Flows of Non-Newtonian Fluids in a Sudden Expansion with Multiple Inlets: A Numerical Study

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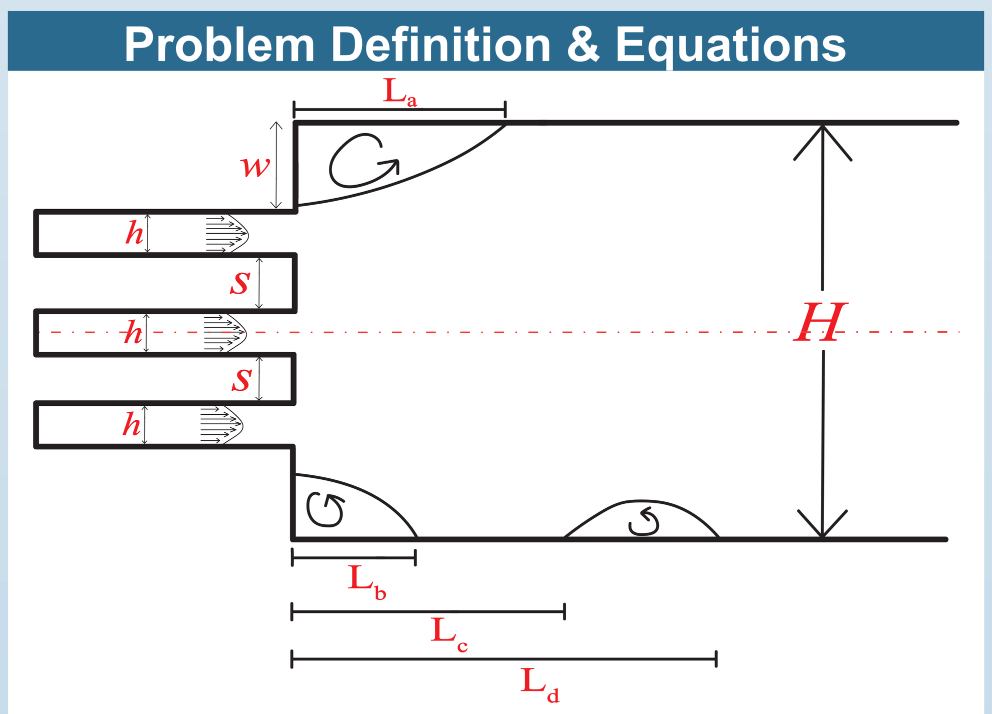
Introduction

- **Sudden expansion** flows are ubiquitous
- **Industrial configurations** can have any number of inlets with **various spacings between inlets**
- This work aims to extend previous work on sudden expansions to a case with multiple (3) inlets
- Ostwald-De Waele (**power-law**) model and **upper-convected Maxwell model** were used to examine the influence of non-Newtonian effects
- Effect of **geometric parameters** (SR, WR), power-law index (n) and dimensionless parameters (Re, Wi, El) on the flow were analysed

Problem Definition & Equations



- Inlets of **equal width and equally spaced**
- Inlet spacing varied but horizontal **symmetry retained**
- Five **geometric cases** examined for 3 inlet configuration with **fixed ER = 9 (ER* = 3)**
- All lengths **normalised** by **one inlet** (h)

Case	SR	WR
A	0.1	2.9
B	0.5	2.5
C (Base)	1.0	2.0
D	1.5	1.5
E	2.0	1.0

Increased inlet spacing ↓

Expansion Ratio
ER = H/h
ER* = H/3h

Spacing Ratio
SR = s/h

Wall Ratio
WR = W/h

Governing Equations

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p - \nabla\cdot\boldsymbol{\tau}$$

Mass and momentum conservation

$$\boldsymbol{\tau} = \eta\dot{\boldsymbol{\gamma}} = \left(K\|\dot{\gamma}\|^{n-1}\right)\dot{\boldsymbol{\gamma}}$$

$$Re_{gen} = \frac{6\rho\bar{u}^{(2-n)}h^n}{K\left[\frac{4n+2}{n}\right]^n}$$

Power-Law Model

$$\boldsymbol{\tau} + \lambda\overset{\nabla}{\boldsymbol{\tau}} = \eta\dot{\boldsymbol{\gamma}}$$

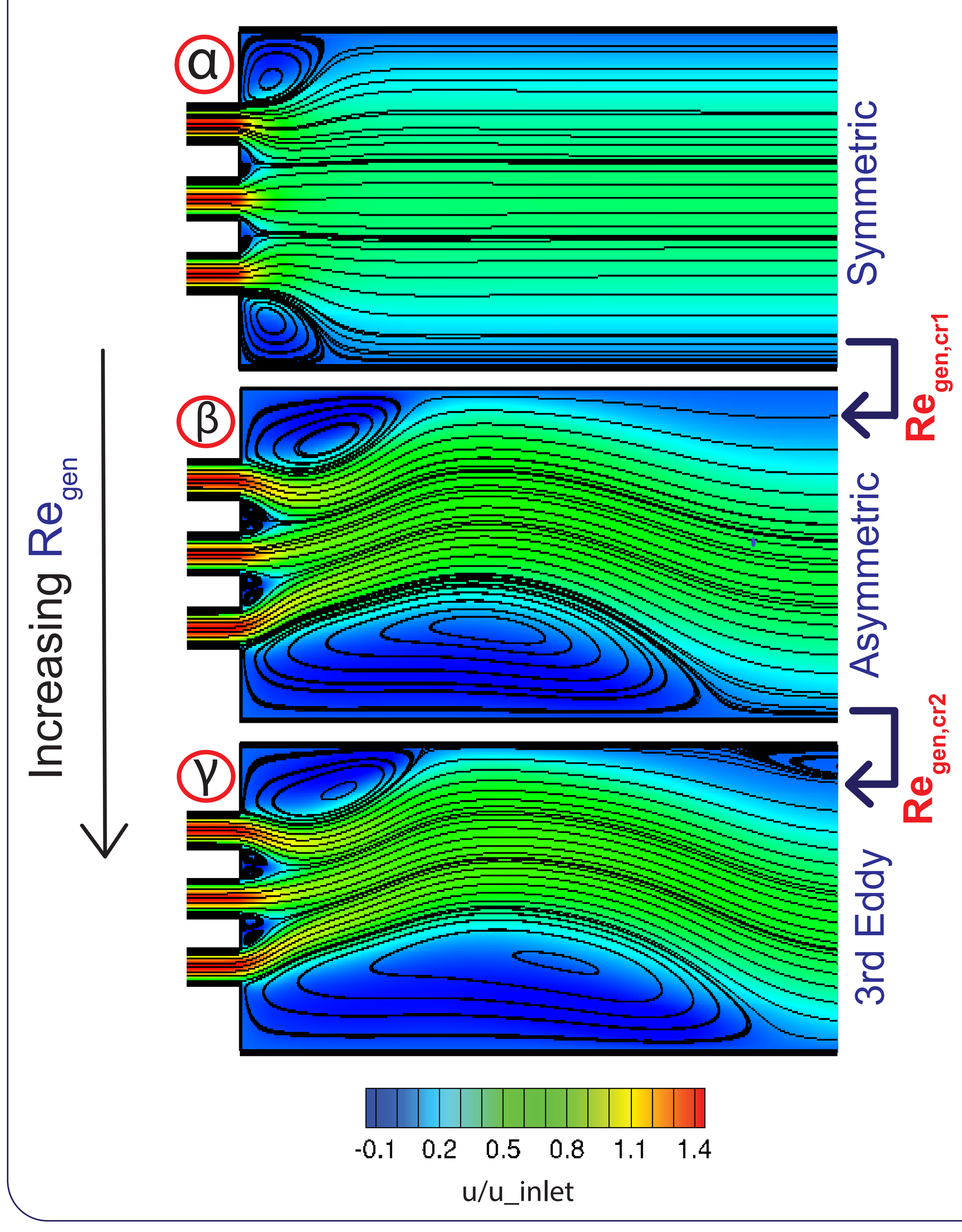
$$Re = \frac{\rho\bar{u}h}{\eta} \quad Wi = \frac{\lambda\bar{u}}{h} \quad El = \frac{Wi}{Re}$$

UCM Model

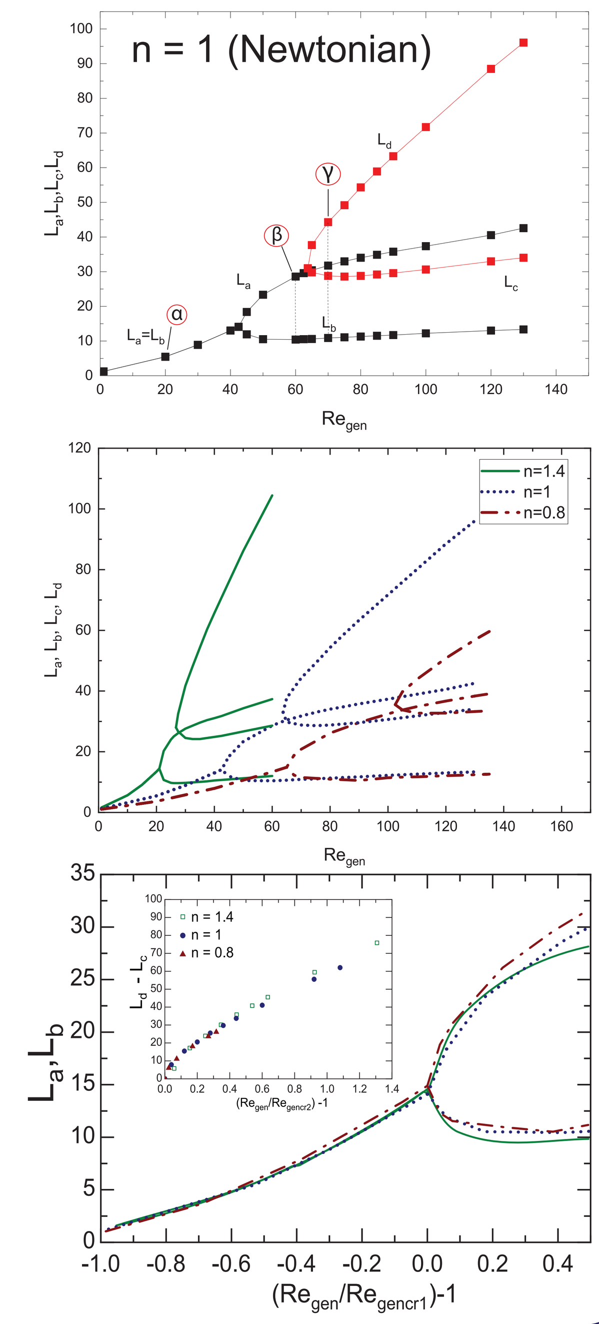
- Solved using an in-house code [1] and OpenFoam RheoTool [2]

Results & Discussion

Base Case: Power-Law Fluids [3]



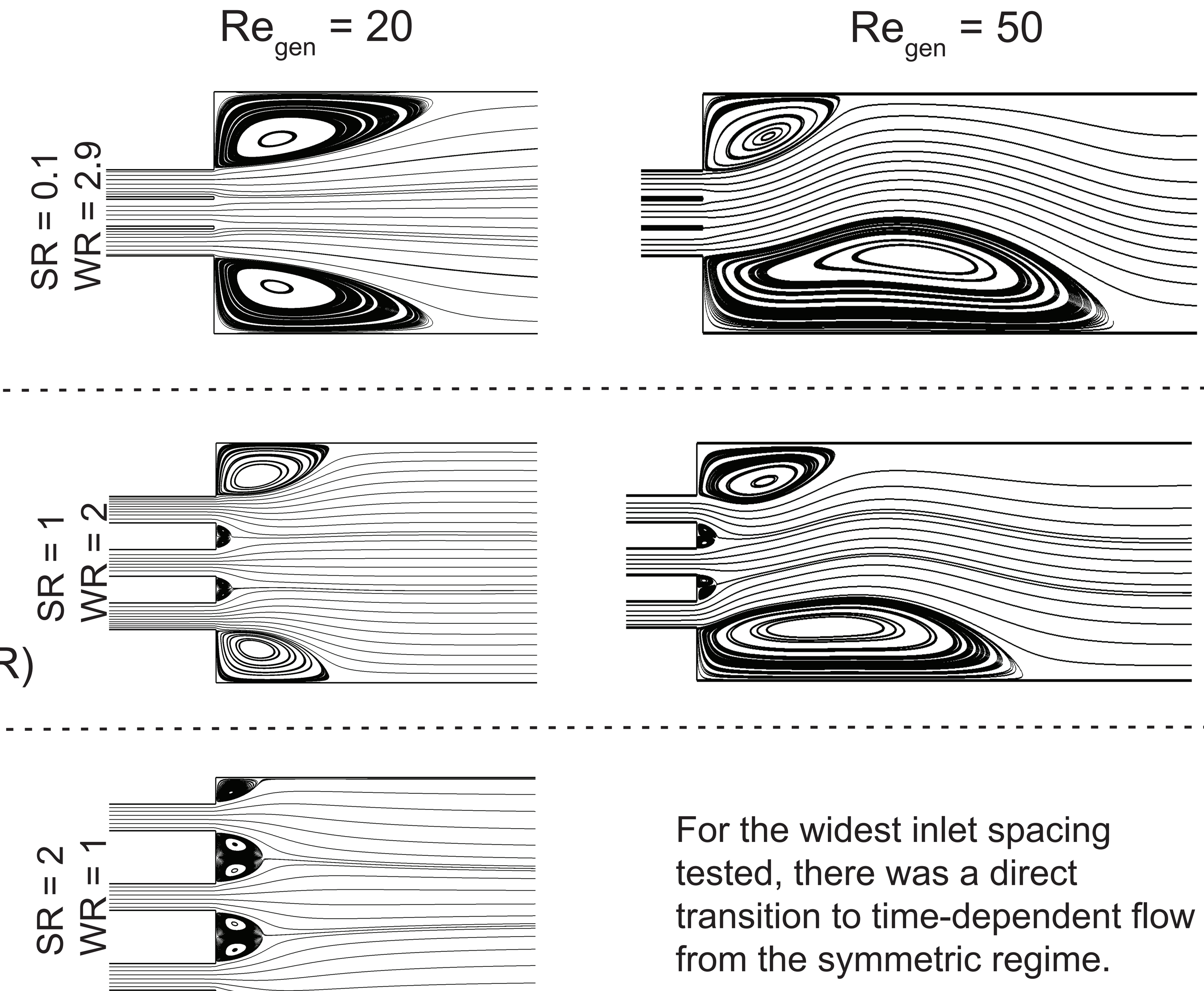
- Corner recirculations and flow regimes **similar to the SI case** [4]
- In the multi-inlet case, we observe inner **recirculations between the inlets** that grow in size within the symmetric regime
- **Shear-thickening**/**Shear-thinning fluids** become asymmetric at **lower**/**higher** Re_{gen}
- At asymmetric transition ($Re_{gen,cr1}$), vortices are approximately equal in length and intensity for all n
- New scalings help collapse the data for various values of n



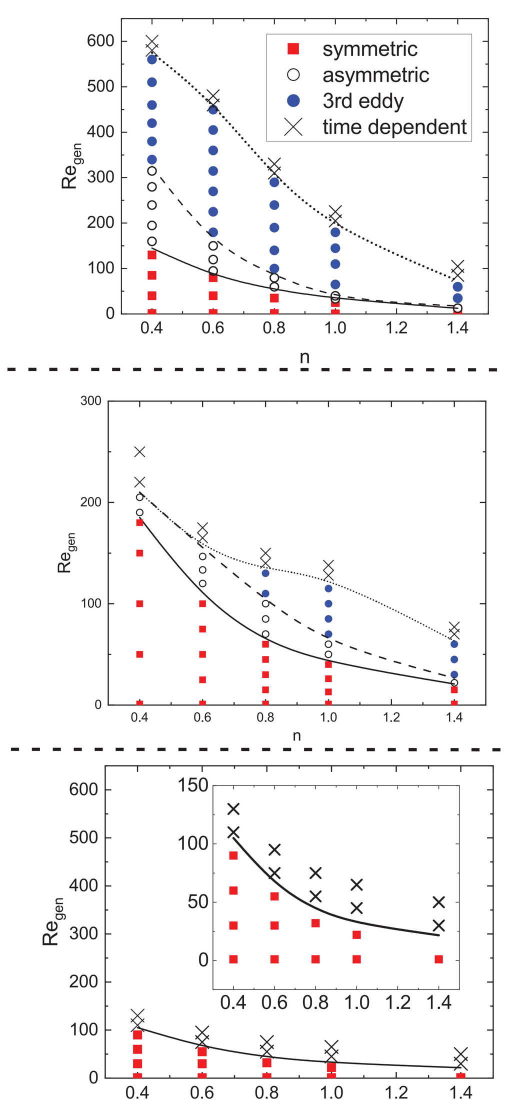
Effect of Inlet Spacing

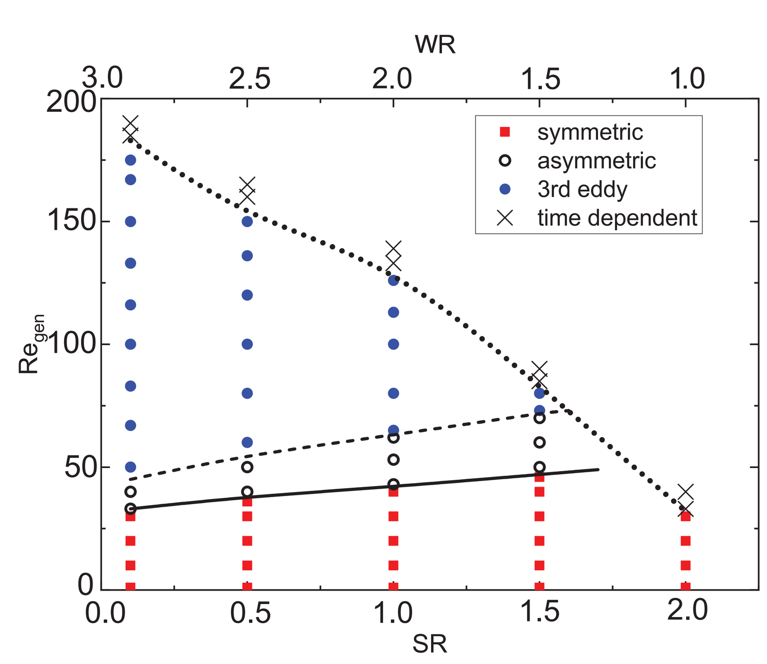
Re_{gen} = 20 Re_{gen} = 50

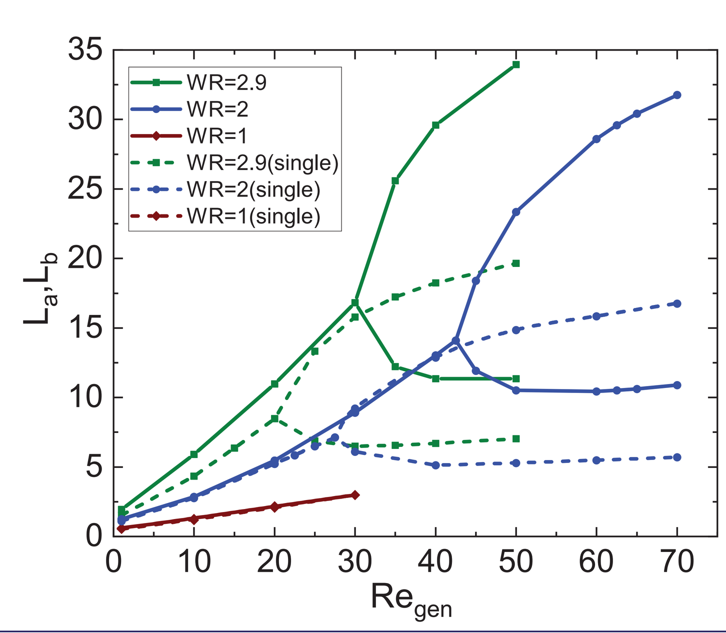
- For smaller values of SR, the intermediate vortices between inlets are attenuated
- Increasing inlet spacing causes both critical Re_{gen} values to increase
- Transition to unsteady flow occurs at lower Re_{gen} for higher SR (or lower WR)

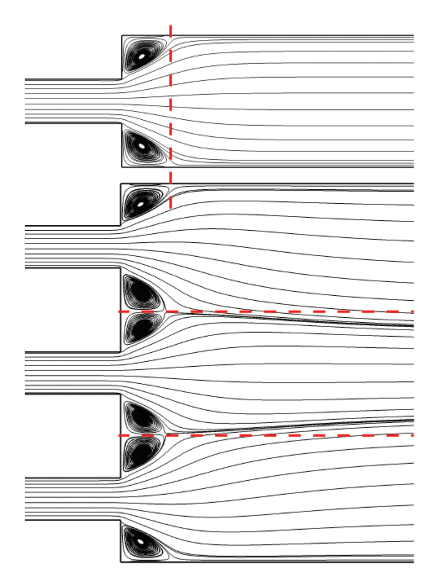


For the widest inlet spacing tested, there was a direct transition to time-dependent flow from the symmetric regime.







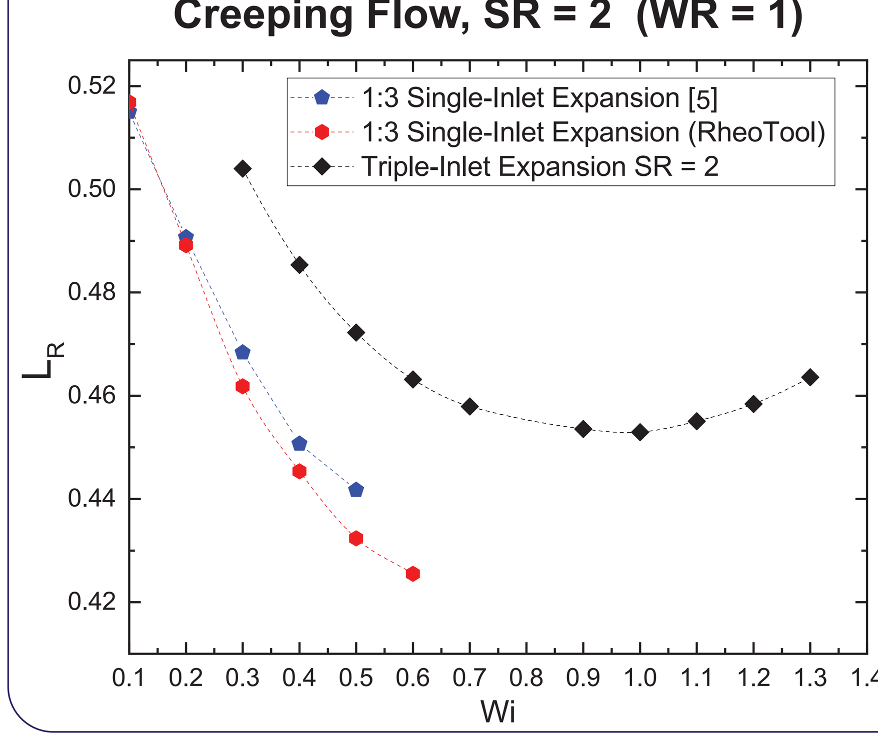


- At the largest SR (or lowest WR), the corner recirculations grow identically to the SI configuration in the symmetric regime

Effect of Viscoelasticity

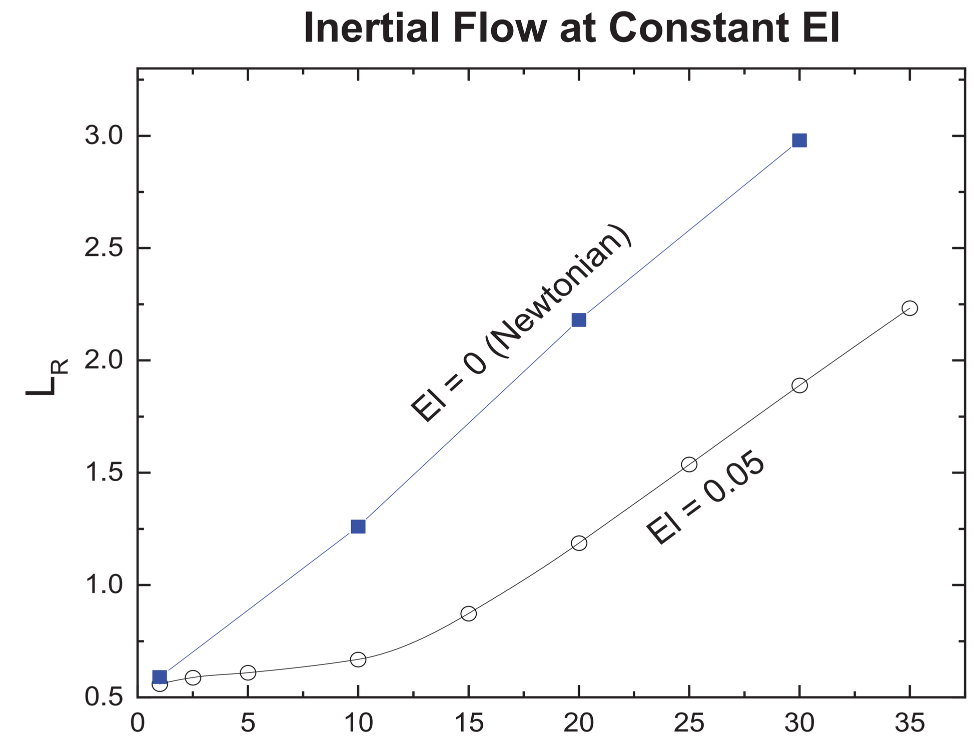
- Viscoelastic code validated with 1:3 SI expansion
- Steady flows obtained for higher Wi than in previous SI studies [5]
- Triple-inlet corner vortices symmetrical and initially decreasing with Wi, as in the SI case

Creeping Flow, SR = 2 (WR = 1)



- Non-monotonic changes in vortex length at higher Wi
- Flow shows a transition to time-dependent behaviour at higher Wi

Inertial Flow at Constant El



- At low Re, corner vortices increase in length very gradually
- As Re is further increased, lengths increase linearly but at a faster rate

References and Acknowledgements

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