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Scalable analytic eigenvalue extraction from a parahermitian matrix

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ABSTRACT

In order to extract the analytic eigenvalues from a parahermitian matrix, the computational cost of the current state-of-the-art method grows factorially with the matrix dimension. Even though the approach offers benefits such as proven convergence, it has been found impractical to operate on matrices with a spatial dimension great than four. Evaluated in the discrete Fourier transform (DFT) domain, the computational bottleneck of this method is a maximum likelihood sequence (MLS) estimation, which probes a set of paths of likely associations across DFT bins, and only retains the best of these. In this paper, we investigate an algorithm that remains covered by the existing method's proof of convergence but results in a significant reduction in computation cost by trading the number of retained paths against the DFT length. We motivate this, and also introduce an enhanced initialisation point for the MLS estimation. We illustrate the benefits of scalable analytic extraction algorithm in a number of simulations.

Video to this article can be found online at https://doi.org/10.1016/j.sctalk.2025.100434.

Figures and tables

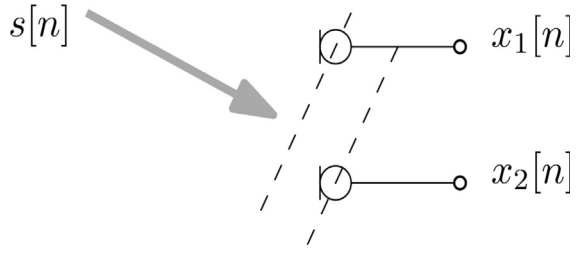

Fig. 1. Space-time covariance and analytic eigenvalue decomposition. For multichannel broadband array data $x[n] \in \mathbb{C}^M$, the space-time covariance matrix [1,2] retains directional information through the inclusion explicit lag component τvia $R[\tau] = E(x[n]x^H[n-\tau])$ with $E(.)$ the expectation operator [3]. Its z -transform $R(z)$ is a parahermitian matrix that satisfies $R^P(z) = (R(1/z^*))^H = R(z)$ and admits an analytic eigenvalue decomposition [4–7] $R(z) = Q(z)\Lambda(z)Q^P(z)$. The EVD factors consist of an analytic paraunitary matrix $Q(z)$ such that $Q^{-1}(z) = Q^P(z)$, and the diagonal matrix $\Lambda(z) = diag(\lambda_1(z), \ldots, \lambda_M(z))$ holding the analytic eigenvalues $\lambda_m(z)$, $m = 1, \ldots, M$. An algorithm to compute the analytic EVD with proven convergence has been suggested in [8,9], but the eigenvalue extraction scales poorly with the spatial dimension M.

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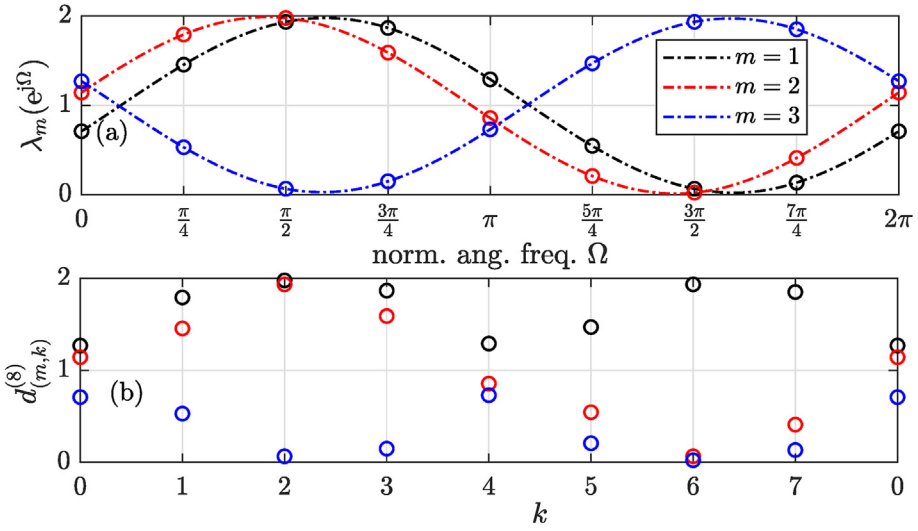

Fig. 2. Example for analytic vs bin-wise eigenvalues. In (a), three analytic eigenvalues are shown with an evaluation in $K = 8$ sample points. In (b), a bin-wise EVD yields three ordered eigenvalues in every bin; however, through evaluating a DFT, the spectral coherence is lost, and the association of sample points across bins is no longer clear. If the association is made w.r.t. spectrally majorised sample points, then spectrally majorised eigenvalues as obtained by time-domain poplynomial EVD algorithm such as in [2,10,11] result. These eigenvalues will converge towards piece-wise analytic functions which can only be approximated by polynomial factor of significantly higher order than may be necessary for the analytic counterparts.

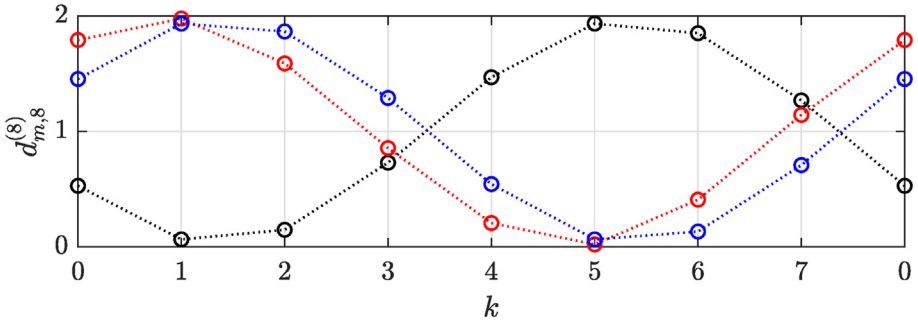

Fig. 3. Extraction of correct association across bins. The analytic eigenvalue extraction approach in [8] works iteratively. Starting with bin index $k = 0$, and retaining possible paths for associations of bins, in its kth iteration, it checks all possible new combinations of including the eigenvalues in the kth bin. The combinations are checked whether they allow a sufficiently smooth interpolation by measuring the power in the pth derivative of a Dirichlet interpolation [8,12] through the sample points in a path. Only a number of N_p best paths are retained. The proposed approach [13] applies two modifications: (I) by increasing the DFT length K, the eigenvalues behave smoother from bin to bin, and it is possible to reduce the number of retained paths; (II) the iteration starts at bin where the eigenvalues maximally separate; thus, w.r.t. Fig. 2, the bins are circularly left-shifted by one position.

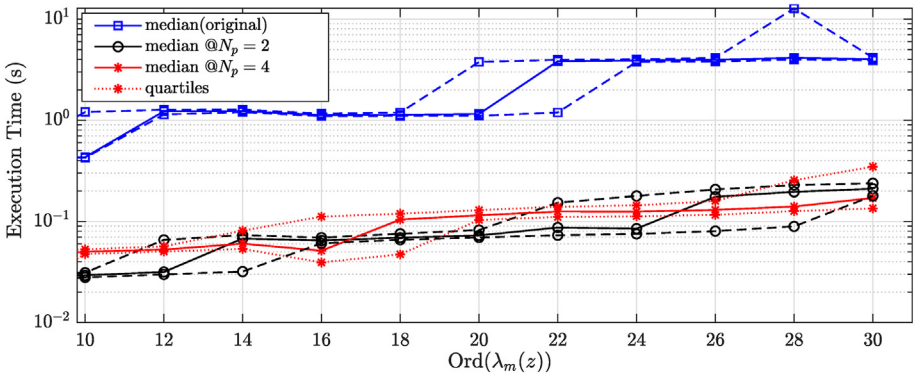

Fig. 4. Ensemble results for the comparison of the proposed approach with [8] for $M = 4$. The proposed approach and the method in [8] are applied to an ensemble of parahermitian matrices of spatial dimension $M = 4$, and over various order to the ground truth eigenvalues. In all cases, both methods are able to extract the correct eigenvalues close to machine precision, but differ on the computational complexity that this takes. For the method in [8], the results are plotted blue — the cost over the order of the eigenvalues shows granularity, with a jump in complexity whenever the DFT length is increase by a power of two. The main cost of the algorithm in [8] is due to the MLS estimation. For the proposed approach, we show two different trade-offs, with only $N_p = 2$ or $N_p = 4$ paths retained. With fewer paths, the algorithm will potentially require a slightly higher DFT length.

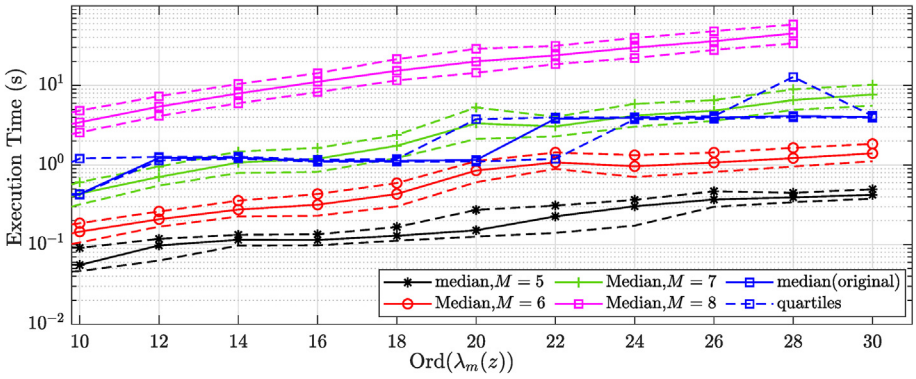

Fig. 5. Ensemble results for scalability. For reference, the figure retains the ensemble results for [8] with $M = 4$, but also compares the cost if the proposed methods for might spatial dimensions. For $M = 7$, the complexity is still comparable to the benchmark of [8] for only $M = 4$. Only when proceeding to $M = 8$ will the complexity of the proposed approach [13] exceed that of the original algorithm in [8].

CRediT authorship contribution statement

Faizan A. Khattak: Conceptualization, Formal analysis, Software, Writing – original draft, Writing – review & editing. **Ian K. Proudler:** Conceptualization, Formal analysis, Writing – review & editing. **Stephan Weiss:** Conceptualization, Formal analysis, Writing – review & editing.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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