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Picture fuzzy large-scale group decision-making approach in a trust relationship-based social network environment

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Abstract: Traditional large-scale group decision-making (LSGDM) methods seldom consider the uncertain relationships between groups or the incompleteness of trust relationship-based social network (SN) information. Picture fuzzy sets (PFSs) have the advantageous capability of comprehensively describing uncertain preference information (from positive, neutral, and negative perspectives). Our purpose is to develop a picture fuzzy LSGDM approach in a trust relationship-based SN environment. The novelty of this study relies on the following two aspects. (1) The incompleteness of trust relationship information in SNs is taken into account and the missing information is estimated using a picture fuzzy trust propagation operator, allowing the construction of a complete picture fuzzy trust relationship-based SN matrix. (2) A picture fuzzy Jensen- a -norm dissimilarity measure is defined. Then, a consensus detection method is developed for LSGDM in a trust relationship-based SN environment. Moreover, based on the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and the proposed picture fuzzy Jensen- a -norm dissimilarity measure, we provide a novel selection process. Finally, a case study of food safety evaluation is provided to demonstrate the effectiveness of the proposed approach: the results of the sensitivity and comparison analyses confirmed that the evaluation results obtained using this method are feasible and credible.

Keywords: Large-scale group decision-making; trust relationship-based social network; picture fuzzy sets; dissimilarity measure; TOPSIS

1 Introduction

Group decision-making (GDM) refers to an analysis process in which individuals choose from several options according to their opinions of two or more decision-makers (DMs), to achieve a certain goal [1]. In recent years, GDM methods have been widely used in various fields, including economic management, operational research, and systems engineering [2-10]. However, with the continuous development of network technology, decision-making problems are been often constrained by many factors and the amount of information for each factor is increasing day by day. Some practical and important decision-making problems often need large-scale groups of experts [11]. Compared with small-scale GDM, large-scale group decision-making (LSGDM) can handle more information and, ultimately, achieve more reliable results by virtue of large-scale group wisdom. Generally speaking, GDM problems involving more than 20 DMs are considered to be LSGDM problems [12].

Previous research into LSGDM methods has mainly focused on the following aspects.

(1) Dimensionality reduction of LSGDM [13-18]. Most LSGDM methods take into account the consistency aspect by reducing the dimensionality (i.e., DMs are clustered according to group opinions, and then the consistency of the decision information within the group or as a whole is explored further). For example, Liu et al. [13] used principal component analysis to reduce the dimension of interval-valued intuitionistic fuzzy LSGDM problems, while Wu and Liu [14] and Du et al. [15] proposed LSGDM clustering methods based on distance and similarity measures, respectively. Xu et al. [19] and Quesada et al. [20] defined a consensus model among DMs for LSGDM problems. Of course, there are some LSGDM methods that do not consider the consistency of the decision-making information, or that directly eliminate DMs who differ greatly from a group's opinions and related decision-making information. However, the actual decision-making process often involves a large number of DMs, and experts often disagree when evaluating projects. If group consistency is not considered or DMs with popular opinions are ignored, there will be certain impacts on the objectivity and rationality of the decision results. Most LSGDM methods better reflect the right of each DM to participate in decision-making by reducing the dimensionality and taking into account group consistency; therefore, these methods are used more widely.

(2) Preference information description [18, 21-22]. Many studies have focused on related ways to reasonably express evaluation information; however, because of the complexity and uncertainty of decision-making problems, as well as the limitation and fuzziness of human cognition, traditional LSGDM methods cannot solve real decision-making problems. After Zadeh [23] proposed the concept of fuzzy sets (FSs) and linguistic terms, some scholars extended the variety of FSs to the LSGDM environment (e.g., intuitionistic fuzzy sets (IFSs) [18] and interval type-2 fuzzy sets (T2FSs) [21-22]) and defined the transformation rules between language terms and their corresponding numerical expressions. Wu et al. [21] and Tian et al. [22] used T2FNs to describe the trust among experts and information about their preferences.

Contrary to the methods mentioned above for the description of uncertain information, picture fuzzy sets (PFSs) [24], as an extension of IFSs, can truly express the opinions of DMs (including yes, abstain, no, and refusal), avoiding the loss of evaluation information in an actual decision-making environment. Recently, a number of studies have focused on the extension of PFSs and their corresponding decision-making methods and applied them to solve various GDM and clustering analysis problems [25-34]. For example, Wang et al. [27] defined the picture fuzzy normalized projection-based VIsekriter-ijumska optimizacija i KOmpromisno Resenje (VIKOR) method and applied it to the risk assessment of construction projects;

additionally, Peng et al. [32] and Tian et al. [33, 34] defined the picture fuzzy analytic network process (ANP) and an Acronym in Portuguese of Interactive and Multi-Criteria Decision-Making (TODIM), the analytic hierarchy process (AHP) and preference ranking organization method for enrichment evaluations (PROTHEE) II decision-making methods, respectively. However, to the best of our knowledge, picture fuzzy information has seldom been used to describe the preference information of large numbers of DMs.

Social networks (SNs) represent collections of “actors” and “relationships”. The actors can be individuals, organizations, or countries, while the relationships can be those between individuals, between organizations and individuals, or between organizations only [35]. The aggregation of these relationships constitutes an entire SN. In such a network, the ranking of individuals or nodes is usually based on their centrality or importance. Each node’s centrality is affected by that of other nodes and this effect is mutual [36-38]. Since the DMs in LSGDM are homogeneous and can be regarded as nodes in an SN, the SN analysis method can be used to determine the centrality or importance of each DM [39-45]. Also, the weight of each DM participating in the decision-making can be determined, to improve the accuracy of decision-making [46-48].

The motivations behind our study can be summarized as follows. (1) Existing methods only consider the existence of a trust relationship among DMs in SNs, while ignoring the trust uncertainty among experts and the incompleteness of relationships in SNs. (2) Most existing methods have used the distance measure, rather than other measures, to determine the consensus in SNs. (3) Contrary to existing means of information expression, PFSs can describe the trust relationships among DMs in SNs and information about DMs’ preferences in LSGDM from three aspects: positive, neutral, and negative memberships. The resulting original decision-making information is particularly comprehensive, effectively improving decision-making accuracy. (4) The existing picture fuzzy decision methods cannot solve LSGDM problems in a trust relationship-based SN environment.

The purpose of this study is hence to propose a picture fuzzy LSGDM approach in a trust relationship-based SN environment. Our contributions can be summarized as follows.

(1) We highlight the advantages of PFSs in considering positive, neutral, and negative memberships when describing uncertain information. Picture fuzzy information is extended to LSGDM problems in an SN environment. Moreover, the transformation rules between linguistic terms and PFSs are established for determining the nature of trust relationships among DMs in an SN, as well as their preference information during the decision-making process. (2) The relationships between incomplete pieces of SN information are considered; moreover, a picture fuzzy trust propagation (PFTP) operator is introduced to estimate the missing trust information in an SN. (3) A picture fuzzy Jensen- a -norm dissimilarity measure is proposed. On this basis, we develop a consensus detection method for LSGDM in an SN environment. (4)

Based on the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [38] and the proposed picture fuzzy Jensen- a -norm dissimilarity measure, we conduct a selection process.

The remainder of this paper is organized as follows. In Section 2, some basic concepts, including those of PFSs and SNs are reviewed, and the picture fuzzy trust score is introduced. In Section 3, a picture fuzzy LSGDM approach is developed in a trust relationship-based SN environment. Then, in Section 4, a case study and results are provided to show the validity and feasibility of the proposed method. Section 5 contains some discussion regarding the sensitivity analysis and comparison analysis results. Finally, Section 6 comprises our conclusions.

2 Preliminaries

This section introduces the related concepts of PFSs and SNs, as well as that of picture fuzzy scores and the corresponding comparison method for two PFSs.

2.1 Picture fuzzy sets

Due to the uncertainty and complexity of the trust relationships between DMs in SNs and the fuzziness of DMs' cognition, ways to describe preference information play an increasingly important role in LSGDM problems. Here, the basic concept of PFSs and their corresponding operations, which will be used for later analyses, are introduced.

Definition 1 [24]. Let X be a universe; then, a PFS $\hat{\psi}=\left\{\left(x,\ Pos_{\hat{\psi}}\left(x\right),\ Neu_{\hat{\psi}}\left(x\right),\ Neg_{\hat{\psi}}\left(x\right)\right)\middle| x\in X\right\}$ is characterized by a positive degree $Pos_{\hat{\psi}}\left(x\right)\in\left[0,1\right]$, neutral degree $Neu_{\hat{\psi}}\left(x\right)\in\left[0,1\right]$, and negative degree $Neg_{\hat{\psi}}\left(x\right)\in\left[0,1\right]$ of the element x to the set $\hat{\psi}$. Here, $0\leq Pos_{\hat{\psi}}\left(x\right)+Neu_{\hat{\psi}}\left(x\right)+Neg_{\hat{\psi}}\left(x\right)\leq 1, \forall x\in X$. For $x\in X$, $\pi_{\hat{\psi}}\left(x\right)=1-\left(Pos_{\hat{\psi}}\left(x\right)+Neu_{\hat{\psi}}\left(x\right)+Neg_{\hat{\psi}}\left(x\right)\right)$ is interpreted as the refusal degree of x in $\hat{\psi}$.

In the following, the picture fuzzy trust score and corresponding comparison method of two PFSs is presented.

Definition 2 [26]. If $\hat{\psi}=\left(x,Pos_{\hat{\psi}}\left(x_i\right),\ Neu_{\hat{\psi}}\left(x_i\right),\ Neg_{\hat{\psi}}\left(x_i\right)\right)$ is a PFS defined as $X=\left\{x_1,x_2,\ldots,x_n\right\}$, then the picture fuzzy trust score $T\left(\hat{\psi}\right)$ of $\hat{\psi}$ can be defined as:

$$T\left(\hat{\psi}\right)=\sum_{i=1}^{n}\frac{Pos_{\hat{\psi}}\left(x_i\right)-Neu_{\hat{\psi}}\left(x_i\right)-Neg_{\hat{\psi}}\left(x_i\right)+1}{3n}. \quad (1)$$

Then, the order relationship of two PFSs $\hat{\psi}_1=\left(x,Pos_1\left(x_i\right),Neu_1\left(x_i\right),Neg_1\left(x_i\right)\right)$ and $\hat{\psi}_2=\left(x,Pos_2\left(x_i\right),Neu_2\left(x_i\right),Neg_2\left(x_i\right)\right)$ is defined as:

(1) if $T(\hat{\psi}_1) > T(\hat{\psi}_2)$, then $\hat{\psi}_1$ is preferred to $\hat{\psi}_2$, i.e., $\hat{\psi}_1 \succ \hat{\psi}_2$;

(2) if $T(\hat{\psi}_1) = T(\hat{\psi}_2)$, then $\hat{\psi}_1$ is indifferent to $\hat{\psi}_2$, i.e., $\hat{\psi}_1 \sim \hat{\psi}_2$;

Here, $0 \le T(\hat{\psi}) \le 1$.

2.2 Social networks

An SN generally uses $V = \{V_1, V_2, \ldots V_r\}$ and $Z = \{Z_1, Z_2, \ldots, Z_p\}$ to represent the set of nodes and the set of edges between the nodes, respectively. Here, the nodes represent experts and the edges represent the relationships among experts [35].

Example 1. Assuming a trust SN exists, then the graph can be illustrated as shown in Fig. 1.

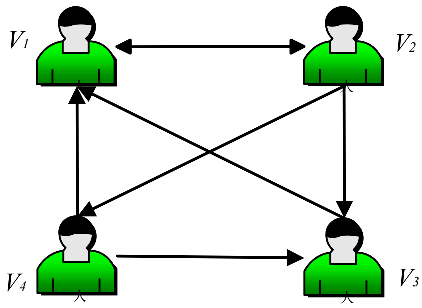

Fig. 1. A trust SN.

Then, the corresponding binary relationship can be denoted in an algebraic form as:

$R = \{(V_1, V_2), (V_2, V_1), (V_2, V_3), (V_2, V_4), (V_3, V_1), (V_4, V_1), (V_4, V_3)\}$,

where $(V_1, V_2), (V_2, V_1) \in R$ indicates that there exists a relationship (directed arc) between expert V_1 and expert V_2, i.e., expert V_1 and expert V_2 trust each other; $(V_1, V_3) \notin R$ indicates that there is no relationship between V_1 and V_3.

The above trust SN can also be denoted by an adjacency matrix as:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}.$$

The corresponding adjacency matrix is also referred to as a sociomatrix and, can also express the relationships between experts.

The approaches presented above, however, can only simply describe directed or undirected relationships among experts; they are unable to describe the uncertain relationships between experts in real trust SNs. To overcome these shortcomings, some researchers have extended IFSs and IT2Ss to the trust SN environment. Nevertheless, compared with IFSs and IT2Ss, PFSs can describe experts' preference information from three different aspects, i.e., trust, neutrality, and distrust, which can more comprehensively express the complex trust relationships that exist among experts in trust SNs. Hence, this paper will construct a trust SN based on the picture fuzzy preferences of experts.

3 Picture fuzzy LSGDM approach in an SN environment

In this section, we describe the development of a picture fuzzy LSGDM approach in a trust relationship-based environment. First, the problem is described, and an SN-based community detection (CD) method employing picture fuzzy preference information is introduced together with the PFTP operator. Second, the method used for determining the weights of experts in each cluster and the weights of clusters is described. Third, the necessary steps for constructing an aggregated decision matrix and obtaining the picture fuzzy consensus index (PFCI) of three levels (based on the weighted PFTP operator and picture fuzzy dissimilarity measure) are presented. Finally, we explain the selection process conducted using TOPSIS and the procedure applied for a picture fuzzy LSGDM approach in a trust relationship-based environment.

3.1 Problem description

We consider a picture fuzzy LSGDM problem in a trust relationship-based SN environment, assuming the presence of $r\left(r \geq 20\right)$ DMs denoted by $V=\left\{V_1, V_2, \ldots V_r\right\}$, n alternatives denoted by $\Lambda=\left\{\alpha_1, \alpha_2, \ldots, \alpha_n\right\}$, and m criteria denoted by $C=\left\{c_1, c_2, \ldots, c_m\right\}$. The weight vector of C is represented by $\varpi=\left(\varpi_1, \varpi_2, \ldots, \varpi_m\right)^T$, where $\varpi_j \geq 0 \quad \left(j=1,2,\ldots,m\right)$, $\sum_{j=1}^{m} \varpi_j = 1$. Considering the incomplete trust relationship-based SN, DMs are able to mutually express their degrees of trust by using the linguistic scales presented in Table 1. The corresponding trust matrix $E_V=\left(E_{V_i,V_j}\right)_{r\times r}$ is obtained as follows:

$$E_V=\begin{pmatrix} & - & E_{V_1,V_3} & \cdots & E_{V_1,V_r} \\ E_{V_2,V_1} & & - & \cdots & E_{V_2,V_r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ E_{Vr,V_1} & E_{V_r,V_2} & - & \cdots & \end{pmatrix}.$$

Here, $-$ denotes the missing trust relationship information, while E_{V_i,V_j} represents the trust relationship between DM V_i and DM V_j.

Based on the transformation rules between the linguistic scales and the PFSs, the above trust matrix can be converted into a picture fuzzy trust matrix. If the trust relationship information is missing, the complete trust relationship-based SN G can be constructed based on the picture fuzzy propagation operator:

$$G=\begin{pmatrix} & G_{12} & G_{13} & \cdots & G_{1r} \\ G_{21} & & G_{23} & \cdots & G_{2r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{r1} & G_{r2} & G_{r3} & \cdots & \end{pmatrix}.$$

We define the evaluation value $E^{(l)}=\left(E_{ij}\right)^{(l)}\left(l=1,2,\ldots,r\right)$ of α_i for criterion c_j provided by expert V_l in the form of linguistic scales (shown in Table 1). The corresponding picture fuzzy evaluation matrix $\overline{E}^{(l)}=\left(\overline{E}_{ij}\right)^{(l)}\left(l=1,2,\ldots,r\right)$ can be determined based on the transformation rules between the linguistic scales and the PFSs:

$$E^{(l)}=\begin{pmatrix} \overline{E}_{11}{}^{(l)} & \overline{E}_{12}{}^{(l)} & \overline{E}_{13}{}^{(l)} & \cdots & \overline{E}_{1m}{}^{(l)} \\ \overline{E}_{21}{}^{(l)} & \overline{E}_{22}{}^{(l)} & \overline{E}_{23}{}^{(l)} & \cdots & \overline{E}_{2m}{}^{(l)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{E}_{n1}{}^{(l)} & \overline{E}_{n2}{}^{(l)} & \overline{E}_{n3}{}^{(l)} & \cdots & \overline{E}_{nm}{}^{(l)} \end{pmatrix}.$$

By considering the weight π_t^l of experts V_l in cluster Com_t, the correspondent aggregated decision matrix can be obtained as follows:

$$\overline{E}^{(t)}=\left(\overline{E}_{ij}\right)^{(t)}=\sum_{V_l\in Com_t}\pi_t^l\left(\overline{E}_{ij}\right)^{(l)}\left(t=1,2,\ldots,q\right). \quad (2)$$

Then, based on the weighted PFTP operator, we obtain the aggregated picture fuzzy decision matrix $\overline{E}=\left(\overline{E}_{ij}\right)_{n\times m}$ for the entire SN:

$$\overline{E}=\begin{pmatrix} \overline{E}_{11} & \overline{E}_{12} & \overline{E}_{13} & \cdots & \overline{E}_{1m} \\ \overline{E}_{21} & \overline{E}_{22} & \overline{E}_{23} & \cdots & \overline{E}_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{E}_{n1} & \overline{E}_{n2} & \overline{E}_{n3} & \cdots & \overline{E}_{nm} \end{pmatrix}.$$

Table 1. Linguistic scales for degrees of trust in the SN and evaluation of the alternatives.

Linguistic Scale for degrees of trust in the SN	Linguistic Scales for evaluation of the alternatives	PFS
Extremely untrusting (EU)	Extremely bad (EB)	$\left(x,0,0,0.95\right)$

Very untrusting (VU)	Very bad (VB)	$(x, 0.05, 0.05, 0.85)$
Untrusting (U)	Bad (B)	$(x, 0.1, 0.1, 0.75)$
Medium (M)	Medium (M)	$(x, 0.55, 0.2, 0.2)$
Trusting (T)	Good (G)	$(x, 0.75, 0.1, 0.1)$
Very trusting (VT)	Very good (VG)	$(x, 0.85, 0.05, 0.05)$
Extremely trusting (ET)	Extremely good (EG)	$(x, 0.95, 0, 0)$

3.2 Picture fuzzy trust propagation operator

In cases of incomplete information in the trust relationship-based SN environment, it is necessary to estimate this information and construct a complete trust relationship-based SN. Victor et al. [49] first developed a trust propagation operator based on t-norms, which is commutative, monotonic, bounded, and associative. On this basis, we introduce an extended PFTP operator based on the weighted picture fuzzy geometric average operator [50], in the following way.

Definition 3. Assume $\hat{\psi}_j = \left(Pos_j(x), Neu_j(x), Neg_j(x) \middle| x \in X\right)(j = 1, 2, \ldots, n)$ is a group of PFSs. Then, the PFTP operator is a mapping *PFTP*: $PFS^n \to PFS$, i.e.,

$$\begin{aligned} &PFTP(\hat{\psi}_1, \hat{\psi}_2, \ldots, \hat{\psi}_n) \\ &= \left(\prod_{j=1}^{n} \left(1 - Neg_j(x) - Neu_j(x)\right) - \prod_{j=1}^{n} \left(1 - Neg_j(x) - Neu_j(x) - Pos_j(x)\right), \right. \\ &\left. \prod_{j=1}^{n} \left(1 - Neg_j(x)\right) - \prod_{j=1}^{n} \left(1 - Neg_j(x) - Neu_j(x)\right), 1 - \prod_{j=1}^{n} \left(1 - Neg_j(x)\right) \right). \end{aligned} \tag{3}$$

The *PFTP* operator satisfies the commutativity, monotonicity, and boundedness. More details about these properties can be found in Ju et al. [50]. Then, the completed picture fuzzy trust sociomatrix can be obtained based on the PFTP operator.

Remark 1. The missing trust information can be determined using the *PFTP* operator. If there are several paths from expert V_i to expert V_j, then the shortest path, i.e., the minimum number of edges, will be selected as the trust propagation path. If the numbers of edges for two or more paths are equal to each other, these paths can be selected using the *PFTP* operator and the average of these aggregated values with respect to three memberships can be calculated as the final estimated trust values.

Example 2. Assume a trust SN comprises of 20 experts; the trust relationships among these experts are shown in Fig. 2. The directed arcs indicate the existence of trust relationships among the experts. The corresponding trust linguistic evaluation information among the experts can be found in Table 2 (see Appendix A), where blanks indicate that there are no direct trust relationships between the corresponding experts. For example, the directed arc between expert V_1 and expert V_2 indicates that expert V_1 trusts expert V_2 completely. From Fig. 2, it can be seen that there is no direct trust relationship between expert V_8 and expert V_9, but there are three indirect paths, i.e., $Pl_1 : V_8 \to V_{14} \to V_7 \to V_9$, $Pl_2 : V_8 \to V_6 \to V_5 \to V_9$, and $Pl_3 : V_8 \to V_6 \to V_{14} \to V_7 \to V_9$. Here the number of edges of path Pl_1 is equal to that of path Pl_2 and is less than that of path Pl_3. Hence, the missing picture fuzzy trust degree between expert V_8 and expert V_9 can be obtained by path Pl_1 and path Pl_2. Based on the PFTP operator, i.e., Eq. (8), we can obtain get $PFTP_{Pl_1} = (x, 0.48, 0.24, 0.28)$ and $PFTP_{Pl_2} = (x, 0.384, 0.3, 0.352)$. Then, the picture fuzzy trust degree between expert V_8 and expert V_9 can be determined as: $\overline{E}_{89} = \left(x, \frac{0.48+0.384}{2}, \frac{0.24+0.3}{2}, \frac{0.28+0.35}{2} \right)$ $= (x, 0.432, 0.27, 0.315)$.

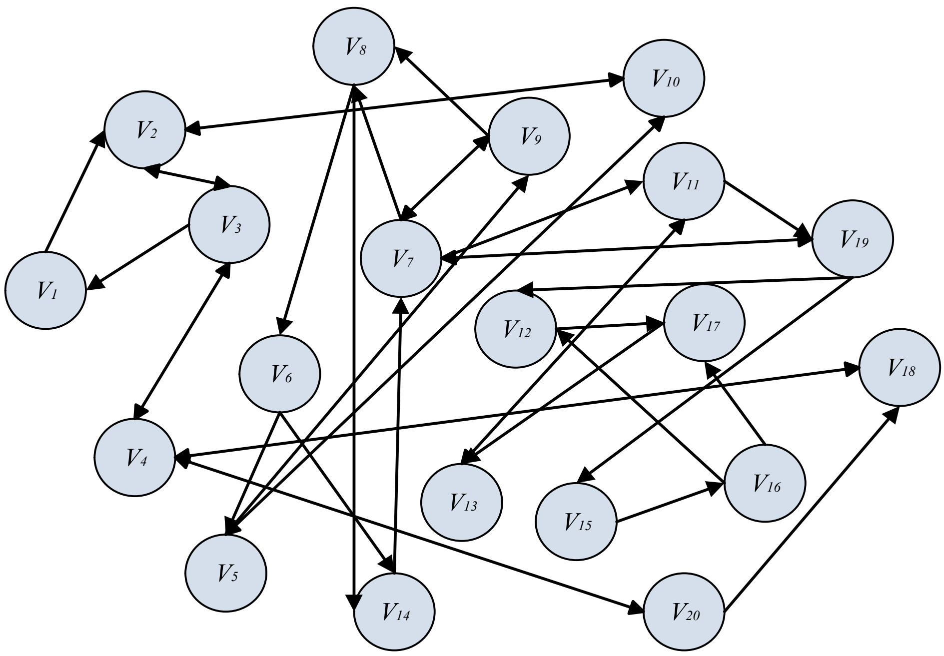

Fig. 2: Original trust SN.

3.3 Community detection in social networks

Based on the concept of community detection (CD) proposed by Newman and Girvan [51], an extended clustering algorithm is defined. Assume an SN comprises of a group of nodes (experts) $V=\{V_1,V_2,\ldots,V_r\}$ and a group of edges $Z=\{Z_1,Z_2,\ldots,Z_p\}$. Each expert provides his/her trust in other experts in the form of the linguistic terms shown in Table 1, which can be translated into a corresponding picture fuzzy sociomatrix $G=\left(G_{ij}\right)_{r\times r}$. Based on the PFTP operator, the missing trust relationship information can be estimated. Then, the picture fuzzy trust degree of expert V_i to expert V_j can be computed using Eq. (1) and denoted as $v_{ij}=T\left(G_{ij}\right)$. $V_i^{in}=\sum_{i\in Com_t} v_{ij}$ and $V_j^{out}=\sum_{j\in Com_t} v_{ij}$, respectively, denote the trust strengths and trusted strengths of the experts in each cluster, Com_t, and $v=\sum_{i\in Com_t} V_i^{in}=\sum_{j\in Com_t} V_j^{out}$. Assuming the entire SN can be divided into two communities, the modularity of the trust SN is [51]:

$$Q=\frac{1}{2v}\sum_{ij}\left(v_{ij}-\frac{V_i^{in}V_j^{out}}{v}\right)\left(y_i y_j+1\right)=\frac{1}{2e}y^T Ay, \quad (4)$$

where $y_i=\begin{cases}1, V_i\in Com_1\\ -1, V_i\in Com_2\end{cases}$ and y denotes the corresponding vector of elements y_i. A denotes the corresponding modularity matrix, and the element of A can be denoted as:

$$A_{ij}=v_{ij}-\frac{V_i^{in}V_j^{out}}{v}. \quad (5)$$

Thus, the gain function ΔQ of an entire SN should be considered when a community $Com_t\left(t=1,2\right)$ within the SN is subdivided. If $\Delta Q>0$, the corresponding community should be changed; otherwise, it is kept as the original community. Let $\sigma_{ij}=\frac{1}{2}\left(y_i y_j+1\right)$, and then ΔQ can be expressed as [51]:

$$\begin{aligned}\Delta Q&=\frac{1}{2v}\left(\sum_{V_i,V_j\in Com_t}\left(A_{ij}+A_{ji}\right)\sigma_{ij}-\sum_{V_i,V_j\in Com_t}\left(A_{ij}+A_{ji}\right)\right)\\&=\frac{1}{4v}\left(\sum_{V_i,V_j\in Com_t}\left(A_{ij}+A_{ji}\right)-\sigma_{ij}\sum_{V_k\in Com_t}\left(A_{ik}+A_{ki}\right)\right)y_i y_j \quad (6)\\&=\frac{1}{4v}y^T\left(A^{(t)}+\left(A^{(t)}\right)^T\right)y.\end{aligned}$$

Here, the elements $A^{(t)}$ can be denoted as: $A_{ij}^{(t)}=A_{ij}-\sigma_{ij}\sum_{V_k\in Com_t}A_{ik}$. $A^{(t)}$ is the submatrix of A for the subgraph Com_t. Thus, the *CD* method with picture fuzzy information can be summarized as Algorithm 1, as follows.

Algorithm 1: The CD method for an SN with picture fuzzy information.

Input: Directed graph $G=(V,Z)$.

Output: Corresponding clusters $Com_1, Com_2, \ldots, Com_q$.

Step 1: Translate the linguistic scales in a trust SN into PFSs using the transformation rules in Table 1 and construct a completed picture fuzzy trust sociomatrix $(G_{ij})_{r \times r}$.

Step 2: Determine the trust strength $v_{ij} = T(G_{ij})$ of expert V_i to expert V_j using Eq. (1).

Step 3: Determine the trust strengths V_i^{in}, V_j^{out} and v.

Step 4: Determine the corresponding modularity matrix $A = (A_{ij})_{r \times r}$ using Eq. (5).

Step 5: Determine the eigenvalue of the symmetric matrix $A + A^T$, and assign communities from the signs of the elements of the eigenvector.

Step 6: ΔQ is repeatedly calculated until no positive value of ΔQ is possible; otherwise, go to Step 3.

Step 7: Output the clusters $Com_1, Com_2, \ldots, Com_q$.

Example 3. From Example 2, based on the transformation rules between linguistic scales and PFSs presented in Table 1, the clusters for 20 experts can be obtained using Algorithm 1, and these 20 experts are divided into three clusters, i.e., $Com_1 = \{V_1, V_2, V_3, V_4, V_{10}, V_{18}, V_{20}\}$, $Com_2 = \{V_5, V_6, V_7, V_8, V_9, V_{14}\}$, and $Com_3 = \{V_{11}, V_{12}, V_{13}, V_{15}, V_{16}, V_{17}, V_{19}\}$.

3.4 Weights of experts in each cluster and the weights of clusters

In LSGDM problems, the similarity of the opinions of experts can reflect well the consensus or consistency of clusters. Thus, the weights of experts in each cluster can be determined by combination with picture fuzzy trust scores.

Definition 4. Assume $\Omega = (V, Z, \lambda)$ is a weighted directed graph, where $V = \{V_1, V_2, \ldots, V_r\}$ is a group of nodes (experts) and $Z = \{Z_1, Z_2, \ldots, Z_n\}$ is a group of directed lines or arcs between pairs of experts and that the corresponding completed picture fuzzy trust information of $\Omega = (V, Z, \lambda)$ is denoted as $(\bar{G}_{ij})_{r \times r}$. Then, the picture fuzzy trust score from expert V_i to expert V_j is:

$$PFTS\left(V_j\right)=\frac{1}{r-1}\sum_{i=1}^{r}T\left(\bar{G}_{ij}\right). \qquad (7)$$

Apparently, the larger the value of $PFTS\left(V_j\right)$, the more important the expert V_j. Then from the order weighted averaging (OWA) operator defined by Yager [52], the weights of experts in the clusters can be determined as follows.

Definition 9. Based on Definition 2, the corresponding weights of experts in each cluster $Com_t\left(t=1,2,\ldots,q\right)$ can be defined as:

$$\pi_{V_i,V_r\in Com_t}^{\delta(i)}=\vartheta\left(\frac{S\left(\delta\left(i\right)\right)}{S\left(\delta\left(r\right)\right)}\right)-\vartheta\left(\frac{S\left(\delta\left(i-1\right)\right)}{S\left(\delta\left(r\right)\right)}\right). \qquad (8)$$

Here, $S\left(\delta\left(i\right)\right)=\sum T_{\delta(i)}$, where δ is a permutation satisfying $T_{\delta(1)}\geq T_{\delta(2)}\geq\ldots\geq T_{\delta(i)}$ and ϑ is a monotone function satisfying: $\vartheta:[0,1]\rightarrow[0,1]$, $\vartheta\left(0\right)=0$, $\vartheta\left(1\right)=1$, and $\vartheta\left(a\right)>\vartheta\left(b\right)$ if $a>b$. Thus, the weight π_t^l of expert V_l in cluster Com_t can be obtained.

The weights of clusters can be obtained using the picture fuzzy trust score, as shown in Algorithm 2.

Algorithm 2: The weights of clusters.

Input: The completed picture fuzzy trust decision matrix $\left(\bar{G}_{ij}\right)_{r\times r}$.

Output: The weight vector of clusters $\varsigma=\left(\varsigma_1,\varsigma_2,\ldots,\varsigma_q\right)$.

Step 1: Calculate the picture fuzzy trust score $T\left(\bar{G}_{ij}\right)$ for each trust value $\bar{G}_{ij}\left(i,j=1,2,\ldots,r\right)$ provided by the experts using Definition 3.

Step 2: Calculate the average picture fuzzy trust score $G=\frac{1}{r}\sum T\left(\bar{G}_{ij}\right)\left(i,j=1,2,\ldots,r\right)$ for the entire evaluation information provided by the experts.

Step 3: Calculate the average picture fuzzy trust score $G_t=\frac{1}{\left|Com_t\right|}\sum_{V_l\in Com_t}T\left(\bar{G}_{ij}\right)$ for each cluster Com_t, where $\left|Com_t\right|$ represents the number of experts in cluster Com_t.

Step 4: Calculate the weight of each cluster Com_t as $\varsigma_t = \frac{1}{|G_t - G|}(t = 1,2,\ldots,q)$.

Step 5: Normalize the corresponding weight as $\bar{\varsigma}_t = \frac{\pi_t}{\sum_{t=1}^{q}\pi_t}(t = 1,2,\ldots,q)$.

Step 6: Output the weight vector of clusters $\bar{\varsigma} = (\bar{\varsigma}_1, \bar{\varsigma}_2, \ldots, \bar{\varsigma}_q)$.

3.5 Picture fuzzy consensus model for LSGDM

Consensus detection plays a key role in LSGDM problems in SN environments. In existing methods, the consensus detection model is always established based on distance measures; however, dissimilarity measures are common quantitative tools for measuring differences between objects. In this study, we defined a picture fuzzy Jensen- a -norm dissimilarity measure to construct a novel consensus detection model.

3.51 Picture fuzzy dissimilarity measure

The Jensen divergence is a very commonly used dissimilarity measure, which has been widely employed in various fields [53]. The Shannon entropy is a special form of a -norm entropy and PFSs can describe uncertain information from multiple perspectives. Here, we replace the Shannon entropy in the Jensen–Shannon divergence with the a-norm divergence, and define a new picture fuzzy Jensen- a -norm dissimilarity measure. The main advantage of this measure is the setting of parameter a , through which it is possible to express the importance of different probability distributions in terms of weight. More details about the a -norm measure can be found in Boekee and Lubbe [54]. The picture fuzzy Jensen- a -norm dissimilarity measure is hence defined based on the Jensen inequality and the a -norm divergence measure.

Definition 5 [53]. Assume $\mathrm{E} = (e_1, e_2, \ldots, e_n)$ and $F = (f_1, f_2, \ldots, f_n)$ are two vectors of complete probability distributions with respective weights $\tau_1, \tau_2 \geq 0$, where $\tau_1 + \tau_2 = 1$. Then, the Jensen- a -norm divergence measure with $a > 0(a \neq 1)$ is given by:

$$JBL_{\tau,a}(\mathrm{E},F) = \frac{a}{1-a}\left(\left(\sum_{i=1}^{n} e_i^a\right)^{1/a} + \left(\sum_{i=1}^{n} f_i^a\right)^{1/a} - \left[\sum_{i=1}^{n}(\tau_1 e_i + \tau_2 f_i)^a\right]^{1/a} - \left[\sum_{i=1}^{n}(\tau_1 f_i + \tau_2 e_i)^a\right]^{1/a}\right). \quad (9)$$

Here $e_i, f_i \geq 0$, $\sum_{i=1}^{n} e_i = 1$ and $\sum_{i=1}^{n} f_i = 1(n \geq 2)$.

$JBL_{\tau,a}(\mathrm{E},F) \geq 0$ and represents a symmetrical divergence measure and a convex function for

$a > 0(a \neq 1)$ [53]. The corresponding proof process is omitted here.

From Definition 1, we get $\pi_{\hat{\psi}}(x) = 1 - (Pos_{\hat{\psi}}(x) + Neu_{\hat{\psi}}(x) + Neg_{\hat{\psi}}(x))$ (i.e., $Pos_{\hat{\psi}}(x) + Neu_{\hat{\psi}}(x) + Neg_{\hat{\psi}}(x) + \pi_{\hat{\psi}}(x) = 1$). Thus, based on Definition 5, we can consider $\hat{\psi}_1 = (Pos_{\hat{\psi}_1}(x_i), Neu_{\hat{\psi}_1}(x_i), Neg_{\hat{\psi}_1}(x_i), \pi_{\hat{\psi}_1}(x_i))$ and $\hat{\psi}_2 = (Pos_{\hat{\psi}_2}(x_i), Neu_{\hat{\psi}_2}(x_i), Neg_{\hat{\psi}_2}(x_i), \pi_{\hat{\psi}_2}(x_i))$ as two probability distributions. The corresponding picture fuzzy Jensen-a-norm dissimilarity measure is defined as follows.

Definition 6. If $\hat{\psi}_1$ and $\hat{\psi}_2$ are two PFSs on a universe of discourse $X = (x_1, x_2, \ldots, x_n)$, then the picture fuzzy Jensen-a-norm dissimilarity measure can be defined as:

$$\begin{aligned}
&PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2) \\
&= \frac{a}{n(1-a)} \sum_{i=1}^{n} \Bigg(\Big((\tau_1 Pos_1(x_i) + \tau_2 Pos_2(x_i))^a + (\tau_1 Neu_1(x_i) + \tau_2 Neu_2(x_i))^a + (\tau_1 Neg_1(x_i) + \tau_2 Neg_2(x_i))^a \\
&+ (\tau_1 \pi_1(x_i) + \tau_2 \pi_2(x_i))^a \Big)^{1/a} + \Big((\tau_1 Pos_2(x_i) + \tau_2 Pos_1(x_i))^a + (\tau_1 Neu_2(x_i) + \tau_2 Neu_1(x_i))^a + (\tau_1 Neg_2(x_i) \\
&+ \tau_2 Neg_1(x_i))^a + (\tau_1 \pi_2(x_i) + \tau_2 \pi_1(x_i))^a \Big)^{1/a} - \Big((Pos_1(x_i))^a + (Neu_1(x_i))^a + (Neg_1(x_i))^a + (\pi_1(x_i))^a \Big)^{1/a} \\
&- \Big((Pos_2(x_i))^a + (Neu_2(x_i))^a + (Neg_2(x_i))^a + (\pi_2(x_i))^a \Big)^{1/a} \Bigg).
\end{aligned} \tag{10}$$

Here, $a > 0(a \neq 1)$, and $\tau_1 + \tau_2 = 1$.

$PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2)$ is non-negative; moreover, it represents a symmetrical dissimilarity measure and a convex function for $a > 0(a \neq 1)$. Its properties are shown as follows.

Theorem 1. $PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2)$ is considered to be a picture fuzzy Jensen-a-norm dissimilarity measure if the following conditions are true:

(1) $PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2) \geq 0$;

(2) $PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2) = PFJBL_{\tau,a}(\hat{\psi}_2, \hat{\psi}_1)$;

(3) $PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2) = 0$ if and only if $\hat{\psi}_1 = \hat{\psi}_2$;

(4) if $\hat{\psi}_1 \leq \hat{\psi}_2 \leq \hat{\psi}_3$, then $PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_3) \geq \max\{PFJBL_{\tau,a}(\hat{\psi}_1, \hat{\psi}_2), PFJBL_{\tau,a}(\hat{\psi}_2, \hat{\psi}_3)\}$.

The proof for **Theorem 1** can be found in **Appendix B**.

The proposed picture fuzzy Jensen-a-norm dissimilarity measure can satisfy other characteristics, as shown in Theorem 2.

Theorem 2. If $\hat{\psi}, \hat{\psi}_1$, and $\hat{\psi}_2$ are three PFSs, then the picture fuzzy Jensen-a-norm dissimilarity measure satisfies the following characteristics:

(1) $PFJBL_{\tau,a}\left(\hat{\psi},\hat{\psi}^{c}\right)=0$ if and only if $Pos_{\hat{\psi}}\left(x_i\right)=Neu_{\hat{\psi}}\left(x_i\right)=Neg_{\hat{\psi}}\left(x_i\right)$ for any $x_i \in X$;

(2) $PFJBL_{\tau,a}\left(\hat{\psi}_1,\hat{\psi}_2\right)=PFJBL_{\tau,a}\left(\hat{\psi}_1^{\ c},\hat{\psi}_2^{\ c}\right)$;

(3) $PFJBL_{\tau,a}\left(\hat{\psi}_1,\hat{\psi}_2^{\ c}\right)=PFJBL_{\tau,a}\left(\hat{\psi}_1^{\ c},\hat{\psi}_2\right)$;

(4) $PFJBL_{\tau,a}\left(\hat{\psi}_1\cup\hat{\psi}_2,\hat{\psi}_1\cap\hat{\psi}_2\right)=PFJBL_{\tau,a}\left(\hat{\psi}_1,\hat{\psi}_2\right)$;

(5) $PFJBL_{\tau,a}\left(\hat{\psi}_1\cap\hat{\psi}_2,\hat{\psi}_1\cup\hat{\psi}_2\right)=PFJBL_{\tau,a}\left(\hat{\psi}_1,\hat{\psi}_2\right)$.

The proof for **Theorem 2** can be found in **Appendix B**.

3.52 Picture fuzzy consensus model

Based on the proposed picture fuzzy Jensen- a -norm dissimilarity measure, we obtain a PFCI comprising three levels.

Level 1. The *PFCI* at the element level. The *PFCI* of expert V_l in cluster Com_t with alternative α_i under criterion c_j is determined as:

$$PFCIE_{ij}^{l,t}=1-PFJBL\left(\overline{E}_{ij}^{l,t},\overline{E}_{ij}^{t}\right), \qquad (11)$$

where $PFJBL\left(\overline{E}_{ij}^{l,t},\overline{E}_{ij}^{t}\right)$ denotes the dissimilarity between $\overline{E}_{ij}^{l,t}$ and $\overline{E}_{ij}^{t}$, which can be calculated using Definition 6.

Level 2. The *PFCI* at the alternative level. The *PFCI* of expert V_l in cluster Com_t with alternative α_i is determined as:

$$PFCIE_{i}^{l,t}=\sum_{j=1}^{m}\varpi_j\cdot PFCIE_{ij}^{l,t}=1-\sum_{j=1}^{m}\varpi_j\cdot PFJBL\left(\overline{E}_{ij}^{l,t},\overline{E}_{ij}^{t}\right). \qquad (12)$$

Level 3: The *PFCI* at the decision matrix level. The *PFCI* of expert V_l in cluster Com_t is determined as:

$$PFCI^{l,t}=\frac{1}{n}\sum_{i=1}^{n}PFCIE_{i}^{l,t}=1-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m}\varpi_j\cdot PFJBL\left(\overline{E}_{ij}^{l,t},\overline{E}_{ij}^{t}\right). \qquad (13)$$

Moreover, the *PFCI* of cluster Com_t at the decision matrix level can be determined as:

$$PFCI^{t}=\sum_{V_l\in Com_t}\pi_t^{l}PFCI^{l,t}=1-\frac{1}{n}\sum_{V_l\in Com_t}\pi_t^{l}\sum_{i=1}^{n}\sum_{j=1}^{m}\varpi_j\cdot PFJBL\left(\overline{E}_{ij}^{l,t},\overline{E}_{ij}^{t}\right). \qquad (14)$$

Thus, the entire *PFCI* of an SN can be determined as:

$$PFCI=\sum_{t=1}^{q}\bar{\varsigma}_t PFCI^t=1-\frac{1}{n}\sum_{t=1}^{q}\bar{\varsigma}_t\sum_{V_l\in Com_t}\pi_t^l\sum_{j=1}^{m}\varpi_j\cdot PFJBL\left(\bar{E}_{ij}^{l,t},\bar{E}_{ij}^{t}\right). \quad (15)$$

The larger the value of $PFCI^t$, the higher the agreement among the experts in cluster Com_t. In particular, if $PFCI^t=1$, then the experts can reach an agreement in cluster Com_t. Moreover, a suitable consensus threshold ξ $(0\le\xi\le1)$ can be determined. If $PFCI^t\ge\xi$, then an acceptable *PFCI* can be obtained. Otherwise, the corresponding evaluation information should be provided again. First, a cluster is identified based on $PFCI^t<\xi$; similarly, from the values of $PFCI^{l,t}$, $PFCIE_i^{l,t}$, and $PFCIE_{ij}^{l,t}$, we determine the corresponding evaluation information $\bar{E}_{ij}^{l,t}$ provided by the experts. Second, we set an adjustment parameter $e^{l,t}=\xi-\min_i\min_j\left\{PFCI_{ij}^{l,t}\right\}$ (for those evaluation values that did not meet the consensus) and $0<e^{l,t}<1$. From this, we can obtain the recommended new evaluation value: $\bar{\bar{E}}_{ij}^{l,t}=e^{l,t}\bar{E}_{ij}^{l,t}+\left(1-e^{l,t}\right)\bar{E}_{ij}^{(t)}$. The consistency level of the decision matrix is discussed next by considering this new evaluation value.

3.6 Selection process with TOPSIS

In this subsection, we introduce a novel selection process that is based on the proposed picture fuzzy Jensen-a-norm dissimilarity measure and the traditional TOPSIS approach.

Algorithm 3. The general aggregated decision matrix.

Input: The picture fuzzy decision matrix $\left(\bar{E}_{ij}^{l,t}\right)_{n\times m}\left(l=1,2,\ldots,r;t=1,2,\ldots,q\right)$, the weights of the criteria are $\varpi=\left(\varpi_1,\varpi_2,\ldots,\varpi_m\right)^T$, and the consensus threshold $\xi\left(0\le\xi\le1\right)$.

Output: The aggregated decision matrix $\bar{E}=\left(\bar{E}_{ij}\right)_{n\times m}$ for the entire SN.

Step 1: Determine the weights π_t^l of experts in each cluster $Com_t\left(t=1,2,\ldots,q\right)$ using Eq. (8).

Step 2: Calculate the aggregated decision matrix $\bar{E}^{(t)}=\left(\bar{E}_{ij}^{(t)}\right)_{n\times m}\left(t=1,2,\ldots,q\right)$ for each cluster Com_t using Eq. (2).

Step 3: Determine the weights $\pi_t\left(t=1,2,\ldots,q\right)$ of the clusters for the entire SN using

Algorithm 2.

Step 4: Calculate the general aggregated decision matrix $\overline{E}=\left(\overline{E}_{ij}\right)_{n\times m}$ using Eq. (3).

Step 5: Determine the $PFCIE_{ij}^{l,t}$, $PFCIE_{i}^{l,t}$, $PFCI^{l,t}$, and $PFCI^{t}$ of expert V_l. If $PFCI^{t}\geq\xi$, skip to Step 7; otherwise, go to Step 6.

Step 6: Find the corresponding evaluation values $\overline{E}_{ij}^{l,t}$ with $PFCI^{t}<\xi$ and recommend the new evaluation values as: $\overline{\overline{E}}_{ij}^{l,t}=e^{l,t}\overline{E}_{ij}^{l,t}+\left(1-e^{l,t}\right)\overline{E}_{ij}^{(t)}$, where $e^{l,t}=\xi-\min_{i}\min_{j}\left\{PFCI_{ij}^{l,t}\right\}$ denotes an adjustment parameter. Then, calculate the updated picture fuzzy decision matrix $\left(\overline{\overline{E}}_{ij}^{l,t}\right)_{n\times m}\left(l=1,2,\ldots,r;t=1,2,\ldots,q\right)$, and assume $\left(\overline{\overline{E}}_{ij}^{l,t}\right)_{n\times m}=\left(\overline{E}_{ij}^{l,t}\right)_{n\times m}\left(l=1,2,\ldots,r;t=1,2,\ldots,q\right)$. Skip to Step 1.

Step 7: Output the final general aggregated picture fuzzy decision matrix $\overline{E}=\left(\overline{E}_{ij}\right)\left(i=1,2,\ldots,n;j=1,2,\ldots,m\right)$, and $PFCI^{t}$ of clusters $Com_t\left(t=1,2,\ldots,q\right)$.

Based on TOPSIS, the following steps are provided to implement the extended picture fuzzy TOPSIS method based on the dissimilarity measure.

Step 1. Determine the picture fuzzy ideal solutions of the general decision matrix $\overline{E}$.

The picture fuzzy positive ideal solution $\overline{E}^{+}$ and the picture fuzzy negative ideal solution $\overline{E}^{-}$ of $\overline{E}$, respectively, can be determined as follows:

$$\overline{E}^{+}=\left\{\overline{E}_1^{+},\overline{E}_2^{+},\ldots,\overline{E}_m^{+}\right\};\tag{16}$$

$$\overline{E}^{-}=\left\{\overline{E}_1^{-},\overline{E}_2^{-},\ldots,\overline{E}_m^{-}\right\};\tag{17}$$

$$\overline{E}_j^{+}=\left(\max_i\left\{Pos_{\overline{E}_{ij}}\right\},\min_i\left\{Neu_{\overline{E}_{ij}}\right\},\min_i\left\{Neg_{\overline{E}_{ij}}\right\}\right)\left(i=1,2,\ldots,n;j=1,2,\ldots,m\right);\tag{18}$$

$$\overline{E}_j^{-}=\left(\min_i\left\{Pos_{\overline{E}_{ij}}\right\},\min_i\left\{Neu_{\overline{E}_{ij}}\right\},\max_i\left\{Neg_{\overline{E}_{ij}}\right\}\right)\left(i=1,2,\ldots,n;j=1,2,\ldots,m\right).\tag{19}$$

Step 2. Calculate the picture fuzzy Jensen-a-norm dissimilarity measures.

Based on Eq. (10), the picture fuzzy Jensen-a-norm dissimilarity measures between $\overline{E}_{ij}$ and $\overline{E}_j^{+}$ are obtained as:

$$\begin{aligned}
&PFJBL_{\tau,a}\left(\bar{E}_{ij},\bar{E}_j^{+}\right)\\
&=\frac{a}{n(1-a)}\Bigg(\Bigg(\left(\tau_1 Pos_{\bar{E}_{ij}}+\tau_2 Pos_{\bar{E}_j^{+}}\right)^a+\left(\tau_1 Neu_{\bar{E}_{ij}}+\tau_2 Neu_{\bar{E}_j^{+}}\right)^a+\left(\tau_1 Neg_{\bar{E}_{ij}}+\tau_2 Neg_{\bar{E}_j^{+}}\right)^a+\left(\tau_1\pi_{\bar{E}_{ij}}+\tau_2\pi_{\bar{E}_j^{+}}\right)^a\Bigg)^{1/a}\\
&+\Bigg(\left(\tau_1 Pos_{\bar{E}_j^{+}}+\tau_2 Pos_{\bar{E}_{ij}}\right)^a+\left(\tau_1 Neu_{\bar{E}_j^{+}}+\tau_2 Neu_{\bar{E}_{ij}}\right)^a+\left(\tau_1 Neg_{\bar{E}_j^{+}}+\tau_2 Neg_{\bar{E}_{ij}}\right)^a+\left(\tau_1\pi_{\bar{E}_j^{+}}+\tau_2\pi_{\bar{E}_{ij}}\right)^a\Bigg)^{1/a}\\
&-\left(\left(Pos_{\bar{E}_{ij}}\right)^a+\left(Neu_{\bar{E}_{ij}}\right)^a+\left(Neg_{\bar{E}_{ij}}\right)^a+\left(\pi_{\bar{E}_{ij}}\right)^a\right)^{1/a}-\left(\left(Pos_{\bar{E}_j^{+}}\right)^a+\left(Neu_{\bar{E}_j^{+}}\right)^a+\left(Neg_{\bar{E}_j^{+}}\right)^a+\left(\pi_{\bar{E}_j^{+}}\right)^a\right)^{1/a}\Bigg).
\end{aligned} \tag{20}$$

Here, $a \geq 0$ and $a \neq 1$.

Similarly, the picture fuzzy Jensen- a -norm dissimilarity measures $PFJBL_{\tau,a}\left(\bar{E}_{ij},\bar{E}_j^{-}\right)$ between $\bar{E}_{ij}$ and $\bar{E}_j^{-}$ can be obtained.

Step 3. Calculate the picture fuzzy relative closeness coefficient.

The picture fuzzy relative closeness coefficient can be determined as:

$$\Delta_i=\frac{\sum_{j=1}^{m}\varpi_j PFJBL_{\tau,a}\left(\bar{E}_{ij},\bar{E}_j^{-}\right)}{\sum_{j=1}^{m}\varpi_j PFJBL_{\tau,a}\left(\bar{E}_{ij},\bar{E}_j^{+}\right)+\sum_{j=1}^{m}\varpi_j PFJBL_{\tau,a}\left(\bar{E}_{ij},\bar{E}_j^{-}\right)}\left(i=1,2,\ldots,n\right). \tag{21}$$

Step 4. Rank all of the alternatives.

The greater the Δ_i value of $\alpha_i (i=1,2,\ldots,5)$ is, the better the alternative.

3.7 The procedure for the proposed approach

From the above analysis, a picture fuzzy LSGDM approach in a trust relationship-based SN environment is proposed and shown in Fig. 3, and the corresponding procedures are provided as follows.

Step 1: Construct completed picture fuzzy trust relationships using the PFTP operator.

Step 2: Implement CD and obtain the SN clusters using Algorithm 1 with the picture fuzzy trust relationship information.

Step 3: Determine the weights of experts in each cluster using Eq. (8) and the weights of the clusters using Algorithm 2.

Step 4: Test the consensus for the SN using three *PFCIs* at three levels and obtain the general aggregated decision matrix for all clusters.

Step 5: Select the process using the picture fuzzy dissimilarity-based TOPSIS method.

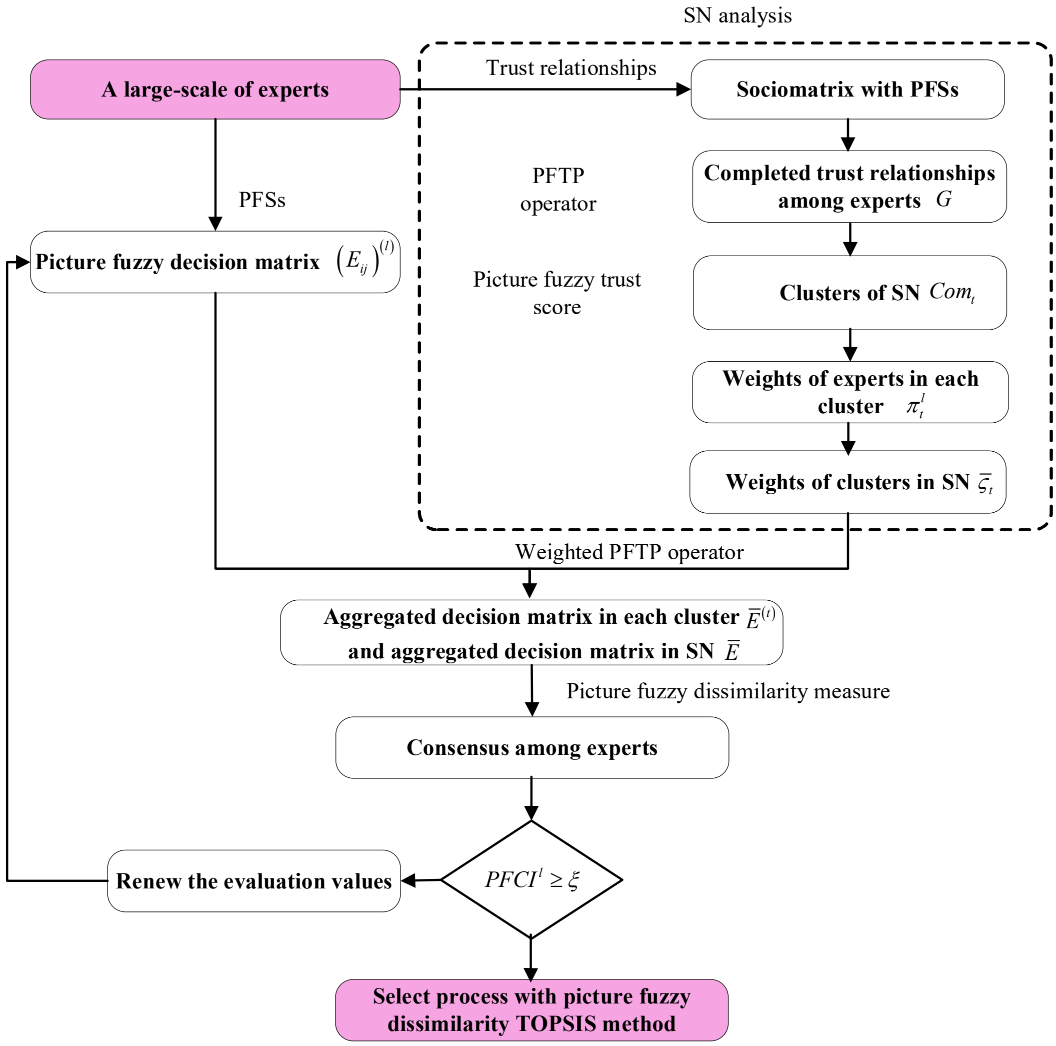

Fig. 3. Procedure for the developed method.

4. Case study and results

In this section, a food safety evaluation problem is discussed as a case study. Suppose the local government plans to evaluate food safety for an annual inspection in one city. Five companies (alternatives), i.e., $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$, are considered. According to the department's previous evaluation projects, six factors should be considered during the evaluation, including c_1 : environment (the quality of the raw materials), c_2 : production (the production line), c_3 : processing (the processing link), c_4 : transport (the packaging, storage, and circulation transportation link), c_5 : sell (the food sale link), and c_6 : monitoring (the food monitoring and safety management link). The corresponding weight vector of these six factors is $\varpi = (0.08,0.17,0.23,0.09,0.26,0.17)^T$. If the SN among 20 experts is as shown in Fig. 2, then the experts can use the linguistic scales to express their trust in other experts, as shown in Table

2 (see Appendix A). The related picture fuzzy trust relationships among 20 experts can be obtained based on the transformation rules shown in Table 1. Moreover, the evaluation results of the five candidates α_i under criterion c_j can be provided by 20 experts in the form of the linguistic scales shown in Table 1. Then the corresponding picture fuzzy decision matrixes $\left(E_{ij}^l\right)_{5\times 6}\left(l=1,2,\ldots,20\right)$ can be obtained based on the transformation rules. For example, the picture fuzzy decision matrix $\left(E_{ij}^1\right)_{5\times 6}$ and $\left(E_{ij}^{20}\right)_{5\times 6}$, which are evaluated by the experts V_1 and V_{20}, are shown in Table 3 (see Appendix A).

The steps for selecting the optimal alternative using the proposed approach are as follows.

Step 1: Construct completed picture fuzzy trust picture fuzzy relationships using the PFTP operator.

From the PFTP operator, the missing trust information can be assessed. For example, there is no direct trust relationship between expert V_7 and expert V_{15}, but there are two indirect paths, i.e., $Pl_1: V_7 \to V_{19} \to V_{15}$ and $Pl_2: V_7 \to V_{11} \to V_{19} \to V_{15}$. Here, the number of edges of path Pl_1 is less than that of path Pl_2. Hence, the missing picture fuzzy trust degree between expert V_7 and expert V_{15} can be obtained by path Pl_1. Based on the PFTP operator, i.e., Eq. (3), we can obtain the picture fuzzy trust degree between expert V_8 and expert V_9 as: $PFTP_{Pl_1}=\left(x,0.1175,0.105,0.775\right)$, i.e., $\overline{E}_{V_7,V_{15}}=\left(x,0.1175,0.105,0.775\right)$.

Step 2: Implement CD and obtain the clusters of the SN.

Based on Algorithm 1, we can obtain three clusters for the SN, i.e., $Com_1=\{V_1,V_2,V_3,V_4,V_{10},V_{18},V_{20}\}$, $Com_2=\{V_5,V_6,V_7,V_8,V_9,V_{14}\}$, and $Com_3=\{V_{11},V_{12},V_{13},V_{15},V_{16},V_{17},V_{19}\}$.

Step 3: Determine the weights of experts in each cluster and the weights of clusters in the SN.

According to the PFTP operator, and assuming $\vartheta(x)=x^3$, the corresponding picture fuzzy trust relationships can be obtained, and the weight vector of experts in each cluster can be determined as: $\pi_1^l=\left(0.1802,0.1513,0.1229,0.0926,0.0926,0.1802,0.1802\right)^T$,

$\pi_2^l=\left(0.1411,0.1308,0.1686,0.1582,0.2214,0.1799\right)^T$,

and $\pi_3^l=\left(0.1937,0.1746,0.0883,0.0883,0.1746,0.1211,0.1594\right)^T$.

Moreover, based on Algorithm 2, the weights of the clusters in the SN can be determined as shown in Table 4.

Table 4. Weights of clusters.

Cluster	Number of experts	Experts	Weight of cluster
Com_1	7	$V_1,V_2,V_3,V_4,V_{10},V_{18},V_{20}$	0.2977

Com_2	6	$V_5, V_6, V_7, V_8, V_9, V_{14}$	0.3719
Com_3	7	$V_{11}, V_{12}, V_{13}, V_{15}, V_{16}, V_{17}, V_{19}$	0.3304

Step 4: Test the consensus for the SN using three *PFCIs* at three levels and obtain the general aggregated decision matrix for all clusters.

Based on Algorithm 3, let $\xi = 0.9$, $\tau_1 = \tau_2 = 0.5$ and $a = 2$, and then the consensus indexes of the clusters, which meet the condition that they are larger than the provided threshold value, can be obtained as: $PFCI^1 = 0.9210$, $PFCI^2 = 0.9322$, and $PFCI^3 = 0.9154$. Thus, the entire $PFCI$ of the SN is: $PFCI = 0.9233 > 0.9$. Moreover, the corresponding general aggregated decision matrix can be obtained as shown in Table 5.

Table 5. General aggregated picture fuzzy decision matrix

	c_1	c_2	c_3	c_4	c_5	c_6
α_1	(0.4021,0.1190,0.4122)	(0.1173,0.2089,0.6203)	(0.5211,0.1431,0.3248)	(0.4219,0.2214,0.2157)	(0.3176,0.2135,0.4016)	(0.3551,0.1103,0.4007)
α_2	(0.4803,0.2067,0.2111)	(0.3056,0.1374,0.4089)	(0.1998,0.1879,0.3321)	(0.6999,0.1034,0.1087)	(0.1995,0.1008,0.5106)	(0.3302,0.2111,0.3201)
α_3	(0.4087,0.1112,0.2006)	(0.3059,0.1798,0.4711)	(0.5018,0.1189,0.2012)	(0.4122,0.2032,0.2167)	(0.3235,0.1764,0.3249)	(0.7268,0.1003,0.1017)
α_4	(0.5440,0.1236,0.3056)	(0.4222,0.1025,0.3021)	(0.7066,0.1111,0.1125)	(0.6236,0.1021,0.2133)	(0.5421,0.0912,0.1211)	(0.5017,0.1103,0.2029)
α_5	(0.3118,0.1033,0.2115)	(0.1211,0.1585,0.6010)	(0.3997,0.1022,0.3777)	(0.6966,0.1011,0.0868)	(0.6214,0.0889,0.1752)	(0.4821,0.1012,0.3022)

Step 5: Select the process by using the picture fuzzy dissimilarity-based TOPSIS method.

Based on the general aggregated evaluation values in Table 5, $\overline{E}_j^+$ and $\overline{E}_j^-$ can be obtained. Then the picture fuzzy Jensen-a-norm dissimilarity measures between $\overline{E}_{ij}$ and $\overline{E}_j^+$, and between $\overline{E}_{ij}$ and $\overline{E}_j^-$, can be obtained, as shown in Tables 6 and 7, respectively.

Table 6. Picture fuzzy Jensen-a-norm dissimilarity measures between $\overline{E}_{ij}$ and $\overline{E}_j^+$

	c_1	c_2	c_3	c_4	c_5	c_6
α_1	0.0520	0.1645	0.0419	0.1063	0.1882	0.1651
α_2	0.0172	0.0183	0.2734	0.0076	0.3281	0.1400
α_3	0.0166	0.0833	0.0316	0.1063	0.1681	0.0000
α_4	0.0167	0.0000	0.0000	0.0227	0.0499	0.0436
α_5	0.0707	0.2157	0.1265	0.0000	0.0136	0.0529

Table 7. Picture fuzzy Jensen- a -norm dissimilarity measures between $\overline{E}_{ij}$ and $\overline{E}_j^-$

	c_1	c_2	c_3	c_4	c_5	c_6
α_1	0.0319	0.0090	0.1698	0.0000	0.0166	0.0183
α_2	0.0506	0.1156	0.0000	0.0674	0.0172	0.0176
α_3	0.0368	0.0493	0.1269	0.0000	0.0891	0.1485
α_4	0.0472	0.2157	0.2734	0.0393	0.2686	0.0891
α_5	0.0561	0.0000	0.0519	0.1063	0.2695	0.0421

Based on Eq. (21), the picture fuzzy relative closeness coefficients can be determined as shown in Table 8.

Table 8. Picture fuzzy relative closeness coefficients

	α_1	α_2	α_3	α_4	α_5
Δ_i	0.2827	0.1737	0.5390	0.8898	0.5515

Since $\Delta_4 > \Delta_5 > \Delta_3 > \Delta_1 > \Delta_2$, the final ranking is $\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$, in which the best alternative is α_4. Thus, the food safety of the fourth company is the highest.

5. Discussion

In this section, sensitivity analysis and comparative analysis were employed to demonstrate the effectiveness and feasibility of the proposed approach.

5.1 Sensitivity analysis

To investigate the robustness of different parameters on the final ranking of alternatives using the picture fuzzy Jensen- a -norm dissimilarity measure, different values of $a \in (0,1) \cup (1,10]$ and $\tau \in (0,1)$ are taken into account. The results are shown in Tables 9 and 10 and Figs. 4 and 5.

(1) When τ is fixed, i.e., $\tau_1 = \tau_2 = 0.5$, and the values of a are changed, the results are shown in Table 9 and Fig. 4. From the results, it can be seen that if $a \in (0, 0.34]$, the final ranking is $\alpha_4 \succ \alpha_5 \succ \alpha_1 \succ \alpha_3 \succ \alpha_2$; if $a \in (0.34, 2.31]$, the final ranking is $\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$; and if $a \in (2.31, 10]$, the final ranking is $\alpha_4 \succ \alpha_3 \succ \alpha_5 \succ \alpha_1 \succ \alpha_2$. Although the positions during α_1, α_3 and α_5 are different, α_4 is always the optimal solution while α_2 is the worst. In other words, the different values of a in the picture fuzzy Jensen- a -norm dissimilarity measure do not influence the final decision-making.

(2) When a is fixed, i.e., $a = 2$, and the values of τ are changed, the results are shown in Table 10. For $\tau_1 = 0.2, 0.4, 0.5, 0.6, 0.8$, the final ranking is always $\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$,

i.e., α_4 is always the optimal solution. Thus, the different values of τ do not influence the final ranking.

(3) When the two parameters τ and a change simultaneously, the corresponding ranking order and the changing tendency of alternatives are shown in Fig. 5. Based on Fig. 5, the value of Δ_3 is larger than that of Δ_5 as the values of τ and a increase. However, the two parameters τ and a do not change the final results and α_4 is always the optimal solution as τ and a change.

From the above analysis, it can be seen that in most cases the two parameters τ and a have no influence on the final decision-making results, indicating that the final results using our proposed LSGDM method are robust and effective for real decision-making processes. In general, DMs can select different parameters according to their preferences. Moreover, the values of the three membership degrees of PFSs are between 0 and 1. If the values of parameters are extremely large, then the difference between the picture fuzzy Jensen- a -norm dissimilarity measure values of alternatives may not be very significant. In view of this, smaller values should be adopted for ease of computation.

Table 9. Final results for different values of parameter a when $\tau_1=\tau_2=0.5$ is fixed.

Parameters	Relative closeness coefficients					Final rankings
$\tau_1=\tau_2=0.5$	α_1	α_2	α_3	α_4	α_5	
$a=2$	0.2827	0.1737	0.5390	0.8898	0.5515	$\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$
$a=4$	0.3376	0.0603	0.6991	0.9851	0.6632	$\alpha_4 \succ \alpha_3 \succ \alpha_5 \succ \alpha_1 \succ \alpha_2$
$a=6$	0.4001	0.0275	0.7507	0.9994	0.7471	$\alpha_4 \succ \alpha_3 \succ \alpha_5 \succ \alpha_1 \succ \alpha_2$
$a=8$	0.4192	0.0208	0.7596	1.0000	0.7708	$\alpha_4 \succ \alpha_3 \succ \alpha_5 \succ \alpha_1 \succ \alpha_2$
$a=10$	0.4260	0.0170	0.7641	1.0000	0.7803	$\alpha_4 \succ \alpha_3 \succ \alpha_5 \succ \alpha_1 \succ \alpha_2$

Table 10. Final results for different values of parameter τ when $a=2$ is fixed.

Parameters	Relative closeness coefficients					Final rankings
$a=2$	α_1	α_2	α_3	α_4	α_5	
$\tau_1=0.2, \tau_2=0.8$	0.2826	0.1756	0.5379	0.8883	0.5505	$\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$
$\tau_1=0.4, \tau_2=0.6$	0.2827	0.1739	0.5389	0.8896	0.5514	$\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$
$\tau_1=\tau_2=0.5$	0.2827	0.1737	0.5390	0.8898	0.5515	$\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$
$\tau_1=0.6, \tau_2=0.4$	0.2827	0.1739	0.5389	0.8896	0.5514	$\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$
$\tau_1=0.8, \tau_2=0.2$	0.2826	0.1756	0.5379	0.8883	0.5505	$\alpha_4 \succ \alpha_5 \succ \alpha_3 \succ \alpha_1 \succ \alpha_2$

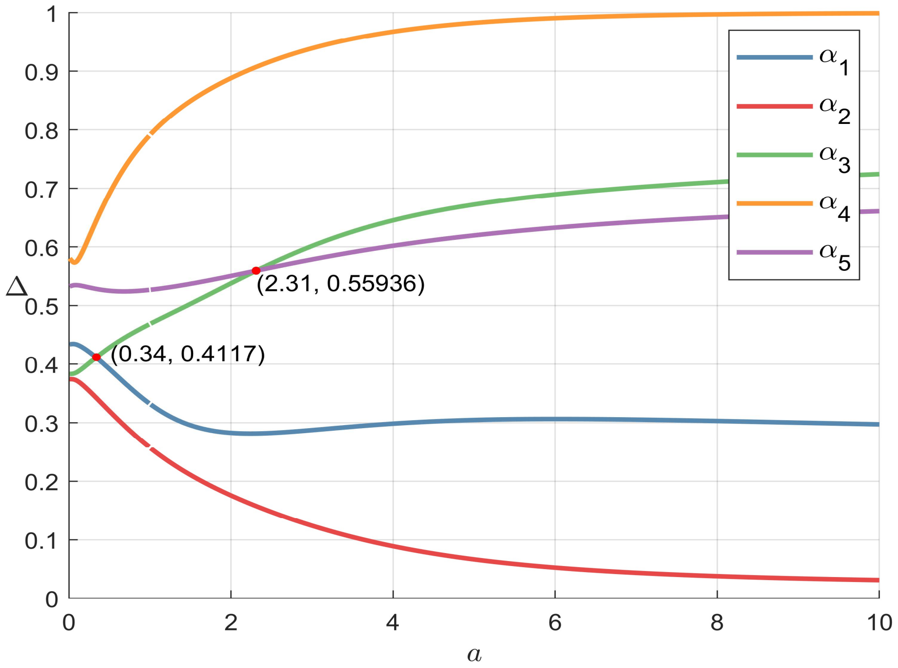

Fig. 4. Final relative closeness coefficients with $a \in (0,1) \cup (1,10]$ and $\tau_1 = \tau_2 = 0.5$

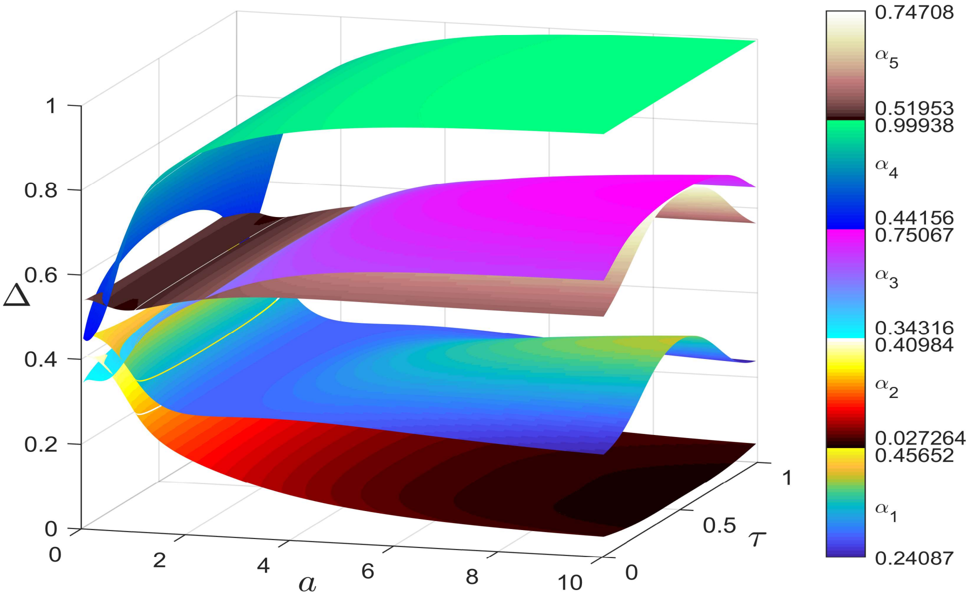

Fig. 5. Final relative closeness coefficients with $a \in (0,1) \cup (1,10]$ and $\tau \in (0,1)$

5.2 Comparative analysis

This subsection compares two different SN LSGDM methods with an interval type-2 fuzzy formation [21-22] to verify the effectiveness of the proposed method.

As the method described by Wu et al. [21] does not take into account the trust relationships among experts in an SN, the directed SN graph in this paper should be revised as an undirected graph in the computation process. The Louvain method is used to directly detect SNs and determine the weights of each cluster of experts and the weights of each cluster in the SN. Then, the aggregated evaluation information can be obtained using the IT2FS weighted arithmetic averaging operator. Finally, the multi-criteria decision-making (MCDM) process is conducted in combination with the IT2FS distance measure and the TOPSIS method. The method developed by Tian et al. [22] is similar to the method proposed in this study. IT2FSs are used to describe the trust relationships among experts in an SN and the preference information of experts. Then the consensus detection of the SN is implemented based on IT2FS distance measures, and the aggregated evaluation information can be obtained by combining these measures with the IT2FS weighted arithmetic operator. Thus, the MCDM process can be carried out successfully. The parameters involved in the following calculation process are calculated using parameter assumptions obtained from the original studies, and the results calculated using the different methods are shown in Table 11.

Table 11. Comparison results.

Alternative	Wu et al. [21]		Tian et al. [22]		The proposed method	
	Final closeness score	Rank	Centroid value	Rank	Final closeness score	Rank
α_1	0.3207	4	0.3011	4	0.2827	4
α_2	0.2659	5	0.2239	5	0.1737	5
α_3	0.6339	2	0.6014	3	0.5390	3
α_4	0.7024	1	0.7162	1	0.8898	1
α_5	0.5814	3	0.5811	2	0.5515	2

The following conclusions are obtained based on the results shown in Table 11.

(1) The rankings obtained using the three methods are only slightly different. The positions of α_3 and α_5 obtained using the method proposed by Wu et al. [21] are opposite to those

obtained using the method proposed by Tian et al. [22] and that of the present study. However, the optimal alternative always appears to be α_4, while the worst alternative is always α_2. These results confirm the validity and feasibility of the method we proposed.

(2) The above results can be mainly explained by the fact that Wu et al.'s method [21] does not consider the trust relationships among experts or the consensus analysis of experts in SNs, resulting in some differences for what influences the second and third positions in the rankings. By contrast, the consensus analysis of experts in SNs is considered in both the proposed methodology and in that of Tian et al. [22]. The same results are obtained using these two methods. Notably, experts always have an impact relationship in LSGDM problems in an SN environment; thus, it is necessary to consider the trust in the SN and simultaneously detect the consensus of the experts.

(3) Although the results obtained using the method of Tian et al. [22] are consistent with those of the method proposed in this study, our method appears to be superior. Table 10 shows that the differences in final closeness score between the two alternatives are larger than the final centroid value obtained using the method of Tian et al. [22] and the closeness score obtained using the method proposed of Wu et al. [21]. For example, the degree of closeness score for the first and second positions (i.e., $\Delta_4 - \Delta_5 = 0.8898 - 0.5515 = 0.3383$) is clearly larger than that obtained using the methods of Tian et al. [22] (i.e., $0.7162 - 0.5811 = 0.1351$) and Wu et al. [21] (i.e., $0.7024 - 0.5814 = 0.1210$). We infer that a greater degree of distinction between the alternatives can be obtained using the method proposed in this study, since it brings DMs to make the necessary choices.

(4) The calculation process used in the method proposed by Tian et al. [22] is more complex than that of the method proposed in this study and involves multiple variable parameters, making it difficult for DMs to select appropriate parameters in real decision-making environments. By contrast, the proposed picture fuzzy LSGDM approach in an SN environment is simpler, more intuitive, and more effective.

6. Conclusions

In this study, we considered the use of PFSs for comprehensively expressing preference information when it is inaccurate, uncertain, or incomplete; we then defined a picture fuzzy LSGDM approach in a trust relationship-based SN environment. First, the trust relationships among experts in an SN were described using the provided linguistic scales and the preference information of DMs was converted into PFSs. Second, a CD was carried out in the SN based on the picture fuzzy trust score. When the trust relationship information in the SN was incomplete, the missing information was estimated using the picture fuzzy propagation operator. Third, we determined the consensus in the SN using the proposed picture fuzzy Jensen- a -norm dissimilarity measure; moreover, based on the TOPSIS [38] method, we selected an optimal alternative. Finally, using a case study of food safety evaluation, we demonstrated the applicability of the proposed LSGDM approach: its evaluation results were feasible and credible.

The advantages of the proposed approach can be summarized as follows. (1) Contrary to most existing LSGDM methods, the developed approach takes into account the complex trust relationships between experts in SNs and can determine the consensus in SNs, which is advantageous when solving LSGDM problems. (2) The picture fuzzy Jensen- a -norm dissimilarity measure proposed in this study is convenient for detecting consensus among experts in SNs; moreover, combined with the classical TOPSIS method, it allows a final selection process of LSGDM problems. (3) Compared with the existing FSs, IFSs, and IT2FSs, the PFSs used in this study can not only describe the trust relationships and preference information from different aspects but also provide more comprehensive and accurate descriptions of uncertainty information. In this case, the trust relationships among experts and their preference information are provided in the form of linguistic scales, which are more convenient than numerical values. The corresponding PFSs can be obtained based on transformation rules, reducing the difficulty for experts to provide evaluation information.

The proposed method has the following two limitations: (1) It assumes that the DMs are completely rational, while in reality they have a bounded rationality; (2) it is suitable for decision-making when there are just a few candidate projects. Future research should therefore address these two limitations.

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Appendix A. Tables for the case study.

Table 2. Linguistic trust evaluation values

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
V_1		VT								
V_2			M							T
V_3	T	T		V						
V_4			M							
V_5									T	VU
V_6					T					
V_7								VT	M	
V_8						M				
V_9							T	VT		
V_{10}					M					
V_{11}										
V_{12}										
V_{13}										
V_{14}							ET			
V_{15}										
V_{16}										
V_{17}										
V_{18}					VT					
V_{19}							EU			
V_{20}										

	V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
V_1										
V_2										
V_3										
V_4								T		VT
V_5										
V_6				ET						
V_7	VU								U	
V_8				T						
V_9										
V_{10}										
V_{11}									ET	
V_{12}							T			
V_{13}	M									
V_{14}										
V_{15}					M					
V_{16}		T					VT			
V_{17}			U							
V_{18}										
V_{19}		T			U					
V_{20}								VT		

Table 3. Evaluation information

V_1	c_1	c_2	c_3	c_4	c_5	c_6
α_1	M	B	M	M	B	B
α_2	M	B	B	VG	VB	B
α_3	M	B	M	B	B	G
α_4	M	M	G	G	M	M
α_5	B	VB	B	VG	G	M
V_{20}	c_1	c_2	c_3	c_4	c_5	c_6
α_1	M	VB	M	B	M	M
α_2	M	B	M	G	B	B
α_3	B	M	M	M	M	VG
α_4	M	VG	M	M	G	G
α_5	M	B	B	G	G	M

Appendix B. The proofs for Theorems 1 and 2.

The process of proof for Theorem 1 is presented in the following.

Proof: Since conditions (2) and (3) can be directly obtained, we will prove conditions (1) and (4) in the following.

(1) Because $\left\{\sum_{i=1}^{n}\left[\lambda p_i+(1-\lambda)q_i\right]^t\right\}^{1/t}\le\lambda\left(\sum_{i=1}^{n}p_i^t\right)^{1/t}+(1-\lambda)\left(\sum_{i=1}^{n}q_i^t\right)^{1/t}$, $t>1$ and $\left\{\sum_{i=1}^{n}\left[\lambda p_i+(1-\lambda)q_i\right]^t\right\}^{1/t}\ge\lambda\left(\sum_{i=1}^{n}p_i^t\right)^{1/t}+(1-\lambda)\left(\sum_{i=1}^{n}q_i^t\right)^{1/t}$, $0<t<1$.

Thus, for $a>1$,

$$\left(\left(\tau_1Pos_1(x_i)+\tau_2Pos_2(x_i)\right)^a+\left(\tau_1Neu_1(x_i)+\tau_2Neu_2(x_i)\right)^a+\left(\tau_1Neg_1(x_i)+\tau_2Neg_2(x_i)\right)^a+\left(\tau_1\pi_1(x_i)+\tau_2\pi_2(x_i)\right)^a\right)^{1/a}\le\tau_1\left(Pos_1^a(x_i)+Neu_1^a(x_i)+Neg_1^a(x_i)+\pi_1^a(x_i)\right)^{1/a}+\tau_2\left(Pos_2^a(x_i)+Neu_2^a(x_i)+Neg_2^a(x_i)+\pi_2^a(x_i)\right)^{1/a}$$

and

$$\left(\left(\tau_1Pos_2(x_i)+\tau_2Pos_1(x_i)\right)^a+\left(\tau_1Neu_2(x_i)+\tau_2Neu_1(x_i)\right)^a+\left(\tau_1Neg_2(x_i)+\tau_2Neg_1(x_i)\right)^a+\left(\tau_1\pi_2(x_i)+\tau_2\pi_1(x_i)\right)^a\right)^{1/a}\le\tau_1\left(Pos_2^a(x_i)+Neu_2^a(x_i)+Neg_2^a(x_i)+\pi_2^a(x_i)\right)^{1/a}+\tau_2\left(Pos_1^a(x_i)+Neu_1^a(x_i)+Neg_1^a(x_i)+\pi_1^a(x_i)\right)^{1/a}$$

i.e.,

$$\sum_{i=1}^{n}\Big(\Big(\big(\tau_1 Pos_1(x_i)+\tau_2 Pos_2(x_i)\big)^a+\big(\tau_1 Neu_1(x_i)+\tau_2 Neu_2(x_i)\big)^a+\big(\tau_1 Neg_1(x_i)+\tau_2 Neg_2(x_i)\big)^a$$
$$+\big(\tau_1\pi_1(x_i)+\tau_2\pi_2(x_i)\big)^a\Big)^{1/a}+\Big(\big(\tau_1 Pos_2(x_i)+\tau_2 Pos_1(x_i)\big)^a+\big(\tau_1 Neu_2(x_i)+\tau_2 Neu_1(x_i)\big)^a$$
$$+\big(\tau_1 Neg_2(x_i)+\tau_2 Neg_1(x_i)\big)^a+\big(\tau_1\pi_2(x_i)+\tau_2\pi_1(x_i)\big)^a\Big)^{1/a}\Big)$$
$$\le\sum_{i=1}^{n}\Big((\tau_1+\tau_2)\big(Pos_1^a(x_i)+Neu_1^a(x_i)+Neg_1^a(x_i)+\pi_1^a(x_i)\big)^{1/a}+(\tau_1+\tau_2)\big(Pos_2^a(x_i)+Neu_2^a(x_i)+Neg_2^a(x_i)+\pi_2^a(x_i)\big)^{1/a}\Big).$$

Since $\tau_1+\tau_2=1$ and $1-a<0$, we can reach $\frac{a}{1-a}<0$. Thus, $PFJBL_{\tau,a}(\psi_1,\psi_2)\ge 0$ can be obtained.

Similarly, for $0<a<1$, we can prove $PFJBL_{\tau,a}(\psi_1,\psi_2)\ge 0$.

(1) Since $\hat{\psi}_1\le\hat{\psi}_2\le\hat{\psi}_3$, then we have $Pos_1(x_i)\le Pos_2(x_i)\le Pos_3(x_i)$, $Neu_1(x_i)\ge Neu_2(x_i)\ge Neu_3(x_i)$, and $Neg_1(x_i)\ge Neg_2(x_i)\ge Neg_3(x_i)$ for any $x\in X$, so $|Pos_1(x_i)-Pos_2(x_i)|+|Neu_1(x_i)-Neu_2(x_i)|+|Neg_1(x_i)-Neg_2(x_i)| \le |Pos_1(x_i)-Pos_3(x_i)|+ |Neu_1(x_i)-Neu_3(x_i)|+|Neg_1(x_i)-Neg_3(x_i)|$ and $|Pos_2(x_i)-Pos_3(x_i)|+|Neu_2(x_i)-Neu_3(x_i)| +|Neg_2(x_i)-Neg_3(x_i)|\le|Pos_1(x_i)-Pos_3(x_i)|+|Neu_1(x_i)-Neu_3(x_i)|+|Neg_2(x_i)-Neg_3(x_i)| \le|Pos_1(x_i)-Pos_3(x_i)|+|Neu_1(x_i)-Neu_3(x_i)|+|Neg_1(x_i)-Neg_3(x_i)|$ are valid. Thus, $PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_2)\le PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_3)$ and $PFJBL_{\tau,a}(\hat{\psi}_2,\hat{\psi}_3)\le PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_3)$, i.e. $PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_3)\ge\max\{PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_2),PFJBL_{\tau,a}(\hat{\psi}_2,\hat{\psi}_3)\}$ can be achieved.

The process of proof for **Theorem 2** is shown in the following.

Proof. According to operations in Definition 2, we have $\hat{\psi}_1{}^c=(Pos_1(x_i),Neu_1(x_i),Neg_1(x_i))$ and $\hat{\psi}_2{}^c=(Pos_2(x_i),Neu_2(x_i),Neg_2(x_i))$. Apparently, property (1) can be obtained directly, therefore, the proof process is omitted here.

(2) $PFJBL_{\tau,a}(\hat{\psi}_1{}^c,\hat{\psi}_2{}^c)$

$$=\frac{a}{n(1-a)}\sum_{i=1}^{n}w_i\Big(\Big(\big(\tau_1 Neg_1(x_i)+\tau_2 Neg_2(x_i)\big)^a+\big(\tau_1 Neu_1(x_i)+\tau_2 Neu_2(x_i)\big)^a+\big(\tau_1 Pos_1(x_i)+\tau_2 Pos_2(x_i)\big)^a+$$
$$\big(\tau_1\pi_1(x_i)+\tau_2\pi_2(x_i)\big)^a\Big)^{1/a}+\Big(\big(\tau_1 Neg_2(x_i)+\tau_2 Neg_1(x_i)\big)^a+\big(\tau_1 Neu_2(x_i)+\tau_2 Neu_1(x_i)\big)^a+\big(\tau_1 Pos_2(x_i)+\tau_2 Pos_1(x_i)\big)^a$$
$$+\big(\tau_1\pi_2(x_i)+\tau_2\pi_1(x_i)\big)^a\Big)^{1/a}-\Big(\big(Neg_1(x_i)\big)^a+\big(Neu_1(x_i)\big)^a+\big(Pos_1(x_i)\big)^a+\big(\pi_1(x_i)\big)^a\Big)^{1/a}$$
$$-\Big(\big(Neg_2(x_i)\big)^a+\big(Neu_2(x_i)\big)^a+\big(Pos_2(x_i)\big)^a+\big(\pi_2(x_i)\big)^a\Big)^{1/a}\Big)=PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_2).$$

(3) $PFJBL_{\tau,a}(\psi_1,\psi_2{}^c)$

$$=\frac{a}{n(1-a)}\sum_{i=1}^{n}w_i\Big(\Big(\big(\tau_1 Pos_1(x_i)+\tau_2 Neu_2(x_i)\big)^a+\big(\tau_1 Neu_1(x_i)+\tau_2 Neu_2(x_i)\big)^a+\big(\tau_1 Neg_1(x_i)+\tau_2 Pos_2(x_i)\big)^a+$$
$$\big(\tau_1\pi_1(x_i)+\tau_2\pi_2(x_i)\big)^a\Big)^{1/a}+\Big(\big(\tau_1 Neg_2(x_i)+\tau_2 Pos_1(x_i)\big)^a+\big(\tau_1 Neu_2(x_i)+\tau_2 Neu_1(x_i)\big)^a+\big(\tau_1 Pos_2(x_i)+$$
$$+\tau_2 Neg_1(x_i)\big)^a+\big(\tau_1\pi_2(x_i)+\tau_2\pi_1(x_i)\big)^a\Big)^{1/a}-\Big(\big(Pos_1(x_i)\big)^a+\big(Neu_1(x_i)\big)^a+\big(Neg_1(x_i)\big)^a+\big(\pi_1(x_i)\big)^a\Big)^{1/a}$$
$$-\Big(\big(Neg_2(x_i)\big)^a+\big(Neu_2(x_i)\big)^a+\big(Pos_2(x_i)\big)^a+\big(\pi_2(x_i)\big)^a\Big)^{1/a}\Big)$$

$$=\frac{a}{n(1-a)}\sum_{i=1}^{n}w_i\Big(\Big(\big(\tau_1 Neg_1(x_i)+\tau_2 Pos_2(x_i)\big)^a+\big(\tau_1 Neu_1(x_i)+\tau_2 Neu_2(x_i)\big)^a+\big(\tau_1 Pos_1(x_i)+\tau_2 Neg_2(x_i)\big)^a+$$
$$\big(\tau_1\pi_1(x_i)+\tau_2\pi_2(x_i)\big)^a\Big)^{1/a}+\Big(\big(\tau_1 Pos_2(x_i)+\tau_2 Neg_1(x_i)\big)^a+\big(\tau_1 Neu_2(x_i)+\tau_2 Neu_1(x_i)\big)^a+\big(\tau_1 Neg_2(x_i)+$$
$$+\tau_2 Pos_1(x_i)\big)^a+\big(\tau_1\pi_2(x_i)+\tau_2\pi_1(x_i)\big)^a\Big)^{1/a}-\Big(\big(Neg_1(x_i)\big)^a+\big(Neu_1(x_i)\big)^a+\big(Pos_1(x_i)\big)^a+\big(\pi_1(x_i)\big)^a\Big)^{1/a}$$
$$-\Big(\big(Pos_2(x_i)\big)^a+\big(Neu_2(x_i)\big)^a+\big(Neg_2(x_i)\big)^a+\big(\pi_2(x_i)\big)^a\Big)^{1/a}\Big)$$

$$=PFJBL_{\tau,a}\left(\hat{\psi}_1^{\,c},\hat{\psi}_2\right).$$

(4) If $Pos_1(x_i)\ge Pos_2(x_i)$ for any $x_i\in X$, then we have $\max\{Pos_1(x_i),Pos_2(x_i)\}=Pos_1(x_i)$ and $\min\{Pos_1(x_i),Pos_2(x_i)\}=Pos_2(x_i)$. Thus, $\tau_1\max\{Pos_1(x_i),Pos_2(x_i)\}+\tau_2\min\{Pos_1(x_i),Pos_2(x_i)\}=\tau_1 Pos_1(x_i)+\tau_2 Pos_2(x_i)$ and $\tau_1\min\{Pos_1(x_i),Pos_2(x_i)\}+\tau_2\max\{Pos_1(x_i),Pos_2(x_i)\}=\tau_1 Pos_2(x_i)+\tau_2 Pos_1(x_i)$ are valid. If $Pos_1(x_i)<Pos_2(x_i)$ for any $x_i\in X$, then we have $\tau_1\max\{Pos_1(x_i),Pos_2(x_i)\}+\tau_2\min\{Pos_1(x_i),Pos_2(x_i)\}=\tau_1 Pos_2(x_i)+\tau_2 Pos_1(x_i)$ and $\tau_1\min\{Pos_1(x_i),Pos_2(x_i)\}$ $+\tau_2\max\{Pos_1(x_i),Pos_2(x_i)\}=\tau_1 Pos_1(x_i)+\tau_2 Pos_2(x_i)$.

Thus,

$$\big(\tau_1\max\{Pos_1(x_i),Pos_2(x_i)\}+\tau_2\min\{Pos_1(x_i),Pos_2(x_i)\}\big)^a+\big(\tau_1\min\{Pos_1(x_i),Pos_2(x_i)\}+\tau_2\max\{Pos_1(x_i),Pos_2(x_i)\}\big)^a$$
$$=\big(\tau_1 Pos_2(x_i)+\tau_2 Pos_1(x_i)\big)^a+\big(\tau_1 Pos_1(x_i)+\tau_2 Pos_2(x_i)\big)^a.$$

Similarly, we have the following results:

$$\big(\tau_1\max\{Neu_1(x_i),Neu_2(x_i)\}+\tau_2\min\{Neu_1(x_i),Neu_2(x_i)\}\big)^a+\big(\tau_1\min\{Neu_1(x_i),Neu_2(x_i)\}+\tau_2\max\{Neu_1(x_i),Neu_2(x_i)\}\big)^a$$
$$=\big(\tau_1 Neu_2(x_i)+\tau_2 Neu_1(x_i)\big)^a+\big(\tau_1 Neu_1(x_i)+\tau_2 Neu_2(x_i)\big)^a;$$

$$\big(\tau_1\max\{Neg_1(x_i),Neg_2(x_i)\}+\tau_2\min\{Neg_1(x_i),Neg_2(x_i)\}\big)^a+\big(\tau_1\min\{Neg_1(x_i),Neg_2(x_i)\}+\tau_2\max\{Neg_1(x_i),Neg_2(x_i)\}\big)^a$$
$$=\big(\tau_1 Neg_2(x_i)+\tau_2 Neg_1(x_i)\big)^a+\big(\tau_1 Neg_1(x_i)+\tau_2 Neg_2(x_i)\big)^a;$$

$$\Big(\tau_1\big(1-\max\{Pos_1(x_i),Pos_2(x_i)\}-\min\{Neu_1(x_i),Neu_2(x_i)\}-\min\{Neg_1(x_i),Neg_2(x_i)\}\big)$$
$$+\tau_2\big(1-\min\{Pos_1(x_i),Pos_2(x_i)\}-\max\{Neu_1(x_i),Neu_2(x_i)\}-\max\{Neg_1(x_i),Neg_2(x_i)\}\big)\Big)^a$$
$$=\Big(\tau_1\big(1-\min\{Pos_1(x_i),Pos_2(x_i)\}-\max\{Neu_1(x_i),Neu_2(x_i)\}-\max\{Neg_1(x_i),Neg_2(x_i)\}\big)$$
$$+\tau_2\big(1-\max\{Pos_1(x_i),Pos_2(x_i)\}-\min\{Neu_1(x_i),Neu_2(x_i)\}-\min\{Neg_1(x_i),Neg_2(x_i)\}\big)\Big)^a$$
$$=\left(\tau_1\pi_1(x_i)+\tau_2\pi_2(x_i)\right)^a.$$

Furthermore,

$$\Big(\big(\max\{Pos_1(x_i),Pos_2(x_i)\}\big)^a+\big(\min\{Neu_1(x_i),Neu_2(x_i)\}\big)^a+\big(\min\{Neg_1(x_i),Neg_2(x_i)\}\big)^a$$
$$+\big(1-\max\{Pos_1(x_i),Pos_2(x_i)\}-\min\{Neu_1(x_i),Neu_2(x_i)\}-\min\{Neg_1(x_i),Neg_2(x_i)\}\big)^a\Big)^{1/a}$$
$$+\Big(\big(\min\{Pos_1(x_i),Pos_2(x_i)\}\big)^a+\big(\max\{Neu_1(x_i),Neu_2(x_i)\}\big)^a+\big(\max\{Neg_1(x_i),Neg_2(x_i)\}\big)^a$$
$$+\big(1-\min\{Pos_1(x_i),Pos_2(x_i)\}-\max\{Neu_1(x_i),Neu_2(x_i)\}-\max\{Neg_1(x_i),Neg_2(x_i)\}\big)^a\Big)^{1/a}$$
$$=\left(Pos_1^a(x_i)+Neu_1^a(x_i)+Neg_1^a(x_i)+\pi_1^a(x_i)\right)^{1/a}+\left(Pos_2^a(x_i)+Neu_2^a(x_i)+Neg_2^a(x_i)+\pi_2^a(x_i)\right)^{1/a}.$$

Therefore, $PFJBL_{\tau,a}(\hat{\psi}_1\cup\hat{\psi}_2,\hat{\psi}_1\cap\hat{\psi}_2)=PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_2)$ can be obtained.

Since picture fuzzy Jensen- a -norm dissimilarity measure satisfies the symmetry, then property (5) is valid, i.e., $PFJBL_{\tau,a}(\hat{\psi}_1\cap\hat{\psi}_2,\hat{\psi}_1\cup\hat{\psi}_2)=PFJBL_{\tau,a}(\hat{\psi}_1,\hat{\psi}_2)$.

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