Erratum: Optimal State Choice for Rydberg-Atom Microwave Sensors [Phys. Rev. Appl. 16, 024008 (2021)]

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I. INSENSITIVITY TO LINEAR POLARIZATION

Recent results from Cloutman et al. [1] highlights a sign error in the matrix elements presented in Appendix B of our original paper [2]. This resulted in an incorrect statement that the EIT spectrum for a transition from $|ns_{1/2}, m_j = \pm 1/2\rangle \rightarrow |n'p_{1/2}, m_j = \pm 1/2\rangle$ is dependent on angle for the case of a linear polarization rotated around the angle of propagation. Below, we provide a corrected derivation which reproduces the result of [1] and verifies that this transition is insensitive to the rotation angle for the case of a linear polarization vector.

Correcting the signs on the matrix elements for transitions $\langle p_{1/2}, m_j = \pm 1/2|er_q|s_{1/2}, m_j = \pm 1/2\rangle$, where q denotes the polarization corresponding to a spherical basis vector, the Hamiltonian for the couplings from $|ns_{1/2}, m_j = \pm 1/2\rangle \rightarrow |n'p_{1/2}, m_j = \pm 1/2\rangle$ becomes

$$H = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & -\Omega_0 & -\Omega_+ \\ 0 & 0 & \Omega_- & \Omega_0 \\ -\Omega_0^* & \Omega_-^* & 0 & 0 \\ -\Omega_+^* & \Omega_0^* & 0 & 0 \end{pmatrix}, \tag{1}$$

where the basis vectors are ordered as $\{|s_{1/2}, m_j = -1/2\rangle, |s_{1/2}, m_j = 1/2\rangle, |p_{1/2}, m_j = -1/2\rangle, |p_{1/2}, m_j = 1/2\rangle\}$. Here $\Omega_0 = dE_0/(\sqrt{6}\hbar)$ and $\Omega_\pm = dE_\pm/(\sqrt{3}\hbar)$, where $E_q = \mathbf{E}\cdot\mathbf{e_q}$ is the projection of the electric field onto the spherical basis and $d = \langle n'p_{1/2}||er||ns_{1/2}\rangle$ is the reduced matrix element for the transition. The corresponding eigenvalues for this matrix can be found as

$$\lambda = \frac{1}{4}\left[\pm(\Omega_+ + \Omega_-) \pm \sqrt{(\Omega_+ - \Omega_-)^2 + 4\Omega_0^2}\right]. \tag{2}$$

For the case of a linearly polarized microwave propagating along the y-axis and rotated by angle θ from the z-axis, the electric field can be written as $\mathbf{E}_\mu = \tilde{E}_\mu(\sin\theta\,\mathbf{e_x} + \cos\theta\,\mathbf{e_z})$ which in spherical coordinates becomes $\mathbf{E}_\mu = \tilde{E}_\mu(\frac{\sin\theta}{\sqrt{2}}\mathbf{e_{-1}} + \cos\theta\,\mathbf{e_0} - \frac{\sin\theta}{\sqrt{2}}\mathbf{e_{+1}})$. Substituting this into the equation above reduces the eigenvalues to $\lambda(\theta) = \pm\hbar\Omega_0/2$ independent of θ, meaning in this case the transition is insensitive to the rotation of the linear polarization. Consequently Fig. 7 in the original paper [2] should look as shown in Fig. 1(a). This insensitivity to polarization angle θ in the presence of a pure linear polarization propagating along y has been confirmed experimentally by Cloutman *et al.* [1] when studying the transition $36S_{1/2} \rightarrow 36P_{1/2}$ at a frequency of 86 GHz.

II. SENSITIVITY TO AN ARBITRARY INCIDENT POLARIZATION

The arguments above show that for any polarization that can be described as a rotated linear polarization we should recover only two eigenstates independent of θ. We now consider the case of a microwave electric field vector of the form $\mathbf{E}_\mu = \tilde{E}_\mu(\frac{\sin\vartheta}{\sqrt{2}}\mathbf{e_{-1}} + \cos\vartheta\,\mathbf{e_0} + \frac{\sin\vartheta}{\sqrt{2}}\mathbf{e_{+1}})$, corresponding to a change in sign of the $\mathbf{e_{+1}}$ component. Plotting the eigenstates as a function of ϑ as shown in Fig. 1(b) we recover the result shown in Fig. 7 of our original Appendix B that it is possible to engineer more than two eigenenergies when varying the polarization angle.

To understand how this effect can be observed on a transition that is insensitive to linear polarization angle, it is instructive to map the fitted field $\mathbf{E}_\mu$ back to Cartesian coordinates using $\mathbf{e}_\pm = \mp(\mathbf{e_x} \pm i\mathbf{e_y})/\sqrt{2}$, from which we find $\mathbf{E}_\mu = \tilde{E}_\mu(-i\sin\vartheta\,\mathbf{e_y} + \cos\vartheta\,\mathbf{e_z})$. This no longer describes a rotated linear polarization, but instead a complex polarization

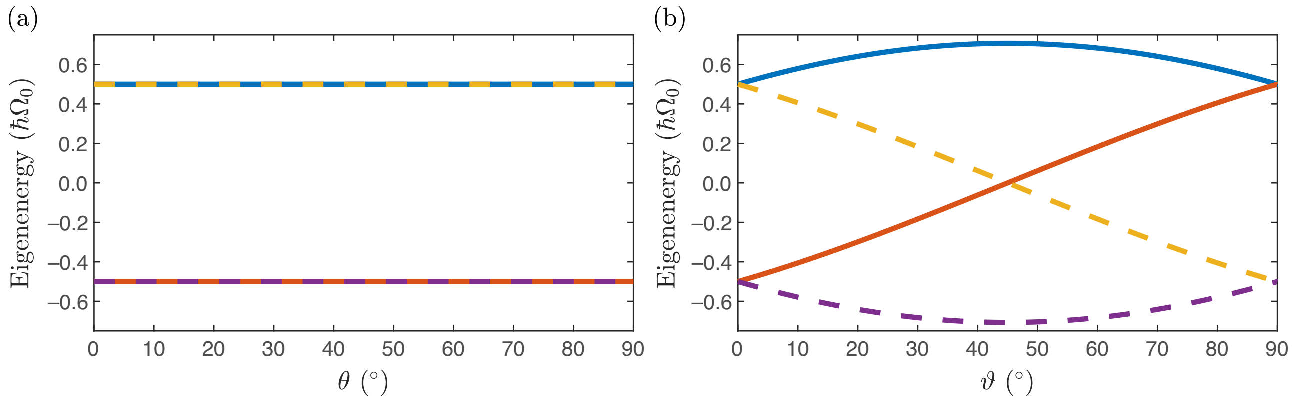

FIG. 1. Autler-Townes eigenenergies in units of $\hbar\Omega_0 = d\tilde{E}_\mu/(\sqrt{6})$ for $nS_{1/2} \rightarrow n'P_{1/2}$ Rydberg levels as a function of microwave polarization. (a) For linearly polarized microwaves propagating along the y-axis with $\mathbf{E}_\mu = \tilde{E}_\mu(\sin\theta\mathbf{e_x} + \cos\theta\mathbf{e_z})$ the splitting is independent of angle θ. (b) For a complex polarization $\mathbf{E}_\mu = \tilde{E}_\mu(-i\sin\vartheta\,\mathbf{e_y} + \cos\vartheta\,\mathbf{e_z})$ the resulting energy levels are sensitive to the polarization angle ϑ with up to four distinct eigen-energies present.

with a 90° phase shift between the components in the y and z planes. For $\vartheta = 0, 90°$ this recovers two eigenenergies as it represents linear polarization along the z and x axes respectively, but at intermediate angles three or four levels will be observed.

Due to the symmetry of this feature about $\vartheta = 45°$ and the fact that it cannot descriminate between a choice of $\pm i\mathbf{e_y}$ it is not possible to extract a unique value of the electric field polarization within the cell, making this poorly suited to polarimetry applications. However, this shows that for an arbitrary incident polarisation the $S_{1/2} \rightarrow P_{1/2}$ is not guaranteed to be polarization insensitive.

III. INTERPRETATION OF EXPERIMENTAL OBSERVATIONS

The motivation for exploring the polarization sensitivity of the $nS_{1/2} \rightarrow nP_{1/2}$ transition comes from data presented in Fig. 5 of the original paper [2] and reproduced here in Fig 2, which shows that in the case where the microwave antenna

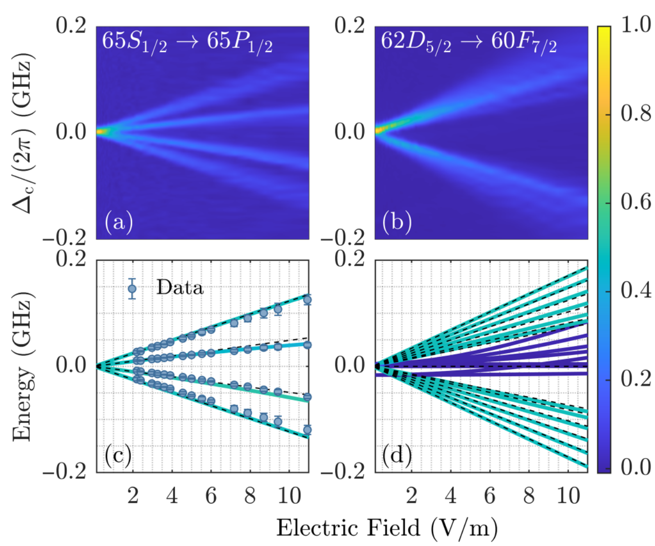

FIG. 2. Impact of microwave scattering. (a) $65S_{1/2} \rightarrow 65P_{1/2}$ and (b) $62D_{5/2} \rightarrow 60F_{7/2}$ spectra as a function of MW field when the antenna is rotated by 5° around the $\hat{y}$-axis. (c)-(d) Corresponding energy lines obtained with our model. The dashed lines indicate the AT splitting expected in the fine structure basis. In (c), the data points represent the AT peaks positions extracted using a four-Gaussian fit. The error bars are the standard error of the fit normalized by the reduced χ^2_v.

was rotated by $\theta = 5^\circ$ about the y-axis, the transition from $65S_{1/2} \rightarrow 65P_{1/2}$ is seen to split into four distinct levels (Fig 2(a)), whilst the $62D_{5/2} \rightarrow 60F_{7/2}$ in Fig 2(b) maintains a two-level structure.

Using our multi-level model we recover excellent quantitative agreement with the experimental data when fitting an electric field polarization of the form $\mathbf{E}_\mu = \tilde{E}_\mu(\frac{\sin\vartheta}{\sqrt{2}}\mathbf{e_{-1}} + \cos\vartheta\,\mathbf{e_0} + \frac{\sin\vartheta}{\sqrt{2}}\mathbf{e_{+1}})$, which as shown above predicts more than two levels can be observed. Results are shown in Fig 2(c,d) for which we obtain $\vartheta = 23.3(3)^\circ$ from fitting to the $65S_{1/2} \rightarrow 65P_{1/2}$ data in (a). blackThe energy levels predicted by the model are identical to those presented in [2], but now calculated by correct propagation of the signs on the spherical polarization vectors and matrix elements.

In the experiment we expect a 5° rotation of the antenna to simply introduce a small percent level component of the field along the $x-$ axis, however we find our results can be predicted using a complex polarization with a phase-shifted component now along the y-axis. The presence of a phase-shifted electric field component along the y-axis in the experiment could arise due to interference of the main microwave field with reflections from the glass cell, mounts and optical table which can have different k-vectors as suggested in the original manuscript. This effect of microwave scattering is more pronounced at lower microwave frequencies (in this case 13 GHz) due to the strong scaling of scattering cross-section with wavelength, and may explain why no scattering effect were observed on the experiments of [1] probing at 86 GHz.

However, this result highlights the fact that for an arbitrary incident microwave polarization vector the resulting spectra can possess more than the two levels predicted for a pure linear polarization, and that in a realistic environment the $S_{1/2} \rightarrow P_{1/2}$ transition can perform worse than $D_{5/2} \rightarrow F_{7/2}$ transitions when seeking to perform microwave sensing, corroborating the conclusion of our original work.

IV. SUMMARY

In summary, we have corrected the derivation presented in Appendix B of our original paper used to explain polarization sensitivity, and verify the results of [1] that the transition from $nS_{1/2} \rightarrow nP_{1/2}$ is inherently insensitive to the rotation of a pure, linear microwave electric field. In the regime of an arbitrary polarization vector, the arguments applied in Sec. I and in [1] to show there can only be two energy eigenvalues no longer apply. Using a complete model of the interacting Rydberg levels with the correct matrix elements, we are able to reproduce our experimental observations by fitting with a complex polarization vector that cannot be described as a rotated linear field.

Thus for microwave sensing in realistic environments, with imperfections arising due to scattered fields and arbitrary electric field polarization, our conclusion that the $62D_{5/2} \rightarrow 60F_{7/2}$ is more robust for sensing than the $65S_{1/2} \rightarrow 65P_{1/2}$ remains a valid assertion as evidenced by the data presented in Fig. 2(a) and (b) independent of our model.

[1] M. Cloutman, M. Chilcott, A. Elliott, J. S. Otto, A. B. Deb, and N. Kjærgaard, Phys. Rev. Appl. **21**, 044025 (2024).
[2] A. Chopinaud, and J. Pritchard, Phys. Rev. Appl. **16**, 024008 (2021).