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Revisiting the Matter of Time Dilation in Special Relativity

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ABSTRACT

There are different interpretations of the Lorentz transformations of the Special Theory of Relativity (SR) concerning the relative rates of two clocks that are moving at constant relative rectilinear velocity (Einstein's definition of conditions necessary for SR). [It is common to talk in terms of how one observer "views" another clock that is moving at constant velocity relative to them. This carries an implication of visual observation. That is a limitation avoided in this contribution by the use of "as determined", without specifying the means employed, but wherever "as viewed" does occur, it is to be considered as "as determined"]. One opinion, which is the settled opinion among physicists, holds that the clocks are "actually" going at different rates. Another opinion holds that it is merely a matter of how observers determine the clocks' rates to be related, with the clocks in fact going at the same rate. The differences in these interpretations are resolved analytically. Five arguments are presented [excluding an inference from the first postulate]. The clocks are determined to have the same rates, i.e. there is no "time dilation" under SR conditions. The related opinion – that the clocks are going at the same rate, but are determined by observers to be going at different rates - is seen to be consistent. Experimental reports, which directly conclude that the clocks are going at different rates, are shown to be seriously flawed, or to not even comply with Einstein's definition of SR.

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1. An Inference from the First Postulate

The Relationship between the Rates of Clocks in Uniform Relative Motion

We consider a series of "stationary" identical clocks that have been set to be synchronised and to have the same rate. "Synchronised" means that they register the same time; "same rate" means that any difference in their displays that there might be between clocks is unchanging with time. Einstein used "synchronous" to indicate that the clocks are for all time synchronised in the sense used here [1]. Where "identical" is used here it means with regard to mechanical, electrical and general physical construction; Einstein used "in all respects alike". For "going at the same rate", we shall, for brevity, use "isotachic".

We now consider one of the clocks to be accelerated rectilinearly to a velocity v, at which point acceleration ceases. It is to be understood that the level of acceleration is not such as will deform the elements (rods, springs, catchment mechanisms etc. for a mechanical clock) in a permanent way, so that the whole mechanism's condition, after the acceleration ceases, is identical to its condition when it was stationary – i.e. there has been only elastic deformation, and the clocks are once more identical, though they may show different times as a result of the temporary effect of any deformation. Equally, during the acceleration, according to the General Theory (GR), the rate of that clock may be altered, but once the acceleration has ceased, the original rate of the clock will be restored, although, again, there will be a difference in the times on the clocks.

Now, Einstein's first postulate says that that mechanism, now travelling at v, must behave in precisely the same fashion as the clocks that have exactly the same mechanism but which have continued to be "stationary". (For example, force produced by the energy source such as a spring – which, we may prima facie presume will not change - will induce the same acceleration of the moving parts.) That is, we might, in a first consideration of clock rates, infer that the moving clock must be isotachic with the stationary ones. Of course, it is possible that during the process of acceleration the clock's rate may alter, due, as described, to temporary deformation of the mechanism, or effects accounted for in the GR ¬- so that at the end of the acceleration period the moving clock is no longer synchronous with the stationary ones. But if the mechanism is again identical, and the acceleration and any temporary deformation have ceased, then its rate should surely be the same as before the acceleration. The concept, then, that a clock at rest in a system of coordinates that is in rectilinear uniform translatory motion with respect to a set of identical clocks at rest in another coordinate system, is somehow, simply by virtue of its velocity, going more slowly than another identical clock at rest in the other, "stationary", coordinate system, is a concept that could be deemed to be at odds with postulate 1, the fundamental Principle of Relativity. It might be considered that a first argument that the rates must be the same is the question "Why-ever should they be different?".

The reason we have for suspecting that a clock moving uniformly rectilinearly at v may not be isotachic with the stationary ones arises from conclusions of SR – the Lorentz transformations (LT's). In this work, the question is asked as to whether that suspicion is well founded. [Of course, the definition of "stationary" is quite arbitrary. We are simply concerned with clocks that are in uniform rectilinear relative translatory motion].

2. The Interpretation of the Lorentz Transformations in respect of Clock Rates

There are two generally recognised interpretations of the LT's with respect to the rates of two identical clocks stationary in different inertial systems in uniform rectilinear relative motion:

2.1 Interpretation a

That the clocks in the two systems are "actually" going at different rates by the factor $[1- v^2/c^2]^{½}$. That is, an observer situated continuously midway between the two clocks which are both moving at the same constant speed towards, or away from, him will actually see (simultaneously by virtue of two 45°mirrors) that they are going at different rates relative to one another (the velocities, the distances, and the time-of-flight delay of light, all being the same for both moving clocks w.r.t the observer, - that is, any effect of time-of-flight delay of light would be the same for both clocks.) This interpretation, – that the clocks are actually going at different rates – has been the opinion of many. Indeed, Einstein in [1] says "Thence we conclude that a balance-clock at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions." It is relevant that he does not use words like "is viewed as going" nor "appears to be going". That is, the equatorial clock must actually go more slowly. [In a note it is explained that he excludes pendulum-type clocks - for which a changed rate would be expected due to the reduced gravitation at the equator. Also, the statement is made some seven years prior to his formulation of the General Theory (GR), so that he is talking without reference to effects of centripetal acceleration, or gravitational potential, which would both be relevant; he is concerned solely with the effect of velocity].

Again, Eddington said, [2a], "it is now known that a clock in motion goes slow in comparison with a fixed clock", and then, [2b], "Thus a clock travelling with finite velocity gives too small a reading – the clock goes slow compared with the time reckoning conventionally adopted." Whereas his first quote might be interpreted as including an accelerating clock, the second quite clearly confines the consideration to the clock's velocity [2].

[It is of relevance that none of their quotes make any reference to any observer who is making the judgement about the clock rates. They are expressing what we may call an "objective" concept of differing rates, based on their interpretation of the conclusions of SR. It surely follows that they must also have had an "objective" concept of two clocks going at the same rate, i.e. without reference as to who is doing the observing].

Another who held to this interpretation was Hawking. His work generally was concerned with advanced concepts, and the matter considered here is so basic that there appear to be no appropriate references in the literature in his name. However, in his video on time travel he clearly holds that the clocks are actually going at different rates due to the relative velocity effect [3]. [At the same time, his proposal would actually work anyway, because it involves the participants travelling round the Earth – i.e. in a circular path, which would result in the stated time dilation, - in accordance with the General Theory (GR) as described at §3.2.2 below. However, he ascribes the effect as being due specifically to the relative velocity].

Further evidence that this interpretation is widely held is the fact that textbooks and digital sources, e.g. Wikipedia, typically continue to introduce the reader to the "clock (or twins) paradox". Although it is then usually pointed out that it is incorrect because the "travelling" twin has to accelerate and decelerate, which precludes consideration under SR, nevertheless, the whole introduction of the concept in the first place is predicated upon the assumption that the clocks are "actually" going at different rates. [The question of "how do we know which is the moving twin?" is answered by the fact that it must be the one who accelerates].

A fourth eminent physicist who expressed this opinion was Møller in his §2.6 [4a]:

"2.6 The Retardation of Moving Clocks. The Clock Paradox

By considering the two events where a moving clock C' (i.e. stationary in S') coincides with one stationary clock in S, and then with another, and applying the LT's he says, [and using {} to indicate his equation numbering]

"………………… we obtain

$\Delta t = t_2 – t_1 = \gamma(t'_2 – t'_1) = (1 – v^2/c^2)^{-½}\Delta t'$ {2.36}

i.e. a clock that is moving with velocity v relative to S will be slow compared to the clocks in S.

………………………… *One could think that that the retardation of moving clocks described by {2.36} was only apparent. However. ......"* [present author's italics]

And, further, he says "According to {2.36} we have the following relation between the increase in proper time dτ and the increase dt of the system S

$d\tau = (1 – v^2/c^2)^{½}\, dt$ {2.38}"

Note that his τ and his t' are synonymous.

That equation will be relevant in §2.5, below. He then goes on to describe the "clock" paradox (i.e. the "twins"), and how it is not consistent in SR, (as described above) and says he will deal with it in GR.

His opinion is very clearly according to interpretation a).

2.2 Interpretation b

This is that there is retardation, but that it is merely a matter of how observers, with clocks moving at constant velocity υ relative to each other, determine each other's clock – they will each determine the other's going slower than their own by the factor $[1- \upsilon^2/c^2]^{-½}$. This interpretation seems also now to be the settled opinion of most. For example, Einstein, in his "popular" text [5a], originally published in 1952 - decades after [1] - now says "As judged from K, the clock is moving with velocity υ; *as judged from this reference-body* [present author's italics], the time which elapses between two strokes of the clock is not one second, but $[1 – \upsilon^2/c^2]^{-½}$ seconds, …. As a consequence of its motion the clock goes more slowly than when at rest". That is rather more equivocal than his statement, from [1], given above at §2.1 and might be interpreted as essentially saying "as viewed", or "appears" or "as determined". Similarly, in considering the length of a moving rod, he first says "The rigid rod is thus shorter when in motion than when at rest, ...", but then further "…the length of the rod *as judged from* K' [present author's italics] would have been $\sqrt{1-\upsilon^2/c^2}$ ", That might well indicate a confirmation of the "as determined" meaning.

[It has to be said, though, that even with interpretation a), one could still say "as viewed from" or "as judged from" the stationary system, and still be technically correct. There is some ambiguity there, and therefore these statements are not necessarily inconsistent with his statement on polar/equatorial clocks at §2.1]

However, with this interpretation b) there is no possibility of a twins paradox, because even though the stationary twin would then view the clock of the travelling one to be retarded all the while when moving, when he comes back and is again stationary, their clocks times would then be seen to coincide (taking it that the clocks were synchronous at the start). That is, a twins paradox would not be possible. We would have to have interpretation a) holding for a twins paradox.

2.2.1 Einstein's Terminology

It is relevant that throughout his original article [1], Einstein repeatedly says things like i) "viewed from the stationary system" and including ii) "It is clear that the same results hold good of bodies at rest in the 'stationary' system, viewed from a system in uniform motion." These words would incline one to interpretation b).

2.3 A First Consideration of Clock Rates: Mutuality

This argument is probably the simplest and most obvious of all: We consider the statement ii) from Einstein [1] as quoted in §2.2.1. That is stating a mutuality of observations, which is a well-accepted feature of SR. Thus, considering each of the two clocks A and B, travelling at υ relative to one another, they must both, according to interpretation a) (and with this mutuality) be "actually" going at a rate which is slower than the other one, and each by the same factor $1/\gamma = [1- \upsilon^2/c^2]^{½}$. That is quite impossible in physics. They cannot both be "actually" going slower than the other. [Of course, the quotation says "viewed from", and that would imply interpretation b). Under interpretation a) it is impossible].

2.4 A Second Consideration of Clock Rates: A Third Clock

A preliminary consideration of clock rates is given above in the context of the first postulate. In this second consideration we take the case of a third clock C, which is identical to the other two, and with respect to which the first two, A and B, are each travelling rectilinearly at υ, that is, by vector addition, (or by relativistic velocity addition) A and B must be going at a constant rectilinear relative velocity, υ' wrt each other, and therefore conforming to the conditions for SR. With respect to both A and B, then, according to interpretation a) of the time relation of SR, the rate of the third clock C must be
$[1- \upsilon^2/c^2]^{½} = 1/\gamma$ times that of each of the other two – that is, logically, those two, A and B traveling at relative υ', must be going at the same rate, as the following shows

$$R_C = R_A/\gamma$$
and
$$R_C = R_B/\gamma$$

where R_X is the rate of clock X. Mathematically that means that $R_A = R_B$.

In a scenario where we consider that C is taken to be the "stationary" clock (which, in empty space, we are allowed to do) we would say, mathematically, $R_A = R_C/\gamma$ and $R_B = R_C/\gamma$ which means again, that $R_A = R_B$. That is, the clocks are "actually" going at the same rate.

Thus, referring to A and B in the above context, interpretation a) is seen to be incorrect.

Therefore, interpretation a) has to be discarded and interpretation b) adopted. Thus, we conclude that even SR itself predicts that the two original clocks, the stationary and the moving, will be isotachic, since their rates are related by the same factor to that of the third clock. (The corollary is, of course, that the third clock must be isotachic with the other two).

There is also, according to SR, the possibility of a dynamic change in the moving clocks, namely that the masses of the elements of the moving clock are increased relative to those of the stationary one, so that the accelerations of the mechanisms will be different, leading to an altered rate. Again, we can consider the third clock C: even according to SR, the relation of that clock's rate to the rates of the other two, A and B, shows that A and B must be isotachic, even including any dynamical effects that there might be according to SR.

2.5 A third consideration of clock rates; the rate relations

We consider the clock rate relations of SR that follow from the LT's. For clocks A and B at rest respectively in two systems S(x,y,z,t) and S'(x'y'z't'), that are in uniform rectilinear translatory motion at v, with S stationary and S' moving in the positive x-x' direction. The relations are derived from the LT's applied to two events, as Moller does for his eqn {2.38} at §2.1 above and we have: first from Møller

$$dt' = dt(1 - \upsilon^2/c^2) \qquad (1)$$

and now by considering Moller's procedure, but by an observer using a clock in S which coincides sequentially with two clocks in S', we obtain

$$dt = dt'(1 - \upsilon^2/c^2)^{½} \qquad (2)$$

where dt and dt' represent the rates of clocks stationary in reference systems S and S' respectively. [Moller's τ is equivalent to his t', the time in the moving system]. Equations (1) and (2) do conform to mutuality.

[The use of the differentials dt and dt' to represent the rates of the clocks is common in the literature – see for example Moller [4a,b]].

In fact, eqns (1) and (2), with the "=" sign, are obviously mutually contradictory. The clocks cannot actually be going slow relative to each other, which provides a mathematical confirmation of the argument from physics in §2.3.

Interestingly, if we take the "=" sign at face value, i.e. "equals", then we may divide (1) by (2), and we get dt'/dt = dt/dt'.

This yields dt' = dt which confirms that the two clocks are going at the same rate. That is, there is no time dilation.

Of course, interpreting them as meaning "as viewed from the other system" is an alternative which is self-consistent, but only with an accompaniment to the interpretation that the clocks are, objectively, isotachic, see §2.6, below. [That interpretation means, of course, always allowing for the effect of the observed rate change due to time-of-flight delay of light, with which SR is not concerned]. The "equals" signs in (1) and (2) cannot otherwise be correct - and they follow immediately from the LT's. We would need to introduce another, different, symbol meaning "equals as viewed", but that is not available to us.

Of course, interpreting them as "equals as viewed" is consistent with Einstein's usage as described at §2.2.1.

Equally, it is also clear that relations (1) and (2) could possibly both be true if $(1 - \upsilon^2/c^2)^{1/2} = 1$, i.e. dt = dt' (however, that implies that v = 0 which is trivial, since under that condition the clocks are by definition going at the same rate).

2.6 A Fourth Consideration: More on "As Viewed".
Two observers A and B, travelling at υ relative to one another must, according to interpretation b) of the LT's, *view* each other's clock as going at a rate which is slower than their own, but each by the same factor $1/\gamma = [1- v^2/c^2]^{1/2}$ in relation to their own. Considering now the possible alternative meaning of the "=" sign in (1) and (2), namely "equals as viewed", it is manifest that the clocks would have to be isotachic, as the following illustrates:
Using the notation R_X^Y to represent the rate of clock X as *viewed* from clock Y, this interpretation of (1) and (2) says that

$$R_B^A = R_A^A\left(\frac{1}{\gamma}\right) \text{ and } R_A^B = R_B^B\left(\frac{1}{\gamma}\right)$$

Where R_X^X represents a clock viewed from its own rest frame, - for example a standard clock will be viewed as unaltered, i.e. going at standard rate.
Thus

$$\frac{R_B^A}{R_A^B} = \frac{R_A^A}{R_B^B}$$

Now if, say, $R_A^A > R_B^B$ (i.e. the clocks are not isotachic) then this equation would be saying that $R_B^A > R_A^B$ (and vice versa) which is not only contrary to mutuality, but means also that the observer with the faster clock A determines the slower clock B to be going faster than the observer with the slower clock B determines the faster clock A to be going (and vice-versa). That would clearly be entirely contrary to any rational expectation. The only consistent relations are that

$$R_B^A = R_A^B \text{ and } R_A^A = R_B^B$$

The first of these is a statement of the mutuality in accordance with Einstein as at §2.3 and the second is saying that the two clocks A and B are isotachic – which confirms the conclusion of §§2.3, 2.4, 2.5, above, - and 2.7 below, - and which is no doubt what workers who hold to interpretation b) would say [1]. That is, there is no time dilation.

2.7 A Fifth Consideration of Clock Rates: Clocks being Accelerated Apart
In a somewhat more physical/mechanical approach, we consider two side-by-side stationary identical clocks restrained by a string from moving apart, but tending to do so by virtue of the force exerted on each by a powerful compressed spring that lies between them, exerting a force on each, directed through their centres of gravity. Being identical, they must be isotachic.

If the string is now cut, each clock must then experience a force of the same magnitude as the other, (the First Law), and since they are identical, they must experience identical but opposite (and decreasing) acceleration all the while as the spring expands. Therefore, whatever effect the acceleration has on the clock rates (due to temporary deformation or to effects as accounted for in GR), must be of the same magnitude for both clocks. (The fact that the accelerations are in opposite directions is of no consequence. The direction of the acceleration does not affect the sense of the change in clock rate, vide clocks in a gravitational field, - the clocks are slowed no matter their disposition w.r.t the mass. Considering the equivalence of gravitation and acceleration, it is seen that the change of the rates of our two accelerating clocks will be in the same sense. Analogously, if we consider the origin to be at the centre of the system, then g and x (see §3.2.1 below) change mutually, (i.e. in this situation a positive x corresponds to a positive g, and vice versa) giving the same sense of rate change for each clock while the spring expands. Therefore, at the end of the spring expansion, as the clocks fly apart at constant relative rectilinear speed, they must still be isotachic.

Although this is only a particular example, it means that we can say that there is no reason to hold that clocks in uniform relative rectilinear motion should be going at different rates.

3. Experimental Evidence
3.1 A statement by Einstein; Defining SR
In his "popular" treatment, Einstein [3b], referring to reference systems K and K', in which "the Galilean law holds" (i.e. inertial systems), says as follows [p61] "But in addition to K, all bodies of reference K' should be given preference in this sense and they should be exactly equivalent to K for the formulation of natural laws provided that they are in a state of *uniform rectilinear and non-rotary motion* [his italics] with respect to K…. The principle of relativity was assumed only for these reference bodies but not for others (i.e. those possessing motion of a different kind). In this sense we speak of the *special* principle of relativity, or special theory of relativity" [The statement is relevant to the effects described in §3.2.1 and §3.2.2 below].

That statement should be borne in mind throughout this present work, since it means that SR is not relevant for any motion which is not rectilinear or which involves acceleration. This will be referred to where appropriate. Many of the motions involved in the experiments considered below do not comply with Einstein's restriction.

3.2 Two Relevant Effects from the General Theory
In the current context, there are two effects of particular interest that are accounted for in the General Theory of Relativity (GR) and not in SR.

3.2.1 Clocks in Linearly Accelerating Frames of Reference
The standard expression for the rate $d\tau$ of a standard clock at rest at position x in a rigid system of reference R(x,y,z,t) that is accelerating with acceleration g in the positive x-direction, compared to the rate dT of a standard clock at rest in an inertial system I(X,Y,Z,T) with x parallel to X is, (Møller [4c])

$$d\tau = dT(1+ gx/c^2) \qquad (5)$$

where x is the position of the clock from the origin of R. Determining where the origin is in any particular situation may be a problem, but even without knowledge of the origin, we can deal with two clocks that are x apart. Thus, for example, for a vehicle accelerating in the x-direction and in which there is a standard clock at the "front" end (i.e. ahead in the direction of the acceleration) and an identical clock at the "rear", a distance h behind, then the forward clock will be going faster than the one behind by the factor

$$d\tau_f = d\tau_r(1 + gh/c^2) \qquad (6)$$

where subscripts f and r refer to the forward and the rearward clock respectively. Similarly, from the Principle of Equivalence of gravity and acceleration, the clock on the mountain top runs faster than the clock in the valley by the same factor where *h* is the height of the mountain top above the valley (near-Earth *g* being taken to be constant over *h*, and ignoring the effect of the different radii of the clocks in the Earth's rotation as described in §3.2.2 below).

This effect will be seen to be relevant in some of the examples of experimental reports examined below.

3.2.2 Clocks following Circular Paths

It is an unfortunate coincidence in relativity theory that the rate of a clock moving in a circular path at velocity v w.r.t the laboratory has, relative to a clock at rest in the laboratory, prima facie, exactly the same expression in both GR and SR - namely in SR we would have [allowing SR relations to be holding in this geometry]

$$d\tau = dt\sqrt{(1 - \upsilon^2/c^2} \qquad (7)$$

where dτ is the rate of a clock moving at constant linear speed *v*, while in GR it is

$$d\tau = dt\sqrt{(1 - \omega^2 r^2/c^2} \qquad (8)$$

where, relative to the laboratory, υ is the velocity of the rotating clock, dτ is the rate of a standard clock at rest at radius r in a rotating reference system whose centre is at rest w.r.t the laboratory and which has constant rotational speed ω, and dt is the rate of an identical clock at the centre, i.e. at rest in the laboratory (and thus also isotachic w.r.t other clocks in the laboratory). That is, the rotating clock is going slow relative to the laboratory clock. Since $\upsilon = \omega r$ it is seen that the two expressions (7) and (8) are identical.

However, there are fundamental differences in the interpretation. The clock at the centre of rotation should be viewed by an observer travelling with the rotating clock (if, again, we were to accept the mutuality of observations in SR in this geometry) to be going slow relative to his own due to their relative velocity whereas according to GR he will, in contrast, observe it to be going faster relative to his own – the larger the ωr the greater the rate difference. The latter effect of the "pseudo-gravity" which acts due to centrifugal action, is well known. These observations by the rotating observer are mutually contradictory as in SR vs GR.

Some workers, for example Møller [4d], seek to explain the identity as demonstrating different aspects of the same effect, but this is incorrect; as explained above. They are two different effects with some different predictions. This "pseudo-gravity" effect of GR will be shown to account for a variety of experimental results previously attributed to effects of SR.

3.3 Experiments using High Precision Clocks

3.3.1 The "Flying Clocks" Experiment

This was an experiment carried out by Hafele and Keating in which a high-precision clock was carried in commercial aircraft on two long-haul flights, one East-West and the other West-East and then compared with a similar clock which had remained on the ground [6]. It was found that the flying clock went more slowly than the ground clock, and this was attributed to the SR effect. However, as Schlegel pointed out, the difference between a flying clock and the ground clock is more correctly explained by a combination of the "mountain-valley" GR effect – see §3.2.1 - (and which was recognised by the original authors) but also together with the pseudo-gravitational effect arising from the GR effect of the circular paths of the flying clocks being at a larger radius than the ground clocks - in accordance with §3.2.2 - rather than an SR effect [7]. Thus, we see that the results of the "flying clocks" experiment have a different explanation, not involving SR. Additionally, Einstein's statement at §3.1 precludes the application of SR for clocks in circular paths. This experiment cannot demonstrate a difference in clock rates according to interpretation a) of SR.

3.3.2 Using Clocks in Satellites

A number of workers e.g. Ashby and Nelson [8], and Wolf & Petit [9], have employed precision clocks in satellites to support the concept of time dilation according to SR. Again, the fact that the clocks are flying in curved orbits means that the correct analysis should employ the GR effect, not SR, as explained in §3.1. *Again, the statement at §3.2.2 is important for these clocks moving in a circular path*. SR is not relevant.

3.3.3 The "Rocket Clock" Experiment

Vessot and Levine [10] describe an experiment in which a probe containing a high-precision clock was launched in a Scout D rocket to an altitude of 10,000km. Also with the clock were a transmitter, which sent a continuous signal at the local clock frequency, and a transponder which could receive a signal from the ground and immediately transmit it back at the frequency at which it was received.

The object of the experiment was to provide demonstration of the effect of gravitation on clocks, but at one point, in the record of the signals received on the ground, there was a zero in the beat frequency between the two signals from the probe, and this was interpreted as being due to the time dilation effect of SR. It has been shown in [11] that this is incorrect, and the experiment does not demonstrate SR. Moreover, the zero beats concerned with in this aspect of the analysis corresponded to a period after the motor thrust had ended, and the probe's velocity had a substantial horizontal component. The probe was then in free fall, accelerating under gravity, and following a curved path until splashing into the ocean. Therefore, the relevant motion *did not comply with Einstein's restrictions for SR as at* §3.1, and the conclusion therefore may not be attributed to an SR effect.

3.3.4 Using Optical Clocks

The work of Chou et al. [12] reports on the time difference between two Al-Mg optical clocks, in one of which the Al+ ion is made to oscillate at 59 MHz. They make an analogy with the so-called twins paradox according to which there should be a clock-rate difference between the twins' clocks due to their relative motion. They say that "In the language of the twin paradox, the moving Al+ ion is the traveling twin, and its harmonic motion amounts to many round trips." The twin paradox is classically set in the context of "uniform translatory motion", and the ions in this experiment are subjected to considerable accelerations – and therefore to GR effects. *With the ions accelerating, the statements at §3.1 and§3.2.1 are particularly important*. SR is not relevant, and the experiment cannot be described as verifying time dilation according to SR.

3.4 Using the Lifetimes of High-Speed Particles

In this second category, typical of the works quoted are the studies of laboratory muons by Bailey et al., and of cosmic ray muons by

Frisch and Smith and by Rossi and Hall [13,15]. Frisch and Smith are regularly quoted as verification of time dilation in accordance with SR [14].

3.4.1 Laboratory Muons

The work of Bailey et al. [13] concerned the lifetimes of muons circulating at $v = 0.999c$ in an orbit of 14m diameter. The extended lifetime noted (in comparison to the rest lifetime) was in close agreement with the value predicted by SR, indicating time dilation.

In these experiments the effects of GR are highly relevant, since the transverse acceleration quoted was ≈ 10^{18}g. Although the result (extended life of the muons) is consistent with the relation of SR, it may not be concluded as demonstrating an effect of SR. Essentially, the result equally demonstrates the pseudo-gravity effect of GR as described above at §3.2.2. Obviously, senior members of the twelve-strong team should have been aware of the pseudo-gravity effect, and it is difficult to comprehend why they chose to describe their results, quite specifically, in terms of "testing the so-called time dilation, or slowing down of moving clocks, predicted by the special theory of relativity".

In addition, because of the rotary motion of the muons, this experiment does not comply with Einstein's statement at §3.1. SR is not relevant. The time dilation which was observed can be attributed to an effect described in GR.

3.4.2 Cosmic Ray Muons

The reports of Frisch and Smith, and of Rossi and Hall, are somewhat alike in principle [14,15]. They measured the survival rate of high-speed cosmic ray muons from mountain-top altitude to a lower level, with an intention of confirming SR time dilation. Rossi and Hall concerned themselves with the relative survival rates of muons of differing momenta and admitted to only a qualitative confirmation. Frisch and Smith determined the survival rate of muons in a particular energy band and reported a quantitative confirmation. In essence, since many more were measured to survive than would be expected from their short lives at rest, it was concluded that this was due to their clocks going slow relative to Earth clocks, i.e. time dilation in accordance with the interpretation a) of SR.

However, it has to be pointed out that since the muons are decelerating as they collide with air molecules, their motion involves acceleration (very large as it happens, – see section §3.4.3, below - and so is not consistent with the Einstein's statement at §3.1. That is, SR is not relevant in these cosmic ray experiments. The time dilation observed may be attributed to an effect described in GR.

3.4.3 The Work of Frisch and Smith

One of the most relevant statements made in Frisch and Smith is that in passing through the air a muon experiences a deceleration on the order of $2\text{x}10^{14}$ ms^{-2} measured *in the frame of reference in which it [the muon]* is at rest [14]. This is deduced from data in Rossi, and from their own data taken at 1950m and 0m. The authors state that "in computing the time dilatation we need apply only special relativistic – rather than general relativistic – transformations ….as long as our calculation itself is made in an inertial frame of reference." That statement demands inspection.

When there are gravitational/acceleration effects, they alter clock rates in an "objective" degree. No matter the inertial system from which an observation is made, an acceleration of another system will be measured to be the same, and the rest system of an accelerating particle is itself, instantaneously, an inertial system travelling at the instantaneous velocity. Considering our own reference system to be quasi-inertial, we will measure the acceleration of the particle to be the same as it is measured in its own rest system. If an effect of the Earth's gravitation is to cause a clock on a mountain-top to go faster than one stationary with respect to it in the valley below, then the *actual* difference between their rates is independent of the inertial reference system from which they might happen to be observed. SR itself says that viewed from an inertial system moving at υ wrt both the mountain and valley clocks, the rates of the two clocks (not moving w.r.t. each other) will both be seen to be altered from their (different) rates by the same factor $[1 - \upsilon^2/c^2]^{½,}$ and *that view will include the different gravitational effects on each*..

The deceleration of the muons is happening piece-meal as the muons collide with atmospheric molecules, and the actual total distance over which the deceleration occurs is almost impossible to estimate. On that basis, it is not possible to provide relevant calculations.

However, the figure given, $2\text{x}10^{14}$ m/s^2, was measured from data obtained on the energy-range relation, and is an average over the period of travel. On that basis, with a distance travelled of 2000m, eqn. (5) gives that the muons' clock rates would at the end be retarded by a time dilation factor of approximately 5.4 compared to their rate at rest. This compares with the experimental factor of 8.8 given by the authors based on velocity. But then again, the deceleration would not be constant, being much higher at lower altitude due to the increasing density of the air, and possibly varying non-linearly, so that taking an average for the deceleration is perhaps a poor approximation. Certainly, the agreement i.e. 5,4 vs 8.8 is perhaps not unreasonable considering the uncertainties in the measurements of the acceleration and the velocity, and in taking the average of the acceleration. All that being said, it is clear that a considerable reduction in surviving numbers of muons is to be expected from GR effects, which is the main point being made. To conclude this sub-subsection, the essence of the situation is that the muons are subjected to enormous decelerations, as measured in any inertial system, and so their motions are not consistent with Einstein's definition of SR – as at §3.1.

3.3.4 The Work of Rossi and Hall

The above conclusion for the work of Frisch and Smith is exactly applicable to the work of Rossi and Hall, which was also done by observing the rate of decay of cosmic ray muons [14,15].

3.4 Doppler Effect

If there is "actual" time dilation, then the standard relation for the Doppler effect is altered due to SR considerations, and there have been numerous experiments carried out where the report claims to have verified the altered relation. These reports are considered exhaustively in McKelvie [16], and it is seen that none of them can so claim.

Discussion

The issue of whether an experiment is examining an "actual" rate difference, or an "as viewed" rate difference requires examination. For example, it might be said that, in the cosmic ray muons experiments, it is merely a matter of how we on Earth view the muons' clocks. However, that would be incorrect, because the fact is that the muons actually survive in larger numbers than their rest life would predict. When brought to rest their clocks would then, with that "as viewed" explanation, be synchronous with earth clocks (taking it that they were synchronous at the start). If it were merely "as viewed", then their clocks would all the while continue to be going at the rest rate and they would survive in appropriately smaller numbers. Of course it is a case of "actually", because of the GR effect of the acceleration. In fact, in every case examined here, it is a case of "actually", but that is because conditions pertaining require analysis using GR and not SR.

Conclusions

There is an implication from the first postulate that two identical clocks that have been synchronized and set to be isotachic at rest and are then arranged to be moving in uniform rectilinear relative motion, will still be isotachic (except if there are any permanent effects on the mechanisms due to earlier acceleration). The mutuality predicted by SR clearly means that they cannot "actually" be going at different rates.

In §2.5 we see that if we take the "=" signs in eqns (1) and (2) to means "equals" then dt = dt' which means that the clocks are isotachic. On the other hand if we take them to mean merely "equals as viewed", then it is seen in §2.6 that that means, again, that the clocks are isotachic.

So, no matter which meaning we put on the "=" sign, there is no possibility of time dilation.

When a third identical clock is considered w.r.t which both clocks of interest are moving at v, then that surmise (equal rates) is seen to be true even if the relations of SR are accepted, (relations which some interpret to mean that the clocks are not isotachic, which would imply time dilation, and which is seen not to be true.). Furthermore, the additional argument in §2.7, involving oppositely accelerated clocks, reinforces the conclusion that the clocks are isotachic.

All these arguments lead to the conclusion that with the conditions of SR, there is no time dilation.

A number of experimental works, which have been used to conclude directly that there is time dilation according to SR, have been seen to not even comply with Einstein's definition of SR, and the results observed are consistent with effects as described by GR due to linear or centripetal acceleration. Reference has been made to work that shows that reports of works purporting to verify the relativistic Doppler effect, are seen to be erroneous.

It has to be said that there are other, modern, theories on time dilation based on the existence of a luminiferous aether and a preferred reference frame e.g. [17,18]. However, we are solely concerned here with conditions of SR, in which those concepts are not relevant.

Of course, time dilation does occur under conditions described in GR.

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