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Convertible Bond Maturity and Debt Overhang *

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Abstract

We develop a dynamic corporate investment model to investigate the determinants of the optimal maturity structure of convertible bonds and its debt overhang effect. Our model predicts that companies opting for short-term (long-term) convertible bonds tend to expedite (delay) their investment activities, thereby alleviating (exacerbating) issues related to underinvestment. This outcome provides a unique theoretical rationale for the observed trend of decreasing convertible bond maturity, as explained by debt overhang theory. We also find that the maturity structure has a heterogeneous impact (speeding up for short-term convertible bonds or delaying for long-term convertible bonds) on optimal conversion timing. Furthermore, contrary to earlier findings, growth firms exhibit a preference for longer-maturity convertible bonds over short straight bonds. These insights, collectively, enhance our understanding of the intricate interactions among debt maturity, conversion options, and corporate investment.

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1 Introduction

Convertible bonds have emerged as a significant financing instrument employed by companies worldwide. In contrast to conventional straight bonds, convertible bonds incorporate a conversion option, granting holders the right to convert the bonds into common shares when the issuer's financial condition is sufficiently robust. This feature is instrumental in mitigating potential adverse selection problems, offering a strategic avenue for backdoor equity financing (Stein, 1992). Consequently, the rise of convertible bonds has spurred not only public discourse on the rationale for their utilization but also academic inquiry into their impact on firm dynamics. In particular, Myers (1977) demonstrates that convertible bonds have the advantage of alleviating companies' underinvestment problem, where shareholders hope to make investment decisions early due to the potential possibilities in which convertible debtholders can quickly share the firm's cash flow. The subsequent literature has extended Myers's discussions and further analyzed the effects of convertible bonds on the firm's real investment decisions; however, limited attention has been given to understanding the interplay between the maturity structure of convertible bonds and a firm's debt overhang problem. This question is significant, as all convertible bonds issued by a firm in reality have a finite term.

In this paper, we investigate the maturity structure of convertible bonds and its connection to the corporate debt overhang problem. To achieve this objective, we employ a framework reminiscent of Diamond and He (2014), integrating dynamic corporate investment into our model. Diverging from Diamond and He's model, which centers on finite-term straight debt, we assume that the firm's initial financing comprises a mix of equity and convertible bonds. Nevertheless, to underscore and compare the core themes of the model, we adhere to Diamond and He (2014)'s approach by introducing the following additional assumptions. First, we assume a zero tax rate, directing our focus toward the tradeoffs introduced by the presence of convertible bonds. Second, we posit the absence of direct exogenous bankruptcy costs but acknowledge that the firm incurs costs through lost future investment opportunities in the event of bankruptcy (i.e., endogenous bankruptcy costs). Within this framework, we then strive to address the following two fundamental questions: How does the maturity structure of convertible bonds influence firms' dynamic investment policies? What factors drive the optimal maturity structure of convertible bonds? To the best of our knowledge, this paper represents the inaugural effort to develop a dynamic corporate investment model that conducts a comprehensive analysis of the optimal maturity of convertible bonds and its associated debt overhang problem.

Empirically, there are a few studies examining the determinants of the maturity structures of convertible bonds. For instance, Verwijmeren and Yang (2020) empirically evaluate the maturity decisions of convertible bonds and finds that the source of capital can influence the design of convertible debt. Verwijmeren and Yang also document that the maturity of convertible bonds varied from 20 years around the mid-2000s to 10 years after 2007, maintaining a decreasing trend in recent years. According to Brown et al. (2012), the most influential determinant, firm quality, might only have a slight effect on the maturities of convertible bonds since arbitrageurs can long convertible bonds and short the firm's common stock to hedge the risk brought about by the potential loss.

On the whole, our main results are consistent with empirical evidence of the traditional debt maturity structure. We find that the optimal maturity can be positively influenced by the return of an asset, asset volatility, and firm size. All these findings find empirical support in Verwijmeren and Yang (2020). Importantly, our model yields the following novel and significant theoretical results.

First, in comparison to the issuance of straight bonds, firms issuing shorter (longer) maturity convertible bonds expedite (delay) investment. This implies that short-term convertible bonds mitigate the debt overhang problem, while long-term convertible bonds exacerbate it. Our model introduces a novel theoretical explanation for the observed decreasing trend in convertible bond maturity based on debt overhang theory. This complements the discussion on"the growth of the convertible arbitrage industry" in Verwijmeren and Yang (2020).

Second, in our model incorporating the debt maturity structure, we observe that only short-term convertible bonds lead to overinvestment, whereas long-term convertible bonds result in underinvestment, hastening the conversion process. These findings stand in contrast to those of Lyandres and Zhdanov (2014), who suggest that the conversion option of *perpetual* convertible bonds enhances equity holders' "overinvestment" incentive, diminishing equity value and aiding in diluting the value accruing to convertible bondholders, thereby postponing optimal conversion. Therefore, our model highlights the crucial role played by the maturity structure in determining the optimal conversion timing.

Third, we identify two opposing effects of the conversion ratio on firm value. This result provides new insights into convertible bond design. First, an increase in the conversion ratio enhances investment incentives, thereby alleviating the debt overhang effect. Second, a higher conversion ratio prompts equity holders to default earlier, diminishing the option value of future investment. In summary, the positive"overinvestment" effect prevails over the negative impact of earlier default, resulting in an overall higher firm value.

Finally, our model suggests that growth firms utilize long-term convertible bonds, contrasting with the findings of Barclay and Smith Jr (1995), where firms are inclined to issue shorter maturity straight bonds when possessing more growth opportunities. Our result aligns with the theoretical discourse in Mayers (1998) and is supported by empirical evidence from Brennan and Schwartz (1988).

From a corporate finance perspective, the existing theoretical literature on convertible bonds has delved into their influence on corporate business management. Notably, Green (1984) demonstrates that convertible bonds can assist in alleviating the asset substitution problem. Brennan and Kraus (1987) and Stein (1992) argue that mitigating the information asymmetry problem may outweigh the advantages of convertible bonds compared to direct equity and straight bond issuances. The duration of the conversion option serves as a period for claim buyers to update information about the company, enabling more informed decisions. Cornelli and Yosha (2003) suggest that replacing straight bonds with convertibles is highly beneficial when companies undertake multistage investment projects. More recently, Yagi and Takashima (2012) discuss how the issuance of convertible bonds influences a firm's timing of investment decisions. Lyandres and Zhdanov (2014) utilize a standard real-options model to examine the impact of convertible bond issuance on a firm's debt overhang problem, as identified by Myers (1977). Nevertheless, these studies solely consider *perpetual* convertible bonds, overlooking their maturity structure.

Our paper is also an extension and complement to theoretical studies of the debt overhang problem in the framework of a continuous-time partial equilibrium model. Among such studies, Mauer and Ott (2000) employ a real options model and analyze the debt overhang resulting from perpetual straight debt financing. Diamond and He (2014) measure the debt overhang cost for finite-term straight debt based on a dynamic structural model with flexible investment. Hackbarth and Mauer (2012) and Sundaresan et al. (2015) explore how the perpetual straight debt structure affects the dynamic debt overhang problem from different perspectives. Chen and Manso (2017) show the debt overhang problem over the business cycle. Wong and Yu (2022) examines the impact of trading in credit default swaps (CDS) on the debt overhang. Gan et al. (2022) analyze the interaction between the choice of heterogeneous debt structure and the debt overhang problem of firms. The common feature of these models is that they focus on the effects of straight debt. By contrast, our model highlights the debt overhang problem when firms choose to issue convertible bonds.

The paper is organized as follows. Section 2 establishes models based on Diamond and He (2014). In Section 3, we solve the model, determining optimal corporate policies and deriving closed-form solutions for the value of convertible bonds. The discussion of our numerical study on the determinants of bond maturity is presented in Section 4. Section 5 provides further discussions and empirical implications. Section 6 concludes the paper, and technical details are consolidated in the Appendix.

2 The model

Based on the partial equilibrium analysis modeling framework of Diamond and He (2014), we develop a dynamic model of a firm's financing and investment decisions with the assumption that its capital structure consists of equity and convertible bonds. The model introduces two standard assumptions. First, there exists a complete financial market in which all investors can freely trade securities to hedge risk. In this complete financial market, we do not take inflation risk into account. This means that all securities are priced in real terms. Second, investors can lend and borrow freely at a real risk-free rate of $r > 0$ after tax.

2.1 Cash flow dynamics and investment opportunities

We consider a firm with assets in place (AIP) and a series of future investment opportunities. The firm's AIP produces cash flows X_t at any time $t \geq 0$, where X_t is governed by a geometric Brownian motion process (Diamond and He, 2014; Wong and Yu, 2022; Gan, Xia, and Zhang, 2022):

$$dX_t = (\mu + \bar{i}(X_t))X_t dt + \sigma X_t dZ_t, \quad X_0 \text{ given}, \tag{1}$$

where μ denotes the baseline growth rate, σ represents the volatility of cash flows, and Z_t is the standard Brownian motion defined on a risk-neutral probability measure space. We can also interpret X_t as firm size. Following Diamond and He (2014), the firm's endogenous investment at each time t is modeled as the investment level $\bar{i}(X_t) \in \{0, i\}$, which takes a binary value, and affects the growth rate of cash flows. For convergence, we impose a parameter restriction $r > \mu + i$. The investment cost is financed with internal funds (i.e., retained earnings) and takes the form $\phi\bar{i}(X_t)X_t$, which is linear in X_t. Here, the constant ϕ measures the degree of investment cost. Equity holders control the investment policy. Whenever the cash flow level X_t reaches an endogenous threshold X_i from below (above), equity holders invest (cut investment), that is,

$$\bar{i}(X) = \begin{cases} i, & X \geq X_i, \\ 0, & X < X_i. \end{cases} \tag{2}$$

The first random time, T_i, that the firm's cash flow level reaches the threshold, X_i, satisfies $T_i = \inf\{t \geq 0, X_t = X_i\}$.

2.2 Convertible bonds and key assumptions

At the initial time $t = 0$, we assume that the firm can raise external funds by issuing a mixture of equity and convertible bonds. After convertible bonds are in place, the firm is subject to two decisions at any time $t > 0$, (1) defaulting on convertible bonds controlled by equity holders and (2) converting convertible bonds to equity. For tractability and better comparison, we employ the same basic framework of Diamond and He (2014), Leland and Toft (1996) and Leland (1998) to model the issuance and maturity of convertible bonds. In the convertible bonds contract, the coupon is zero, and the associated par value is given by P, which remains unchanged before conversion or bankruptcy. Let f be the firm's refinancing frequency. At each instant, when a portion fdt of the total amount of convertible bonds matures in the infinitesimal time interval $(t, t + dt)$, the firm should be refinanced to hold the total par value of outstanding debt constant. Thus, we can show that the mean maturity of convertible bonds is $m = 1/f$.

The conversion threshold X_c of convertible bonds from debt to equity is determined by debt holders, who maximize their own interests. Following Lyandres and Zhdanov (2014), we assume that the par value of convertible bonds and the firm's equity share can be infinitely subdivided. We normalize the total amount of outstanding shares issued by the firm at the initial time as 1. It is further assumed that the conversion option is exercised by all debt holders at the same time. Convertible bonds can be converted into η newly issued shares at the time of conversion. Therefore, upon full conversion, convertible debt holders obtain a fraction $\eta/(1+\eta)$ of the firm. [1]

To better highlight and compare the central themes of the model, we further follow Diamond and He (2014) by providing the following additional assumptions. First, we assume a zero tax rate so that we can focus on the tradeoffs purely caused by the presence of convertible bonds. Second, we assume that there are no direct exogenous bankruptcy costs, but future investment opportunities are lost (i.e., endogenous bankruptcy costs) when the firm's cash flow level decreases to the endogenously determined threshold X_d. At that instant, the absolute priority principle is implemented, and debt holders take over the entire firm and obtain

$$L(X_d) = \mathbb{E}_{T_d}\left[\int_{T_d}^{\infty} e^{-r(s-T_d)} X_s ds\right] = U_0 X_d, \tag{3}$$

where $U_0 = 1/(r-\mu)$; T_d is the first random time that the firm's cash flow level reaches the default threshold X_d, i.e., $T_d = \inf\{t \geq 0, X_t = X_d\}$; and $\mathbb{E}_{T_d}[\cdot]$ is the conditional expectation operator at time T_d. Notably, the qualitative results and their intuition are not sensitive to either of these two assumptions. Third, we do not focus our analysis on the case of an optimal capital structure (or optimal convertible bond issuance policy). This is because it is well documented that the firm's capital structure often deviates from the optimal level, which may be due to the debt overhang problem (see, e.g., Chen and Manso, 2017) . Finally, we abstract from issues related to information asymmetries between shareholders and debt holders (see, e.g., Morellec and Schürhoff, 2011).

[1] In the following discussion, we assume that the initial value of the convertible bonds is unchanged; thus, a fixed conversion rate is adopted.

3 Model solution

We solve the model by using the standard dynamic programming method. The solution is dependent on the order of the optimal thresholds, which is assumed to be given by:

$$X_d < X_i < X_0 < X_c. \tag{4}$$

This ordering can be easily satisfied with reasonable model parameters from the literature. Intuitively, shareholders cut investment when the firm's AIP deteriorates sufficiently. Debt holders thus prefer to exercise the conversion option in the investment region because investment enhances equity value.

When a firm is financed with pure equity and always invests, its present value of AIP at any time t satisfies (see, e.g., Gan et al., 2022)

$$A(X_t) = \mathbb{E}_t\left[\int_t^{\infty} e^{-r(s-t)}\left(X_s - \phi i X_s\right) ds\right] = U_i X_t, \tag{5}$$

where $U_i \equiv \frac{1-\phi i}{r-(\mu+i)}$. As shown in Wong and Yu (2022) and Gan et al. (2022), we impose the parametric restriction to ensure that $\Pi \equiv U_i/U_0 > 1$. Then, Π captures the marginal value of investment, which can measure the profitability of the firm's growth opportunities.

3.1 Valuation of corporate securities

This subsection provides the value functions of convertible bonds, equity and the total firm.

3.1.1 Value of convertible bonds

Let $D(X_t)$ be the market value of convertible bonds at time t. As discussed above, the firm continuously issues new convertible bonds with market value $(D(X_t)/m)dt$ at each instant to retire the corresponding maturing bonds before conversion or bankruptcy. Because retired convertible bonds should be paid $(P/m)dt$, which results in the valuation change in convertible bonds $\frac{1}{m}[P - D(X_t)]dt$. Based on standard contingent claim models (see Diamond and He, 2014), given the equity holders' investment policy X_i and default policy X_d, we can show that the market value of convertible bonds satisfies the following ordinary differential equation (ODE):

$$rD(X) = \frac{1}{m}(P - D(X)) + (\mu + \bar{i}(X))XD'(X) + \frac{\sigma^2}{2}X^2 D''(X). \tag{6}$$

The interpretation of Equation (6) is standard. That is, the no-arbitrage rule implies that the required return by debt holders (the left-hand side) should be equal to the sum of the change in debt value due to rollover (the first term on the right-hand side) and the expected change in debt value (the last two terms).

We solve the ODE (6) by using the following boundary conditions:

$$\begin{cases} lim_{X\uparrow X_c} D(X) = \eta/(1+\eta)U_i X_c, \\ lim_{X\downarrow X_d} D\left(X_d\right) = L(X_d), \\ lim_{X\downarrow X_i} D\left(X\right) = lim_{X\uparrow X_i} D\left(X\right), \\ lim_{X\downarrow X_i} D'\left(X\right) = lim_{X\uparrow X_i} D'\left(X\right). \end{cases} \tag{7}$$

First, when a firm's cash flow level is sufficiently large, debt holders choose to exercise the conversion option. Upon full conversion, debt holders receive $\eta/(1+\eta)$ fraction of equity shares. We thus have $lim_{X\to X_c} D(X) = \eta/(1+\eta)U_i X_c$. Second, when a default event occurs, the value-matching condition at the default threshold $X = X_d$ holds. Third, debt value should satisfy the value-matching and nonarbitrage condition at the investment threshold X_i.

Solving the ODE (6) with the above boundary conditions, we have

$$D(X) = \begin{cases} \eta/(1+\eta)U_i X, & \text{if} \quad X > X_c, \\ \frac{P}{1+mr} + A_1 X^{\alpha_1} + A_2 X^{\alpha_2}, & \text{if} \quad X_i \leq X \leq X_c, \\ \frac{P}{1+mr} + A_3 X^{\beta_1} + A_4 X^{\beta_2}, & \text{if} \quad X_d < X < X_i, \end{cases} \tag{8}$$

where α_1, α_2, β_1, and β_2 satisfy

$$\alpha_1 = 0.5 - (\mu+i)/\sigma^2 - \sqrt{\left(1/2 - (\mu+i)/\sigma^2\right)^2 + 2(r+1/m)/\sigma^2}, \tag{9}$$

$$\alpha_2 = 0.5 - (\mu+i)/\sigma^2 + \sqrt{\left(1/2 - (\mu+i)/\sigma^2\right)^2 + 2(r+1/m)/\sigma^2}, \tag{10}$$

$$\beta_1 = 0.5 - \mu/\sigma^2 - \sqrt{\left(1/2 - \mu/\sigma^2\right)^2 + 2(r+1/m)/\sigma^2}, \tag{11}$$

$$\beta_2 = 0.5 - \mu/\sigma^2 + \sqrt{\left(1/2 - \mu/\sigma^2\right)^2 + 2(r+1/m)/\sigma^2}. \tag{12}$$

and A_1, A_2, A_3 and A_4 are given in the Appendix.

Equation (8) is defined in three regions and can be interpreted as follows. In region $(X > X_c)$, debt holders have already exercised the conversion option, and debt value therefore equals a fraction $\eta/(1+\eta)$ of equity shares. In region $(X_i \leq X \leq X_c)$, the first term denotes the expected present discounted value of the perpetual coupon payment of convertible bonds. The last two terms reflect the change in the debt value when considering both the future possibility of entering the no-investment region and conversion option exercise. Finally, in region ($X_d < X < X_i$), the last two terms reflect the adjustment of the debt value when considering both the future probability of entering the investment region again and final default.

3.1.2 Values of equity and total firm

For shareholders, let $E(X)$ be the equity value. Because shareholders obtain/suffer rollover gains/losses of $\frac{1}{m}[D(X_t) - P]dt$ at each time t before the conversion of convertible bonds or firm bankruptcy, the net cash flow of the investment cost paid to them in such a situation is given by

$$X_t dt - \phi X_t \bar{i}(X_t) dt + \frac{1}{m}\left[D\left(X_t\right) - P\right] dt. \tag{13}$$

Then, we can show that the equity value satisfies the following ODE:

$$rE(X) = \max_{\bar{i}(X)\in[0,i]} X - \phi\bar{i}(X)X + \frac{1}{m}(D(X) - P) + (\mu + \bar{i}(X))XE'(X) + \frac{1}{2}\sigma^2 X^2 E''(X). \tag{14}$$

The total firm value is defined by $V(X) = E(X) + D(X)$. By the standard variation principle, the total firm value yielding a cash flow of $X_t - \phi\bar{i}(X_t)X_t$ at each time t satisfies the following ODE:

$$rV(X) = X - \phi\bar{i}(X)X + (\mu + \bar{i}(X))XV'(X) + \frac{1}{2}\sigma^2 X^2 V''(X). \tag{15}$$

taking the equity holders' investment policy X_i and default policy X_d as given. Similarly, the ODE (15) is subject to the following boundary conditions:

$$\begin{cases} lim_{X\uparrow X_c} V(X) = U_i X_c, & \text{the value-matching condition at } X_c; \\ lim_{X\downarrow X_d} V\left(X_d\right) = L(X_d), & \text{the value-matching condition at } X_d; \\ lim_{X\downarrow X_i} V\left(X\right) = lim_{X\uparrow X_i} V\left(X\right), & \text{the value-matching condition at } X_i; \\ lim_{X\downarrow X_i} V'\left(X\right) = lim_{X\uparrow X_i} V'\left(X\right), & \text{the non-arbitrage condition at } X_c. \end{cases} \tag{16}$$

Then, using Equations (15)–(16), we can obtain the solutions for the market value of the firm:

$$V(X) = \begin{cases} U_i X, & \text{if} \quad X > X_c, \\ \frac{X(1-\phi i)}{r-\mu-i} + B_1 X^{\gamma_1} + B_2 X^{\gamma_2}, & \text{if} \quad X_i \le X \le X_c, \\ \frac{X}{r-\mu} + B_3 X^{\delta_1} + B_4 X^{\delta_2}, & \text{if} \quad X_d < X < X_i. \end{cases} \tag{17}$$

where

$$\gamma_1 = 0.5 - (\mu + i)/\sigma^2 - \sqrt{\left(1/2 - (\mu + i)/\sigma^2\right)^2 + 2r/\sigma^2}, \tag{18}$$

$$\gamma_2 = 0.5 - (\mu + i)/\sigma^2 + \sqrt{\left(1/2 - (\mu + i)/\sigma^2\right)^2 + 2r/\sigma^2}, \tag{19}$$

$$\delta_1 = 0.5 - \mu/\sigma^2 - \sqrt{\left(1/2 - \mu/\sigma^2\right)^2 + 2r\sigma^2}, \tag{20}$$

$$\delta_2 = 0.5 - \mu/\sigma^2 + \sqrt{\left(1/2 - \mu/\sigma^2\right)^2 + 2r\sigma^2}. \tag{21}$$

The constants B_1, B_2, B_3 and B_4 are determined by the above boundary conditions and provided in the Appendix.

Finally, we can obtain the solutions for the market value of equity $E(X)$ by $V(X) - D(X)$, in that:

$$E(X) = \begin{cases} 1/(1+\eta)U_i X, & \text{if} \quad X > X_c, \\ \frac{X(1-\phi i)}{r-\mu-i} - \frac{P}{1+mr} + B_1 X^{\gamma_1} + B_2 X^{\gamma_2} - A_1 X^{\alpha_1} - A_2 X^{\alpha_2}, & \text{if} \quad X_i \leq X \leq X_c, \\ \frac{X}{r-\mu} - \frac{P}{1+mr} + B_3 X^{\delta_1} + B_4 X^{\delta_2} - A_3 X^{\beta_1} - A_4 X^{\beta_2}, & \text{if} \quad X_b < X < X_i. \end{cases} \tag{22}$$

The interpretation of (17) and (22) is similar to the discussion in the value of convertible bonds.

3.2 Endogenous corporate decisions

Now, we focus on endogenous corporate decisions. The endogenous investment threshold X_i is controlled by shareholders, and it can be characterized by the first-order condition in Equation (14):

$$E'(X_i) = \phi. \tag{23}$$

As in standard capital structure models, the endogenous bankruptcy policy X_d controlled by shareholders also satisfies a smooth-pasting condition, which is given by

$$E'(X_d) = 0. \tag{24}$$

The endogenous conversion threshold X_c is controlled by debt holders, which satisfies the smooth-pasting condition:

$$D'(X_c) = \eta/(1+\eta)U_i. \tag{25}$$

Utilizing conditions (23) − (25), we can show that the endogenous thresholds are obtained uniquely by simultaneously solving (numerically) the following three equations:

$$\begin{cases} \frac{X_i(1-\phi i)}{r-\mu-i} + B_1\gamma_1 X_i^{\gamma_1} + B_2\gamma_2 X_i^{\gamma_2} - A_1\alpha_1 X_i^{\alpha_1} - A_2\alpha_2 X_i^{\alpha_2} = \phi X_i, \\ B_3\delta_1 X_b^{\delta_1} + B_4\delta_2 X_b^{\delta_2} - A_3\beta_1 X_b^{\beta_1} - A_4\beta_2 X_b^{\beta_2} + X_b/(r-\mu) = 0, \\ A_1\alpha_1 X_c^{\alpha_1} + A_2\alpha_2 X_c^{\alpha_2} = \eta/(1+\eta)U_i X_c. \end{cases} \tag{26}$$

We will provide the solution of the model using numerical calculation methods, as there is no analytical expression.

4 Model implications

In this section, we conduct numerical analysis to better illustrate our model implications, which are novel to the literature (see, e.g. Lyandres and Zhdanov, 2014; Diamond and He, 2014; Verwijmeren and Yang, 2020). Specifically, we first investigate the corporate underinvestment problem (debt overhang) in Subsections 4.1 and 4.2, focusing on convertible bond maturity and the conversion ratio. Then, in Subsection 4.3, we explore key determinants of the optimal convertible bond maturity. In addition, optimal corporate policies (investment and default) have been reexamined when firms issue convertible bonds with optimal maturity.

We choose the baseline parameter values as follows, and whenever applicable, all parameters are annualized. Based on Diamond and He (2014) and Wong and Yu (2022), we set the risk-free rate $r = 8\%$, the cash flow volatility $\sigma = 20\%$, the investment rate $i = 1.5\%$, the degree of the investment cost $\phi = 9$, and the baseline growth rate $\mu = 0$. Furthermore, in the base case scenario, the assumption of $\eta = 0.65$ is adopted as in Lyandres and Zhdanov (2014). We choose the initial firm size X_0 to be 1, which can be set freely. Finally, following Diamond and He (2014), we set the value of the risky convertible debt at its issuance time (i.e., at the initial time $t = 0$) as $D(X_0) = 10$. We search for the par value P so that $D(X_0) = 10$ always when varying debt maturity m.

4.1 Effects of debt maturity

Figure 1 presents optimal corporate investment (Panel A) and default (Panel B) policies and the optimal conversion timing (Panel C) for different debt maturities. The blue dash-dotted line represents the case in which the firm issues straight bonds (‘SB’ for short in the

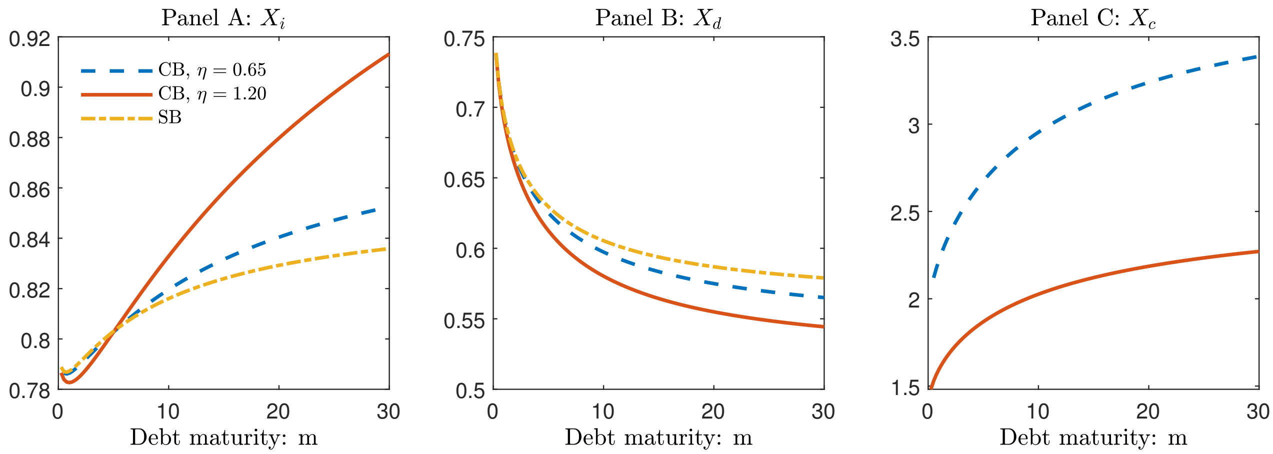

Figure 1: Optimal investment policy (Panel A), default policy (Panel B), and conversion policy (Panel C) for various debt maturities. The blue dash-dotted line presents corporate policies under the baseline scenario (straight bonds), while the red dashed line and the solid pink line represent the convertible bond (CB) case with two different conversion ratios, $\eta = 0.65$ and $\eta = 1.2$, respectively.

figure), which is similar to the model in Diamond and He (2014), i.e., $\eta = 0$ in our model such that convertible bonds ('CB' for short in the figure) are never converted into shares. Consistent with the central finding in Diamond and He (2014) that firms with shorter debt maturity of straight bonds expedite corporate investment, we observe that equity holders accelerate investment when the firm issues short-term convertible bonds. Intriguingly, firms issuing shorter (longer) maturity convertible bonds expedite (delay) investment (i.e., a lower investment threshold) compared to straight bond issuance. This implies that short-term convertible bonds alleviate the current debt overhang, while long-term convertible bonds exacerbate it. The intuition is straightforward: holders of long-term convertible bonds accrue more gains for positive news upon investment due to the option characteristic. Consequently, equity holders postpone investment, leading to a more pronounced debt overhang problem.

In contrast to the explanation provided in Verwijmeren and Yang (2020) that firms tend to replace callable long-term convertible bonds with short-term straight convertible bonds, our model introduces a new theoretical argument for the decreasing convertible bond maturity based on debt overhang theory. Moreover, Panel A indicates that the higher the conversion ratio is, the more severe the debt overhang induced for longer convertible bonds and the less severe that for shorter convertible bonds.

Now, let us delve into the corporate default policy. Panel B of Figure 1 reveals that firms issuing short-term convertible bonds default earlier (indicated by a higher default threshold), consequently forgoing more future investment opportunities. This tendency is partially attributed to the fact that short-term bonds incur higher financing costs when firms face financial distress. Furthermore, akin to Diamond and He (2014), the value of short-term convertible bonds is less resilient to firm risks, rendering the equity value more volatile and, in turn, enhancing the value of the default option.

Additionally, our findings indicate that firms issuing convertible bonds default later than those issuing straight bonds, corroborating the equity-friendly nature of convertible bonds. As discussed in Lyandres and Zhdanov (2014), the conversion option of *perpetual* convertible bonds promotes an "overinvestment" incentive among equity holders, reducing equity value and diluting the value accruing to convertible bond holders, thereby delaying optimal conversion. However, in our model with the debt maturity structure, we observe that only short-term convertible bonds induce overinvestment, while long-term convertible bonds lead to underinvestment, thus expediting the conversion process. Furthermore, in Panel C of Figure 1, holders of long-term convertible bonds delay the conversion of their claims due to the option value of waiting.

4.2 Effects of the conversion ratio

Figure 2 illustrates the optimal investment policy (Panel A), default policy (Panel B), and firm value (Panel C and D) for different conversion ratios with two distinct debt maturities, $m = 2.5$ and $m = 25$. To facilitate comparison, we include the results for straight bonds (blue dashed dotted line and black dotted line) as benchmarks.

First, as depicted in Panel A of Figure 2, for a short-term debt maturity $m = 2.5$, firms issuing convertible bonds invest earlier than those issuing straight bonds. Conversely, for a long-term debt maturity $m = 25$, the opposite holds true. These findings align with the earlier discussion that short-term convertible bonds alleviate the debt overhang effect, while long-term convertible bonds exacerbate it. In comparison to short-term convertible bonds, when the AIP deteriorates, the lower repricing frequency of long-term convertible bonds results in them sharing more losses with shareholders. Moreover, they also share more profits in response to positive news. Consequently, to prevent future investment income flows from

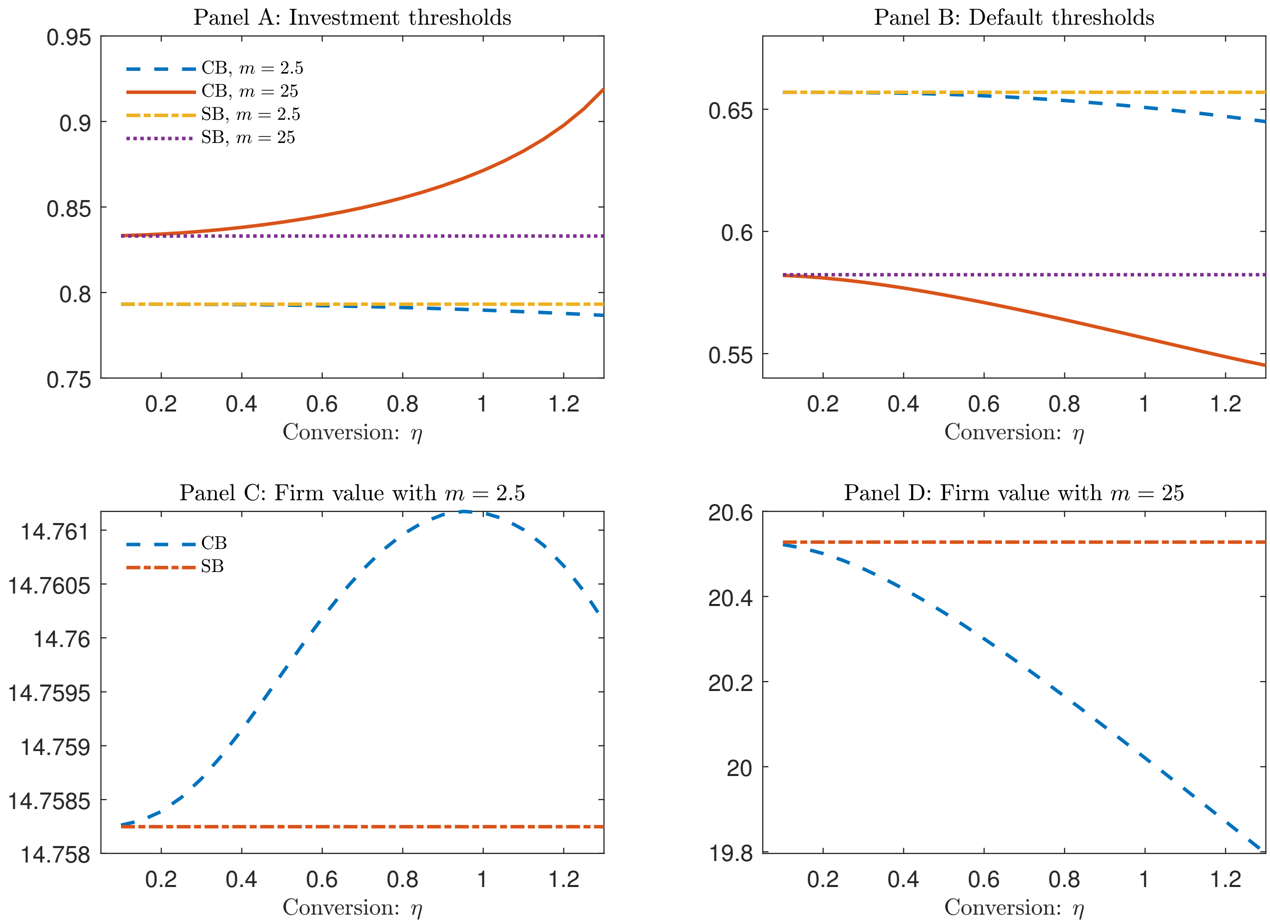

Figure 2: This figure represents corporate investment policy (Panel A), default policy (Panel B), and firm value (Panel C and D) as a function of the conversion ratio with two different debt maturities, $m = 2.5$ and $m = 25$. For better comparison, we add straight bonds (blue dash-dotted line and black dotted line) as benchmarks in all panels.

benefiting convertible bond holders, shareholders establish a higher investment trigger point for long-term convertible bonds, thereby delaying corporate investment.

Second, as the conversion ratio increases, the investment trigger point under the financing of short-term convertible bonds becomes lower than that of straight bonds. This is attributed to the higher repricing frequency of short-term convertible bonds and the greater willingness of short-term convertible creditors to convert early (as evidenced by the lower trigger point than long-term bonds, as indicated in Panel C of Figure 1), thereby reducing the duration of bond interest payments.

Third, in comparison to long-term convertible bonds, short-term convertible bonds result in lower investment trigger points and higher bankruptcy trigger points. However, the proportion of increased bankruptcy trigger points is higher than that of decreased investment trigger points. Consequently, the enterprise value of short-term convertible bonds is lower than that of their long-term counterparts, as seen in Panels C and D of Figure 2. Due to the lower risk-sharing nature of short-term convertible bonds, they prompt earlier defaults, leading to the destruction of future investment opportunities and a negative impact on firm value due to short-term overhang.

Furthermore, as the conversion ratio increases, the optimal exercise trigger point for convertible creditors is advanced. The exercise trigger point of short-term convertible bonds is lower than that of long-term convertible bonds. As the conversion ratio rises, the enterprise value under short-term convertible bond financing undergoes an inverted U-shaped transformation, while the enterprise value under long-term convertible bond financing exhibits a monotonically decreasing trend.

4.3 Optimal convertible bond maturity

In our dynamic model, short-term convertible bonds have two opposing effects on firm valuation. On the one hand, shorter-term convertible bonds result in larger rollover losses for equity holders when the firm faces financial distress, hastening default and thereby eroding more future investment opportunities, ultimately reducing firm value. On the other hand, firms issuing short-term convertible bonds expedite current investment, alleviating the debt overhang effect and increasing firm value. Consequently, firms issuing convertible bonds must choose an optimal debt maturity to balance these two opposing forces. Following Diamond and He (2014), we define the optimal convertible bond maturity m^* as the maturity that maximizes the initial firm value, as follows:

$$m^*(X_0) = \arg\max_m V(X_0; m), \tag{27}$$

given the risky convertible debt value of $D(X_0) = 10$ at the issuance time $t = 0$, ex post optimal investment, default and conversion option exercise policies.

Figure 3 presents numerical results of optimal corporate policies (Panels A and B), optimal debt maturity (Panel C), and firm value (Panel D) for various conversion ratios when debt maturities are optimally chosen. Similar to the discussion of the debt maturity structure, we identify two opposite effects of the conversion ratio on firm value. First, increasing the

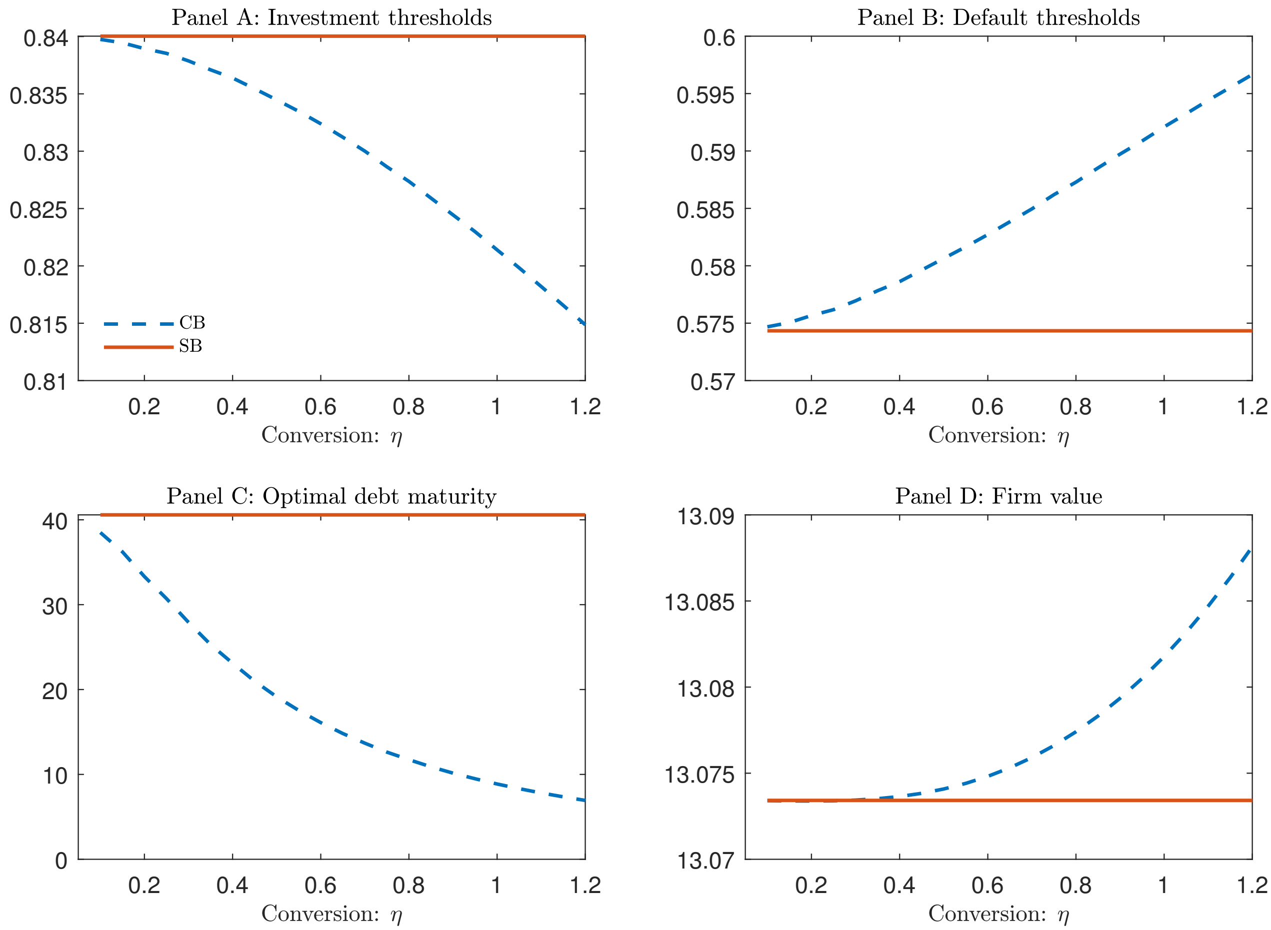

Figure 3: This figure displays optimal corporate investment policy (Panel A), default policy (Panel B), optimal debt maturity (Panel C), and firm value (Panel D) as a function of the conversion ratio. For better comparison, we add straight bonds (blue solid line) as benchmarks in all panels.

conversion ratio improves the investment incentive (a lower investment threshold as in Panel A of Figure 3), thus mitigating the debt overhang effect. Second, a higher conversion ratio induces equity holders to default earlier (as shown in Panel B of Figure 3), reducing the option value of future investment. Overall, when the conversion ratio increases, the positive "overinvestment" effect dominates the negative effect of earlier default, leading to a higher firm value (Panel D of Figure 3).

Figure 4 illustrates the impact of the baseline growth rate μ, risk term σ, size of firms X_0, and firm's profitability of a growth opportunity Π on the optimal debt maturity m^*. In Panel A of Figure 4, it is observed that the optimal debt maturity can be positively influenced by

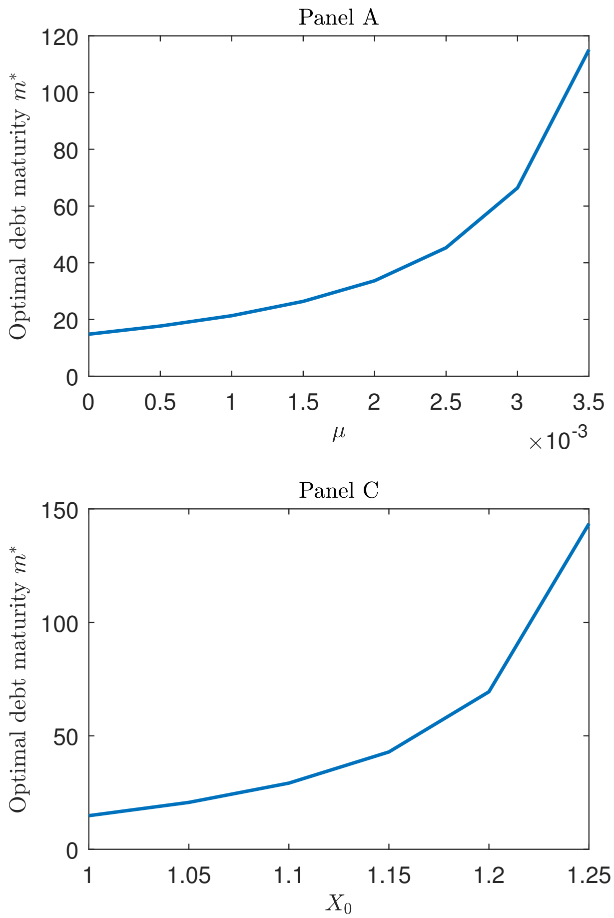

Figure 4: This figure presents the optimal convertible bond maturity as a function of corporate growth rate (Panel A), cash flow volatility (Panel B), initial firm size (Panel C), and the profitability of future investment opportunities.

the returns on assets, aligning with the notion that the quality of firms plays a crucial role in their access to long-term borrowing markets (Diamond, 1991). Verwijmeren and Yang (2020) also propose that rated companies are more inclined to issue longer-maturity convertibles because longer maturities help firms avoid substantial rollover losses.

As depicted in Panel B of Figure 4, an increase in volatility significantly and continuously reduces the optimal convertible bond maturity, albeit with a diminishing marginal effect. This negative correlation between these two variables may stem from the preference of buy-side arbitrageurs, such as hedge funds, who favor high-volatility and short-maturity convertible bonds to enhance the profitability of their hedging strategies (Brown et al., 2012). Similarly, Panel C of Figure 4 demonstrates that larger firms prefer to issue longer-term convertible bonds, with the importance of agency costs possibly explaining this observation.

Panel D of Figure 4 illustrates the effects of the profitability of growth opportunities on the optimal maturity of convertible bonds. Our model suggests that firms with fewer growth opportunities lean toward short-term convertible bonds (Panel D of Figure 4), in contrast to the findings of Barclay and Smith Jr (1995), where firms are inclined to issue shorter-maturity straight bonds when they have more growth opportunities. One interpretation from Mayers (1998) is that growth firms are more likely to issue long-term convertible bonds due to the higher possibility of reducing leverage for future investment opportunities. In other words, firms with fewer growth opportunities might be more inclined to avoid agency problems, and managers have incentives to control these agency problems by using short-term convertible bonds, as they imply a lower risk of overinvestment and excessive risk-taking. Additionally, our result is consistent with Brennan and Schwartz (1988), suggesting that issuing long-term convertible securities can positively influence firms with more growth opportunities.

5 Further discussions

In this section, we provide nuanced discussions regarding the potential ramifications of relaxing model assumptions or expanding our model to encompass additional pivotal factors, shedding light on their influence on our principal findings. Specifically, Subsection 5.1 discusses the implications of market incompleteness on the maturity structure of convertible bonds and the associated debt overhang predicament. Moving forward, Subsection 5.2 embarks on a preliminary qualitative exploration, contemplating how the interplay between supply and demand for convertible bonds within a dynamic stochastic general equilibrium model (DSGE) might exert an influence on our core outcomes. Drawing inspiration from Benchimol et al. (2023), Subsection 5.3 extends our discourse to encompass the impact of uncertainty on convertible bond pricing, potentially instigating modifications to established models and unraveling the mechanisms steering such alterations. Lastly, Subsection 5.4 encapsulates our model's implications, culminating in the formulation of discernible empirical hypotheses ready for testing.

5.1 Incomplete markets

In line with the standard Black-Merton-Scholes option-pricing framework and a substantial body of subsequent literature, our model initially assumes complete financial markets. However, in reality, financial markets are characterized by imperfections. A natural inquiry arises: what implications would arise for the maturity structure of convertible bonds and their impact on debt overhang if we introduce these imperfect market frictions?

Drawing inspiration from Chen et al. (2010), we augment our baseline model to account for market incompleteness. In this extended framework, we posit that inside shareholders can only engage in dynamic and frictionless trading involving a risk-free asset and a stock market portfolio. Consequently, shareholders encounter incomplete markets due to the absence of hedging contracts to diversify their idiosyncratic risk.

As suggested by Chen et al. (2010), market incompleteness leads to shareholders underinvesting compared to the complete market scenario, with risky straight bonds providing partial alleviation of this issue. Extending this insight, the equity dilution effect implies that convertible bonds may not be as effective as ordinary bonds in mitigating underinvestment or debt backlog resulting from incomplete markets. Furthermore, focusing solely on convertible bond financing, a lengthier term structure offers shareholders greater returns for employing convertible bonds to diversify idiosyncratic risks. This, in turn, mitigates shareholders' inclination toward underinvestment and debt overhang problems. Consequently, when contemplating the optimal term structure, shareholders in incomplete markets are inclined to issue convertible bonds with longer maturities compared to the conditions in complete markets.

5.2 Convertible bond supply and demand

In this study, we employed a partial equilibrium model to streamline the analysis and underscore essential economic mechanisms. Within this model, we assumed that the supply of bonds and firms' financing needs through bond issuance are unconstrained exogenous variables. This simplification enabled us to concentrate exclusively on the impact of convertible bond issuance on firms' investment decisions, overlooking the intricate dynamics stemming from fluctuations and changes in bond market supply and demand.

However, we acknowledge the disparity between this assumption and the real-world scenario. Indeed, shifts in supply and demand within the bond market are pivotal determinants of a firm's financing costs and its fundraising capacity, directly influencing its financing risks. Consequently, firms must comprehensively consider the bond market's supply and demand landscape when formulating financing and investment strategies.

On a parallel note, the study by Benchimol and Ivashchenko (2021) underscores the significance of international business cycles and volatility shocks in shaping bond market supply and demand. This naturally prompts the question: What implications would the inclusion of bond supply and demand, along with considerations for international business cycles and volatility shocks, have on the maturity structure of convertible bonds and their subsequent impact on debt overhang in our study?

To address this question, a modification of our research methodology is imperative. Specifically, we propose in future research by expanding the existing partial equilibrium model and, drawing inspiration from the insights of Benchimol and Ivashchenko (2021), constructing a DSGE model. This extended model would encompass multiple sectors, including government, households, banks, firms, bond markets, and the international sector. The introduction of volatility shocks and international business cycles would further alter bond demand and supply dynamics.

Within the framework of this extended DSGE model, we anticipate that the presence of international business cycles and volatility shocks could exacerbate the adverse impact of convertible debt on a firm's current and future investment decisions (a stronger debt overhang effect), compared to scenarios where these factors are not considered. Illustrating this point, aligned with the discourse in Chen and Manso (2017), shareholders facing macroeconomic risks such as international business cycles and volatility shocks may exhibit reduced incentive to exercise current investments, potentially defaulting earlier and foregoing future investment opportunities. This caution arises from the fear that uncertain circumstances might lead to significant increases in financing risks in the future, thereby impacting the financial stability and long-term development prospects of the firm. Consequently, shareholders become more circumspect in decision-making to mitigate potential financing risks. Therefore, we anticipate that the introduction of international business cycles and volatility shocks would accentuate the debt overhang problem arising from convertible bonds.

However, a nuanced, case-by-case analysis is imperative when evaluating how international business cycles and volatility shocks influence the supply-demand relationship of convertible bonds and, consequently, firms' decisions regarding the maturity structure of these bonds. Motivated by the insights in Chen et al. (2021), the presence of international business cycles and volatility shocks exposes firms to financing risks, leading to a trade-off in selecting the maturity structure of convertible bonds: stability of long-term debt versus flexibility of short-term debt. This decision is influenced not only by the intensity of international business cycles and volatility shocks but also by expected bankruptcy costs, rollover risks, and liquidity costs. Firms more exposed to international business cycles and volatility shocks may lean towards long-term convertible debt to mitigate short-term refinancing risks and bankruptcy costs. In contrast, firms less exposed to these factors may opt for short-term convertible debt to gain greater flexibility and lower financing costs.

While our analysis thus far represents a preliminary qualitative exploration, a more in-depth investigation is required to understand how key factors such as fluctuations in the international business cycle and volatility shocks specifically impact the dynamics of supply and demand for convertible bonds. Additionally, exploring how these changes in supply and demand influence firms' investment decisions and debt maturity structure choices within the convertible bond financing framework is an avenue we reserve for future research.

5.3 Uncertainty

Uncertainty is a central concept in finance, closely related to risk, information and market volatility. Inspired by the recent study by Benchimol et al. (2023) that explores the nexus of monetary policy, uncertainty, and stock market reactions, we now provide further discussions on how uncertainty affects the pricing of convertible bonds, potentially altering the results of existing models and uncovering the mechanisms behind such changes.

Firstly, the research by Benchimol et al. (2023) underscores the pivotal role of uncertainty in investigating the impact of monetary policy on the stock market. Extending this insight to the convertible bond market, we discuss how uncertainty directly influences these financial instruments. Unlike traditional bonds, convertible bonds have the latent value of conversion into equity, intensifying investor focus on conversion value and potential returns during uncertain times. Heightened uncertainty tends to render investors' expectations of future cash flows increasingly uncertain, potentially leading to an augmented demand for risk premia on convertible bonds. This increased demand for risk premia could ultimately result in a potential decline in the market prices of convertible bonds.

Secondly, uncertainty may indirectly impact convertible bond pricing by shaping the behaviour of market participants. During periods of elevated uncertainty, investors tend to be more cautious, reducing their acquisition of newly issued convertible bonds or opting to hold cash instead of riskier assets. These shifts in market sentiment can alter the supply and demand dynamics in the convertible bond market, thereby influencing prices. A decrease in demand amid constant or increasing supply often leads to a decline in prices. Furthermore, owing to the dual nature of convertibles as both equities and bonds, investors may emphasise their equity-like characteristics, particularly the potential upside upon conversion, further affecting pricing dynamics.

Thirdly, uncertainty can interact with other factors to shape the convertible bond market. In an open economy, international capital flows and exchange rate fluctuations may elevate uncertainty in the domestic convertible bond market. Simultaneously, volatility shocks, stemming from unexpected events or policy changes, can suddenly heighten market uncertainty. These external factors can indirectly impact convertible bond pricing by influencing investors' expectations and altering the supply-demand balance in the market.

Lastly, the changes induced by uncertainty in the pricing of convertible bonds have a direct impact on companies' investment decisions and their choice of debt maturity structure. For instance, in a scenario of low interest rates in the convertible bond market, the cost of financing through convertible bond issuance decreases, potentially incentivizing firms to increase debt financing for their investment activities. Concurrently, companies may opt for longer-term convertible bonds to leverage lower financing costs over an extended period. Conversely, rising interest rates may impose higher financing costs on firms, potentially tempering their investment inclination, leading them to favour shorter-term convertible bonds to mitigate financing expenses. Understanding this influential mechanism is critical for comprehending firms' investment and financing decisions in the face of uncertainty. However, it is crucial to note that uncertainty is a complex and multifaceted concept with the potential to affect convertible bond prices and corporate investment decisions through multiple channels. Investigating how these factors interact and influence corporate investment and financing decisions is worthwhile and fruitful, which we leave for future research.

5.4 Empirical predictions

Our model's implications offer a means to reconcile existing empirical evidence concerning the determinants of convertible bond maturity. As noted by Verwijmeren and Yang (2020), firms exhibiting lower stock return volatility, higher profitability, and larger size tend to issue convertibles with longer maturities, aligning with our model's predictions.

However, it is noteworthy that, to the best of our knowledge, the empirical literature lacks exploration into the relationship between convertible bond maturity and the debt overhang problem, along with its dynamic consequences. Consequently, our model introduces novel and testable empirical hypotheses that address these gaps in the literature. The hypotheses generated are as follows:

H1. In the sample, firms issuing short-term (long-term) convertible bonds expedite (delay) investment, thereby mitigating (exaggerating) debt overhang problems.

H2. In comparison to identical benchmark firms issuing straight bonds, the issuance of short-term convertible bonds is anticipated to alleviate the debt overhang problem, while the issuance of long-term convertible bonds is expected to exacerbate it.

H3. Firms with more profitable growth opportunities are anticipated to issue convertibles with longer maturities than firms with less profitable growth opportunities.

These hypotheses provide a foundation for empirical investigations and have the potential to contribute valuable insights to the literature on convertible bonds and debt overhang.

6 Conclusion

Building upon the framework proposed by Diamond and He (2014), our continuous-time model delves into the intricacies of convertible bonds' optimal maturity and their ensuing debt overhang effect. While a substantial body of empirical literature has examined the determinants of convertible bond maturity, the theoretical exploration of convertible bonds and their debt overhang effect has been notably scarce. To the best of our knowledge, our study represents the first theoretical undertaking, offering a thorough analysis of the determinants of optimal convertible bond maturity and the consequential debt overhang problem.

Our primary findings align with established theories and empirical observations regarding traditional debt maturity structures. Crucially, our model contributes novel insights. First, in comparison to straight bonds, firms issuing shorter (longer) maturity convertible bonds hasten (delay) their investment decisions. This implies that short-term convertible bonds alleviate the debt overhang problem, while long-term convertible bonds exacerbate it. Our model furnishes a unique theoretical rationale for the observed decline in convertible bond maturity, complementing the discourse on"the growth of the convertible arbitrage industry" elucidated by Verwijmeren and Yang (2020).

Second, our study reveals that only short-term convertible bonds induce overinvestment, while their long-term counterparts cause underinvestment, consequently expediting the conversion process. This finding diverges from Lyandres and Zhdanov (2014), which posits that the conversion option of perpetual convertible bonds enhances equity holders' "overinvestment" tendencies, leading to delayed optimal conversion. Thus, our model underscores the pivotal role of maturity structure in determining optimal conversion timing.

Third, we identify two conflicting effects of the conversion ratio on firm value, offering fresh insights into convertible bond design. While an increased conversion ratio enhances investment incentives, mitigating the debt overhang effect, it also prompts earlier default by equity holders, diminishing the option value of future investment. Despite this, the positive "overinvestment" effect outweighs the negative impact of early default, resulting in higher overall firm value.

Last, our model posits that growth-oriented firms prefer long-term convertible bonds. This stands in contrast to studies such as Barclay and Smith Jr (1995) and Lewis et al. (1998), which suggest that firms with growth opportunities tend to opt for shorter straight bonds. Our findings align with the theoretical discourse in Stein (1992) and empirical evidence from Brennan and Schwartz (1988) and Mayers (1998).

We provide some new insights into corporate business decisions with convertible bonds by highlighting the difference between convertible and straight bonds. However, firms often simultaneously use other various types of debt in practice (e.g., private debt, market debt and guaranteed debt). Therefore, it is worthwhile to extend our model to firms simultaneously issuing these types of debt and to further study the relevant issues. Moreover, our model lacks an endogenous consideration of the dynamics inherent in convertible bond demand and supply. The fluctuations in demand, driven by diverse hedging demands from convertible arbitrageurs, and the varying supply across convertible bonds with different maturities may exhibit distinct structures, exerting an influence on the determination of optimal convertible bond maturity (see Kim, 1990; de Jong et al., 2011). Additionally, we can contextualize our study within the framework of Benchimol and Ivashchenko (2021) and Benchimol et al. (2023), exploring the potential impact of uncertainty, international business cycles (from an open economy perspective), and volatility shocks on bond demand and supply, thus exploring how these factors may shape our results and methodology. We acknowledge that addressing this challenge represents a promising avenue for future research.

Appendix A Appendix for Section 3

In this section, we provide technical details in deriving convertible bond value, equity value and total firm value (defined by the sum of convertible bond value and equity value). The value function of convertible bonds involves value matching at X_c, X_i and X_d, which yields

$$\frac{P}{1+mr} + A_1 X_c^{\alpha_1} + A_2 X_c^{\alpha_2} = \frac{\eta}{1+\eta} U_i X_c, \tag{A.1}$$

$$A_1 X_i^{\alpha_1} + A_2 X_i^{\alpha_2} = A_3 X_i^{\beta_1} + A_4 X_i^{\beta_2}, \tag{A.2}$$

$$A_3 X_d^{\beta_1} + A_4 X_d^{\beta_2} + \frac{P}{1+mr} = \frac{X_d}{r-\mu}. \tag{A.3}$$

The no-arbitrage condition for $D(X)$ at X_i yields

$$A_1 \alpha_1 X_i^{\alpha_1} + A_2 \alpha_2 X_i^{\alpha_2} = A_3 \beta_1 X_i^{\beta_1} + A_4 \beta_2 X_i^{\beta_2}. \tag{A.4}$$

We then find that the constants A_1, A_2, A_3 and A_4 satisfy

$$A_1 = \frac{(A_3 X_i^{\beta_1} + A_4 X_i^{\beta_2}) X_c^{\alpha_2} - (\eta/(1+\eta) U_i X_c - P/(1+mr)) X_i^{\alpha_2}}{X_c^{\alpha_2} X_i^{\alpha_1} - X_c^{\alpha_1} X_i^{\alpha_2}}, \tag{A.5}$$

$$A_2 = \frac{(\eta/(1+\eta) U_i X_c - P/(1+mr)) X_i^{\alpha_1} - (A_3 X_i^{\beta_1} + A_4 X_i^{\beta_2}) X_c^{\alpha_1}}{X_c^{\alpha_2} X_i^{\alpha_1} - X_c^{\alpha_1} X_i^{\alpha_2}}, \tag{A.6}$$

$$A_3 = \frac{(\alpha_1 - \alpha_2 - \beta_2)(U_0 X_d - P/(1+mr)) X_i^{\beta_2} X_c^{\alpha_1} + \alpha_2 (\eta/(1+\eta) U_i X_c - P/(1+mr)) X_i^{\alpha_1} X_d^{\beta_2}}{(\alpha_1 - \alpha_2 - \beta_2) X_i^{\beta_2} X_c^{\alpha_1} X_d^{\beta_1} - (\alpha_1 - \alpha_2 - \beta_1) X_i^{\beta_1} X_c^{\alpha_1} X_d^{\beta_2}}, \tag{A.7}$$

$$A_4 = \frac{-\alpha_2 (\eta/(1+\eta) U_i X_c - P/(1+mr)) X_i^{\alpha_1} X_d^{\beta_1} - (\alpha_1 - \alpha_2 - \beta_1)(U_0 X_d - P/(1+mr)) X_d^{\beta_1} X_c^{\alpha_1}}{(\alpha_1 - \alpha_2 - \beta_2) X_i^{\beta_2} X_c^{\alpha_1} X_d^{\beta_1} - (\alpha_1 - \alpha_2 - \beta_1) X_i^{\beta_1} X_c^{\alpha_1} X_d^{\beta_2}}. \tag{A.8}$$

Similarly, the value function of the firm involves value matching at X_c, X_i and X_d, which yields

$$B_1 X_c^{\gamma_1} + B_2 X_c^{\gamma_2} = U_i X_c, \tag{A.9}$$

$$B_1 X_i^{\gamma_1} + B_2 X_i^{\gamma_2} + U_i X_i = U_0 X_i + B_3 X_i^{\delta_1} + B_4 X_i^{\delta_2}, \tag{A.10}$$

$$B_3 X_d^{\delta_1} + B_4 X_d^{\delta_2} = U_0 X_d. \tag{A.11}$$

The no-arbitrage condition for $V(X)$ at X_i yields

$$B_1\gamma_1 X_i^{\gamma_1} + B_2\gamma_2 X_i^{\gamma_2} + U_i X_i = U_0 X_i + B_3\delta_1 X_i^{\delta_1} + B_4\delta_2 X_i^{\delta_2}. \tag{A.12}$$

We then find that the constants B_1, B_2, B_3 and B_4 satisfy

$$B_1 = \frac{((U_0 - U_i)X_i + B_3 X_i^{\delta_1} + B_4 X_i^{\delta_2})X_c^{\gamma_2} - U_i X_c X_i^{\gamma_2}}{X_c^{\gamma_2} X_i^{\gamma_1} - X_c^{\gamma_1} X_i^{\gamma_2}}, \tag{A.13}$$

$$B_2 = \frac{U_i X_c X_i^{\gamma_1} - ((U_0 - U_i)X_i + B_3 X_i^{\delta_1} + B_4 X_i^{\delta_2})X_c^{\gamma_1}}{X_c^{\gamma_2} X_i^{\gamma_1} - X_c^{\gamma_1} X_i^{\gamma_2}}, \tag{A.14}$$

$$B_3 = \frac{M_d U_0 X_d - X_d^{\delta_2} M_f}{X_d^{\delta_1} M_d - X_d^{\delta_2} M_c}, \tag{A.15}$$

$$B_4 = \frac{X_d^{\delta_1} M_f - M_c U_0 X_d}{X_d^{\delta_1} M_d - X_d^{\delta_2} M_c}. \tag{A.16}$$

where

$$M_f = (X_i^{\gamma_2} X_c^{\gamma_1} - X_i^{\gamma_1} X_c^{\gamma_2})(\gamma_1 - 1)(U_0 - U_i)X_i + (\gamma_1 - \gamma_2)[U_i X_c X_i^{\gamma_1+\gamma_2} - (U_0 - U_i)X_c^{\gamma_1} X_i^{\gamma_1+1}], \tag{A.17}$$

$$M_c = (\gamma_1 - \gamma_2)X_i^{\gamma_2+\delta_1} X_c^{\gamma_1} - (\gamma_1 - \delta_1)X_i^{\delta_1}(X_i^{\gamma_2} X_c^{\gamma_1} - X_i^{\gamma_1} X_c^{\gamma_2}), \tag{A.18}$$

$$M_d = (\gamma_1 - \gamma_2)X_i^{\gamma_2+\delta_2} X_c^{\gamma_1} - (\gamma_1 - \delta_2)X_i^{\delta_2}(X_i^{\gamma_2} X_c^{\gamma_1} - X_i^{\gamma_1} X_c^{\gamma_2}). \tag{A.19}$$

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