PROPAGATING UNCERTAINTY USING IEEE STD C57.104-2019 DISSOLVED GAS ANALYSIS METHODOLOGY FOR TRANSFORMERS

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Abstract

Dissolved Gas Analysis is a well-established tool for transformer health monitoring with published Standards to help with its interpretation. However, even though it is known that there is measurement uncertainty regarding the true value of sampled gas, there is less available guidance regarding the practical implications. This paper proposes a method for propagating the measurement uncertainties through the methodology presented in the IEEE Std C57.104-2019 to provide the degree of support for its potential outputs. This is done relying on the simplifying assumption that the measurement uncertainty can be expressed as a symmetric triangular distribution for a given gas sample, and that gas samples are independent. The joint probability function is derived in general terms analytically and then a stratified sampling approach is proposed to numerically solve the function. In addition, a modification is made to allow for a more constrained sampling space by deriving and using a simplifying marginal probability without impacting accuracy. These are presented via a use of a case study to demonstrate the efficacy of the proposed approaches.

1 Introduction

Dissolved Gas Analysis (DGA) is one of the most common approaches used for the health monitoring of medium and high voltage transformers given the wide range of potential issues it can detect [1, 2]. Captured within the liquid insulation, gases generated by various processes within the transformer can be quantified and analysed to infer its state. The specific gases and their quantities will depend on the mode of degradation principally affected by the ensuing temperature / energy involved [1, 2, 3, 4, 5]. The specific interpretations are subject to active research but there are two established standards from the IEC 60599-2015 and IEEE Std C57.104-2019 for this topic [4, 5]. [5] was published more recently and so can be thought to represent a more current understanding of the subject and thus is the focus of this research.

The methodology presented in [5] outputs a Status Level equal to either 1, 2, or 3 with associated recommended actions. These Status Levels can simplistically be interpreted as things being probably okay, things being possibly not okay, and things being probably not okay, respectively. They are derived through the use of four tables, each comparing a metric to a given threshold. Although, it is emphasised in [5] that the thresholds should be tailored to a given asset fleet where possible. The values provided for the thresholds differ depending on the gas, and more generally, the age range and type of the transformer. [5] is intended for mineral oil immersed transformers of either sealed or free-breathing type.

There are seven gases covered: hydrogen (H_2), methane (CH_4), ethane (C_2H_6), ethylene (C_2H_4), acetylene (C_2H_2), carbon monoxide (CO), and carbon dioxide (CO_2). Within [5], Table 1 uses the absolute gas level (PPM) from the most recent sample and compares to the 90th percentile from their dataset. Table 2 uses the same metric but compares to the 95th percentile from their dataset. Table 3 considers the relative change in gas levels from the previous sample relative to the 95th percentile from their dataset. Lastly, Table 4 considers the rate of gas change normalised to a year based on 4 – 6 samples, according to specific criteria, relative to the 95th percentile from their dataset. These may range from a 4 to 24-month period. The Status Level is then derived from the outputs of these checks against the tables for each gas, and then the worst-case Status Level across the gases is selected to represent the current sample.

[5] references [6] and highlights that there may be measurement uncertainty especially as samples are measured at near the lower detection limit. [4] suggests a 15% uncertainty on measurements ten times above the analytical detection limit, increasing to 30% below that, if no further information is known. Measurement uncertainty can be presented in different ways. For example, [2] suggests simply outputting the minimum, mean, and maximum. [7] suggested a triangle distribution, and it seems intuitive given it represents a distribution where the least is known of it except its range and its mean being assumed also its mode. Although this is the distribution assumed henceforth, other alternatives such as a normal distribution are equally valid. Using this assumption, and the assumptions that the gases and each sample are all independent, this paper explores the propagation of the uncertainty through the methodology presented in [5] such that the output would be the degree of support for each Status Level to better inform decision-makers.

The analytical derivation of the problem will be outlined for the Status Level stemming from the tables of each gas. Then, the marginal probability distributions for the tables will be derived. At this point, the focus is shifted to a numerical approach to solve the problem as the analytical form grows complicated. A stratified sampling approach is proposed and presented with a case study to demonstrate its efficacy. In addition, the approach is augmented using a simplifying marginal distribution for a subset of the samples to greatly reduce the search space without hindering accuracy.

2. Methodology

2.1 Combined Status Level

One of the final outputs of the IEEE Standard is a Status Level, L, representing the state of the transformer at the current time. Each gas, g, outputs a Status Level and the worst-case is assumed to represent the cohort of k gases. Therefore, the probability of an output of Status Level 1, L_1, is that of all gases outputting a Status Level 1. Similarly, the probability of Status Level 3, L_3, would be that of any gas outputting a Status Level 3. Status Level 2, L_2, is easiest defined as simply the residual probability from these mutually exclusive outcomes. This is shown in equations (1), (2), and (3) for each Status Level, respectively.

$$P(L_1) = \prod_{g=1}^{k} P\left(L_{1,g}\right) \tag{1}$$

$$P(L_3) = 1 - \prod_{g=1}^{k} 1 - P\left(L_{3,g}\right) \tag{2}$$

$$P(L_2) = 1 - P(L_1 \cap L_3) \tag{3}$$

The subsequent equations are for a given gas, but for simplicity the subscript g for gases will be omitted henceforth.

2.2 Individual Status Level

For a given gas, the Status Level is determined by the outputs of four Tables, T_n ($n = 1,2,3,4$). The relationships between the Tables and the Status Levels are shown in (4), (5), and (6), respectively.

$$P(L_1) = P(T_1 \cap T_2 \cap T_3 \cap T_4) \tag{4}$$

$$P(L_2) = P(T'_1 \cap T_2 \cap T_3 \cap T_4) \cup (T'_3 \cap T_2 \cap T_4) \tag{5}$$

$$P(L_{3,m}) = P\left((T'_2 \cap T_4) \cup T'_4\right) \tag{6}$$

Each probability definitions for the individual Tables are shown in (7), (8), (9), and (10), respectively. Table 1, T_1, is the probability that the newest sample, Y_1, has an absolute value less than the threshold, τ_1. Similarly, Table 2, T_2, is the probability that Y_1 has an absolute value less than the threshold, τ_2. Table 3, T_3, compares the remainder of the newest sample, Y_1, subtracted from the previous sample, Y_2, to the threshold, τ_3. Lastly, Table 4, T_4, uses the slope of a linear regression, β_1, of a set of samples, $Y_1, Y_2, \dots, Y_n$, to compare against a threshold, τ_4. This represents the change in gas levels normalised to a one-year interval.

$$P(T_1) = P(Y_1 < \tau_1) \tag{7}$$

$$P(T_2) = P(Y_1 < \tau_2) \tag{8}$$

$$P(T_3) = P\left((Y_1 - Y_2) < \tau_3\right) \tag{9}$$

$$P(T_4) = P(\beta_1(Y_1, Y_2, \dots, Y_n) < \tau_4) \tag{10}$$

Perhaps the key takeaway is that although Y_1 can be considered independent of the other samples, the combined probabilities of the tables needed to determine the Status Level are not simply the product of one another that would have been the case were they all independent events. In particular, the probabilities $P(T_3)$ and $P(T_4)$ require joint probability distributions of the other samples.

2.3 Marginal Probability Derivations

The subsequent derivations will not be generalisable to cases where the samples' probability distributions are other than a symmetric triangular distribution. This assumption is intrinsic throughout and will not be reiterated for each equation.

2.3.1 Sample Distributions: For a given gas's sample, Y_i, its probability density function, $f_i(y_i)$, and corresponding cumulative distribution function, $F_i(y_i)$, can be expressed as shown in (11) and (12), respectively.

$$f_i(y_i) = \begin{cases} 0 & \text{for } y_i < \check{Y}_i, \\ 4\left(y_i - \check{Y}_i\right)(W_i)^{-1} & \text{for } \check{Y}_i \le y_i < \bar{Y}_i, \\ 2(W_i)^{-1} & \text{for } y_i = \bar{Y}_i, \\ 1 - 4\left(\hat{Y}_i - y_i\right)(W_i)^{-1} & \text{for } \bar{Y}_i < y_i \le \hat{Y}_i, \\ 1 & \text{for } \hat{Y}_i < y_i. \end{cases} \tag{11}$$

$$F_i(y_i) = \begin{cases} 0 & \text{for } y_i < \check{Y}_i, \\ 2\left(y_i - \check{Y}_i\right)^2(W_i)^{-1} & \text{for } \check{Y}_i < y_i \le \bar{Y}_i, \\ 1 - 2\left(\hat{Y}_i - y_i\right)^2(W_i)^{-1} & \text{for } \bar{Y}_i < y_i < \hat{Y}_i, \\ 1 & \text{for } \hat{Y}_i \le y_i. \end{cases} \tag{12}$$

where $\check{Y}_i$, $\bar{Y}_i$, and $\hat{Y}_i$, represent the minimum, mean, and maximum potential value for the sample, y_i, respectively. W_i is simply the range between the minimum and maximum potential values for the sample.

2.3.2 Tables 1 and 2: Equation (12) allows for the simple evaluation of the probability of a value being less than a threshold as shown in (13) using Table 1, T_1, as an example.

$$P(T_1) = P(Y_1 < \tau_1) = 1 - F_1(\tau_1) \tag{13}$$

2.3.3 Table 3: For Table 3, T_3, the same form as (13) can be used once the values for $\check{Y}_{1-2}$, $\bar{Y}_{1-2}$, and $\hat{Y}_{1-2}$ for use in (13) are first calculated using (14), (15), and (16), respectively.

$$\check{Y}_{1-2} = \check{Y}_1 - \hat{Y}_2 \tag{14}$$

$$\bar{Y}_{1-2} = \bar{Y}_1 - \bar{Y}_2 \tag{15}$$

$$\hat{Y}_{1-2} = \hat{Y}_1 - \check{Y}_2 \tag{16}$$

2.3.4 Table 4: For Table 4, T_4, the distribution of the potential values is not triangular and requires a different process. The metric being compared to the threshold, τ_4, is the slope of a linear regression line, β_1, normalised to a one-year duration using a pre-defined set of samples, $Y_1, Y_2, \dots, Y_n$, where n can be between 3 and 6, inclusively. The generic equation for a linear regression line is shown in (17).

$$y = \beta_0 + \beta_1 x \tag{17}$$

where β_0 is the intercept point, β_1 is the slope coefficient, and x is the time the sample was taken relative to sample Y_n. In this context, x can be assumed fixed known values, with only the gas values, y, having uncertainty. Given they are unbiased estimates, the mean expected estimate of the slope, $\bar{\beta}_1$, can be obtained using (18).

$$\bar{\beta}_1 = \frac{\sum_{i=1}^{n}[(x_i - \bar{x})(y_i - \bar{y})]}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{18}$$

(18) shows that for $P(T_4)$ specifically, β_0 is not relevant given it is not needed to evaluate $\bar{\beta}_1$, nor is it used when comparing to the threshold, τ_4. This allows for an alternative form shown in (19).

$$\bar{\beta}_1 = \sum_{i=1}^{n} \frac{\Delta\beta_1}{\Delta y_i}\, \bar{y}_i \tag{19}$$

For brevity, $\Delta\beta_1/\Delta y_i$ is henceforth termed c_i and represents the change in β_1 as y_i is changed. To calculate the value for a given c_i, gas values at points except y_i can be taken as zero as in (20).

$$\frac{\Delta\beta_1}{\Delta y_i} = \frac{\bar{\beta}_1\left(\delta_{ij}(\bar{y}_1), \delta_{ij}(\bar{y}_2), \dots, \delta_{ij}(\bar{y}_n)\right)}{\bar{y}_i} \tag{20}$$

where δ_{ij} is the Kronecker delta function explained in (21) that simply equates all gas values except the point in question to zero. Please note, the times, x_i, and thus, $\bar{x}_i$, remain unaffected. This process can then be repeated for each c_i in turn.

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases} \tag{21}$$

The maximum value for β_1, $\hat{\beta}_1$, can be determined using (22) where the function, $\hat{h}(y_i)$, picks the worst-case value for $y_i$to maximise β_1, as shown in (23). The minimum value, $\check{\beta}_1$, can similarly be determined using the reversed logic denoted by $\check{h}(y_i)$.

$$\hat{\beta}_1 = \sum_{i=1}^{n} c_i \times \hat{h}(y_i) \tag{22}$$

$$\hat{h}(y_i) = \begin{cases} \check{y}_i & \text{for } x_i < \bar{x}_i, \\ \hat{y}_i & \text{for } x_i \geq \bar{x}_i. \end{cases} \tag{23}$$

Although this defines the extents and mean of the expected value for the slope β_1, the distribution is non-trivial to calculate. It is the convolution of each component distribution of $c_1 \times f_1(y_1)$. Given this must be repeated $n-1$ times where n can be up to six samples, this creates a non-trivial problem to analytically solve.

However, given that samples, $y_3, \dots, y_n$, are used only once for the calculation of $P(T_4)$ and are assumed independent of one another, they can be combined into a single marginal distribution, demarcated with the subscript, N. This results in a joint distribution shown in (24).

$$\beta_1 = \int_{\check{Y}_1}^{\hat{Y}_1} c_1 \times f_1(y_1) \times \int_{\check{Y}_2}^{\hat{Y}_2} c_2 \times f_2(y_2) \times \int_{\check{Y}_N}^{\hat{Y}_N} c_N \times f_N(y_N)\, dY_N\, dy_2\, dy_1 \tag{24}$$

To obtain the values for N, samples, $y_3, \dots, y_n$, must be combined. (25) and (26) rename terms for brevity. In this case, $g_N(y_N)$ and so $G_N(y_N)$, are no longer triangular distribution functions described by $f_i(y_i)$ and $F_i(y_i)$. The combined form can be determined by convolving each of the distributions. The convolving for each iteration is shown in (27).

$$g_i(y_i) = c_i \times f_i(y_i) \tag{25}$$

$$G_i(y_i) = \int_{\check{Y}_i}^{\hat{Y}_i} c_i \times f_i(y_i)\, dy \tag{26}$$

$$(g_i * g_j)(y) = \int_{-\infty}^{\infty} g_i(z) \times g_j(y - z)\, dz \tag{27}$$

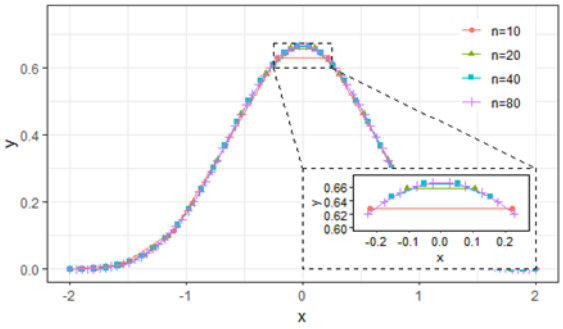

Fig. 1 Plot of convolving a symmetric triangle using Riemann Sum with varying number of samples.

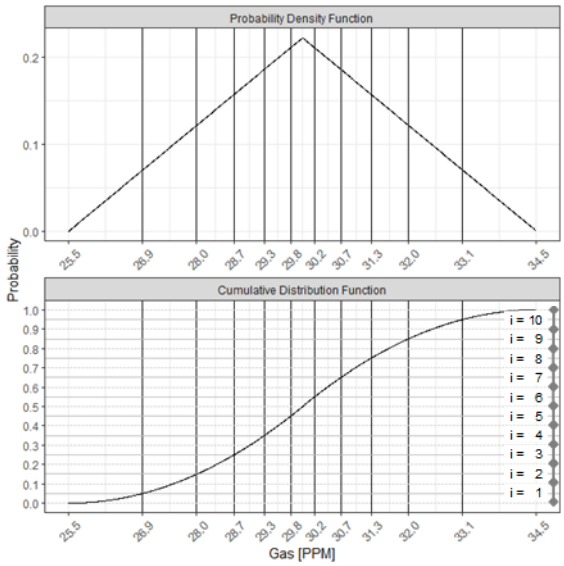

Fig. 2 Plot of sampling protocol using 10 samples.

2.4 Joint Probability

Once samples, $y_3, \dots, y_n$, have been convolved as per (27), this then provides (28) which when integrated spans the entire probability space. This can then be modified using appropriate limits to find the probabilities of certain events. For example, the lower bound for the first sample could be set to the higher of $\check{Y}_1$ and the threshold for Table 1, τ_1, and the upper bound set to the lower of $\hat{Y}_1$ and the threshold for Table 2, τ_2, to represent $P(T'_1 \cap T_2)$.

$$\beta_1 = \int_{\check{Y}_1}^{\hat{Y}_1} g_1(y_1) \times \int_{\check{Y}_2}^{\hat{Y}_2} g_2(y_2) \times \int_{\check{Y}_N}^{\hat{Y}_N} g_N(y_N)\, dY_N\, dy_2\, dy_1 \tag{28}$$

However, (28) represents a piecewise integration with many parts depending on the scenario and requires careful consideration of the limits for each of the integrations that results in a non-trivial analytical solution. Furthermore, the convolved distribution, $g_N(y_N)$, can also be complicated to analytically solve. Therefore, an alternative approach is proposed.

2.4.1 Alternative Numerical Estimation: Rather than analytically convolving the relevant distributions to obtain $g_N(y_N)$ as shown in (27), the distribution can be estimated for each point along its relevant range such that $y_k \in [\check{Y}_i + \check{Y}_j, \hat{Y}_i + \hat{Y}_j]$. To calculate this, an approximation via a Riemann Sum was used as shown in (29).

$$(g_i * g_j)(y_k) \approx \sum_{l=1}^{m} g_i(z_l) \times g_j(y_k - z_l)\, \Delta z \tag{29}$$

where the range being summed across, m, covers the entirety of both distributions, g_i and g_j, such that $z_l \in [min(\check{Y}_i|\check{Y}_j), max(\hat{Y}_i|\hat{Y}_j)]$. These points are at a suitably small interval, Δz, to provide a reasonable estimate of the true value. An illustrative example is shown in Fig 1, where a simple symmetric triangle ranging from -1 to 1 is convolved with itself using various values for the number of intervals. It demonstrates as the number of intervals increase, Δz decreases, and the expected function is better represented.

The output of one convolving is then used for the next iteratively to combine all distributions across $y_3, \dots, y_n$. Prior to each iteration, the output can be rescaled by dividing each point, y_k, by the total trapezoidal area to make a well-defined probability density function. For the case study, this is done using the "trapz" function from the "pracma" library in R [8]. The "cumtrapz" function from the same library is then used to find the approximate $G_{i*j}(y_{i*j})$, acting as the cumulative distribution function. Care should be taken regarding the numerical stability of these imprecise operations to avoid excessive accumulated errors if the intervals are too small or if c_i approaches zero.

A stratified sampling approach is used to then find the final joint probabilities. Depending on the desired number of samples, s, for a given gas, the probability space is segmented into equidistant intervals, p_i, using (30). This is then used in (31) to obtain the gas value, y_i, to represent the probability interval, p_i.

$$p_i = \left(\frac{2i-1}{2s}\right) \tag{30}$$

$$S(y_i) = \begin{cases} \check{Y}_i + \sqrt{\dfrac{W_i.p_i}{2}} & \text{for } 0 \le p_i < 0.5, \\ \hat{Y}_i - \sqrt{\dfrac{W_i(1-p_i)}{2}} & \text{for } 0.5 < p_i \le 1. \end{cases} \tag{31}$$

where i is the index of the interval such that $i \in [1, s]$. An illustrative example is shown in Fig 2, where the first DGA output for the gas, CH_4, is sampled ten times. This low value is to avoid visual clutter. In the case studies, 50 or 1,000 samples were used.

The probability can then be approximated by taking the sum of the probability for every given combination of samples, y_i, from the set S_i generated using (31). (32) shows $P(L_1)$ as an example.

$$P(L_1) = \sum_{y_i \in S_i} P(T_1) \cup P(T_2) \cup P(T_3) \cup P(T_4) \tag{32}$$

3. Case Study

3.1 Data

To demonstrate the proposed approach, five sets of DGA outputs for three gases are used as detailed in Table 1. Three sets of results are presented representing different approaches. The first is labelled "A" and uses the sampling approach shown in (30) for every DGA output using 50 samples. In this approach, $g_N(y_N)$, and so (29) is irrelevant as every point is being represented.

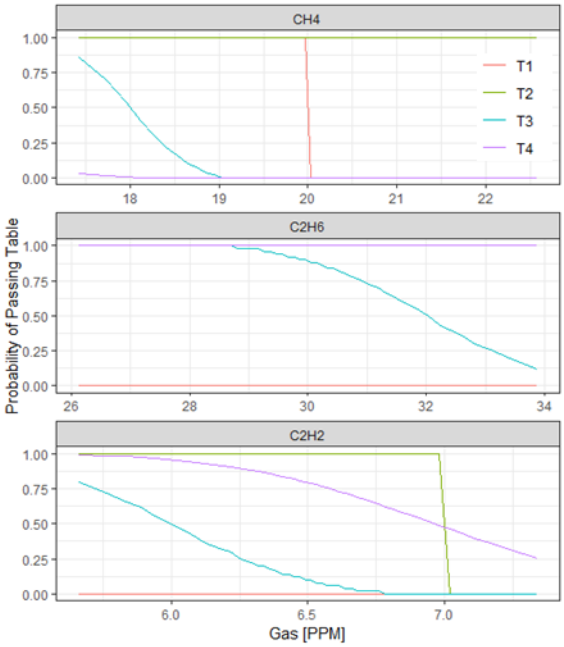

Fig. 3 Plot of the probability of passing each Table for a given sample value.

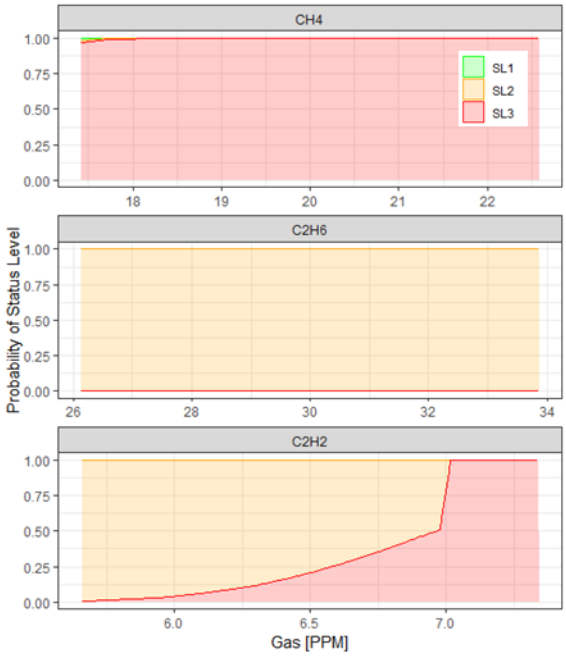

Fig. 4 Plot of the probability of the Status Level for a given sample value.

The second and third approaches, use the sampling approach for only the first and second DGA output, (Y_1, Y_2), using 50 and 1,000 samples and are labelled "B" and "B+", respectively. When estimating $g_N(y_N)$, 1,000 samples were used. Approach B is most directly comparable to Approach A, and Approach B+ demonstrates the added scope for increasing the number of samples given the improved scalability. This is because although Approaches B and B+ add some complexity prior to starting the sweep by requiring the distribution, $g_N(y_N)$, to be estimated, it caps the number of dimensions to sweep across to two, reducing the time to compute relative to Approach A. The approaches represent 50^5, $3 \times 1{,}000 + 50^2$, and $3 \times 1{,}000 + 1{,}000^2$ samples, respectively. Indicatively, Approach A took an order of magnitude longer than approach B, although this is highly dependent on the coded implementation and hardware used. The difference would have been even starker were there to have been the full potential 6 DGA outputs to consider.

As mentioned, Approaches B and B+ use (29) to represent the remaining DGA outputs as a marginal distribution. Then, going through each sample for (Y_1, Y_2) iteratively, the probability for Table 4 is calculated by summing (33).

$$P(T_4|Y_1, Y_2) = 1 - G_N(\tau_4 - c_1 \times Y_1 - c_2 \times Y_2) \qquad (33)$$

It is reemphasised that the location of x_i relative to $\bar{x}$ must be accounted for to determine the sign of c_i as discussed earlier.

3.2 Results

From the results of case study, Fig 3 shows the probabilities of each $P(T_i|Y_1)$ or the gases and demonstrates the non-linear nature of the distributions. For all Approaches, the probabilities for the Tables must then be combined into Status Levels as discussed earlier using (1) – (6). The outputs are shown in Table 2 and Table 3. In this case, despite the $P(T_i|Y_1)$ varying along Y_1, the resulting $P(L_i|Y_1)$ did not show similarly gradual changes, as shown in Fig 4. Given the selection criteria for the final Status Level being picking the worst case, it is then unsurprising that in this case, the combined $P(L_i|Y_1)$ for all gases is near 1 as shown in Table 3.

Table 1 Case study data.

Gas:		C_2H_2		CH_4		C_2H_6	
i	x_i	$\bar{Y}_i$	W_i	$\bar{Y}_i$	W_i	$\bar{Y}_i$	W_i
1	180	6.5	1.95	20.0	6.0	30.0	9.0
2	150	6.0	1.80	8.0	2.4	25.0	7.5
3	120	6.0	1.80	11.0	3.3	40.0	12.0
4	90	6.5	1.95	12.0	3.6	38.0	11.4
5	60	6.5	1.95	10.0	3.0	50.0	15.0

4 Conclusion

Propagating uncertainty through the IEEE Std C57.104-2019 methodology is surprisingly complicated largely due to the metric used for Table 4. This can create a joint probability with numerous samples. The problem was greatly simplified by assuming symmetric triangular distributions and that the gases are independent. It is argued that for the application of measurement uncertainty alone, these assumptions are reasonable. This allows for all but the two most recent samples can be combined into single distribution. This can reduce the computation time for numerically solving the probability spaces via sampling methods.

The results demonstrated near identical outputs but at a greatly reduced computation time. However, there is potentially a simple analytical solution for the convolving of the samples

when creating the combined distribution that further work could derive. This would remove the need for estimating the distribution numerically and would simplify the process further. Furthermore, the results also highlight the onerous Status Level combination approach potentially losing information. This suggests there may be a more granular combination approach available utilising the derived probabilities that could be explored in further work. Lastly, other distributions such as the Normal Distribution could be investigated given its simpler additive properties.

Table 2 Probability for each Table for each Method.

Gas	Method:	$P(T_{1,m})$	$P(T_{2,m})$	$P(T_{3,m})$	$P(T_{4,m})$
C_2H_2	A	0.000	0.689	0.184	0.734
	B	0.000	0.687	0.184	0.733
	B+	0.000	0.688	0.185	0.733
CH_4	A	0.500	0.001	0.068	0.002
	B	0.500	0.001	0.068	0.002
	B+	0.500	0.001	0.069	0.002
C_2H_6	A	0.000	1.000	0.791	1.000
	B	0.000	1.000	0.792	1.000
	B+	0.000	1.000	0.791	1.000

Table 3 Probability for each Status Level for each Method, including combined Status Level.

Gas	Method	$P(L_{1,m})$	$P(L_{2,m})$	$P(L_{3,m})$
C_2H_2	A	0.000	0.689	0.311
	B	0.000	0.688	0.313
	B+	0.000	0.687	0.312
CH_4	A	0.001	0.001	0.998
	B	0.001	0.001	0.998
	B+	0.001	0.001	0.998
C_2H_6	A	0.000	1.000	0.000
	B	0.000	1.000	0.000
	B+	0.000	1.000	0.000
Total	**A**	**0.000**	**0.001**	**0.999**
	B	**0.000**	**0.001**	**0.999**
	B+	**0.000**	**0.001**	**0.999**

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