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Quantum correlations and laser threshold

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Abstract. We present a model of nanolasers that includes the effect of all two-particle quantum correlations. We find that the lasing starts with a finite coherent amplitude and that lasing and non-lasing solutions coexist.

1. Introduction

Nanolasers have been intensively studied because of their technologically promising properties [1], extreme miniaturization, minimal energy requirements, and small numbers of emitters and field modes coupled to the quantum dots. They are also very interesting from a fundamental point of view, since they are very far from the semiclassical limit that applies to macroscopic lasers while remaining too complex for the resolution of quantum master equations. In this paper we derive a model of quantum dot nanolasers that includes all two-particle quantum correlations and present numerical results for the simple case of identical, single-electron quantum dots.

2. Model

We start from the fully quantized total Hamiltonian $H = H_{\mathrm{E}} + H_{QD} + H_{int}$, where the electromagnetic energy is

$$H_{\mathrm{E}} = h \sum_q \nu_q \Big(b_q^\dagger b_q + \frac{1}{2} \Big), \tag{1}$$

with ν_q frequency of a q-th mode photon, and the charge carrier energy is

$$H_{QD} = \sum_n \Big(\varepsilon_{c,n} c_n^\dagger c_n + \varepsilon_{v,n} v_n^\dagger v_n \Big), \tag{2}$$

with $\epsilon_{c,n}$, $\epsilon_{v,n}$ energies and $c_n, c_n^\dagger$ and $v_n, v_n^\dagger$ annihilation and creation operators for conduction and valence electrons of the n-th quantum dot, respectively. The light-matter interaction term in the dipole approximation is

$$H_{int} = -ih \sum_{n,q} \Big[g_{nq} b_q c_n^\dagger v_n - g_{nq}^* b_q^\dagger v_n^\dagger c_n \Big], \tag{3}$$

where g_{nq} is the strength of light-matter coupling of q-th mode in the n-th quantum dot.

We consider nanolasers with good cavities of high quality factors, e.g. $Q \sim 10^4$, operating at low temperatures so that we can model the Coulomb interactions as carrier dephasing [2]. We assume that the pump is incoherent [3] and include the effect of the external environment through the Born-Markov approximation assuming that nanolaser and environment are not correlated. The environment dynamics is indeed very fast and thus there are no significant memory effects. The Hamiltonian H and the Lindblad diffusors, included to take into account the losses [4], give rise to an infinite hierarchy of coupled equations of motion for the expectation values. These can be truncated to a finite set by assuming that all quantum correlations above a certain order are negligible and can be set to zero [5]. This procedure leads to a finite-dimensional model of quantum dots in a nanocavity.

By neglecting two–particle quantum correlations with fast oscillations at the frequency of the laser field – such as the correlation between laser field and population in semiclassical theories –, we previously derived a semiclassical Coherent and Incoherent Model (CIM) with a fast dynamics of the coherent variables (the expectation values of field and medium polarization, $\langle b\rangle, \langle v^\dagger c\rangle$), and a slow dynamics of the incoherent variables (the expectation values of population, photon assisted polarization and number of photons, $\langle c^\dagger c\rangle, \langle bc^\dagger v\rangle, \langle b^\dagger b\rangle$) [6, 7, 8].

Here, we compare the semiclassical CIM with a new Two Particles Model (TPM) that includes all two–particle quantum correlations. The new variables in this model are the expectation values of two photon processes, $\langle bb\rangle, \langle b^\dagger b^\dagger\rangle$, photon absorption (emission) and electron excitation (de-excitation), $\langle b^\dagger c^\dagger v\rangle, \langle bv^\dagger c\rangle$, and light induced processes involving two fermionic particles, $\langle c_l^\dagger v_n^\dagger c_n v_l\rangle, \langle c_l^\dagger c_n^\dagger c_n c_l\rangle$, $\langle c_l^\dagger v_n^\dagger c_n c_l\rangle$, $\langle v_l^\dagger v_n^\dagger c_n c_l\rangle$, where the indices l, n, with $l \neq n$, identify different quantum dots.

For the case of identical quantum dots in a single mode nanolaser, the TPM equations read

$$d_t\langle b\rangle = -(\gamma_c + i\nu)\langle b\rangle + Ng^*\langle v^\dagger c\rangle, \tag{4a}$$

$$d_t\langle v^\dagger c\rangle = -(\gamma + i\nu_\varepsilon)\langle v^\dagger c\rangle + g\left(2\langle bc^\dagger c\rangle - \langle b\rangle\right). \tag{4b}$$

$$d_t\langle c^\dagger c\rangle = -\gamma_{nr}\langle c^\dagger c\rangle - \left(g\langle bc^\dagger v\rangle + \text{h.c.}\right) + \left[-\gamma_{nl}\langle c^\dagger c\rangle + r(1 - \langle c^\dagger c\rangle)\right], \tag{4c}$$

$$d_t\langle b^\dagger b\rangle = -2\gamma_c\langle b^\dagger b\rangle + N(g\langle bc^\dagger v\rangle + h.c.), \tag{4d}$$

$$\begin{aligned} d_t\langle bc^\dagger v\rangle = &-(\gamma + \gamma_c + i\Delta\nu)\langle bc^\dagger v\rangle \\ &+ g^*\left[\langle c^\dagger c\rangle + 2\langle(b^\dagger bc^\dagger c)\rangle - \langle b^\dagger b\rangle\right] + (N-1)g^*\langle c^\dagger v^\dagger cv\rangle, \end{aligned} \tag{4e}$$

$$\begin{aligned} d_t\langle bc^\dagger c\rangle = &-(\gamma_c + \gamma_{nr} + i\nu)\langle bc^\dagger c\rangle - g\langle(bbc^\dagger v)\rangle - g^*\langle(b^\dagger bv^\dagger c)\rangle + (N-1)g^*\langle c^\dagger v^\dagger cc\rangle \\ &+ \langle b\rangle\left[-\gamma_{nl}\langle c^\dagger c\rangle + r\left(1 - \langle c^\dagger c\rangle\right)\right], \end{aligned} \tag{4f}$$

$$d_t\langle bb\rangle = -2(\gamma_c + i\nu)\langle bb\rangle + 2Ng^*\langle bv^\dagger c\rangle \tag{4g}$$

$$d_t\langle bv^\dagger c\rangle = -[\gamma_c + \gamma + i(\nu + \nu_\varepsilon)]\langle bv^\dagger c\rangle + g\left[2\langle(bbc^\dagger c)\rangle - \langle bb\rangle\right] + (N-1)g^*\langle v^\dagger v^\dagger cc\rangle \tag{4h}$$

$$\begin{aligned} d_t\langle c^\dagger v^\dagger cv\rangle = &-2\gamma(1+\mu)\langle c^\dagger v^\dagger cv\rangle + g^*\left[2\langle(b^\dagger v^\dagger c^\dagger cc)\rangle - \langle b^\dagger v^\dagger c\rangle\right] \\ &+ g\left[2\langle(bc^\dagger c^\dagger cv)\rangle - \langle bc^\dagger v\rangle\right], \end{aligned} \tag{4i}$$

$$\begin{aligned} d_t\langle v^\dagger c^\dagger cc\rangle = &-(\gamma(1+\mu) + \gamma_{nr} + i\nu_\varepsilon)\langle v^\dagger c^\dagger cc\rangle + g\left[2\langle(bc^\dagger c^\dagger cc)\rangle - \langle bc^\dagger c\rangle\right] \\ &- g\langle(bc^\dagger v^\dagger cv)\rangle - g^*\langle(b^\dagger v^\dagger v^\dagger cc)\rangle + \langle v^\dagger c\rangle\left[-\gamma_{nl}\langle c^\dagger c\rangle + r\left(1 - \langle c^\dagger c\rangle\right)\right], \end{aligned} \tag{4j}$$

$$d_t\langle c^\dagger c^\dagger cc\rangle = -2\gamma_{nr}\langle c^\dagger c^\dagger cc\rangle - \left[2g\langle(bc^\dagger c^\dagger cv)\rangle + \text{h.c.}\right] \tag{4k}$$

$$+ 2\langle c^\dagger c\rangle \left[-\gamma_{nl}\langle c^\dagger c\rangle + r\left(1 - \langle c^\dagger c\rangle\right)\right],$$

$$d_t\langle v^\dagger v^\dagger cc\rangle = -2\left[\gamma(1+\mu) + i\nu_\varepsilon\right]\langle v^\dagger v^\dagger cc\rangle + 2g\left[2\langle(bv^\dagger c^\dagger cc)\rangle - \langle bv^\dagger c\rangle\right], \quad (41)$$

where N is the number of identical quantum dots coupled to the laser mode, r is the pump rate per emitter, γ is the polarization decay rate, γ_c is the laser mode decay rate, γ_{nl} is the excited state decay rate into non-lasing modes, γ_{nr} is the excited state's non-radiative decay rate, $\gamma_l = \frac{2}{h}\frac{|g|^2}{\gamma_c+\gamma}$ is the excited state decay rate into the laser mode, $\beta = \gamma_l/(\gamma_l + \gamma_{nl})$ is the ratio between the decay rate into the laser mode and the total radiative decay, $\gamma\mu$, with $\mu \geq 0$, is the dephasing rate due to phonon scattering [9], ν is the frequency of the laser mode, $\nu_\varepsilon = (\epsilon_c - \epsilon_v)/h$ is the transition frequency and $\Delta\nu = \nu - \nu_\varepsilon$ is the detuning. For simplicity, we have dropped the indices for the laser modes and the quantum dots.

3. Numerical results

In this section we present results of the TPM for two nanolasers, one with $N = 30$ and the other with $N = 100$. All other parameters are the same for both devices: $\gamma = 10^{13}$ Hz, $\gamma_c = 10^{10}$ Hz, $\gamma_{nl} = 0$, $\gamma_{nr} = 10^9$ Hz and $\Delta\nu = 0$.

We focus first on the bifurcation behavior of the lasing solution. In the semiclassical CIM the non-lasing solution with $\langle b\rangle = 0$ exists for all values of the control parameters and becomes unstable via a pitchfork bifurcation (Hopf bifurcation in the general case of $\Delta\nu \neq 0$) at a critical value of the pump parameter r, where a lasing solution branches off from the non-lasing solution, see figure 1(a,b). Note that the amplitude of the coherent field of the lasing solution

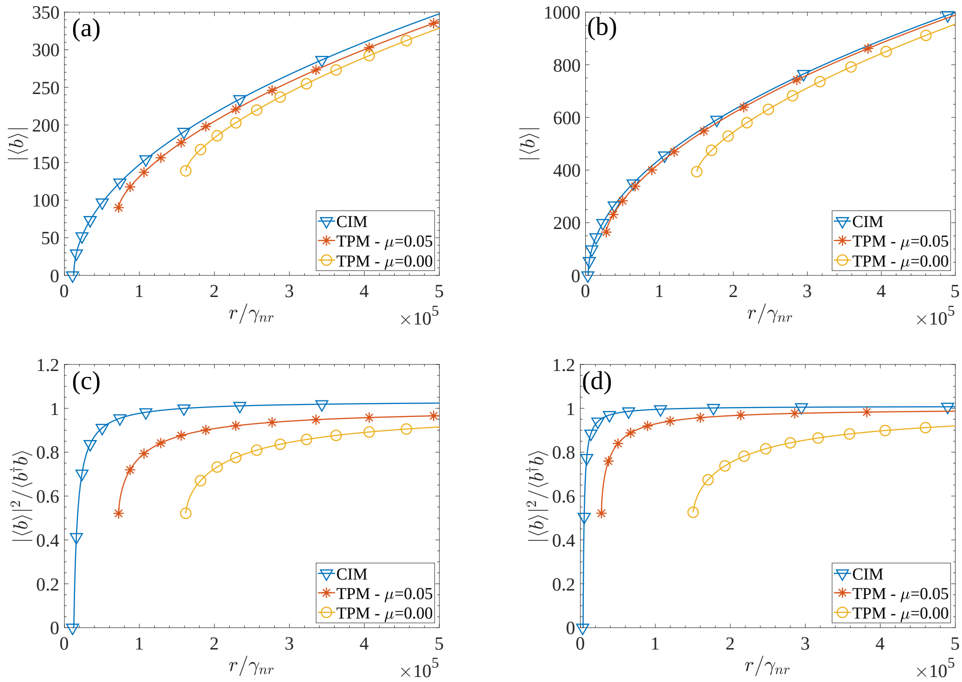

Figure 1. Coherent field amplitude $|\langle b\rangle|$ in non dimensional units (top row) and $|\langle b\rangle|^2/\langle b^\dagger b\rangle$ ratio (bottom row) vs non dimensional pump rate r/γ_{nr} for single mode nanolasers containing $N = 30$ (left column) and $N = 100$ (right column) quantum dots. In all cases only the stable lasing solutions are plotted. They were obtained by integrating until equilibrium equations (4) (TPM) and equations (4a)-(4e), suitably truncated (CIM).

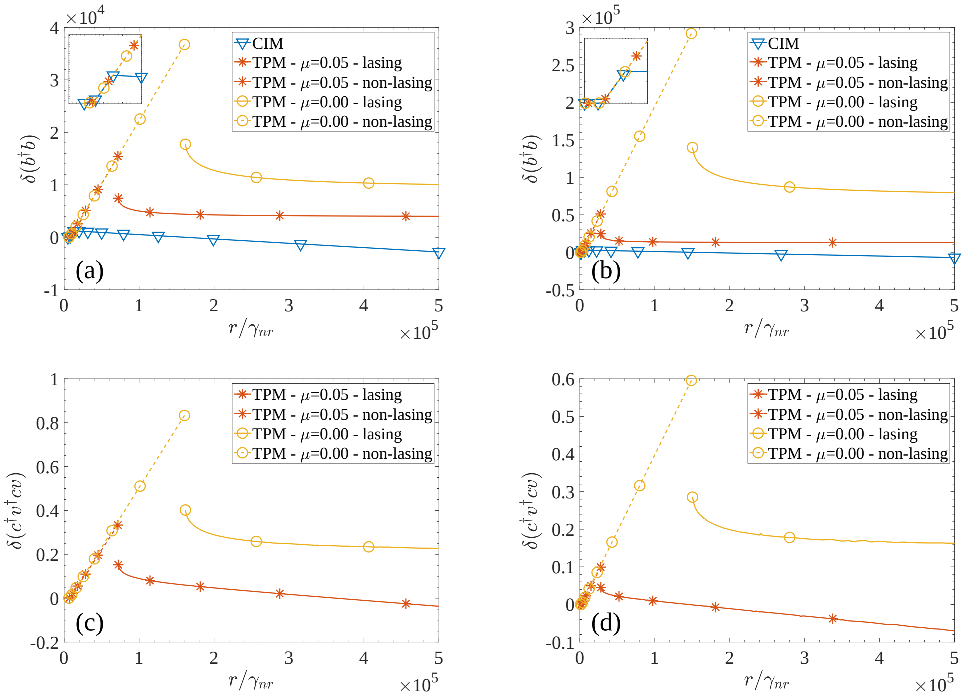

Figure 2. Photon-photon quantum correlation $\delta(b^\dagger b)$ (top row) and electron-electron quantum correlation $\delta(c^\dagger v^\dagger cv)$ – defined only for the TPM – (bottom row) vs r/γ_{nr} for single mode nanolasers containing $N = 30$ (left column) and $N = 100$ (right column) quantum dots. All parameter values as in figure 1. The CIM laser threshold is identified by the change of slope of the two-particle correlations, see inserts in panels (a) and (b) which show an enlargement of the low pump region. At the TPM threshold the correlations have a jump discontinuity that reflects the one in $|\langle b\rangle|$.

grows continuously from zero as r increases, a scenario compatible with a classical field, but not with a quantized field. A non-lasing solution exists also for the TPM but it is stable for all pump values. Two lasing solutions, one stable and one unstable, appear with a non-zero amplitude at a threshold pump value through a saddle node bifurcation. We plot the stable lasing solution in figure 1(a,b). This can be achieved only if triggered by a small, but finite, burst of photons. By definition, the unstable solution cannot be reached for any generic value of the initial conditions. However, it may still play a significant role in the laser dynamics because its stable manifold may act as a barrier to stable lasing and reduce the size of the basin of attraction of the stable lasing solution. A more detailed analysis of the phase space of the lasing solutions which would answer quantitatively these questions is, however, outside the scope of this paper.

The presence of weak phonon scattering, i.e. setting $\mu \neq 0$ but small, lowers the TPM laser threshold and brings the coherent amplitude very close to that of the semiclassical CIM (see the curves $\mu = 0.00$ and $\mu = 0.05$ in figures 1(a,b)). This effect is strengthened by an increase in the number of quantum dots, as can be seen by comparing figures 1(a,c) and 1(b,c).

In figures 1(c,d) we plot the ratio $|\langle b\rangle|^2/\langle b^\dagger b\rangle$. In the TPM case this is always smaller than one, indicating that there are photons generated by spontaneous emission even for pump values higher than the laser threshold. In the CIM case the ratio becomes slightly larger than 1 for sufficiently large pump values. This reflects the fact that the CIM photon-photon correlations become negative at high pump, see figure 2(a-b).

The effect of two-particle quantum correlations can be seen in figure 2. Here we plot quantum

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correlations as a function of r/γ_{nr} for the same lasers as in figure 1. The correlation between photon absorption and emission, $\delta(b^\dagger b)$, is represented in panels (a) and (b), while that between upward and downward electron transitions $\delta(c^\dagger v^\dagger cv)$ is in panels (c) and (d). The latter is defined only for the TPM. The laser threshold for the CIM is identified by the change of slope of the two particle correlations [8], while for the TPM by their discontinuous jump. Comparing this figure with figure 1, we can see that lasing is associated with a reduction of the correlations, which is discontinuous for the TPM. In particular both correlations are reduced by the phonon scattering and the electron-electron correlation is reduced also by increasing the number of quantum dots.

Conclusions

We have derived nonlinear models of nanolasers which allow one to investigate transitions from thermal, anti-bunching and lasing emission. All models show that lasing is possible, but in models with two-particle quantum correlations lasing can be achieved only if triggered by a finite amplitude fluctuation of the coherent variables. In addition, stable lasing and non-lasing solutions can coexist.

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