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Establishing a Relationship Between Particle Size Distribution & Thaw Weakening Susceptibility in Soils

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Abstract

Susceptibility to frost action in soils is an important consideration in cold and seasonably cold regions. Whilst the particle size distribution (PSD) is commonly used to measure susceptibility to frost action, this method is typically seen as unreliable. Moreover, susceptibility criteria are generally specific to frost heave despite heave and thaw weakening both being described by the term frost action. Therefore, the effect of PSD on thaw weakening has not been fully explored. This study sets out to establish a relationship between PSD and thaw weakening susceptibility considering a better set of PSD descriptors than those traditionally used (e.g. d_{50}, C_u and C_c) towards the entire PSD. By examining available experimental data in the literature, in particular studies used to establish existing thaw weakening susceptibility criteria and testing set out in ASTM D5918-13, it was found that PSD influences thaw weakening susceptibility. PSDs located to the right of the stability line in the normalised entropy diagram (used to characterise PSDs) were found to be largely non-susceptible, whereas finer PSDs to the left the stability line were found to be highly susceptible to thaw weakening. It was also found that PSD greatly affects changes in bearing capacity after thawing. The analyses presented here demonstrate that the grading entropy stability criteria has significant potential as a method of predicting thaw weakening susceptibility.

1. Introduction

Frost action describes two behaviours: frost heave and thaw weakening. Both are associated with damage to infrastructure. Therefore, determining whether a soil will experience frost action is important in cold regions due to instability in transportation infrastructure (e.g. Konrad, 1989, Lee et al., 1995; Chamberlain, 1989; Zhang and Shijie, 2001; Hazirbaba and Gullu, 2010). The simplest method of assessing frost action susceptibility (FAS) is based on the particle size distribution (PSD). However, more recent studies have found PSD is not an ideal means of FAS assessment (e.g. Dagli 2017; Sheng, 2021; Hao et al. 2023). There are a number of suggestions for this. For example, plasticity in finer soils is known to be important regarding the formation of ice lenses which the PSD cannot quantify (Chamberlain, 1981). Loranger, et al (2022) also discuss the significance of mineralogy (of the fines fractions, in particular) on frost susceptibility of soils. Moreover, many FAS criteria based on PSD are dependent on the uniformity coefficient (C_u), which excludes fines and gravel content. Fines in particular have been shown to be important in the development of ice lenses (e.g. Ćwiąkała et al. 2016; Niggemann and Fuentes, 2023). Also, susceptibility criteria are generally specific to frost heave rather than thaw weakening. Therefore, the effects of thaw weakening are underrepresented/neglected in many FAS criteria. Chamberlain (1981) drew attention to this

> *"Both seem to be major indicators of frost-susceptible soils. However, for decades there has been an almost universal tendency to define frost susceptibility in terms of frost heaving alone, i.e. a frost-susceptible soil was one which heaved when frozen." - Chamberlain (1981)*

Whilst some criteria include thaw weakening tests in conjunction with PSD analysis (e.g. U.S Army Corp frost tests), PSD in these criteria is only relevant to the determination of frost heave susceptibility and not explicitly related to thaw weakening. Recently, Gülen et al (2024) suggested a link between increased thaw weakening and the average grain size (d_{50}). However, d_{50}, and C_u, are insufficient parameters to adequately describe the PSD. This study re-investigates experimental data readily used to establish current thaw weakening susceptibility criteria to understand the effects of PSD on thaw weakening susceptibility. Grading entropy coordinates are used to characterise the entire PSD and relate thaw weakening behaviours to the internal stability criteria proposed by Lörincz (1986). In summary, it is currently accepted that frost heave is affected by PSD, and FAS criteria are predominately related to frost heave which are generally unlinked from thaw weakening effects when determining frost action susceptibility. The present study postulates that both frost heave and thaw weakening may be linked, and that analysis of the entire PSD curve will provide better assessment of frost action. Note however that the current study sets out to establish a relationship between PSD and thaw weakening susceptibility using the grading entropy stability criteria so that future works may assess these two behaviours under one unified criteria.

A fundamental disadvantage of traditional PSD descriptors such as C_u, C_c, d_{50}, etc. is that they are defined in terms of individual diameters (such as d_{10}, d_{30}, d_{60}). Hence by definition, these descriptors ignore the influence of fines and gravel content on PSD quantification. The use of grading entropy coordinates to describe PSD has become more widely accepted as it provides a more complete quantification of the PSD considering all size ranges (e.g. Imre et al., 2012). This is the first attempt to relate thaw weaking to PSD by means of descriptors that account for the entire PSD. It is shown by using existing experimental data used to establish the criteria set out in ASTM D5918-13, that in fact these two phenomena (i.e. thaw and heave) related to frost susceptibility are linked and can be successfully assessed when considering grading entropy coordinates.

2. Frost action

Frost susceptible soils undergoing freezing are subject to significant changes that influence engineering properties. When exposed to freezing temperatures, pore water in soils freezes, creating ice lenses. These temperatures enable cryogenic-suction which attracts ground water from surrounding areas towards the freezing front - feeding the growth of ice lenses (Blanchard and Fremond, 1982). Therefore, frozen soils often contain more water after the formation of ice, which contribute to excess pore pressures after thaw (Chamberlain, 1981; 1986; Simonsen et al. 2002). It has been discussed by Chamberlain & Gow (1979) that significant changes to the internal structure of soils takes place at the location of ice lenses, altering soil stability, as has been seen experimentally with SEM by Chen, et al (2024). Furthermore, due to expansion induced via freezing; soils undergo notable changes in density. As expected, loose soils tend to densify, whereas soils with higher density loosen according to the classical critical state soil mechanics framework. In, fact, Qi et al (2008) suggested that a critical dry density (γ_{dry}) exists corresponding to the minimum value of thawing settlement. When the original γ_{dry} is at critical, soil density remains unchanged after freeze-thaw (FT). If the original γ_{dry} is larger than critical, then density decreases. Whereas if the original γ_{dry} is less than critical, density increases after FT. Changes in density alter the engineering properties and induce damaging behaviours like thaw induced settlements (Xie et al., 2015; Liu et al., 2016).

Frost susceptible soils are typically fine-grained (tills, silts, and clays). When exposed to freezing temperatures ice crystals form within the void space of finer particles and can cumulatively create ice lenses in larger voids (e.g. Ćwiąkała et al. 2016). These soils are considered frost susceptible due their mineralogy (Bilodeau et al., 2008), capillary action (Dagli, 2017), and permeability (Chamberlain & Gow, 1979). Capillary action, especially in clays and tills, enable water to migrate towards the freezing front, even in relatively impermeable soils. However, during thaw, capillary action (and changes in soil structure) enables the retention of water against gravity (Rufaidah and Bashir, 2022), the separation of particles/platelets, and excessive swelling in clays (Chamberlain, 1989). These behaviours create instability within susceptible soils which result in excess pore water pressures after thaw, soil collapse, and excessive settlements - which collectively contribute to losses in soil bearing capacity. Hence, a means to predict these behaviours is crucial for cold regions infrastructure. In summary, whilst thaw weakening and frost susceptibility are affected by other factors in addition to PSD, arguably, these additional factors are also related to PSD (e.g. density, permeability, etc.). As a result, there is significant potential on defining criteria that account for accurate (and complete) PSD descriptors.

3. Thaw weakening susceptibility criteria

Jessberger and Carbee (1970) first investigated the effect of thaw weakening on bearing capacity on the basis of California Bearing Ratio (CBR) tests with values taken before and after freeze thaw (FT). This testing procedure, and its later iterations, are now common procedures to measure susceptibility to thaw weakening. The use of the CBR is beneficial as it is a well-established procedure in transportation infrastructure maintenance and design, which are amongst the most commonly affected by frost action (e.g. ASTM5918-13). Hence their work is of note and their data will be investigated in this study. Notably, Chamberlain (1986) developed an improved and updated testing procedure to measure frost heave and thaw weakening susceptibility. The data from these seminal sources together with more recent data by Gülen et al (2024) are used in this study to assess the suitability of a new PSD criterion. For comparison purposes, the thaw weakening susceptibility criteria set out in ASTM D5918-13 is adopted in this study to quantify the effect of thaw weakening, as shown in Table 1:

Table 1. Thaw weakening susceptibility criteria according to the ASTM D5918-13:

Susceptibility	**CBR Ranges (%)**
Negligible	> 20
Very low	20 – 15
Low	15 – 10
Medium	10 – 5
High	5 – 2
Very High	< 2

4. Particle size distributions & CBR data

All PSD curves used in this work are shown in Figure 1. Table 2 shows the CBR data for all soils investigated with their corresponding citation:

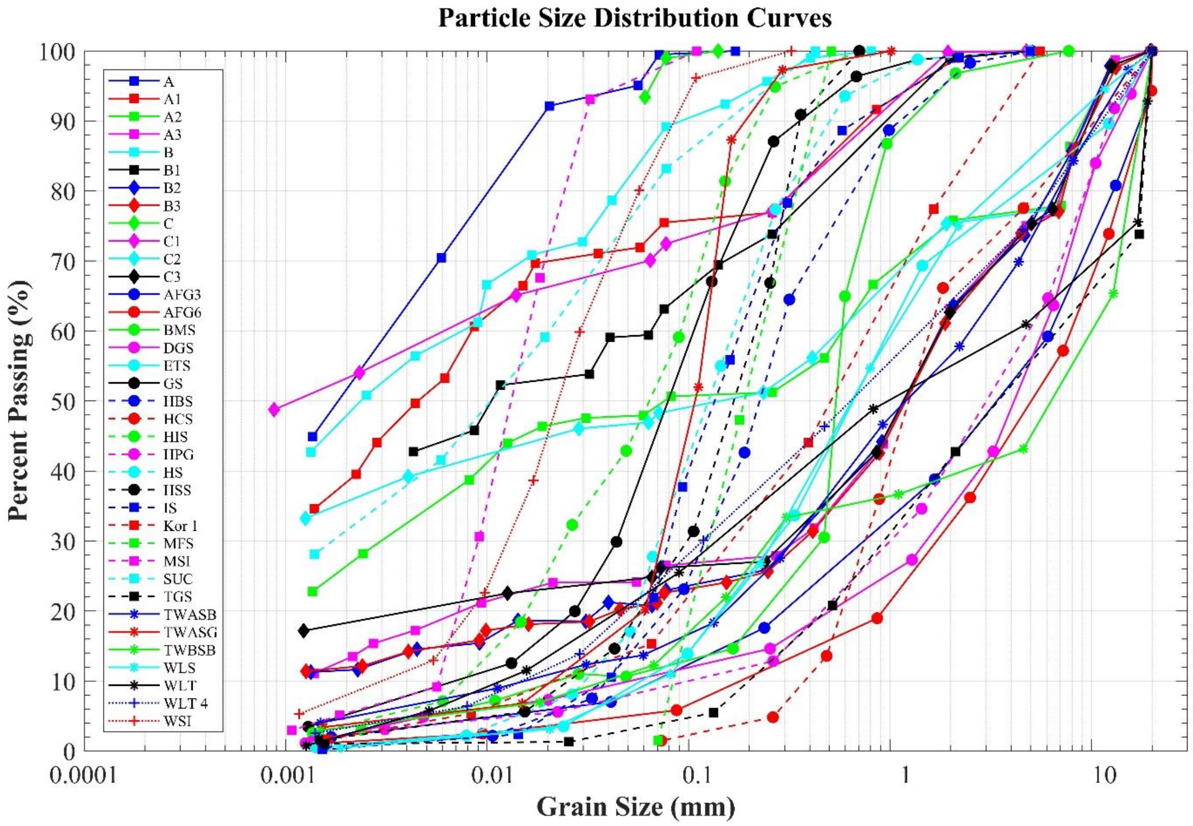


Figure 1. Particle size distribution curves from Jessberger & Carbee (1970, Chamberlain (1986), and Gülen et al (2024).

Table 2. CBR properties of soils before and after thaw.

Soil ID	Non-frozen CBR (%)	Thawed CBR (%)	Change in CBR (%)	Reference
MSI	50.5	0.63	49.87	Jessberger & Carbee (1970)
HPG	97.2	31.7	65.5	Jessberger & Carbee (1970)
HCS	51.1	38.8	12.3	Jessberger & Carbee (1970)
HIS	37.8	7.3	30.5	Jessberger & Carbee (1970)
WLT	141.1	7.3	133.8	Jessberger & Carbee (1970)
AFG-3	74.2	66.7	7.5	Jessberger & Carbee (1970)
AFG-6	78.1	69.7	8.4	Jessberger & Carbee (1970)
MFS	32.4	23.5	8.9	Jessberger & Carbee (1970)
Kor-1	66.2	8.2	58	Jessberger & Carbee (1970)
HIS	70	0.8	69.2	Jessberger & Carbee (1970)
WSI	48.7	0.7	48	Jessberger & Carbee (1970)
SUC	38.7	9	29.7	Jessberger & Carbee (1970)
WLT-4	133.3	2.45	130.85	Jessberger & Carbee (1970)
BMS	37.8	26.1	11.7	Jessberger & Carbee (1970)
TCS	97	73.8	23.2	Jessberger & Carbee (1970)
ETS	97	11.7	85.3	Jessberger & Carbee (1970)
WSL	74	45.7	28.3	Jessberger & Carbee (1970)

Dense Graded Stone (DGS)	10	9	1	Chamberlain (1986)
Graves Sand (GS)	5	2	3	Chamberlain (1986)
Hart Bros Sand (BBS)	8	4	4	Chamberlain (1986)
Hyannis sand (HS)	7	6	1	Chamberlain (1986)
Ikalanian sand (IS)	8	2	6	Chamberlain (1986)
Taxiway A subbase (TWASB)	24	12	12	Chamberlain (1986)
Taxiway A subgrade (TWASG)	15	12	3	Chamberlain (1986)
Taxiway B subbase (TWBSB)	24	12	12	Chamberlain (1986)
Taxiway B subgrade (TWBSG)	36	20	16	Chamberlain (1986)
A	2.9	1.1	1.8	Gülen et al (2024)
A1	3.8	1.3	2.5	Gülen et al (2024)
A2	4.6	2.2	2.4	Gülen et al (2024)
A3	7.1	4.3	2.8	Gülen et al (2024)
B	2.8	1	1.8	Gülen et al (2024)
B1	3.5	1.4	2.1	Gülen et al (2024)
B2	4.5	1.9	2.6	Gülen et al (2024)
B3	6.7	3.4	3.3	Gülen et al (2024)
C	2.6	0.8	1.8	Gülen et al (2024)
C1	3.1	1.3	1.8	Gülen et al (2024)
C2	4.2	1.7	2.5	Gülen et al (2024)
C3	6	3	3	Gülen et al (2024)

5. Grading entropy coordinates

Proposed by Lörincz (1986), grading entropy coordinates are able to represent any PSD by a single point on Cartesian plane. The grading entropy concept has been used to understand other geotechnical phenomena, and hence only a brief description is provided here. However, a key incentive for using grading entropy coordinates is the ability to quantify the entire PSD beyond the traditional nominal particle diameters (d_{60}, d_{50}, and d_{30}, and d_{10}) which are used in traditional PSD descriptors (such d_{50}, as C_u and C_c). The total entropy (S) of any grading curve may be split into two components which describe the entire PSD, and form the coordinate pair:

$$S = \Delta S + S_0 \tag{1}$$

Where, *ΔS* is the entropy increment, and S_0 is the base entropy. The entropy increment (*ΔS*), normally on the y-axis, is defined as:

$$\Delta S = -\frac{1}{\ln{(2)}}\sum_{i=1}^{N} x_i ln\, x_i \tag{2}$$

Where *N* is the number of the fractions in the PSD, x_i is the relative frequency which in essence is the probability of a given configuration occurring over any ith fraction, where:

$$\sum_{i=i_1}^{i_N} x_i = 1, \quad x_i \geq 0 \tag{3}$$

The base entropy (S_0), normally on the x-axis, is given by:

$$S_0 = \sum_{i=1}^{n} x_i S_{oi} \tag{4}$$

S_{oi} is known as the intrinsic entropy, which is an integer which increases relative to standard sieve sizes and can be seen in Table 3. For the calculation of the grading entropy coordinates, the sieve mesh diameters increase by a magnitude of 2 (i.e. 0.25, 0.5, 1, 2, 4, 8 mm). The fractions are numbered by increasing integers, as seen in Table 3, where 'd' is the mesh diameter and S_0 the base entropy:

Table 3. Fractions, their numbering and equivalent eigen-entropies, S_{oi}.

Fraction	0	…	22	23	24
d (mm)	2^{-22} - 2^{-21}	…	1 - 2	2 - 4	4 - 8
S_{0i}	0	…	22	23	24

In Table 3, the limiting diameter values for the i^{th} fraction are given in terms of d_{min} as follows:

$$2^{i+1} d_{min} \geq d > 2^i d_{min} \tag{5}$$

In this study, normalised versions of these coordinates are used to relate thaw weakening to the internal stability criteria proposed by Lörincz (1986):

$$A = \frac{S_0 - S_{0min}}{S_{0max} - S_{0min}} \tag{6}$$

$$B = \frac{\Delta S}{\ln (N)} \tag{7}$$

where A is the normalised base entropy and B is the normalised entropy increment, forming the Cartesian coordinate pair (A, B). Note that by definition, grading entropy coordinates consider all particle size ranges for any PSD. Traditional PSD descriptor such as C_c and C_u only account for particle diameters between d_{10} and d_{60}, therefore fines and gravel content are explicitly ignored. Entropy coordinates have simple physical meanings, A (or S_0) are logarithmic means of the average grain diameter and relate to the skewness of the PSD. B (or ΔS) are a measure of how much a PSD is influenced by all its fractions and relates to the kurtosis of the PSD, i.e. a measure of dispersity. However, these measures of skewness/kurtosis include all size ranges.

5.1. Stability criteria

Lörincz (1986) established a stability criteria with the normalised coordinates. Based on a comprehensive set of laboratory suffusion/filtration tests, he found that for A<0.667 soils become internally unstable. Whereas for A=0.667 and A>0.667, soils may be considered transitionally and internally stable, respectively. Figure 2 shows a normalised entropy diagram illustrating such a concept/criterion:

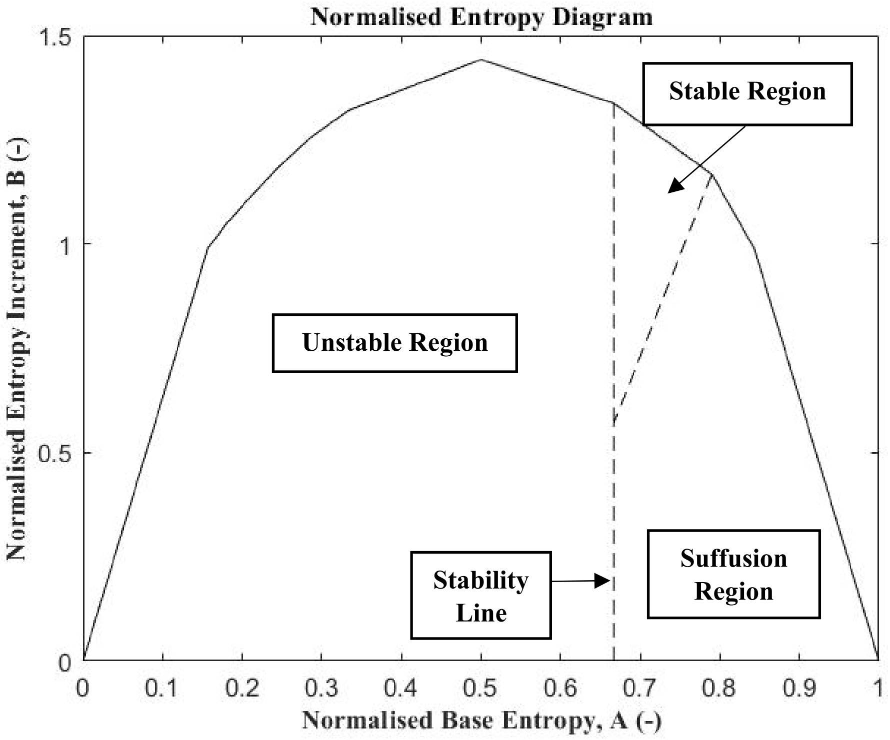


Figure 2. Normalised entropy diagram demonstrating the regions of soil internal stability, as proposed by Lörincz (1986).

As a PSD moves from A=0.667 in the direction of decreasing grain size (decreasing A-coordinates) towards the unstable region, coarser-grains become separated and may float in a matrix of finer particles. The transition from a coarse-grained soil skeleton to a fine-grained matrix is suggested to take place at the stability line (Nagy et al. 2012; Leak, 2024). A>0.667 soils that are coarse grain dominated and are usually internally stable. Recent numerical research using DEM simulations have also explored the relationship between A, B, and the role of finer fractions acting as perpendicular support to strong force network that underlie this observed macro-scale stability (Leak, 2024). For a detailed definition of grading entropy, the reader is referred to Imre et al. (2012).

It is important to understand Lörincz' stability criterion in terms of physical mechanisms at the particle level. A stable material is one in which there is a strong network of inter-particle contacts (i.e. strong force chains) which are well supported by an orthogonal network of weak(er) inter-particle contacts. Such a model has been used by many researchers to understand evolution of soil's strength and stiffness (e.g. Tordesillas et al 2010; Barreto & O'Sullivan; 2012). In the unstable or suffusion region, contact networks can be more easily disrupted. Fine particles can be easily removed through pore constrictions as observed in internal erosion phenomena. The existence and strength of these networks of inter-particles contacts also govern the behaviour under frost heave and thaw weakening. Soils with unstable contact networks may easily disrupted during FT cycles, hence producing thaw weakening. Similarly, frost susceptibility has been associated to permeability, water content and capillarity effects, all of which being known to be related to PSD properties. It is therefore not surprising that as shown in the next section, a clear relationship between thaw weakening and (accurate) PSD descriptors can be made.

6. Results

6.1. Normalised grading entropy diagram

All PSDs shown in Figure 1 are illustrated on the normalised entropy diagram in Figure 3, where each soil is classified by their thaw weakening susceptibility according to the ASTM D5918-13 standard.

The key benefit of using the normalised entropy diagram is that it enables to explain susceptibility to thaw weakening (and other soil behaviours) within the same framework established by Lörincz (1986). This framework has been validated on the basis of a comprehensive set of laboratory suffusion tests by Lörincz (1986) as well as by additional in-situ and laboratory tests by many other researchers (e.g. Barreto et al 2019; Nagy et al 2012; Feng et al 2019; Leak 2024):

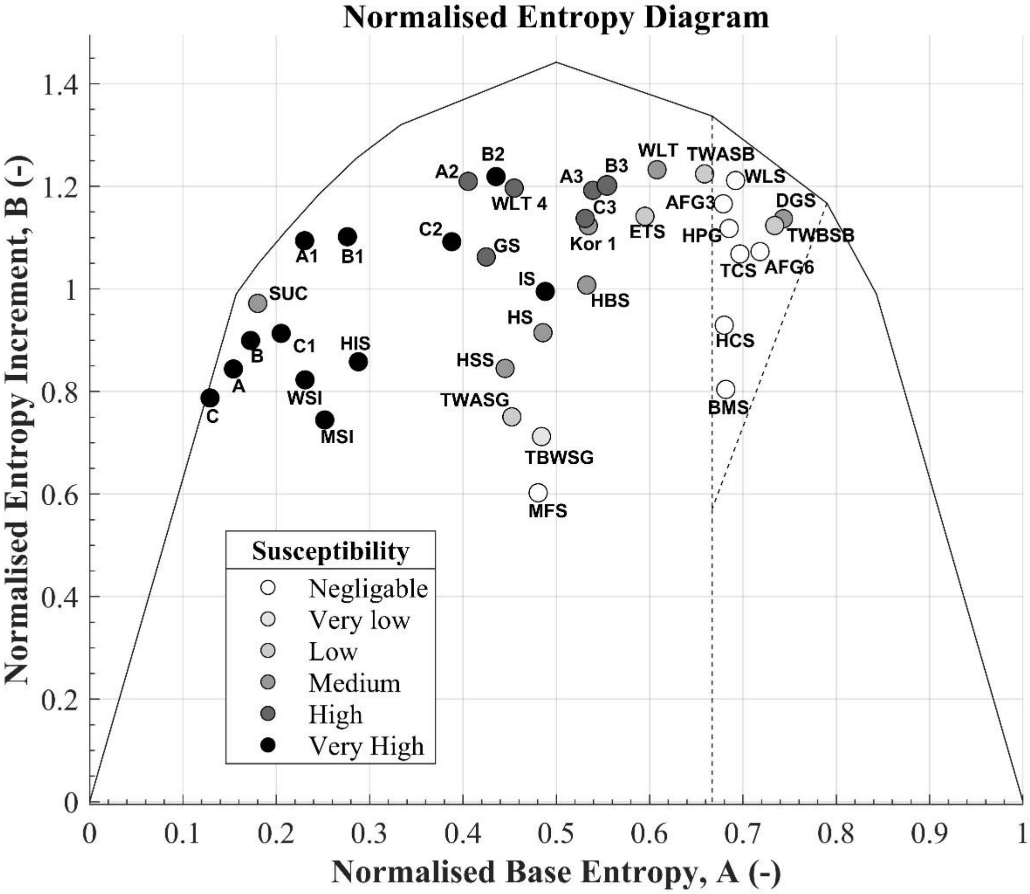


Figure 3. All PSDs investigated in this work characterised by the criteria set out by ASTM D5918-13 displayed on normalised entropy diagram.

The stability criteria shows good agreement with the criteria set out by ASTM D5918-13 and appears well suited to identifying susceptibility to thaw weakening. Soils classified as Very High, High, and Medium susceptibility are located to the left of the stability line in the unstable region (A<0.667), with the exception of **DGS** (Medium). Soils with Negligible susceptibility to thaw weakening are found primarily in the stable region (with exception of soil MFS i.e. Negligible). It is unclear why these discrepancies exist, but in the case of MFS, Table 2 shows a thawed CBR value of 23.5%, which is the weakest of soils amongst the same (Negligible) category. As mentioned previously soil parameters such as particle minerology are known to affect frost action. Whilst further research is needed to fully describe the discrepancies exhibited by soils **MFS**, and **TWBSB**, for soil **DGS** the following description is provided by Chamberlain (1981):

"The parent rock material is a very hard, shale-like formation that breaks up in a plate-like manner when crushed. The fines are non-plastic."

Stone itself is known to be a frost susceptible material due to its porous structure. Moreover, it may be speculated that the broken shale rock fines may contribute to the instability, particularly as the soil underwent loading in the CBR, which might have induced some particle breakage during testing. This in turn, may have contributed to its classification as High despite being located in the stable zone. In other words, a changing grain size during testing may have contributed to this discrepancy and the initial PSD used to calculate the entropy coordinates may not be same after testing, hence its location on the entropy diagram.

From Figure 3, it is also observed that as grain size decreases, thaw weakening susceptibility increases. Finer particles are well established as being frost susceptible, hence agreement with increased susceptibility in soils with A<0.667 appears reasonable given that the stability line represents a transition from coarse to fine dominated soils (Nagy, 2012). The susceptibility for a soil to present unstable behaviours following thaw may therefore be characterised by grading entropy coordinates and the stability criteria.

6.2. Compaction behaviours

Susceptibility to thaw weakening may be better understood by studying basic soil properties, particularly those important in construction. Figure 4 shows the dry unit weight (γ_{dry}) and optimum water content (w_{opt}) for all soils considered in Table 2. The same data shown in Figure 3 is plotted in terms of compaction test data:

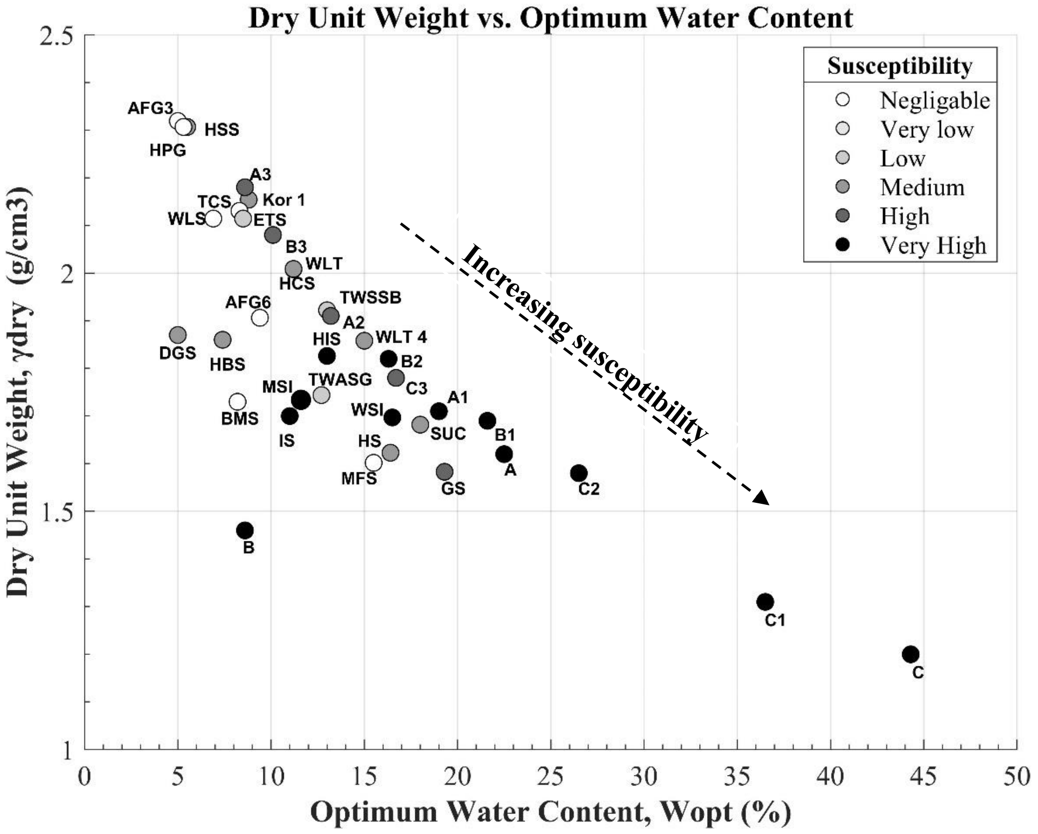
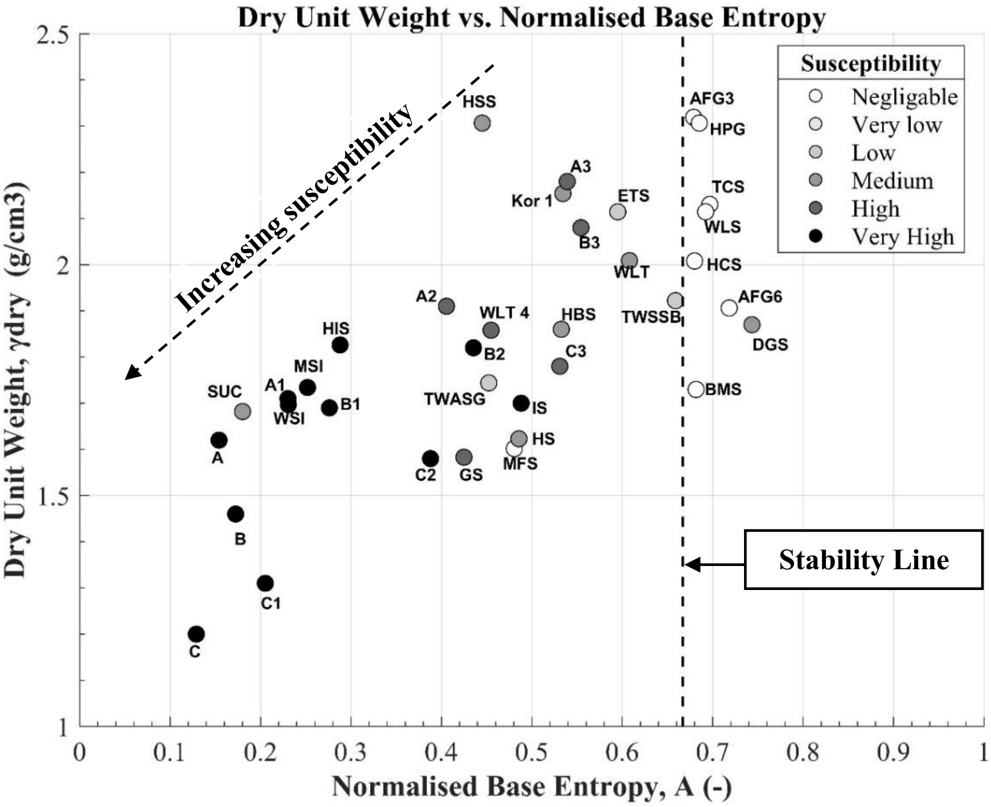
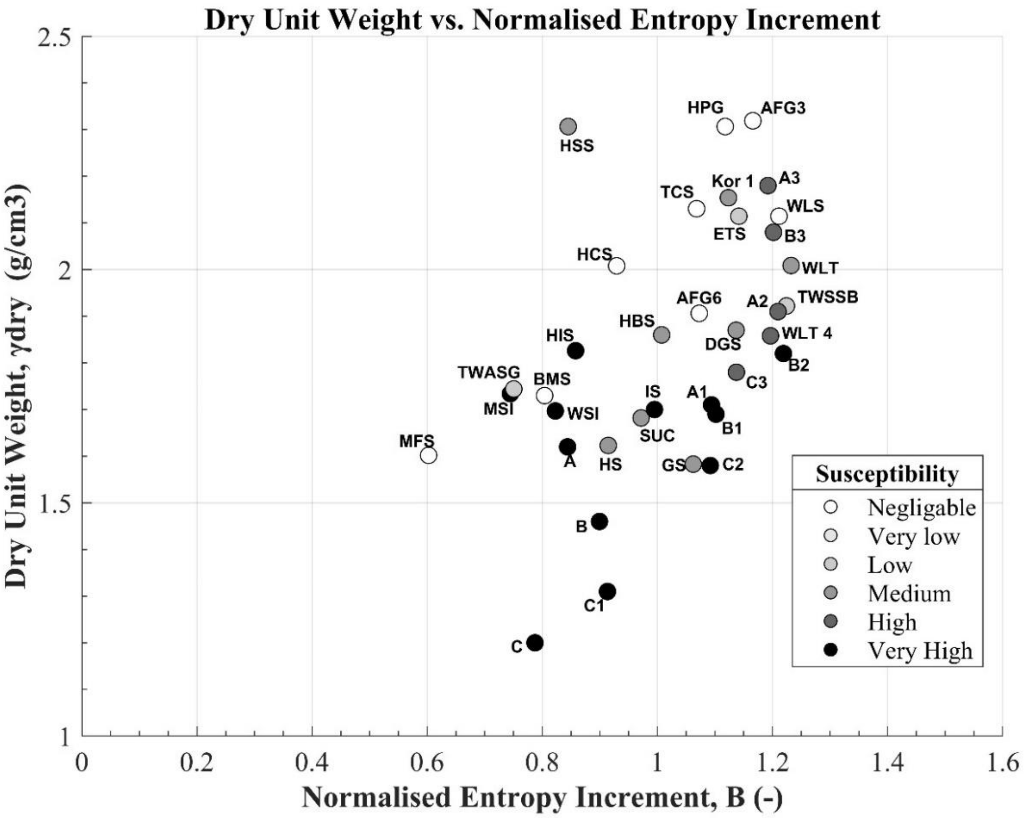


Figure 4. Dry unit weight plotted with optimum water content for all soils investigated. All soils have been classified by the criteria set out by ASTM D5918-13.

Figure 4 shows that susceptibility to thaw weakening increases as w_{opt} increases. Intuitively, the higher wopt, the higher the water holding capacity and hence a larger likelihood of ice-lenses/crystal formation (which increases thaw susceptibility). Soils with “Negligible” susceptibility have some of the highest values of γ_{dry} (e.g. **AFG3**, **HPG**, **TCS**, **WLS**). Whilst increased density reduces void ratio (and water holding capacity), an alternative perspective is that that density can also increase capillarity rise, potentially increasing further water ingress, and hence also increasing frost susceptibility. This is an aspect that warrants further research but is not the focus of the present study. However, the results in Figure 4 are surprising, given the established effects that γ_{dry} has on FT (e.g. Qi et al. 2008). To provide further insight, the effect of PSD on the compaction behaviours of these soils is shown in Figure 5 a) and b) show γ_{dry} with each of the entropy coordinates, i.e. normalised base entropy (A) and normalised entropy increment (B), respectively:



a)



b)

Figure 5. The dry unit weight for all soils investigated plotted with the a) normalised base entropy and the b) normalised entropy increment coordinates. All soils have been classified as per ASTM D5918-13.

The results of Figure 5 a) offer a possible explanation for the results in Figure 4. Soils with higher values of γ_{dry} are those with larger A-coordinates (i.e. larger grain sizes) and are also well-graded (large B-coordinates). These soils likely have lower capillarity due to their larger average grain sizes. As discussed earlier, it may be expected that denser soils promote capillarity by having more compact packings. However, Figure 5 a) suggests that grain size influences γ_{dry}, and that in soils with a range of sizes (e.g. **AFG3**, **AFG6**, **BMS**, and **HCS)** it is PSD that also influences susceptibility to thaw weakening and not γ_{dry} - even if soils have high values of γ_{dry}. In other words, although these soils may exhibit higher values of γ_{dry} than highly susceptible soils, their pore networks are not able to create effective capillary networks, which appears to be PSD dependant. This may be important in soils classified as tills and other construction materials which often contain a range of particle sizes. This is an interesting finding given that initial γ_{dry} has been shown to influence the change in density during FT and suggests that this may also be notably influenced by PSD. Note that the rate at which water will flow through a soil via capillary action during FT has also been shown to be related to PSD (e.g. Gao et al. 2024) and these results confirm the explanation provided here. Furthermore, the trend in Figure 5 a) can also be related to the fines content (Fc), which in turn also affects capillary rise. With reference to Figure 1, Negligible, Low and some Medium soils do not have a large Fc. Whereas most Medium, High and Very High susceptible soils have large Fc (i.e. particles <0.0745 mm). This is highlighted clearly in Figure 6 which shows the Fc against the A-coordinate:

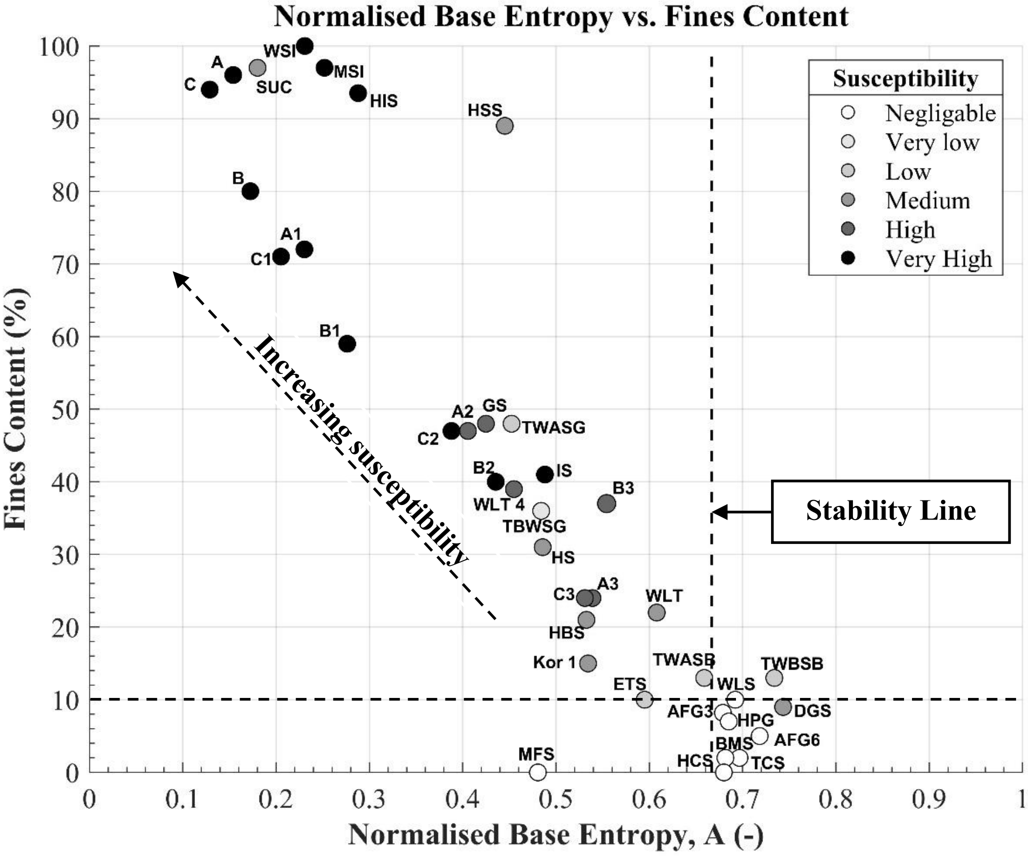
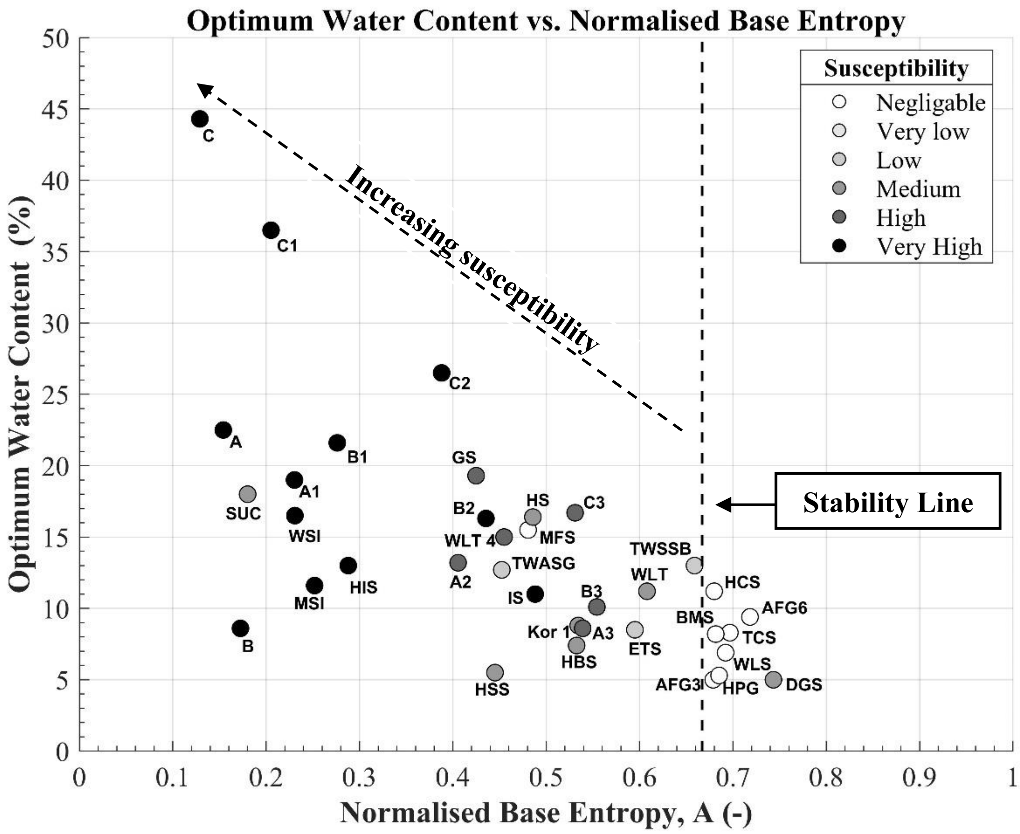


Figure 6. Normalised base entropy plotted with fines content (percentage smaller than 0.074 mm). All soils have been classified by ASTM D5918-13.

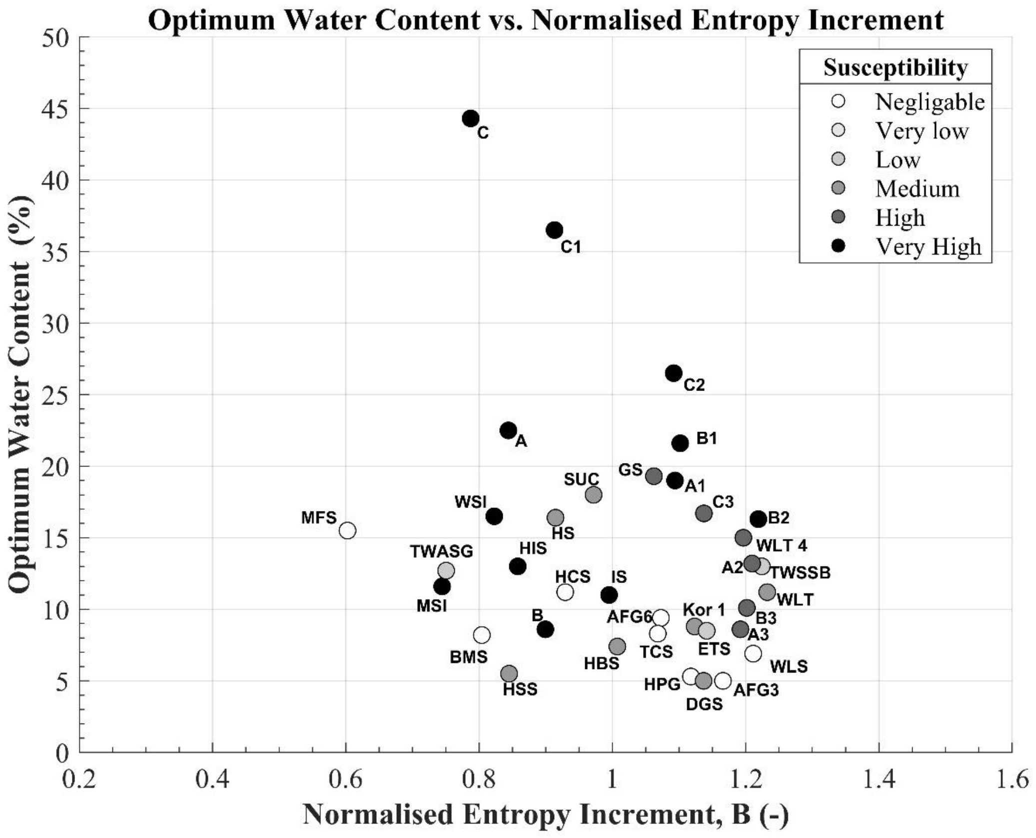
There is a linear increase in susceptibility with increasing Fc. Soils with Negligible susceptibility have generally less than 10% Fc (highlighted with a dashed line). This emphasises the importance of accounting for the entire and accurate PSD (especially fines) when considering thaw weakening. These results are also interesting given that many frost heave criteria (e.g. Casagrande's criteria) define susceptible soils as those with more than 10% passing the 0.075 mm sieve. Whilst further research is warranted, this suggests that grading entropy coordinates may provide a criteria able to predict susceptibility to both frost heave and thaw weakening (i.e. frost action). The need for such a criteria was also noted by Chamberlain (1981):

"...frost heaving, and thaw weakening have been treated as though they were unrelated. Some link should be developed between these two damaging results of frost action. Realistically, until we are successful in reliably determining the susceptibility of soil to frost heave and thaw weakening separately, it is fruitless to attempt to combine the two in a single scheme." - *Chamberlain (1981)*

In Figure 5 b) the relationship is less clear. However, it seems that γ_{dry} increases with the B-coordinate which makes sense given that γ_{dry} is associated with well-graded soils. Figure 5 a) and b) indicate that PSD seems to influence compaction behaviours which impact thaw weakening susceptibility. This is further supported by observing Figure 7 a) and b) which show the w_{opt} with each coordinate:


Optimum Water Content vs. Normalised Base Entropy
Optimum Water Content (%)
Normalised Base Entropy, A (-)
0
5
10
15
20
25
30
35
40
45
50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Susceptibility
Negligable
Very low
Low
Medium
High
Very High
Increasing susceptibility
Stability Line
C
C1
C2
A
B1
A1
SUC
WSI
MSI
HIS
B
GS
B2
WLT 4
A2
HS
MFS
TWASG
C3
IS
B3
Kor 1
A3
HBS
HSS
TWSSB
WLT
ETS
HCS
BMS
AFG6
TCS
WLS
AFG3
HPG
DGS

a)


Optimum Water Content vs. Normalised Entropy Increment
Optimum Water Content (%)
Normalised Entropy Increment, B (-)
0
5
10
15
20
25
30
35
40
45
50
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Susceptibility
Negligable
Very low
Low
Medium
High
Very High
C
C1
C2
A
B1
GS
A1
SUC
WSI
HS
C3
B2
MFS
TWASG
HIS
WLT 4
A2
TWSSB
MSI
HCS
IS
WLT
B
AFG6
Kor 1
B3
A3
BMS
HBS
TCS
ETS
WLS
HSS
HPG
DGS
AFG3

b)

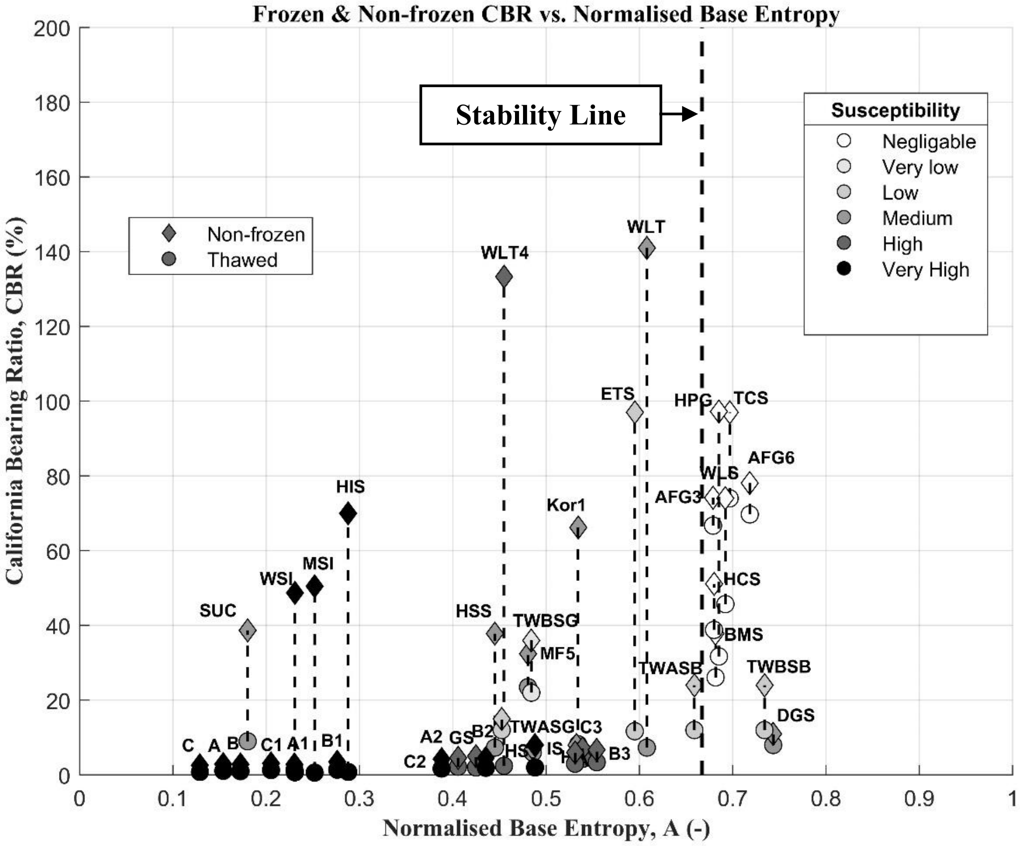
Figure 7. Optimum water content for all soils investigated plotted with the a) normalised base entropy and the b) normalised entropy increment coordinates. All soils have been classified ASTM D5918-13.

In Figure 7 a) as the A-coordinate decreases, w_{opt} increases and susceptibility to thaw weakening increases. As may be expected, soils with larger A-coordinates have the lowest values of w_{opt}, likely due to their larger void space. Note also that as soils pass below A=0.667, w_{opt} increases notably with susceptibility. The figure also highlights that greater water content is needed in finer soils to reach w_{opt}, which increases susceptibility to thaw weakening. Furthermore, higher water content increases the risk of unfrozen water, which may offer further explanation for the behaviours seen in Figure 7 a). Not all water in soil freezes during frost action (Feng et al. 2024) and unfrozen water content has been shown to alter the mechanical properties of soils, including strength and structure, which promotes instability (Bi et al., 2023). This has also been shown to be influenced by PSD (Qiu et al. 2020).

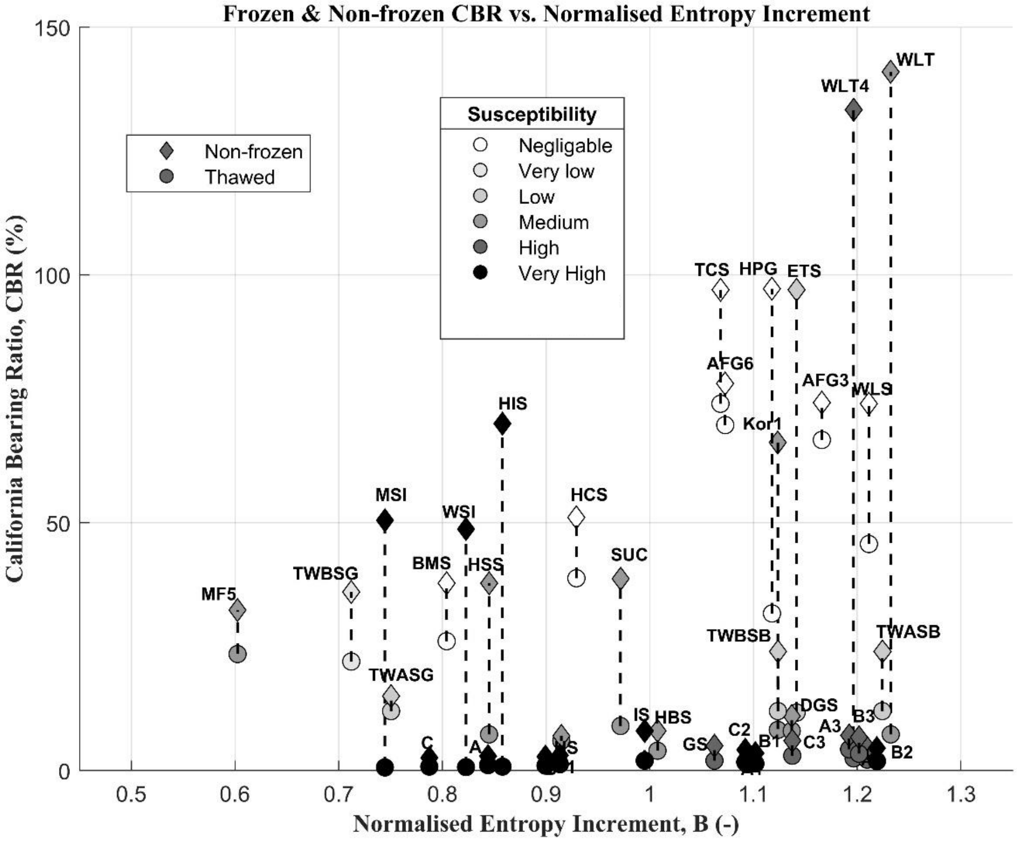
In Figure 7 b), no clear trend is observed with the B-coordinate. These figures suggest that thaw weakening is largely affected by grain size as opposed to the amount of material within the fractions. This is however expected. It is well recognised that polydispersity (i.e. whether a soil is well-graded, represented by a large B value, or uniformly graded, represented by a low B-value) has a bigger effect on maximum γ_{dry}, rather than w_{opt}. Well-graded soils can achieve a much wider range of densities than uniform soils if the same range of compaction water contents are considered. This offers explanation as to why clear trends can be observed in Figure 4-7 a), but not in Figure 7 b).

6.3. Effect of thaw weakening in CBR results

To investigate the effect of thaw weakening in CBR results, Figure 8 a) and b) show the non-frozen and thawed CBR values plotted against each grading entropy coordinate:


Frozen & Non-frozen CBR vs. Normalised Base Entropy
Stability Line
Susceptibility
○ Negligable
○ Very low
● Low
● Medium
● High
● Very High
◆ Non-frozen
● Thawed
California Bearing Ratio, CBR (%)
Normalised Base Entropy, A (-)
0
20
40
60
80
100
120
140
160
180
200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
WLT4
WLT
ETS
HPG
TCS
AFG6
WLS
AFG3
Kor1
HIS
MSI
WSI
SUC
HSS
TWBSG
MF5
HCS
BMS
TWASB
TWBSB
DGS
C
A
B
C1
A1
B1
A2
GS
B2
C2
HS
TWASG
C3
IS
B3

a)



b)

Figure 8. California bearing ratio results for all soils before and after freeze-thaw plotted with the a) normalised base entropy and the b) normalised entropy increment coordinates.

In Figure 8 a), as the A-coordinate decreases there are frequent examples of large changes in CBR before and after FT. Soils with A>0.667 have higher non-frozen CBR values and experienced fewer changes after thawing. Note that some soils (e.g. **WLT, HIS, and WLT 4)** with A<0.677 experienced large non-frozen CBR values (i.e. Negligible), despite these being classified earlier as Highly Susceptible. In Figure 8 b) whilst soils with higher B-coordinates appear to be those which experienced the greatest change in CBR after FT, no clear relationship is apparent. Also note that stable soils (such as **HPG** and **WLS**) experienced a large CBR reduction after thawing, however these were still deemed as soils with Low or Negligible susceptibility. In other words, whilst large changes in CBR may be associated with increasing susceptibility, thaw weakening is not explicitly related to the CBR reduction itself, rather to the most stable condition that can be achieved upon thawing. The data in Figure 8 a) shows that such a condition is strongly affected by PSD as also observed by Lörincz' stability criterion in Figure 3.

Nevertheless, Figure 8 highlights that changes in internal stability and CBR values during thaw weakening are notably influenced by PSD. Changes in CBR between non-frozen and thawed soils are often very large for soils with A<0.667. Soil **WLT** for instance is deemed Negligible but undergoes a change in CBR of 28.3%, indicating that changes in CBR between non-frozen and thawed soils may also be an important factor when assessing stability during thaw weakening. Note that soils with A<0.677 that did not experience large changes in CBR were due to already exhibiting very low non-frozen CBR values, suggesting that these soils were inherently unstable prior to FT. To fully interpret the effect of PSD on the change in CBR, the normalised change in CBR against the A-coordinate is presented in Figure 9, where the normalised CBR is defined as follows:

$$CBR_{normilised} = \frac{Non\,frozen\,CBR - Thawed\,CBR}{Non\,frozen\,CBR} \tag{8}$$

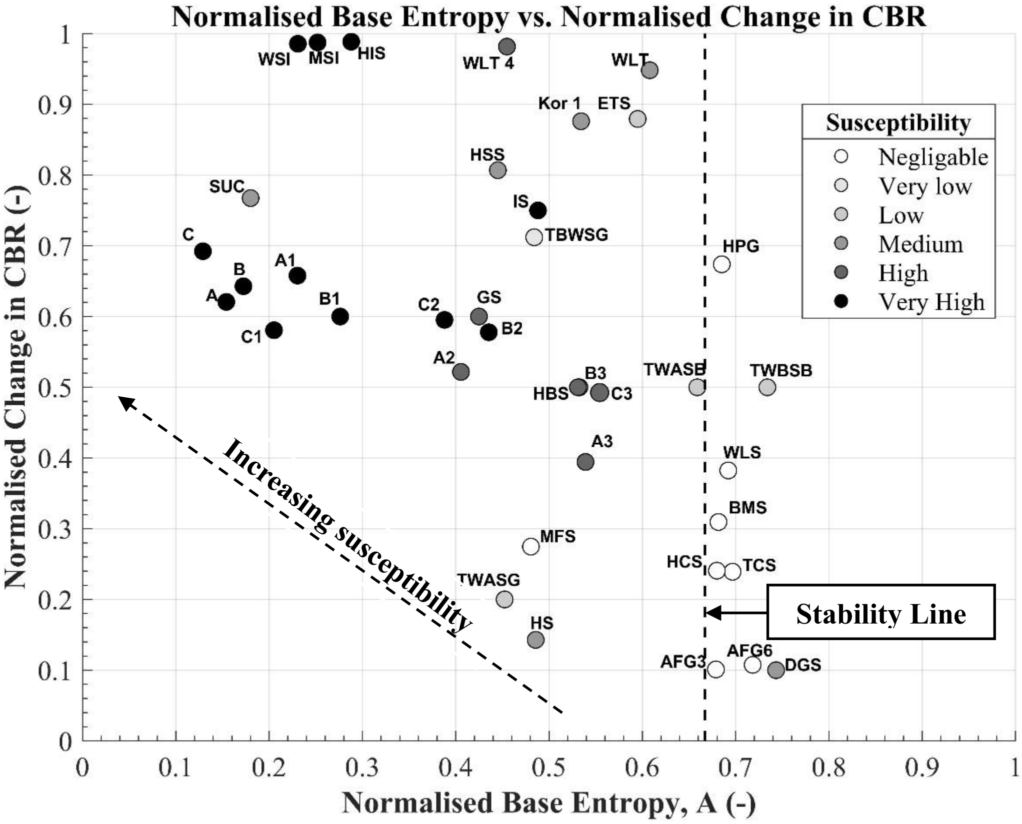


Figure 9. Normalised change in CBR plotted with the normalised base entropy.

Figure 9 shows clearly that as the A-coordinate reduces the change in CBR increases, particularly below the stability line. Similar findings were obtained by Gülen et al (2024) who found that changes in CBR increased with reducing d_{50}. Notably soils with Very High susceptibility experienced the greatest changes further suggesting that FT can alter the internal structure of soils, and that this may be PSD dependant.

Thus far all data has been assessed via the grading entropy coordinates and clear relationships have been seen throughout. It is therefore worth comparing the CBR data in Figure 8 with commonly used grading descriptors such as d_{50} and C_u, as shown in Figures 10 a) and b).

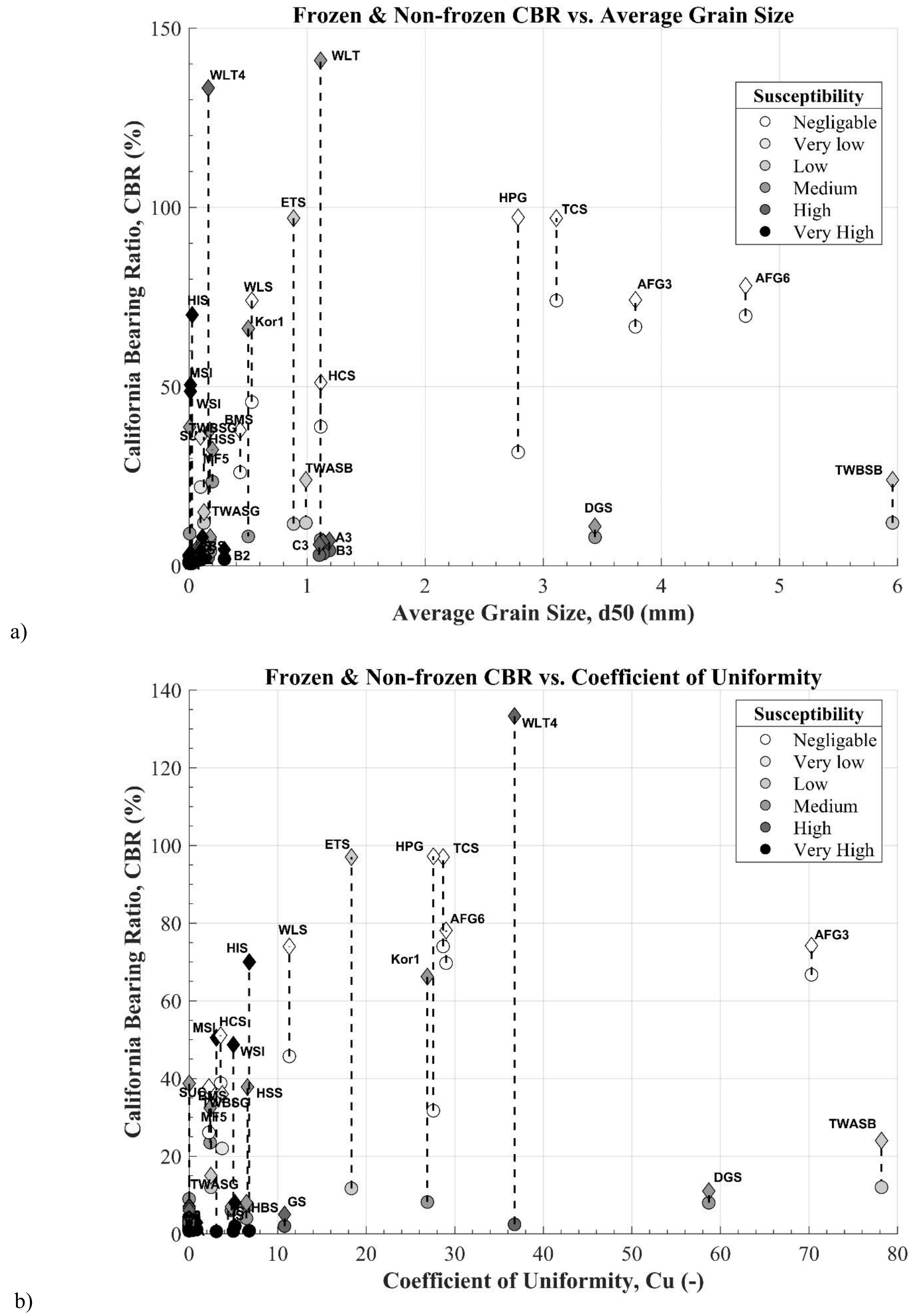


Figure 10. California bearing ratio results for all soils before and after freeze-thaw plotted with the a) average grain size, d_{50} and the b) coefficient of uniformity, C_u.

Comparing d_{50} to the A-coordiante is appropriate as they both measure average grain size – but differ in their approach. A key advantage of the A-coordinate is its ability to explain susceptibility to thaw weakening within the framework of the stability criteria (Lörincz, 1986). This is made clear in Figure 8 a) where susceptible and non-susceptible soils are clearly distinguished by the stability line (A=0.667). In contrast, although it may be seen in Figure 10 a) that susceptibility to thaw weakening does increase as d_{50} reduces, clear distinctions between susceptible and non-susceptible soils are difficult to be seen.

In Figure 10 b), examining the C_u as a means of determining susceptibility to thaw weakening, there are clear limitations and no clear observable relationship. For instance, when comparing multiple PSDs the large range in C_u values (smallest being $C_u = 3.07$ and largest being $C_u = 78$) make comparisons difficult and therefore physical relationships hard to observe. Note that the same limitation may be seen in Figure 10 a). Whereas the entropy coordinates have a narrower range of values enabling easier comparisons with larger datasets and physical relationships easier to observe. More importantly, C_u excludes Fc, which has already been highlighted in Figure 6 to be important regarding thaw weakening susceptibility. Hence, it is unsurprising that no clear relationship can be seen in Figure 10 b). Finally, note that all soils from the dataset of Gülen et al (2024) do not contain particle diameters corresponding to d_{10}. Grading entropy coordinates, in comparison have no reliance on individual diameters and hence can interpret all PSD curves regardless of nominal particle diameters.

Conclusions

This work has investigated existing data used to establish existing thaw weakening susceptibility criteria in ASTM D5918-13. A relationship between thaw weakening susceptibility and PSD using the grading entropy stability criteria was clearly found. The key findings from this study are as follows:

1. Thaw weakening is related to PSD and susceptibility to thaw weakening may be characterised by the grading entropy stability criteria. The use of grading entropy coordinates to assess the susceptibility to frost heave and thaw weakening is promising.
2. The effect of PSD on the susceptibility to thaw weakening is likely related to the capillarity of the soils and the pore networks created by the grain size. Even in soils with high values of γ_{dry} (which would be expected to promote capillarity), grain size appeared to determine susceptibility to thaw weakening.
3. PSD influences γ_{dry}. This means that the PSD not only affects susceptibility to thaw weakening but might also influence density changes that take place during FT. Future work should seek to harmonise this with the critical γ_{dry}.
4. Finer soils require greater water contents to reach w_{opt}. Higher w_{opt} in this study correlated with increased susceptibility to thaw weakening and was PSD dependant. This may greatly influence construction in cold regions.
5. Large changes in CBR between non-frozen and thawed soils were dependent on PSD. This indicated that notable changes in the soil structure took place during thaw greatly affecting bearing capacity in susceptible soils. Moreover, soils with A>0.667 had larger CBR values prior to FT and experienced fewer changes in CBR after thaw when compared to those with A<0.667.

It has been demonstrated here that when grading entropy coordinates are used, the effect of moisture content, capillary effects and the stability of inter-particle contact networks hypothesised by Lörincz can all be linked and explained as a function of the PSD descriptors. Such a framework also enables PSD characteristics to be linked to physical mechanisms that underlie thaw weakening susceptibility. Furthermore, it can be emphasised that using grading entropy coordinates within the context of well-accepted thaw weakening susceptibility is possible for both laboratory and in-situ conditions, hence readily applicable in geotechnical practice. Grading entropy theory presents an alternative method to quantify PSD characteristics and provides a context to interpret its effect of thaw weakening susceptibility criteria.

The findings described above indicate that both frost heave and thaw weakening may be linked effects i.e. as frost heave as already been linked to PSD (e.g. Casagrande 1932; Chamberlain 1981), and that they may be assessed together if appropriate descriptors of the soils PSD are employed. This investigation is part of a wider study to establish a new unified frost action susceptibility criterion, hence future works will incorporate both frost heave and thaw weakening into the grading entropy stability criteria.

Competing interests

We confirm there are no competing interests or financial relationships which may bias this work.

Data availability

All data may be provided upon reasonable request to the authors.

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