

Robust Backstepping Controllers for Linear Motor Drives

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Abstract—This work presents robust backstepping controllers to achieve position tracking control of a linear motor drive system with parameter uncertainties, discontinuous frictional force, and unknown external disturbance. First, a robust control scheme is developed to provide asymptotic stability under tracking control of the linear drive system. The assumption of a constant upper bound on the disturbance during the control design can lead to an overly conservative controller. To avoid this, an adaptation mechanism has been proposed to adapt the upper bound based on the current state measurement and the delayed state and input variables. The developed control scheme was shown to provide a global asymptotic stability of the closed-loop linear motor drive system. The control strategies are evaluated by numerical simulation of the linear motor drive. The simulation results show that the use of the adaptive disturbance upper bound adjustment technique results in significant performance improvement.

Keywords: backstepping control, linear motor drives, robust control.

I. INTRODUCTION

The linear motor drive is widely applied in modern manufacturing processes, including automatic machine inspection, machine tools, and semiconductor manufacturing [1]. This is motivated by its many advantages compared to the rotary type such as the elimination of the need for gears between the motion device and motor, mechanical loss reduction, high-speed operation, silent operation, and high initial thrust force [2]. Despite this merit of linear drive motors, there exists a significant interaction between direct drives and the machining process [3]. This makes it important to develop drive control systems capable of providing high tracking performance. Achieving good tracking performance is challenging because the motor parameters vary strongly in reaction to the dynamics of the air gap, phase unbalance, rail resistivity, and saturation of magnetization [4]. Besides, uncertain parameter variations, unmodelled dynamics, and external load disturbance also affect the performance of the control system in practical applications [5], [6], [7]. Frictional force effects cannot be avoided because the operation of the drive motor involves contact between two bodies and this poses a major drawback to control systems since they may result in steady state error. Traditional methods such as adaptive and variable-structure control [8], [9] address this challenge by developing a friction model to estimate and compensate for the effects of frictional forces. However,

a precise friction model is difficult to obtain because it is usually not completely understood [10] and is highly non-linear. A great deal of research [11], [12], [13], [14] has been conducted in the past three decades to address these challenges associated with linear drive motor position tracking control.

Different robust control schemes have been developed for the linear drive motor context. The authors in [9] proposed an optimal control scheme based on H_∞ measure to provide robust tracking in the presence of disturbances like cutting forces. In [5], it was highlighted that H_∞ methods give a conservative performance which is not very suitable for high-accuracy tracking control. As an alternative method, disturbance observer (DOB) based controllers were proposed to improve the tracking performance despite the impact of system uncertainties [15], [16]. An experimental study of the linear drive control was used [17] to demonstrate that the DOB approach cannot adequately deal with the discontinuity associated with Coulomb friction. Another challenge that has also motivated the development of position controllers for linear motor drives is the need to minimize the nonlinear ripple and cogging effects. For instance, a first-order approximation of ripple effects was experimentally obtained based on which a feed-forward controller was developed to cancel its effects to improve position tracking [18]. However, this offline technique of identifying a compensation model can be of limited utility since this may change due to operating conditions and a particular model may only be useful for a specific linear drive motor. In consideration of this, a feed-forward controller that relies on neural networks for the estimation of uncertainties was proposed to improve positional accuracy [19]. However, this scheme did not provide a theoretical guarantee of closed-loop stability.

Given the limitations associated with the above control methods, adaptive robust control (ARC) methods received significant interest in position-tracking control of linear motors. These ARC schemes mostly rely on backstepping techniques that rely on Lyapunov functions to enforce the asymptotic stability of the closed-loop response of the drive motors. Adaptive backstepping is achieved via a recursive and systematic design procedure for nonlinear feedback control, and the method offers direct means to handle system uncertainties and nonlinearities [20]. For instance, an ARC scheme was developed for high-speed and high-accuracy position control of machine tools driven by rotary AC motors [17]. The study compared the ARC controller with the DOB approach and found that the former outperformed the latter, especially in dealing with discontinuity and improvement of tracking accuracy. Additionally, significant disturbances

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and parameter variations are allowed in the ARC technique as opposed to the DOB method whose performance can deteriorate significantly when estimation performance deteriorates. In [5], ARC was developed for iron-core linear motors which face greater parameter variations and other difficulties when compared to their rotary counterparts. The effectiveness of the scheme was demonstrated via extensive simulation studies. In [3], the robustness performance of ARC was further improved by the use of a recurrent neural network (RNN) to estimate lumped uncertainties. The control scheme contained a discontinuous sign function which was then approximated by a mathematical function developed by [21] to minimize the chattering phenomena resulting from the discontinuity. In [22], a dual-loop control strategy was developed to improve the speed and tracking accuracy of the linear drive system. The outer loop uses an online trajectory replanning strategy that forces the replanned trajectory to converge to the actual trajectory in minimum time under system constraints. The inner loop then uses an ARC to improve tracking performance in the presence of disturbances. The control framework was experimentally validated [22]. Like the previously discussed ARC technique [3], the control input also includes a discontinuous function.

In light of the reviewed literature, this study proposes a robust backstepping controller to guarantee global asymptotic stability for position tracking by a linear drive motor. By acknowledging that the use of a constant upper bound can lead to conservative performance, the study proposes an approach to adaptively adjust the disturbance upper bound. This is achieved by approximating the disturbance upper bound based on the current state measurement and the delayed values of the input and state variables. The performance of the robust controller with constant upper bounds and adaptively changing upper bounds are compared via numerical simulations. The rest of this paper is organized as follows. Section II presents the problem formulation by describing the mathematical model of the linear motor and stating the control objective. Section III provides the development of a robust backstepping controller with a sign function and the neural-network-based adaptive robust backstepping scheme without any discontinuous function. A simulation study is presented in Section IV. Finally, Section V provides concluding remarks.

II. PROBLEM FORMULATION

A. Linear Drive Motor

The linear drive system with parameter deviations and external disturbances force can be modeled as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ M\dot{x}_2 &= k_f u - (B + \Delta B)x_2 - f(x_2) - f_d \end{aligned} \quad (1)$$

where x_1 [m] is the position of the mover, x_2 [m/s] is the mover velocity, M [kg] is the total mass of the moving element, B [N.s/m] represents the combined viscous friction coefficient and load damping, ΔB is a parametric uncertainty, f_d [N] represents the external disturbance forces, and u [V] is the control input voltage of the motor while k_f [N/V] is the input constant. In addition, $f(x_2)$ [N] is the

combined stiction and Coulomb friction. By considering the Coulomb, stiction, and Stribeck effect, this is expressed as [3]:

$$f(x_2) = f_c \text{sgn}(x_2) + (f_s - f_c) e^{-(x_2/\dot{x}_s)^2} \text{sgn}(x_2) + K_v x_2 \quad (2)$$

where f_c is the Coulomb friction, f_s is the static friction, \dot{x}_s denotes the Stribeck velocity parameter, $\text{sgn}(\cdot)$ is the sign function, K_v is the viscous coefficient. The model (1) can be re-written with lumped uncertainties and disturbances as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \bar{A}x_2 + \bar{B}u + \bar{C}f(x_2) + F_e \\ y &= x_1 \end{aligned} \quad (3)$$

where y is the controlled output, $\bar{A} = -B/M$, $\bar{B} = k_f/M$, $\bar{C} = -1/M$ and $F_e = \bar{C}(\Delta Bx_2 + f_d)$ is the system uncertainty. The system uncertainty is unknown and shall be observed. Observing the unknown disturbance is underpinned by the assumption that the disturbance is constant during the observation period. This is practical for real applications if the update rate of the observer is sufficiently fast compared to changes in the system uncertainty, F_e .

B. Control Objective

The position reference is given as y^d and the initial reference may differ from the initial mover position. The control objective is to steer the uncertain, nonlinear model of the linear drive system (3) output y to track the desired reference, y^d . Mathematically, this means that

$$\lim_{t \rightarrow \infty} (y - y^d) \rightarrow 0, \quad (4)$$

despite the presence of model uncertainties and external disturbances affecting the system.

III. ADAPTIVE ROBUST BACKSTEPPING CONTROL DESIGN

This section describes the adaptive backstepping control system to attain position tracking as follows.

A. Robust Backstepping Control System

The objective of the position tracking can be achieved by steering the tracking error z_1 defined as

$$z_1 = y - y^d \quad (5)$$

towards zero. The derivative of the tracking error is

$$\dot{z}_1 = x_2 - \dot{y}^d. \quad (6)$$

In (6), the variable x_2 is viewed as a virtual control signal. Let the following stabilizing function be defined:

$$\eta = \dot{y}^d - k_1 z_1 \quad (7)$$

where $k_1 > 0$ is a design constant. Assume that the virtual control signal, i.e. the velocity of the mover, is given as $x_2 = \eta$, then we have

$$\begin{aligned} \dot{z}_1 &= x_2 - \dot{y}^d \\ &= -k_1 z_1 \end{aligned} \quad (8)$$

Define the first Lyapunov function as

$$V_1 = \frac{1}{2}z_1^2. \quad (9)$$

The derivative of this Lyapunov function is

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 \\ &= -k_1 z_1^2 \end{aligned} \quad (10)$$

Thus, the derivative of the Lyapunov function V_1 is negative definite for all values of z_1 , that is, it is globally non-increasing. Since the state vector x_2 may not be exactly equal to the stabilizing function η , let the following error be defined:

$$z_2 = x_2 - \eta \quad (11)$$

The derivative of z_2 yields

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\eta} \\ &= \bar{A}x_2 + \bar{B}u + \bar{C}f(x_2) + F_e - \dot{\eta} \end{aligned} \quad (12)$$

In order to obtain the backstepping controller, the lumped disturbance is assumed to have an upper bound, that is, $|F_e| \leq \bar{F}_e$, and the second Lyapunov function of the linear drive system is defined as

$$V_2(z_1(t), z_2(t)) = V_1 + \frac{1}{2}z_2^2. \quad (13)$$

By noting that $x_2 = z_2 + \eta$, the derivative of the second Lyapunov function gives

$$\begin{aligned} \dot{V}_2(z_1(t), z_2(t)) &= -k_1 z_1^2 + z_2 \dot{z}_2 \\ &= -k_1 z_1^2 + z_2 [\bar{A}(z_2 + \eta) + \bar{B}u \\ &\quad + \bar{C}f(x_2) + F_e - \dot{\eta}] \end{aligned} \quad (14)$$

According to (14), a backstepping control law can be designed as

$$\begin{aligned} u &= \bar{B}^{-1} [-k_2 z_2 - \bar{A}(z_2 + \eta) \\ &\quad - \bar{C}f(x_2) - \bar{F}_e \text{sgn}(z_2) + \dot{\eta}] \end{aligned} \quad (15)$$

where $k_2 > 0$ and by substituting (15) into (14) we obtain

$$\dot{V}_2(z_1(t), z_2(t)) = -k_1 z_1^2 - k_2 z_2^2 + z_2 F_e - |z_2| \bar{F}_e. \quad (16)$$

which is globally negative definite for all values of z_1, z_2 and F_e because $z_2 F_e - |z_2| \bar{F}_e \leq 0$, indicating that the Lyapunov function is globally non-increasing. Let the following be defined

$$D(t) = k_1 z_1^2 + k_2 z_2^2 \leq -\dot{V}_2(z_1(t), z_2(t)) \quad (17)$$

This means that we can write

$$\int_0^t D(\tau) d\tau \leq -V_2(z_1(t), z_2(t)) + V_2(z_1(0), z_2(0)) \quad (18)$$

By noting that $V_2(z_1(0), z_2(0))$ is bounded and $\dot{V}_2(z_1(t), z_2(t))$ is bounded and globally negative definite, it follows that

$$\lim_{t \rightarrow \infty} \int_0^t D(\tau) d\tau < \infty \quad (19)$$

Besides, it is noted that $\dot{D}(t)$ is also bounded which implies that $D(t)$ is uniformly continuous. Based on Barbalat's lemma [23], the following results hold:

$$\lim_{t \rightarrow \infty} D(t) = 0. \quad (20)$$

Therefore, it can be concluded that the errors z_1 and z_2 will converge to zero as $t \rightarrow \infty$. This means that the backstepping controller (15) is asymptotically stable even in the presence of parametric uncertainty and external disturbances. It is noted that the convergence of the errors means that $\lim_{t \rightarrow \infty} y(t) = y^d$ and $\lim_{t \rightarrow \infty} x_2(t) = \dot{y}^d$.

B. Robust Adaptive Backstepping Control System

It is noted that in the robust controller (15), an upper bound \bar{F}_e is arbitrarily defined for the system uncertainty. This means that the worst-case scenario is used which can result in a conservative performance of the controller. Hence, we propose a simple technique to adapt the disturbance bounds used in the controller to be less conservative. To do this, the delayed input and state variables are employed as follows.

$$\begin{aligned} \hat{F}_e &= c - \bar{A}x_2(t - \tau) + \bar{B}u(t - \tau) \\ &\quad + \bar{C}f(x_2(t - \tau)) \end{aligned} \quad (21)$$

where x_2 denote the current measurement of the mover velocity, $x_2(t - \tau)$ and $u(t - \tau)$ are delayed velocity and input variables. The intuition behind the use of τ is to capture how the time-varying disturbance changes based on the evolution of the linear motor dynamics. The selection of this delay is guided by a consideration of the system's transient response, ensuring that τ sufficiently captures the evolving dynamics over time. Based on this approximation, the upper bound on the lumped uncertainty is given by

$$\bar{F}_e^a = \begin{cases} \hat{F}_e, & \text{for } |\hat{F}_e| \leq \bar{F}_e \\ \bar{F}_e, & \text{for } |\hat{F}_e| > \bar{F}_e \end{cases} \quad (22)$$

where $|F_e| \leq \bar{F}_e$ represent the user-defined disturbance upper bound. For simplicity, the upper bound can be chosen arbitrarily as the value that gives the least acceptable tracking performance under the robust control law. This ensures that the worst-case scenario defined by the user is not exceeded as this may lead to a control signal that is too conservative to provide meaningful tracking performance. Consequently, the robust control law is modified to obtain the adaptive law as

$$\begin{aligned} u^a &= \bar{B}^{-1} [-k_2 z_2 - \bar{A}(z_2 + \eta) \\ &\quad - \bar{C}f(x_2) - \bar{F}_e^a \text{sgn}(z_2) + \dot{\eta}] \end{aligned} \quad (23)$$

Note that the asymptotic stability of (15) remains intact through the utilization of the adaptive mechanism (22). This is ensured by the fact that the stability condition $z_2 F_e - |z_2| \bar{F}_e \leq 0$ is always satisfied.

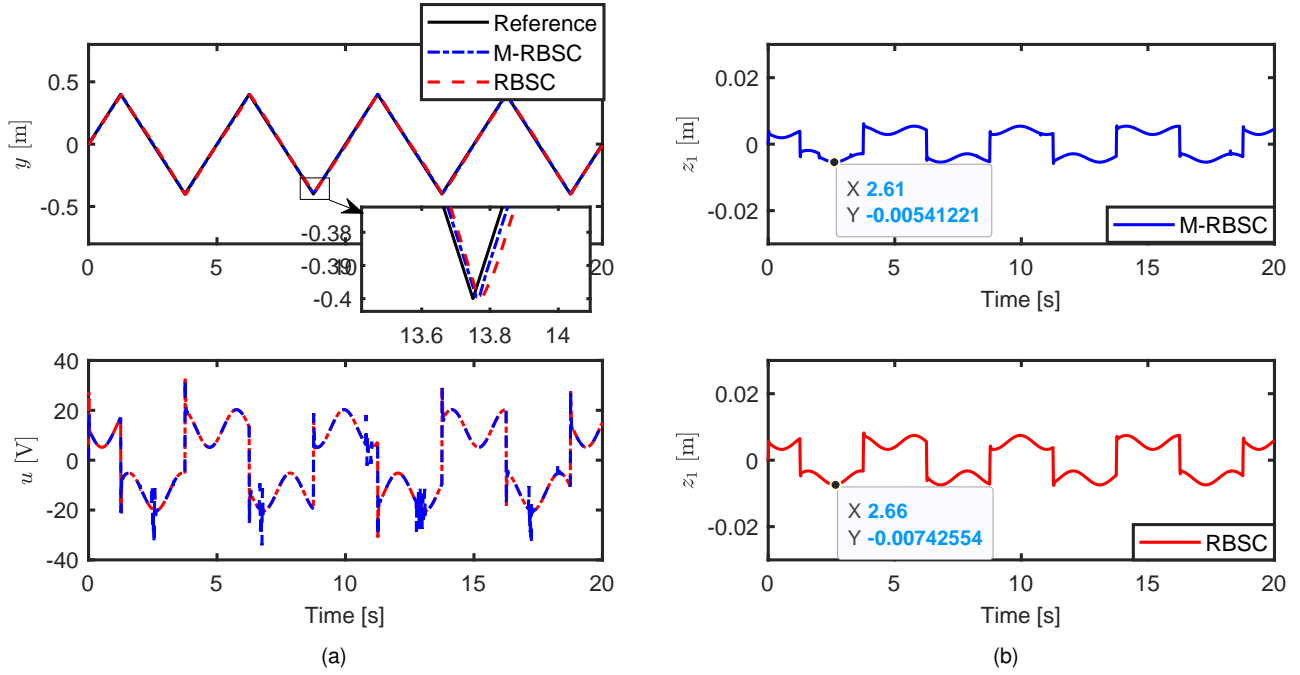


Fig. 1. Triangular position reference tracking with external disturbances. (a) Results showing the position tracking (top) and control signal (bottom). (b) Tracking error of the compared controllers.

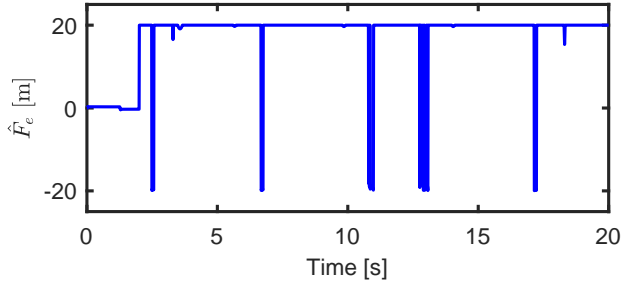


Fig. 2. Triangular position reference tracking scenario: disturbance bounds based on (22) used to implement M-RBSC

IV. NUMERICAL SIMULATION

The linear drive system is simulated using the robust backstepping controller (RBSC) with a constant upper bound (15) and the robust backstepping controller (23) in which the upper bounds are adaptively determined using (M-RBSC) (22) under different conditions in the presence of frictional force. The system is assumed to be affected by model uncertainty and external disturbance given by

$$\Delta B = 4B, \quad f_d = 15 \sin(2t) \text{ N.} \quad (24)$$

The parameters of the linear drive system with a mover are given as follows [2]: $M = 0.3$ kg, $B = 0.7954$ N.s/m and $k_f = 1$ N/V. The frictional force parameters are given as $f_c = 0.006$, $f_s = 0.01$, $\dot{x}_s = 0.1$, $K_v = 5$. A unit delay is used to implement (22). The backstepping controllers are implemented using the parameters:

$$k_1 = 100, \quad k_2 = 80, \quad \bar{d} = 20 \quad (25)$$

The sign function in the backstepping control law (15) makes it discontinuous. Since a continuous control signal is desired for the practical application of the control law, an approximation of this function is used. Different functions can be employed. In this work, the following approximation of the sign function [2] is proposed for the nominal controller (15) as follows:

$$\text{sign}(x_2) = \frac{2}{\pi} \arctan \left(900 \frac{2}{\pi} x_2 \right) \quad (26)$$

The results obtained under two different reference signals are presented in the next subsection.

A. Simulation Results

Two types of reference signal tracking problems are considered. First, a triangular periodic reference trajectory is considered for the mover position. The simulation results from the backstepping controllers under the triangular reference signal are presented in Fig. 1. In Fig. 1a, it is seen that the M-RBSC scheme provides better performance, especially around the point of change in direction of the reference signal. It is noted that the RBSC with a constant upper bound has a smoother input voltage compared to the M-RBSC because the latter uses an upper bound that is changed online as shown in Fig. 2.

The overall tracking error of the controllers is shown in Fig 1b and it is seen that the M-RBSC shows reduced tracking error. The absolute maximum error of the M-RBSC is 0.0062 m while that of the RBSC is 0.0082 m. The tracking error measured in terms of the root mean square error of the M-RBSC is 0.0041 m which represents a 31% improvement

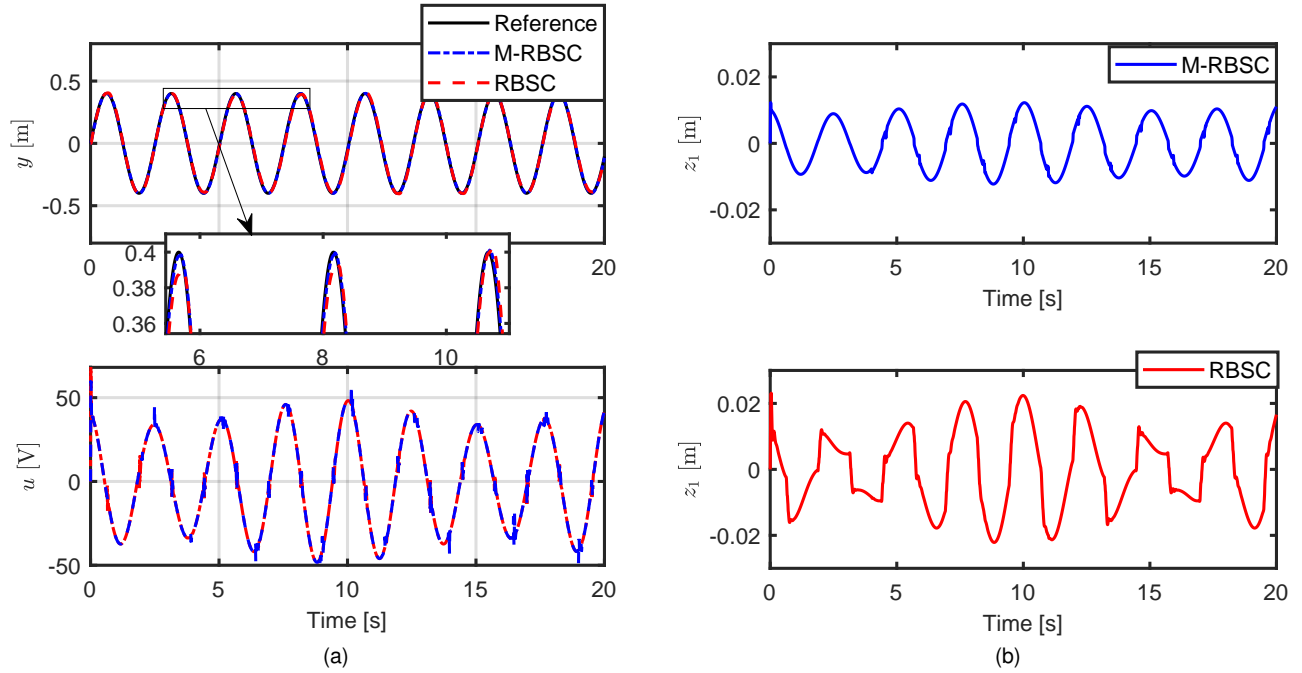


Fig. 3. Sinusoidal position reference tracking. (a) Results showing the position tracking (top) and control signal (bottom). (b) Tracking error of the compared controllers.

over the RBSC with a root mean square error of 0.0054 m. The superiority of the M-RBSC control stems from its ability to modify the upper bound during the operation of the control system based on the current statement measurement and the delayed input and state variables.

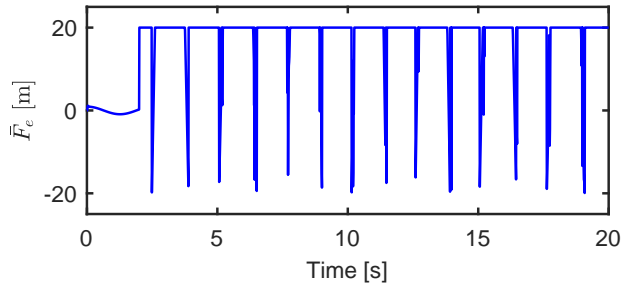


Fig. 4. Sinusoidal reference tracking scenario: disturbance bounds based on (22) used to implement M-RBSC

In Fig. 3, the performance of the studied controllers tracking sinusoidal reference trajectory is shown. Similar to the previous case, the M-RBSC and RBSC are compared. The controllers both provided good tracking performance under the effects of the disturbances but the zoomed-in section shows that the M-RBSC scheme was able to more closely follow the reference signal compared to the RBSC with a constant upper bound considering the worst-case scenario. The variation of the upper bound used by the M-RBSC scheme is shown in Fig. 4. It is, however, noted that this variation in the upper bound does cause some random changes in the control input voltage generated by

the M-RBSC, making it less smooth compared to the RBSC scheme.

The tracking errors for both control schemes are presented in Fig. 3b. Whereas the tracking error from the RBSC scheme exceeded 0.02 m, the M-RBSC scheme stayed below 0.02 m. Specifically, the root mean square error of the M-RBSC scheme is 0.0077 m while that of the RBSC is 0.0115 m. Also, the maximum absolute error of the RBSC scheme is 0.022 m and that of the M-RBSC strategy is 0.012 m. Therefore, the results show that a significant performance improvement is achieved through the use of adaptive upper bounds.

V. CONCLUSIONS

This paper has presented a robust backstepping control system with a stability guarantee for the linear drive system position control. The model of the linear drive system with parameter uncertainty, discontinuous frictional force, and external disturbance is first presented. Then, a robust control scheme in which an upper bound needs to be defined for its implementation is designed. By noting that the use of this upper bound can lead to conservative performance in practical applications, we propose a simple technique to adaptively adjust the upper bounds of the disturbances based on the current state measurement and the delayed input and state of the system. The simulation results demonstrated that the use of an adaptively varying disturbance upper bound can result in significant performance improvement measured in terms of improved position tracking accuracy measured by both the root mean square error and the absolute maximum error.

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