

# ROBUST BACKSTEPPING CONTROL OF INDUCTION MOTOR DRIVES USING ARTIFICIAL NEURAL NETWORKS AND SLIDING-MODE FLUX OBSERVERS

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**Abstract** In this paper, using the three-phase induction motor fifth order model in a stationary two axis reference frame with stator current and rotor flux as state variables, a conventional backstepping controller is first designed for speed and rotor flux control of an induction motor drive. Then in order to make the control system stable and robust against all electromechanical parameter uncertainties as well as to the unknown load torque disturbance, the backstepping control is combined with artificial neural networks in order to design a robust nonlinear controller. It will be shown that the composite controller is capable of compensating the parameters variations and rejecting the external load torque disturbance. The overall system stability is proved by the Lyapunov theory. It is also shown that the method of artificial neural network training, guarantees the boundedness of errors and artificial neural network weights. Furthermore, in order to make the drive system free from flux sensor, a sliding-mode rotor flux observer is employed that is also robust to all electrical parameter uncertainties and variations. Finally, the validity and effectiveness of the proposed controller is verified by computer simulation.

**Keywords** Artificial Neural Network, Backstepping, Induction Motor, Observer, Robust, Sliding-Mode

**چکیده** در این مقاله، با استفاده از مدل مرتبه پنجم موتور القایی در مختصات مرجع ساکن استاتور، ابتدا یک کنترل کننده گام به گام به عقب متعارفی برای کنترل سرعت و شار استاتور درایو موتور القایی طراحی می شود. سپس برای مقاوم سازی سیستم کنترل در برابر همه نامعینی های پارامتری الکتریکی و مکانیکی و گشتاور بار نامشخص، کنترل گام به گام به عقب با شبکه های عصبی ترکیب می شود تا یک کنترل مقاوم ترکیبی حاصل شود. نشان داده می شود که کنترل کننده ترکیبی قادر به جبران تغییرات پارامتری و تحمل گشتاور بار نامشخص است. پایداری سیستم کلی به کمک تئوری لیاپانف اثبات می شود. همچنین نشان داده می شود که روش آموزش شبکه عصبی، تضمین کننده کراندار بودن خطاها و وزن های شبکه عصبی است. به علاوه، برای رفع نیاز سیستم کنترل درایو به حسگر شار، یک مشاهده گر مد لغزشی شار رتور به کار گرفته می شود که نسبت به تغییرات همه پارامترهای الکتریکی مقاوم است. در نهایت، عملکرد مناسب کنترل کننده پیشنهادی با شبیه سازی مورد آزمایش قرار می گیرد.

## 1. INTRODUCTION

In the last two decades, nonlinear control methods such as input-output feedback linearization and Sliding-Mode(SM) control have been applied to the Induction Motor (IM) drive [1-2]. Especially in recent years, in the field of adaptive and robust control, there has been a tremendous amount of activity on a special control scheme known as

“backstepping” [3-5]. A major problem of the backstepping control approach is that certain function must be “linear in the unknown system parameters” and in addition, some very tedious analysis is needed to determine a “regression matrix” [5]. It must be noted that in adaptive backstepping control, the problem of finding a regression matrix is more difficult in comparison with conventional backstepping method.

Artificial Neural Network (ANN) has been applied to system identification or identification-based control. Uncertainty on how to initialize the ANN weights leads to the necessity for “preliminary off-line tuning”. Recently many ANN controllers have been proposed for various control applications that can provide closed-loop stability [6-8].

To overcome the above problems, in [9], a combination of backstepping control and ANN has been proposed. According to this method, in the process of backstepping controller design, two ANN are used to estimate two nonlinear functions. Therefore there is no need to find the regression matrix for on-line estimation of unknown parameters.

In [9], using the ANN, the theory of robust backstepping control has been presented for strictly feedback nonlinear systems. This method has been applied to a single arm robot in [10] and to a rotor flux Field Oriented Control (FOC) IM drives in [11].

One may note that the FOC methods are in fact a type of partial feedback linearization control technique in which the zero dynamic stability can not be proven. As a result, through this method, it can not be guaranteed that the system model preserves its robustness against the parameter variations. In addition in these control methods, the field orientation can be achieved only in the system steady state conditions. Moreover, the control method of [11] is only robust with respect to the rotor resistance variation.

To overcome the above problems, in this paper, using the fifth order model of IM in a fixed stator reference frame, with stator currents and rotor fluxes as state variables, using the nonlinear method described in [9], a composite nonlinear controller is designed that makes the IM drive system control robust and stable against the motor parameter uncertainties and external load torque disturbance. In this control approach, a two level SVPWM inverter feeds the IM drive.

Furthermore, the rotor flux is estimated by a SM observer which is also robust to all electrical parameter uncertainties.

## 2. IM MODEL

The IM fifth order model in a fixed two axis

reference frame with rotor fluxes and stator currents as state variables [12], is given by

$$\frac{d\omega}{dt} = \frac{3n_p M}{2JL_r} (\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) - \frac{T_l}{J} \quad (1)$$

$$\frac{d\psi_{ra}}{dt} = -\frac{R_r}{L_r} \psi_{ra} - n_p \omega \psi_{rb} + \frac{R_r}{L_r} M i_{sa} \quad (2)$$

$$\frac{d\psi_{rb}}{dt} = -\frac{R_r}{L_r} \psi_{rb} + n_p \omega \psi_{ra} + \frac{R_r}{L_r} M i_{sb} \quad (3)$$

$$\frac{di_{sa}}{dt} = \frac{MR_r}{\sigma L_s L_r^2} \psi_{ra} + \frac{n_p M}{\sigma L_s L_r} \omega \psi_{rb} - \left\{ \frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \right\} i_{sa} + \frac{1}{\sigma L_s} u_{sa} \quad (4)$$

$$\frac{di_{sb}}{dt} = \frac{MR_r}{\sigma L_s L_r^2} \psi_{rb} - \frac{n_p M}{\sigma L_s L_r} \omega \psi_{ra} - \left\{ \frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \right\} i_{sb} + \frac{1}{\sigma L_s} u_{sb} \quad (5)$$

where  $i_{sa}, i_{sb}, \psi_{ra}, \psi_{rb}, u_{sa}, u_{sb}$  are the stator currents, rotor fluxes and stator voltages, respectively. Subscripts a,b indicate vector components in the fixed stator reference frame. Subscripts r,s indicate rotor and stator components.  $\omega$  is the rotor angular mechanical speed and  $\sigma = 1 - M^2 / (L_s L_r)$ .

$L_s, L_r$  are per-phase stator and rotor special inductances, respectively.  $M$  is the per phase magnetizing inductance.  $n_p$  is number of pole pairs.  $R_s, R_r$  are stator and rotor resistances, respectively.

## 3. ROBUST BACKSTEPPING CONTROL

**3.1. ANN Basics** Define  $W$  as the collection of

ANN weights, then the net output is [4]

$$y = W^T \phi(x) \quad (6)$$

Let  $S$  be a compact simply connected set of  $\mathfrak{R}^n$ , with map  $f : S \rightarrow \mathfrak{R}^n$ , define  $C^m(S)$  the functional space such that  $f$  is continuous. A general nonlinear function  $f(x) \in C^m(S)$ ,  $x(t) \in S$  can be approximated by a neural network as

$$f(x) = W^T \phi(x) + \varepsilon(x) \quad (7)$$

with  $\varepsilon(x)$  an ANN functional reconstruction error vector and  $\phi(x)$  is sigmoid activation function.

### 3.2. Robust Backstepping Control of IM Using ANN

Using the well known fifth order IM model in a stator two axis reference frame where the rotor fluxes and stator currents are assumed as state variables [12], the robust nonlinear controller is designed in the following way.

Dividing the above IM model into two nonlinear sub-systems, where  $i_{sa}, i_{sb}$  are the outputs for the first sub-system which are simultaneously assumed the fictitious inputs of the second sub-system. It is assumed that:

- The reference trajectories  $\omega^r$  and  $\psi_r^r$  are differentiable and bounded.
- The load torque is an unknown constant and resistances, inductances and moment of inertia are unknown and bounded.

In the first step of the controller design,  $i_{sa}, i_{sb}$  are assumed as fictitious controls for the second sub-system. The main objective is to obtain these controls so that the desired rotor speed and rotor flux amplitude signals are perfectly tracked in spite of machine parameters and external load torque uncertainties.

Considering  $\omega^r$  and  $\psi_r^r$  as references for  $\omega$  and  $\psi_r$ , tracking error equations are

$$\begin{aligned} e_1 &= \omega - \omega^r \\ e_2 &= \psi_{ra}^2 + \psi_{rb}^2 - \psi_r^{r2} = \psi_r^2 - \psi_r^{r2} \end{aligned} \quad (8)$$

then

$$\begin{aligned} D_1 \dot{e} &= F_1 + G_1 i, F_1 = \begin{bmatrix} -\frac{T_l L_r}{M} - J \frac{L_r}{M} \dot{\omega}^r \\ -\frac{2}{M} \psi_r^2 - 2 \frac{L_r}{R_r M} \psi_r^r \dot{\psi}_r^r \end{bmatrix} \\ G_1 &= \begin{bmatrix} -\frac{3n_p}{2} \psi_{rb} & \frac{3n_p}{2} \psi_{ra} \\ \psi_{rb} & \psi_{ra} \end{bmatrix}, D_1 = \begin{bmatrix} J \frac{L_r}{M} & 0 \\ 0 & \frac{L_r}{R_r M} \end{bmatrix} \end{aligned} \quad (9)$$

Where

$$e = [e_1, e_2]^T.$$

It is clear that  $G_1$  is known and invertible. By treating  $i$  as a fictitious input, a controller for the ideal  $i(\bar{i})$  is designed as

$$\bar{i} = G_1^{-1} [\hat{F}_1 - K_1 e], K_1 > 0 \quad (10)$$

where  $K_1$  a design parameter and  $\hat{F}_1$  the estimate of  $F_1$  which will be estimated in the next section with a two layer ANN. Substituting 10 into 9 gives

$$D_1 \dot{e} = F_1 - \hat{F}_1 - K_1 e + G_1 \eta, \eta = i - \bar{i} \quad (11)$$

In the second step, the control  $u(u_{sa}, u_{sb})$  are obtained in such a way that  $\eta$  in Equation 11, becomes as small as possible. Differentiating  $\eta$  with respect to time  $t$ , yields

$$D_2 \dot{\eta} = F_2 + G_2 u \quad (12)$$

Where

$$\begin{aligned} u &= \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix}, G_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_2 = \sigma L_s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ F_2 &= \dots + D_2 \{ G_1^{-1} (\hat{F}_1 + K_1 e) + G_1^{-1} \dot{\hat{F}}_1 \\ &+ G_1^{-1} K_1 D_1^{-1} (F_1 - \hat{F}_1 - K_1 e + G_1 \eta) \} \end{aligned} \quad (13)$$

To make  $\eta$  as small as possible, the following control is chosen

$$u = G_2^{-1}[-\hat{F}_2 - K_2\eta - G_1^T e] \quad (14)$$

In 14,  $\hat{F}_2$  is an estimate of  $F_2$  in that like the first step, a two layer ANN is used to estimate it. In addition a term  $-G_1^T e$  is added in 14 which is necessary to cancel the effect of  $G_1\eta$  in 11.

Combining 12 and 14 gives

$$D_2 \dot{\eta} = F_2 - \hat{F}_2 - K_2\eta - G_1^T e \quad (15)$$

**3.3.  $F_1, F_2$  Approximation** In this section, functions  $F_1, F_2$  are approximated two-by-two layer ANN. In adaptive backstepping control, it is assumed that functions  $F_1, F_2$  are linear in terms of known regression matrices, however in the ANN method there is no limitation for these functions. Using ANNs approximation property,  $F_1, F_2$  as outputs of two two-layer ANN with constant weights  $w_i$ , is assumed to be as follows

$$\begin{aligned} F_1 &= W_1^T \phi_1 + \varepsilon_1, \quad \|\varepsilon_1\| < \varepsilon_{1N} = \text{cte} \\ F_2 &= W_2^T \phi_2 + \varepsilon_2, \quad \|\varepsilon_2\| < \varepsilon_{2N} = \text{cte} \end{aligned} \quad (16)$$

where  $\phi_1, \phi_2$  provide suitable basis functions. From 16, one can find that net reconstruction error

$\varepsilon_i(x)$  is bounded by a known constant  $\varepsilon_{iN}$ .

- The ideal weights are bounded by known positive values so that

$$\|W_1\|_F \leq W_{1M}, \quad \|W_2\|_F \leq W_{2M} \quad (17)$$

or equivalently:

$$\|Z\|_F \leq Z_M, \quad Z = \text{diag}\{W_1, W_2\} \quad (18)$$

The actual inputs to ANN1 are  $\psi_r, \omega^r, \psi_r^r, \psi_r^r$  and

actual inputs to ANN2 are  $\omega, \omega^r, \omega^r, \psi_r, \psi_r^r, \psi_{ra}, \psi_{rb}, \psi_r^r, i_{sa}, i_{sb}, e_1, e_2$ .

Considering  $F_1, F_2$ , it is clear why these inputs are selected for each neural network. For example,  $F_1$  contains signals  $\psi_r$  (that is measured) and  $\omega^r, \psi_r^r, \psi_r^r$  which are known as desired trajectories. Uncertainties such as  $T_1, L_r, M, R_r, J$  also exist in  $F_1$  that should be estimated by NN1. As shown in Figure 1,  $W_1$  and  $W_2$  are second layer weights of NN1 and NN2, respectively. For first weights of these NNs, small positive constants are selected. Each NN has two outputs that are first and second

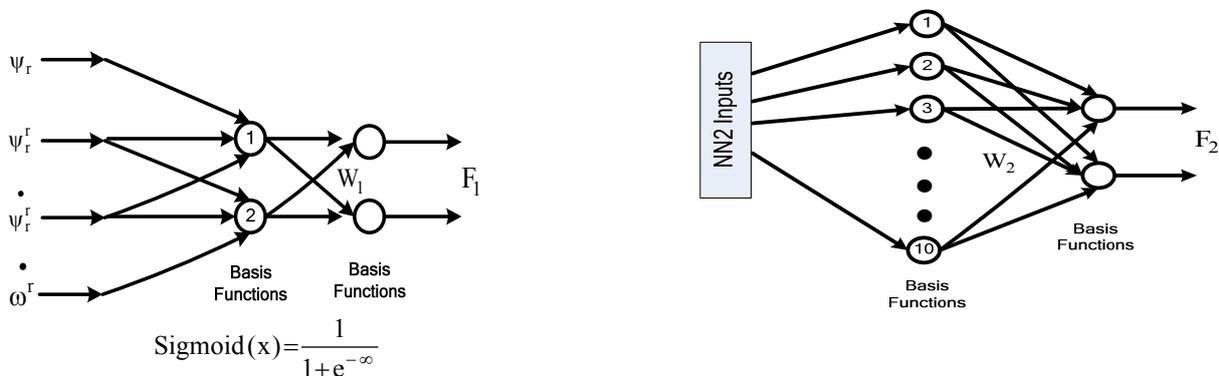


Figure 1. Neural networks structure with inputs and outputs.

elements of estimated matrix functions  $F_1$  and  $F_2$ . For example, first output of NN1 is estimate of

$$\left( \frac{-T_1 L_r}{M} - J \frac{L_r}{M} \dot{\omega}^r \right)$$

and second output of NN1 is

$$\text{estimate of } \left( -\frac{2}{M} \psi_r^2 - 2 \frac{L_r}{R_r M} \psi_r^r \psi_r^r \right).$$

On line ANN approximation of  $F_1$  is

$$\hat{F}_1 = \hat{W}_1^T \phi_1 \quad (19)$$

Then the error dynamic Equation of 11 becomes

$$D_1 \dot{e} = \tilde{W}_1^T \phi_1 - K_1 e + G_1 \eta + \varepsilon_1 \quad (20)$$

where  $\tilde{W}_1 = W_1 - \hat{W}_1$  is the error in weight estimation. Similarly, approximation of  $F_2$  is assumes as

$$\hat{F}_2 = \hat{W}_2^T \phi_2 \quad (21)$$

then the error dynamic 15 will be

$$D_2 \dot{\eta} = \tilde{W}_2^T \phi_2 - K_2 \eta - G_1^T e + \varepsilon_2 \quad (22)$$

Note that there is a term  $G_1 \eta$  in 20 and a term  $-G_1^T e$  in 22. This means there are couplings between the error dynamics 20 and 22.

**3.4. Updating ANNs Weights** In this part, the stability of the proposed controller is proven based on Lyapunov's stability theory. This analysis shows that tracking errors and updated weights are Uniformly Ultimately Bounded (UUB).

Theory: Let the desired trajectories  $\omega$ ,  $\psi_r$  be bounded. Take the control input 14 with weight updates be provided by

$$\begin{aligned} \dot{\hat{W}}_1 &= \Gamma_1 \phi_1 e^T - k_\omega \Gamma_1 \|\zeta\| \hat{W}_1 \\ \dot{\hat{W}}_2 &= \Gamma_2 \phi_2 e^T - k_\omega \Gamma_2 \|\zeta\| \hat{W}_2 \end{aligned} \quad (23)$$

with any constant matrices  $\Gamma_1 = \Gamma_1^T > 0$ ,  $\Gamma_2 = \Gamma_2^T > 0$  and scalar positive constant  $k_\omega$ . Then the errors  $\eta(t), e(t)$  are UUB. ANN updated weights are bounded. The errors  $\eta(t), e(t)$  can be kept as small as desired by increasing gains  $K_i$ .

**Proof:** Define the following Lyapunov function

$$V = \frac{1}{2} \zeta^T \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \zeta + \frac{1}{2} \text{tr} \left( \tilde{Z}^T \Gamma^{-1} \tilde{Z} \right)$$

Where

$$\begin{aligned} \zeta &= \begin{bmatrix} e^T, \eta^T \end{bmatrix}^T, Z = \text{diag} \left[ W_1, W_2 \right], \Gamma = \text{diag} \left[ \Gamma_1, \Gamma_2 \right] \\ K &= \text{diag} \left[ K_1, K_2 \right]. \end{aligned}$$

Derivation of  $V$  with using of 20 and 22 yields

$$\begin{aligned} \dot{V} &= \zeta^T \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \dot{\zeta} + \text{tr} \left( \tilde{Z}^T \Gamma^{-1} \dot{\tilde{Z}} \right) = \\ &= \zeta^T \left( \tilde{Z}^T \phi + \varepsilon - K \zeta \right) + \text{tr} \left( \tilde{Z}^T \Gamma^{-1} \dot{\tilde{Z}} \right) = \\ &= -\zeta^T K \zeta + \zeta^T \varepsilon + \zeta^T \tilde{Z}^T \phi + \text{tr} \left( \tilde{Z}^T \Gamma^{-1} \dot{\tilde{Z}} \right) \end{aligned}$$

We know  $\zeta^T \tilde{Z}^T \phi = \text{tr} \left( \tilde{Z}^T \phi \zeta^T \right)$ , then

$$\dot{V} = -\zeta^T K \zeta + \zeta^T \varepsilon + \text{tr} \left( \tilde{Z}^T \phi \zeta^T + \tilde{Z}^T \Gamma^{-1} \dot{\tilde{Z}} \right)$$

Considering Equation 23 note that

$$\dot{\tilde{Z}} = \dot{Z} - \dot{\hat{Z}} = -\dot{\hat{Z}}$$

Then

$$\begin{aligned} \dot{V} &= -\zeta^T K \zeta + \zeta^T \varepsilon + \text{tr} \left( k_\omega \tilde{Z}^T \|\zeta\| \dot{\hat{Z}} \right) = \\ &= -\zeta^T K \zeta + k_\omega \|\zeta\| \text{tr} \left( \tilde{Z}^T (Z - \tilde{Z}) \right) + \zeta^T \varepsilon \end{aligned}$$

Using Schwartz inequality

$$\begin{aligned} \dot{V} &\leq -\|\zeta\| \\ &\left[ \lambda_{\min} \|\zeta\| + k_{\omega} \|\tilde{Z}\|_F \left( \|\tilde{Z}\|_F - Z_M \right) - \varepsilon_N \right] \end{aligned}$$

Where  $\lambda_{\min}$  is the minimum eigenvalue of  $K$ .

If  $\|\zeta\| > \left[ k_{\omega} Z_M^2 / 4 + \varepsilon_N \right] / \lambda_{\min}$  or  $\|\tilde{Z}\|_F > Z_M / 2 + \sqrt{Z_M^2 / 4 + \varepsilon_N / k_{\omega}}$ , then  $\dot{V}$  is negative outside a compact set. Therefore, the control gain  $K$ , which is contained in  $\lambda_{\min}$ , can be selected large enough so that

$$\left[ k_{\omega} Z_M^2 / 4 + \varepsilon_N \right] / \lambda_{\min} < b_{\zeta}.$$

According to a standard Lyapunov theorem, this demonstrates the UUB of both  $\|\zeta\|, \|\tilde{Z}\|_F$ . [9]

• Small tracking error bounds may be achieved by selecting large control gain  $K$ . The parameter  $k_{\omega}$  offers a design tradeoff between the relative eventual magnitudes of  $\|\zeta\|$  and  $\|\tilde{Z}\|_F$ , smaller  $k_{\omega}$  yields smaller  $\|\zeta\|$  and larger  $\|\tilde{Z}\|_F$ , and vice versa.

• If  $\hat{W}_1(0)$  are taken as zeroes the linear proportional control term- $K\xi$  stabilizes the system on an interim basis.

#### 4. SM-FLUX OBSERVER

In this section, a SM rotor flux observer is employed that is robust subject to the IM electrical parameter variations and uncertainties. This method has already been applied to a primary type Linear Induction Motor (LIM) for low speed operation [13].

From (4,5), the IM current model can be

represented by

$$\begin{aligned} \begin{bmatrix} \dot{i}_{sa} \\ \dot{i}_{sb} \end{bmatrix} &= C \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + F \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} + E \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} = \\ &(\bar{C} + \Delta C) \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + (\bar{F} + \Delta F) \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} + \\ &(\bar{E} + \Delta E) \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} \equiv \bar{C} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + \bar{F} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} + \bar{E} \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} + \begin{bmatrix} \lambda_{ia} \\ \lambda_{ib} \end{bmatrix} \end{aligned} \quad (24)$$

where  $\bar{C}, \bar{E}, \bar{F}$  are the nominal values of  $C, E, F$  respectively;  $\Delta C, \Delta E, \Delta F$  denote the lumped uncertainty functions introduced by system parameters and

$$\begin{aligned} E &= \begin{bmatrix} \frac{MR_r}{\sigma L_s L_r^2} & \frac{n_p M \omega}{\sigma L_s L_r} \\ \frac{n_p M \omega}{\sigma L_s L_r} & \frac{MR_r}{\sigma L_s L_r^2} \end{bmatrix}, F = \frac{1}{\sigma L_s} \\ C &= \begin{bmatrix} -\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} & 0 \\ 0 & -\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \end{bmatrix} \end{aligned} \quad (25)$$

The current model lumped uncertainty vector is

$$\begin{bmatrix} \lambda_{ia} \\ \lambda_{ib} \end{bmatrix} = \Delta C \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + \Delta F \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} + \Delta E \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} \quad (26)$$

The current model uncertainties are assumed to be bounded i.e.  $\lambda_{ia} \leq \eta_{ia}, \lambda_{ib} \leq \eta_{ib}$ . Now,

$$E \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} = \frac{M}{\sigma L_s L_r} \begin{bmatrix} \frac{R_r}{L_r} & n_p \omega \\ -n_p \omega & \frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} \equiv K_B \begin{bmatrix} V_a \\ V_b \end{bmatrix} \quad (27)$$

Where  $K_B = M/\sigma L_s L_r$ . According to the definition as shown in Equation 27, the IM current model can be rewritten as

$$\begin{bmatrix} \dot{i}_{sa} \\ \dot{i}_{sb} \end{bmatrix} = \bar{C} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + \bar{F} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} + \bar{K}_B \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \end{bmatrix} + \begin{bmatrix} \lambda_{ia} \\ \lambda_{ib} \end{bmatrix} \quad (28)$$

where  $\bar{K}_B, \bar{V}_a, \bar{V}_b$  are the nominal parameters of  $K_B, V_a, V_b$ . To design a SM current observer the switching surfaces are defined as follows

$$S(t) = \begin{bmatrix} S_{ia}(t) \\ S_{ib}(t) \end{bmatrix} = \begin{bmatrix} \left( \frac{d}{dt} + \zeta \right) \int_0^t e_{ia}(\tau) d\tau \\ \left( \frac{d}{dt} + \zeta \right) \int_0^t e_{ib}(\tau) d\tau \end{bmatrix} = \begin{bmatrix} e_{ia}(t) - e_{ia}(0) + \zeta \int_0^t e_{ia}(\tau) d\tau \\ e_{ib}(t) - e_{ib}(0) + \zeta \int_0^t e_{ib}(\tau) d\tau \end{bmatrix} \quad (29)$$

where  $\zeta, \omega$  are positive values (note that  $\omega$  in 29 is the constant positive value not rotor speed). The current estimation error vector is defined as

$$\begin{bmatrix} e_{ia} \\ e_{ib} \end{bmatrix} = \begin{bmatrix} \hat{i}_{sa} - i_{sa} \\ \hat{i}_{sb} - i_{sb} \end{bmatrix} \quad (30)$$

The SM current observer is proposed in the following form

$$\begin{bmatrix} \dot{\hat{i}}_{sa} \\ \dot{\hat{i}}_{sb} \end{bmatrix} = \bar{C} \begin{bmatrix} \hat{i}_{sa} \\ \hat{i}_{sb} \end{bmatrix} + \bar{F} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} + \bar{K}_B \begin{bmatrix} \hat{V}_a \\ \hat{V}_b \end{bmatrix} + \begin{bmatrix} K_{ia} \operatorname{sgn}(S_{ia}(t)) \\ K_{ib} \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} \quad (31)$$

$\hat{V}_a, \hat{V}_b$  are from 27. Subtracting 28 from 31, gives

$$\begin{bmatrix} \dot{e}_{ia} \\ \dot{e}_{ib} \end{bmatrix} = \bar{C} \begin{bmatrix} e_{ia} \\ e_{ib} \end{bmatrix} + \bar{K}_B \begin{bmatrix} \hat{V}_a - \bar{V}_a \\ \hat{V}_b - \bar{V}_b \end{bmatrix} + \begin{bmatrix} K_{ia} \operatorname{sgn}(S_{ia}(t)) \\ K_{ib} \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} - \begin{bmatrix} \lambda_{ia} \\ \lambda_{ib} \end{bmatrix} \quad (32)$$

Differentiating  $S(t)$  with respect to time, using Equation 32, yields

$$\begin{aligned} \dot{S}(t) &= \begin{bmatrix} \dot{S}_{ia}(t) \\ \dot{S}_{ib}(t) \end{bmatrix} = \begin{bmatrix} \dot{e}_{ia} + \zeta e_{ia}(t) - \zeta e_{ia}(0) \\ \dot{e}_{ib} + \omega e_{ib}(t) - \omega e_{ib}(0) \end{bmatrix} \\ &= \bar{C} \begin{bmatrix} e_{ia} \\ e_{ib} \end{bmatrix} + \bar{K}_B \begin{bmatrix} \hat{V}_a - \bar{V}_a \\ \hat{V}_b - \bar{V}_b \end{bmatrix} + \begin{bmatrix} K_{ia} \operatorname{sgn}(S_{ia}(t)) \\ K_{ib} \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} + \\ &\quad \begin{bmatrix} \zeta e_{ia}(t) - \zeta e_{ia}(0) - \lambda_{ia} \\ \omega e_{ib}(t) - \omega e_{ib}(0) - \lambda_{ib} \end{bmatrix} \end{aligned} \quad (33)$$

Candidating Lyapunov function as

$$V_{ab}(S(t), \tilde{\eta}_{ia}(t), \tilde{\eta}_{ib}(t)) = \frac{1}{2} S^T(t) S(t) + \frac{1}{2k} \tilde{\eta}_{ia}^2(t) + \frac{1}{2v} \tilde{\eta}_{ib}^2(t) \quad (34)$$

Where  $k, v$  are positive constants and

$$\tilde{\eta}_{ia}(t) = \hat{\eta}_{ia}(t) - \eta_{ia}, \quad \tilde{\eta}_{ib}(t) = \hat{\eta}_{ib}(t) - \eta_{ib} \quad (35)$$

Taking the derivative of the Lyapunov function and using 31, one can obtain that

$$\begin{aligned} \dot{V}_{ab} &= S^T(t) \dot{S}(t) + \frac{1}{k} \tilde{\eta}_{ia}(t) \dot{\hat{\eta}}_{ia} + \frac{1}{v} \tilde{\eta}_{ib}(t) \dot{\hat{\eta}}_{ib} = \\ &= S^T(t) \left\{ \bar{C} \begin{bmatrix} e_{ia} \\ e_{ib} \end{bmatrix} + \bar{K}_B \begin{bmatrix} \hat{V}_a - \bar{V}_a \\ \hat{V}_b - \bar{V}_b \end{bmatrix} + \begin{bmatrix} K_{ia} \operatorname{sgn}(S_{ia}(t)) \\ K_{ib} \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} \right\} + \\ &\quad \begin{bmatrix} \zeta e_{ia}(t) - \zeta e_{ia}(0) - \lambda_{ia} \\ \omega e_{ib}(t) - \omega e_{ib}(0) - \lambda_{ib} \end{bmatrix} \} + \\ &= \frac{1}{k} (\hat{\eta}_{ia}(t) - \eta_{ia}) \dot{\hat{\eta}}_{ia} + \frac{1}{v} (\hat{\eta}_{ib}(t) - \eta_{ib}) \dot{\hat{\eta}}_{ib} \end{aligned} \quad (36)$$

The control gains are designed as

$$\begin{bmatrix} K_{ia} \\ K_{ib} \end{bmatrix} = -\bar{C} \begin{bmatrix} e_{ia} \operatorname{sgn}(S_{ia}(t)) \\ e_{ib} \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} - \bar{K} \begin{bmatrix} (\hat{V}_a - \bar{V}_a) \operatorname{sgn}(S_{ia}(t)) \\ (\hat{V}_b - \bar{V}_b) \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} + \begin{bmatrix} \zeta e_{ia}(0) \operatorname{sgn}(S_{ia}(t)) - \zeta e_{ia}(0) \operatorname{sgn}(S_{ia}(t)) - \hat{\eta}_{ia}(t) \\ \omega e_{ib}(0) \operatorname{sgn}(S_{ib}(t)) - \omega e_{ib}(0) \operatorname{sgn}(S_{ib}(t)) - \hat{\eta}_{ib}(t) \end{bmatrix} \quad (37)$$

Where

$$\tilde{\eta}_{ia} = \hat{\eta}_{ia} - \eta_{ia}, \tilde{\eta}_{ib} = \hat{\eta}_{ib} - \eta_{ib},$$

$$e_{ia} = \hat{i}_a - i_a, e_{ib} = \hat{i}_b - i_b.$$

$$\dot{\hat{\eta}}_{ia}(t) = K |S_{ia}(t)|, \dot{\hat{\eta}}_{ib}(t) = K |S_{ib}(t)|$$

Then Equation 36 can be rewritten as follows

$$\begin{aligned} \dot{V}_{ab} &= -S^T(t) \begin{bmatrix} \hat{\eta}_{ia}(t) \operatorname{sgn}(S_{ia}(t)) + \lambda_{ia} \\ \hat{\eta}_{ib}(t) \operatorname{sgn}(S_{ib}(t)) + \lambda_{ib} \end{bmatrix} + \\ & \left( \hat{\eta}_{ia}(t) - \eta_{ia} \right) |S_{ia}(t)| + \left( \hat{\eta}_{ib}(t) - \eta_{ib} \right) |S_{ib}(t)| = - \\ & \hat{\eta}_{ia}(t) |S_{ia}(t)| - \lambda_{ia} |S_{ia}(t)| - \hat{\eta}_{ib}(t) |S_{ib}(t)| - \lambda_{ib} |S_{ib}(t)| \\ & + \left( \hat{\eta}_{ia}(t) - \eta_{ia} \right) |S_{ia}(t)| + \left( \hat{\eta}_{ib}(t) - \eta_{ib} \right) |S_{ib}(t)| \\ & = -\lambda_{ia} |S_{ia}(t)| - \lambda_{ib} |S_{ib}(t)| - \tilde{\eta}_{ia} |S_{ia}(t)| - \tilde{\eta}_{ib} |S_{ib}(t)| \\ & \leq \left| \lambda_{ia} |S_{ia}(t)| - \tilde{\eta}_{ia} |S_{ia}(t)| \right| + \left| \lambda_{ib} |S_{ib}(t)| - \tilde{\eta}_{ib} |S_{ib}(t)| \right| \\ & = |S_{ia}(t)| \left( \eta_{ia} - \lambda_{ia} \right) - |S_{ib}(t)| \left( \eta_{ib} - \lambda_{ib} \right) \leq 0 \end{aligned} \quad (38)$$

Defining the following term

$$\begin{aligned} P_{ab}(t) &\equiv |S_{ia}(t)| \left( \eta_{ia} - \lambda_{ia} \right) + |S_{ib}(t)| \\ & \left( \eta_{ib} - \lambda_{ib} \right) \leq -\dot{V}_{ab}(S(t), \hat{\eta}_{ia}(t), \hat{\eta}_{ib}(t)) \end{aligned} \quad (39)$$

Then

$$\int_0^t P_{ab}(\tau) d\tau \leq V_{ab}(S(0), \hat{\eta}_{ia}(0), \hat{\eta}_{ib}(0)) - V_{ab}(S(t), \hat{\eta}_{ia}(t), \hat{\eta}_{ib}(t)) \quad (40)$$

The following result can be concluded

$$\lim_{t \rightarrow \infty} \int_0^t P_{ab}(\tau) d\tau < \infty \quad (41)$$

By Barbalat's Lemma [14]

$$\lim_{t \rightarrow \infty} P_{ab}(t) = 0 \quad (42)$$

That is  $S_{ia}(t) \rightarrow 0, S_{ib}(t) \rightarrow 0$  as  $t \rightarrow \infty$ . As a result, the proposed sliding mode current observer is asymptotically stable, even if system uncertainties exist. Moreover, the current estimation errors will converge to zero according to  $S(t) = 0$ . Consequently, the estimated flux can be derived according to 27 and 31 as follows

$$\begin{aligned} \begin{bmatrix} \hat{\psi}_{ra} \\ \hat{\psi}_{rb} \end{bmatrix} &= \bar{E}^{-1} \bar{K}_B \begin{bmatrix} \hat{V}_a \\ \hat{V}_b \end{bmatrix} = \bar{E}^{-1} \left\{ \begin{bmatrix} \dot{\hat{i}}_{sa} \\ \dot{\hat{i}}_{sb} \end{bmatrix} - \bar{C} \begin{bmatrix} \hat{i}_{sa} \\ \hat{i}_{sb} \end{bmatrix} \right\} - \\ & \bar{F} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} - \begin{bmatrix} K_{ia} \operatorname{sgn}(S_{ia}(t)) \\ K_{ib} \operatorname{sgn}(S_{ib}(t)) \end{bmatrix} \end{aligned} \quad (43)$$

where  $\dot{\hat{i}}_{sa}, \dot{\hat{i}}_{sb}$  are from 31,  $K_{ia}, K_{ib}$  from 37 and  $S_{ia}, S_{ib}$  from 29.

## 5. SYSTEM SIMULATION

Based on the proposed control strategy described in the previous section, the overall block diagram of IM drive control is shown in Figure 2. In this scheme, two ANNs are used to estimate nonlinear functions  $F_1, F_2$  that contain uncertainties. Estimated functions  $(\hat{F}_1, \hat{F}_2)$  are delivered to controllers to produce controls  $\bar{i}, u$ .



and motor electromechanical parameters assumed to be twice their nominal values at  $t = 1$  sec.

Figure 4 shows the simulated results obtained for an exponential reference flux rising up from zero to  $1.3W.t$  at  $t = 0$  sec and an exponential reference speed rising up from zero to  $220$  rad/s at  $t = 0.3$  sec, a load torque profile which is also shown in Figure 4 and motor electromechanical parameters assumed to be twice their nominal values at  $t = 0$  sec.

The IM rotor flux control is obtained for an exponential reference speed, rising up from zero to

$220$  rad/s at  $t = 0.3$  sec and an exponential flux reference from zero to  $1.3W.t$  at  $t = 0$  sec, down to  $0.8 W.t$  at  $t = 2$  sec and rising up to  $1.3 W.t$  at  $t = 3.5$  sec, a step up load torque from zero to  $40N.m.$  at  $t = 1$  sec is shown in Figure 5. In addition the IM speed control is obtained for an exponential reference flux rising up from zero to  $1.3W.t$  at  $t = 0$  sec and an exponential reference speed from zero to  $220$  rad/s at  $t = 0.3$  sec, down to  $-220$  rad/s at  $t = 2$  sec, rising up to  $220$  rad/s at  $t = 3.5$  sec, a step load torque from zero to  $40N.m.$  at  $t = 1$  sec is shown in Figure 6. In flux and speed control

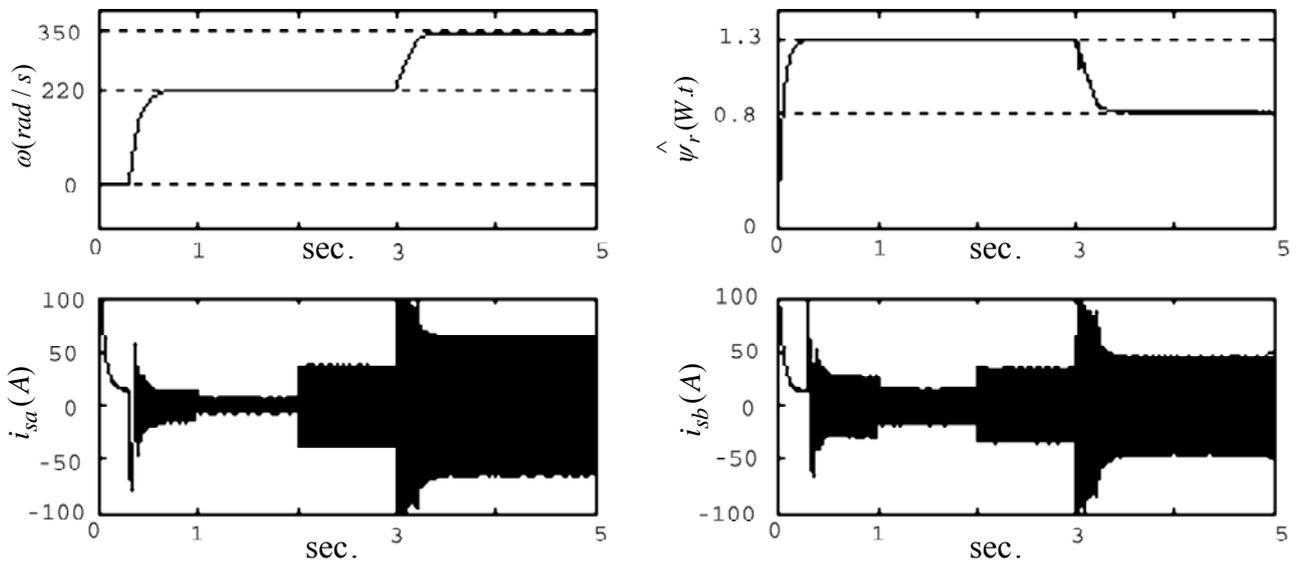


Figure 3. IM speed and flux control using robust backstepping controller.

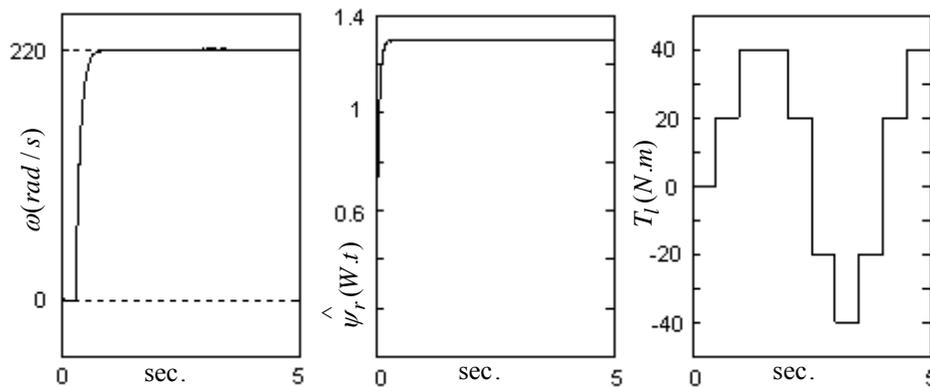


Figure 4. IM torque control.

performance, the motor electromechanical parameters are assumed to be twice their nominal values at  $t = 0$  sec and the rest of the conditions are assumed the same as described for Figure 4.

## 6. CONCLUSIONS

In this paper, a composite nonlinear controller has been proposed for the IM flux and speed control that is robust to all electromechanical parameter variations and uncertainties. First a backstepping controller is designed for a three-phase IM. This controller provides speed and flux tracking for desired trajectories, but it needs electrical and mechanical parameters to be known and if some uncertainties exist in the control system, the controller can not do its rule properly. So, to

overcome this problem, a method is used that estimates nonlinear functions through uncertainties. Artificial neural networks have the ability to do this. Combining backstepping controller with ANNs, a composite controller that is robust to all electromechanical uncertainties is obtained. Stability of this composite controller is proved by the Lyapunov theory. Also small tracking errors are obtained through selecting large control gains. The above method has advantages in comparing conventional methods such as Adaptive Backstepping. It is simple, it includes less and simpler equations but more computer calculations. In addition, to making a control system free from physical flux sensor, a SM flux observer is designed that is also robust with respect to electrical parameter uncertainties. Computer simulation results show that by this control method, a perfect speed and rotor flux tracking control can be

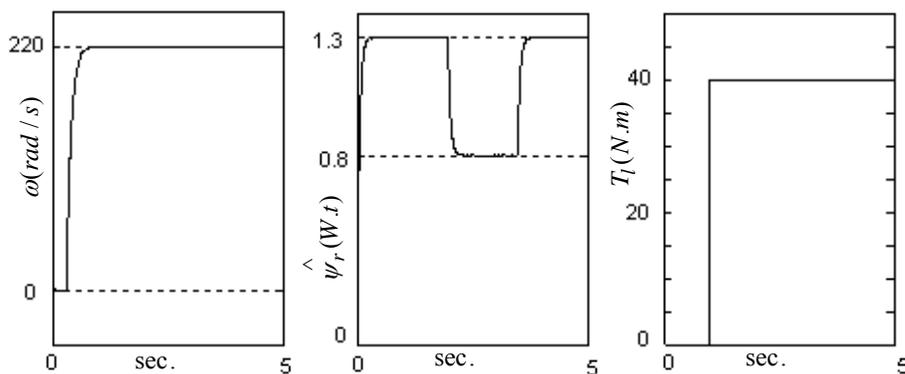


Figure 5. IM flux control.

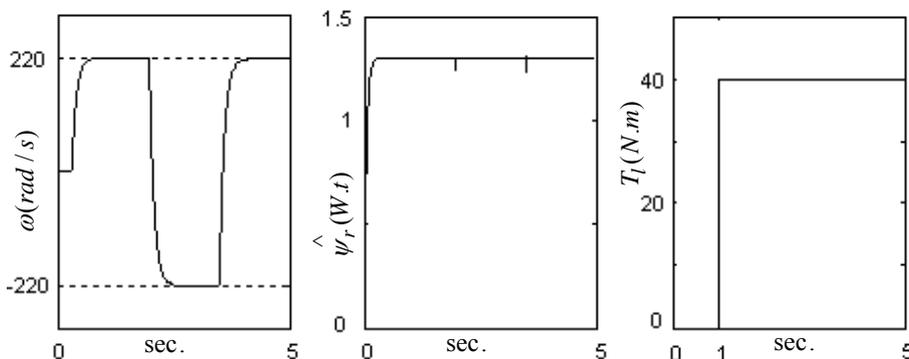


Figure 6. IM speed control.

achieved in spite of parameter uncertainties and external load torque disturbances.

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