	2,971,148	0.38%	0.86%	0.20%	18,000	11	1,083
		High	2,958,043	2,979,157	2,961,053	0.52%	0.71%	0.10%	18,000	14	2,791
20	15,000	Low	2,977,364	3,037,701	2,978,659	0.32%	2.03%	0.04%	18,000	11	1,304
		Medium	2,972,517	3,028,342	2,968,996	0.59%	1.88%	-0.12%	18,000	11	2,637
		High	2,951,783	3,010,172	2,956,152	0.34%	1.98%	0.15%	18,000	10	10,679
	25,000	Low	2,934,083	2,975,280	2,917,315	0.92%	1.40%	-0.57%	18,000	13	1,224
		Medium	2,910,587	2,969,588	2,912,659	0.20%	2.03%	0.07%	18,000	10	2,056
		High	2,909,076	2,967,523	2,911,968	0.21%	2.01%	0.10%	18,000	12	2,514
	35,000	Low	2,885,713	2,941,520	2,885,508	0.40%	1.93%	-0.01%	18,000	13	781
		Medium	2,886,377	2,941,520	2,885,847	0.37%	1.91%	-0.02%	18,000	11	1,173
		High	2,884,831	2,941,489	2,887,499	0.32%	1.96%	0.09%	18,000	11	2,302
50	15,000	Low	2,916,049	2,984,562	2,918,267	0.14%	2.35%	0.08%	18,000	14	2,208
		Medium	2,912,347	2,980,748	2,915,647	0.13%	2.35%	0.11%	18,000	12	2,331
		High	2,906,811	2,969,487	2,910,695	0.18%	2.16%	0.13%	18,000	13	2,203
	25,000	Low	2,872,075	2,939,868	2,872,343	0.26%	2.36%	0.01%	18,000	10	2,121
		Medium	2,871,215	2,939,161	2,874,113	0.24%	2.37%	0.10%	18,000	9	2,033
		High	2,875,199	2,939,161	2,874,736	0.32%	2.22%	-0.02%	18,000	9	1,071
	35,000	Low	2,856,446	2,919,790	2,858,740	0.42%	2.22%	0.08%	18,000	10	586
		Medium	2,855,932	2,919,856	2,857,904	0.45%	2.24%	0.07%	18,000	11	595
		High	2,855,049	2,919,789	2,858,859	0.20%	2.27%	0.13%	18,000	7	463
Average			**2,935,936**	**2,982,713**	**2,934,976**	**0.42%**	**1.61%**	**-0.03%**	**18,000**	**13**	**2,293**

Table A1. Results (*Cont.*)

Network size: Extra Large											
Fleet	Fleet	CC	Objective value			Optimality gap			Solution time (seconds)		
size	capacity	capacity	GU	R&F	F&O	Gurobi	Gu-R&F	Gu-F&O	Gu	R&F	F&O
10	15000	Low	4,433,725	4,372,464	4,346,879	2.79%	-1.38%	-1.96%	18,000	29	1,946
		Medium	4,393,475	4,349,237	4,300,813	2.79%	-1.01%	-2.11%	18,000	28	4,196
		High	4,353,222	4,437,752	4,435,999	2.83%	1.94%	1.90%	18,000	32	706
	25000	Low	4,372,349	4,355,052	4,356,289	3.35%	-0.40%	-0.37%	18,000	29	784
		Medium	4,374,202	4,392,105	4,364,379	3.75%	0.41%	-0.22%	18,000	31	861
		High	4,344,973	4,293,879	4,296,038	3.58%	-1.18%	-1.13%	18,000	30	1,417
	35000	Low	4,353,290	4,420,523	4,382,003	3.90%	1.54%	0.66%	18,000	31	1,121
		Medium	4,422,963	4,310,566	4,235,555	5.59%	-2.54%	-4.24%	18,000	28	6,556
		High	4,474,227	4,250,926	4,252,564	8.28%	-4.99%	-4.95%	18,000	27	2,756
20	15000	Low	4,261,865	4,359,597	4,251,174	2.16%	2.29%	-0.25%	18,000	116	9,905
		Medium	4,285,247	4,298,372	4,254,420	2.84%	0.31%	-0.72%	18,000	158	3,868
		High	4,381,858	4,530,936	4,475,401	7.13%	3.40%	2.13%	18,000	138	3,486
	25000	Low	4,340,642	4,288,347	4,236,715	6.22%	-1.20%	-2.39%	18,000	135	5,043
		Medium	4,335,083	4,227,070	4,176,159	5.31%	-2.49%	-3.67%	18,000	20	954
		High	4,286,415	4,436,442	4,383,446	4.36%	3.50%	2.26%	18,000	143	3,423
	35000	Low	4,247,031	4,230,912	4,170,067	3.96%	-0.38%	-1.81%	18,000	128	5,847
		Medium	4,235,217	4,308,204	4,235,053	3.82%	1.72%	0.00%	18,000	131	5,605
		High	4,262,301	4,159,447	4,161,130	4.47%	-2.41%	-2.37%	18,000	133	4,501
50	15000	Low	4,170,463	4,404,417	4,209,966	1.63%	5.61%	0.95%	18,000	35	6,501
		Medium	4,243,870	4,421,736	4,202,370	3.37%	4.19%	-0.98%	18,000	35	7,580
		High	4,285,535	4,308,510	4,296,831	4.32%	0.54%	0.26%	18,000	34	4,764
	25000	Low	4,151,537	4,357,777	4,204,329	2.31%	4.97%	1.27%	18,000	33	918
		Medium	4,283,866	4,343,310	4,189,111	6.39%	1.39%	-2.21%	18,000	26	915
		High	4,263,283	4,396,762	4,241,381	6.03%	3.13%	-0.51%	18,000	36	1,181
	35000	Low	4,305,091	4,274,513	4,111,640	7.28%	-0.71%	-4.49%	18,000	32	1,284
		Medium	4,280,244	4,279,191	4,115,460	6.37%	-0.02%	-3.85%	18,000	23	1,099
		High	4,268,086	4,274,686	4,113,162	6.46%	0.15%	-3.63%	18,000	19	717
Average			**4,311,484**	**4,336,397**	**4,259,198**	**4.49%**	**0.61%**	**-1.20%**	**18,000**	**61**	**3,257**