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Balancing the Great British Electricity System – **Bulk Dispatch Optimisation**

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Abstract—The Balancing Mechanism, managed by the British electricity system operator, National Grid ESO, is designed to maintain a balance between electricity generation and demand. It achieves this by purchasing extra generation (or demand reduction) through accepted offers and extra demand (or generation reduction) through accepted bids from Balancing Mechanism Units (BMUs) in real-time. The current manual approach for instructing BMUs becomes increasingly challenging as access to the market widens and the number of BMUs grows. This paper introduces a proof-of-concept optimisation model to assist Control Room engineers in making optimal decisions for a large number of BMUs. We outline the requirements for instructing BMUs, provide their mathematical formulation, and illustrate this using simple examples. Computational performance results based on test cases with up to 500 units are presented.

Index Terms-Power systems analysis; electricity system modelling; power dispatch; mixed integer linear programming.

NOTATION

Sets

${\cal G}$	Units, indexed by g								
${\mathcal T}$	Time intervals, indexed by t								
$\mathcal{G}_A\subseteq\mathcal{G}$	All-or-nothing units, indexed by g								
$\mathcal{G}_{\mathrm{C}}\subseteq\mathcal{G}$	Units with closed instructions, indexed by g								
Parameters	Parameters								
$\mathrm{P}_{g,t}$	Committed power level of unit g (MW)								
$\mathbf{U}_t^{\mathrm{UB}}, \mathbf{U}_t^{\mathrm{LB}}$	$^{\mathrm{B}}, \mathrm{U}_{t}^{\mathrm{LB}}$ Upper/lower MW requirement (in MW)								
$S_{g,t}^{\mathrm{UB}}, S_{g,t}^{\mathrm{LB}}$	Upper/lower bound of unit g (MW)								
$R_g^{\mathrm{U}},R_g^{\mathrm{D}}$	ramp-up/ramp-down rates of unit g (MW/min)								
N_g^B, N_g^O	Notice to bid/offer of unit g (min)								
F_g	Minimum flat top time of unit g (min)								
$\mathbf{M}_g^{\mathrm{Z}}, \mathbf{M}_g^{\mathrm{NZ}}$	Minimum zero/non-zero time of unit g (min)								
$\mathrm{V}_g^O,\mathrm{V}_g^B$	Maximum delivery volume of accepting of-								
	fers/bids from unit g (MWh)								
M_g^P	Maximum delivery period of accepting								
	bids/offers from unit g (min)								

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MAppropriately chosen big-M value

Variables

$x_{g,t}^{\mathrm{U}}, x_{g,t}^{\mathrm{D}}$	Binary: 1 if unit g is redispatched up/down on
	interval $\left[t,t+1\right]$ and 0 otherwise
$x_{g,t}^+, x_{g,t}^-$	Binary: 1 if unit g is exporting/importing in
	interval $[t, t+1]$ and 0 otherwise
$p_{g,t}$	Target level of unit g at time t (MW)
$p_{g,t}^{\rm U}, p_{g,t}^{\rm D}$	Upward/downward dispatch from unit g (MW)

Acronyms

NGESO	National Grid Electricity System Operator
OBP	Open Balancing Platform
BM(U)	Balancing Mechanism (Unit)
GB	Great Britain
BOA	Bid-offer Acceptance
FPN	Final Physical Notification
CL, CCL	Committed Level, Capped Committed Level

I. INTRODUCTION

National Grid ESO (NGESO) is the electricity transmission system operator of Great Britain (GB). It plays a key role in ensuring a reliable electricity supply across the high voltage GB transmission network. To maintain a balance between electricity generation and demand while adhering to operational standards, NGESO operates an hour-ahead market known as the Balancing Mechanism (BM). In this pay-as-bid market, participants, known as balancing mechanism units (BMUs), receive bid-offer acceptance (BOA) instructions from NGESO to adjust their electricity output, each instruction incurring a specific cost. NGESO's objective is to make cost-effective decisions while upholding statutory limits on frequency, voltage and power flow as outlined in the Security and Quality of Supply Standards [1].

Traditionally, the process of balancing electricity generation and demand primarily relied on large-scale generation units with capacities of 50 MW or more. Control Room engineers had relatively few generators to manage, making instructions based on a merit-order straight forward. However, recent regulatory changes have opened the door to the BM market for smaller generators with capacities as low as 1 MW [2], and discussions on the barriers to entry to the BM for sub 1 MW providers and decimal bids are ongoing [3]. While this expansion brings potential cost savings in the BM, it also introduces a substantial increase in the number of participating balancing units, and a greater complexity due to the range of static and dynamic constraints involved. This potentially renders control engineers' manual dispatch instructions impractical for efficient and cost-effective balancing actions.

There is a wealth of academic literature on optimisation of dispatch and redispatch costs, and related topics, notably the unit commitment problem [4]. In [5], a methodology is presented for solving the energy balancing problem with replacement reserves. [4] presents a stochastic optimisation model for balancing, incorporating uncertainty from renewable generation, while [6] focuses on integrating demand response into the balancing problem. Furthermore, [7] discusses dispatch optimisation for ancillary services. [8] formulates a redispatch problem that models the minimum up and down time constraints following a commitment of a thermal power plant. [9] examines the impact of dynamic line ratings on redispatch costs and [10] explores how HVDC transmission technology can reduce redispatch costs. [11] proposes an optimisation technique that employs storage devices to minimise redispatch costs, with ramp rate constraints as part of the formulation. The existing literature, including the aforementioned studies, primarily deals with generic system-level constraints that are not directly applicable to the Balancing Mechanism market. For instance, these studies represent ramp rate constraints as inequality constraints, allowing units to ramp up or down within a defined bound. This differs from BM rules, which require units to ramp at specified rates.

This paper presents the outcomes of a collaborative research project between the University of Strathclyde and NGESO, aimed at designing a proof-of-concept decision-support optimisation model for efficient power dispatch among a large number of diverse market participants. Building on prior research [12], the project resulted in developing a proofof-concept optimisation model, prototype tool and approach to assisting Control Room engineers in formulating energy requirements and making economical power redispatch decisions. The model identifies cost-effective actions for redispatching electricity generation to ensure energy balance in the GB system. This model has been subsequently extended and enhanced further for a wider set of requirements by NGESO as part of the Balancing Programme, which facilitated implementation of bulk dispatch capability of NGESO's Open Balancing Platform (OBP) launched in December 2023 [13], [14]. While the optimisation model outline in this paper serves as the foundation for the bulk dispatch optimisation model in NGESO's OBP, it is important to note that the NGESO's OBP model has undergone enhancements and improvements throughout its development. Therefore, while similar in concept, the model described here does not precisely match the version utilised by OBP.

The contributions of this paper are three-fold:

- Elicitation and documentation of requirements for redispatch, including but not limited to BM participants.
- 2) Framing and formulation of the problem as a mathematical programming problem.
- 3) Tests and validation of the developed framework.

The remainder of the paper is organised as follows. Section II discusses the bulk dispatch problem and articulates the functional requirements of redispatch within the BM. Section III presents the formulation of a mathematical programming model using the techniques of mixed-integer linear programming (MILP), and Section IV demonstrates application of the developed model to a range of test scenarios. The paper concludes with a discussion in Section VI.

II. REQUIREMENTS OF INSTRUCTING BALANCING MECHANISM UNITS

In Great Britain, electricity trading primarily occurs in forward wholesale markets. Generators and suppliers enter contracts for specific half-hour periods known as settlement periods, often spanning years into the future. Trading continues until one hour before real-time operations, a stage known as *gate-closure*. By gate-closure, all BMUs will have submitted their final physical notifications (FPN) detailing their intended power generation and consumption levels, along with prices for offers to sell energy (increasing generation or decreasing demand) and bids to buy energy (decreasing generation or increasing demand).

After gate-closure, National Grid ESO assumes control to balance demand and generation in real-time by accepting bids and offers in a cost-optimal manner and sending corresponding BOAs. Accepted BOAs will result in units' power output being altered and FPN profiles translating into CL/CCL profiles. The latter may be altered further, should units' maximum export or import limits change, setting CCL to a value different than CL. The need for balancing may arise because of power imbalance where market may not have achieved an equilibrium and/or due to transmission constraints. In this paper, our focus is on the power balancing problem only.

Figure 1 presents a typical BMU instruction. In this example, the BMU submitted a final physical notification (FPN) to generate 50 MW, with the maximum export limit (MEL) of 120 MW and the minimum stable export limit (SEL) of 20 MW. Control Room instructed the unit to increase generation by 30 MW from 08:56 to 09:06. The BMU's rampup and ramp-down rate are 5 MW/min each. To meet this instruction, the unit begins to ramp up at 08:50. The BMU has a notice period of 5 minutes, and the latest time to receive and accept the instruction is therefore 08:45. At 09:06, the unit starts to ramp down returning to its pre-instruction level at 09:12. The energy delivered by the BMU is equal to the area under the curve describing the instruction (solid line in

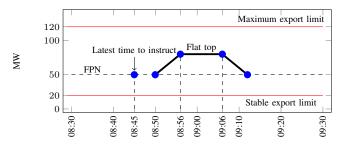


Fig. 1. A BMU has submitted an FPN of 50 MW. The BMU is instructed to provide extra 30 MW from 08:56 to 09:06. To deliver this additional power, the unit starts ramping up at 08:50. The solid line describes the shape of the instruction (which is a four-point BOA).

Figure 1), and the total cost of this energy is a function of this unit's offer prices that cover the time interval at hand.

Table I lists the requirements for instructing market participants who provide balancing or ancillary services, along with brief explanations. A more detailed explanation, including mathematical formulations, can be found in the next section. Although our primary focus is on Balancing Mechanism Units (BMUs), it is worth noting that the instructions issued by control room for other service providers, such as Short Term Operating Reserve (STOR) or Frequency Reserve (FR), often share similar requirements.

The control room employs real-time monitoring of electricity generation, demand and system frequency, coupled with short-term forecasting of demand and renewable generation. These decision-support tools empower control room engineers to anticipate and address potential imbalances. Figure 2 presents a positive extra power/energy requirement that a control room engineer may want to achieve. In this example, there is a need for an additional 100 MW of power between 09:00 and 09:30, as indicated by the lower bound. The upper bound provides a degree of flexibility in meeting this target, which is important, considering that exactly meeting the target may prove costly or even infeasible in certain cases. This flexibility permits a variety of units-export import and bi-directional units—including those with slower ramping capabilities to respond and fulfil the requirement. Typically, redispatch to deliver extra power incurs a positive cost, and the optimal solution tends to align with the lower bounds of the energy requirement. For negative requirements, it is the upper bound that determines the minimal amount of generation reduction or demand increase, whereas the lower bound serves as a tolerance on delivering an excess of power change that would be delivered by redispatching the units.

III. MATHEMATICAL MODEL

Control Room's dispatch instructions to BMUs have costs associated —these can be calculated from units' FPN, BOD1, CL/CCL. The aim is to describe a mathematical program that provides a cost-optimal solution for a given power/energy

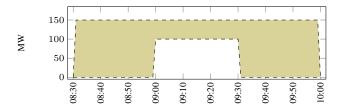


Fig. 2. An example of energy requirement from Control Room. 100 MW of extra generation is required from 09:00 to 09:30. The requirement is described using a lower and upper bounds. This defines the requirement as a combination of a minimum amount of extra power needed and tolerance on exceeding it.

requirement. Let $\mathcal G$ be a set of BMUs and $\mathcal T$ be the set of optimisation window time points. The set \mathcal{T} is defined as $\{1, 2, \dots, T\}$, where T is the final time point of optimisation window. In what follows, we shall assume that each constraint is added $\forall g \in \mathcal{G}, t \in \mathcal{T}$. Let $x_{g,t}^{U}$ and $x_{g,t}^{D}$ be binary variables that model the redispatch (ramp-up or ramp-down) of a unit q in time period [t, t+1]:

$$x_{g,t}^{\mathsf{U}} = \begin{cases} 1 & \text{unit } g \text{ ramps up in } [t,t+1], \\ 0 & \text{otherwise,} \end{cases}$$
 (1a)

$$x_{g,t}^{\mathsf{U}} = \begin{cases} 1 & \text{unit } g \text{ ramps up in } [t,t+1], \\ 0 & \text{otherwise,} \end{cases}$$
 (1a)
$$x_{g,t}^{\mathsf{D}} = \begin{cases} 1 & \text{unit } g \text{ ramps down in } [t,t+1], \\ 0 & \text{otherwise.} \end{cases}$$
 (1b)

The following constraints ensure that only one of the upward/downward redispatch services is provided at any given time step:

$$x_{g,t}^{\mathrm{U}} + x_{g,t}^{\mathrm{D}} \le 1.$$
 (2)

Let $p_{g,t}$ denote power output and $\mathbf{P}_{g,t}$ be CCL of unit gin time period [t, t+1], respectively. Thus, $P_{g,t}$ is g's preoptimisation power output and $p_{g,t}$ is g's optimised power output on interval [t, t+1]. If unit g is not instructed, then we have $p_{g,t} = P_{g,t} \ \forall t \in \mathcal{T}$. Let $p_{g,t}^{U}$ and $p_{g,t}^{D}$ be the upward and downward redispatch of unit g during [t,t+1], respectively. Unit q's optimised output is modelled as follows:

$$p_{g,t} = p_{g,t-1} + \left(p_{g,t}^{U} - p_{g,t}^{D}\right),$$
 (3)

where $p_{g,1}=P_{g,1}$, $p_{g,t}^{\rm U}$ and $p_{g,t}^{\rm D}$ are the positive and negative parts of the total redispatch of unit g.

A. Control Room user's requirement

Control Room user's requirement for power/energy is described by two power curves: an upper and lower bound profiles. The objective of our optimisation problem is to determine a least cost solution that meets the generation requirement, i.e. the total redispatch across all BMUs lies within the feasible region described by the two bound profiles:

$$\mathbf{U}_{t}^{\mathrm{LB}} \leq \sum_{g \in \mathcal{G}} p_{g,t} \leq \mathbf{U}_{t}^{\mathrm{UB}}. \tag{4}$$

Modelling generation requirement via lower and upper bounds provides some flexibility, as it allows to (i) represent both the requirement and the tolerance on exceeding (or under-

¹Bid-Offer Data is submitted by market participants to indicate the amount by which they can increase or decrease output in return for payment in any given settlement period.

 $\label{eq:table in the bulk dispatch optimisation} TABLE\ I$ A list of requirements for the bulk dispatch optimisation.

Requirement	Units	Explanation
Ramp rate	MW/min	All instructed units must respect their ramp rates
Minimum flat top time	min	There is a limit on the minimal lengths of flat tops
Notice to offer/bid min		Issuing instructions should respect notice times
Stable export/import limits	min	Units' capacities must be respected in instructions
Min zero/non-zero time	min	Minimum times on zero and non-zero operation must be respected when instructing units
Maximum delivery period	min	Temporal constraint on delivering volume
Maximum delivery volume	MWh	Maximum volume that can be delivered over a given time
All or nothing units	0 or 1	A unit that if instructed can only offer full capacity
Closed instructions	0 or 1	A unit that if instructed must be brought back to its pre-instruction operating level

delivering) it, and (ii) leaves room to treating these bounds as either hard or soft constraints.

B. Ramp rates

Ramp rates describe the maximum rates of upward/downward MW change, in units' outputs per unit of time. A BMU may submit multi-elbow ramp rates, however, in this paper we only consider the case of a single elbow ramp rate. Let $\mathbf{R}_g^{\mathbf{U}}$ and $\mathbf{R}_g^{\mathbf{D}}$ denote the ramp-up and ramp-down rate of a unit g, respectively. These are modelled as follows:

$$p_{q,t}^{U} = x_{q,t}^{U} R_{q}^{U},$$
 (5a) $p_{q,t}^{D} = x_{q,t}^{D} R_{q}^{D}.$ (5b)

C. Unit output bounds

Let $\mathbf{S}^{\mathrm{LB}}_{g,t}$ and $\mathbf{S}^{\mathrm{UB}}_{g,t}$ denotes the lower and upper bounds of a BMU g. The following constraints model the bounds on a units output:

$$\mathbf{S}_{g,t}^{\mathrm{LB}} \le p_{g,t} \le \mathbf{S}_{g,t}^{\mathrm{UB}}.\tag{6}$$

D. Notice to offer (NTO) and Notice to bid (NTB)

Each BMU g submits notice-to-offer (NTO) and notice-to-bid (NTB) times (in minutes), $N_g^{\rm O}$ and $N_g^{\rm B}$. These are the minimal amounts of notice times to receive offer/bid acceptances, respectively. The following constraints ensure that redispatch actions honour the notice periods:

$$\sum_{t=1}^{N_g^{O}} p_{g,t}^{U} = 0, \qquad (7a) \qquad \sum_{t=1}^{N_g^{B}} p_{g,t}^{D} = 0. \qquad (7b)$$

E. Minimum flat top times

Flat-tops are described as intervals of constancy—these are periods during which the units maintains a constant power level that sits between two ramps moving in the opposite directions. An example of a flat top is shown in Figure 1. Control Room may desire to dispatch a unit at certain MW level and keep it at that level for a certain minimal amount of time. This requirement is modelled as follows:

$$\sum_{k=t+1}^{t+\mathcal{F}_g} x_{g,k}^{\mathcal{D}} \le \left(1 + x_{g,t+1}^{\mathcal{U}} - x_{g,t}^{\mathcal{U}}\right) M, \tag{8a}$$

$$\sum_{k=t+1}^{t+F_g} x_{g,k}^{U} \le \left(1 + x_{g,t+1}^{D} - x_{g,t}^{D}\right) M, \tag{8b}$$

where F_g is the flat-top time in minutes for a BMU g and M is an appropriately chosen big-M value.

F. Minimum zero and non-zero times

Minimum zero time is the minimum time that a BM Unit which has been exporting (importing) must operate at zero or be importing (exporting), before returning to exporting (importing). We define two sets of binary variables $x_{g,t}^+$ and $x_{g,t}^-$ to model the export and import status of a unit g, respectively. The indicator variables are modelled as follows:

$$-(1 - x_{q,t}^+)M \le p_{q,t} - \epsilon \le x_{q,t}^+M,$$
 (9a)

$$-x_{q,t}^- M \le p_{q,t} + \epsilon \le (1 - x_{q,t}^-) M,$$
 (9b)

$$x_{a,t}^+ + x_{a,t}^- \le 1,$$
 (9c)

where $[-\epsilon, \epsilon]$ is a small interval within which the power output of a unit is considered zero. The minimum zero requirement is modelled by the following two sets of constraints:

$$\sum_{k=t+1}^{t+\mathcal{M}_{g}^{Z}} x_{g,k}^{+} \le \left(1 + x_{g,t+1}^{+} - x_{g,t}^{+}\right) M, \tag{10a}$$

$$\sum_{k=t+1}^{t+M_g^Z} x_{g,k}^- \le \left(1 + x_{g,t+1}^- - x_{g,t}^-\right) M, \tag{10b}$$

where M_g^Z is the minimum zero time of a generator g in minutes. M is an appropriately chosen big-M value for the constraints and in this case the big-M value can be M_g^Z as that is a bound of the left hand side of inequalities (10).

Minimum non-zero time is the minimum amount of time that a BMU must export (import) as a result of a dispatch instruction. This requirement is modelled as follows:

$$\mathbf{M}_{g}^{NZ} - \left(1 + x_{g,t}^{+} - x_{g,t+1}^{+}\right) M \le \sum_{k=t+1}^{t+\mathbf{M}_{g}^{NZ}} x_{g,k}^{+},$$
 (11a)

$$M_g^{NZ} - (1 + x_{g,t}^- - x_{g,t+1}^-) M \le \sum_{k=t+1}^{t+M_g^{NZ}} x_{g,k}^-,$$
 (11b)

where $\mathbf{M}_g^{\mathrm{NZ}}$ is the minimum non-zero time of unit g. If unit g is switching to exporting (importing) at time t, the right hand sides of inequalities (11) ensure that the unit continues to export (import) for a given $\mathbf{M}_g^{\mathrm{NZ}}$ minimum non-zero time period.

G. Maximum delivery period and volume

Maximum delivery volume is the amount of energy in offers (or bids) that a BMU can deliver over a specified period known as *maximum delivery period*. This constraint is modelled using the following inequalities:

$$\sum_{k=t}^{t+\mathcal{M}_g^P - 1} (p_{g,k} - P_{g,k})^+ \le 60 \mathcal{V}_g^O, \tag{12a}$$

$$\sum_{k=t}^{t+M_g^P-1} (p_{g,k} - P_{g,k})^- \le 60 V_g^B,$$
 (12b)

where w^+ and w^- represent the positive and negative parts of w, \mathbf{M}_g^P is the maximum delivery period, \mathbf{V}_g^O and \mathbf{V}_g^B are the maximum delivery volume for offers and bids for unit g, respectively.

H. All-or-nothing units

All-or-nothing units are a special kind of BMUs that cannot be part dispatched. This means that if such units are instructed, their full capacity must be employed. This requirement is modelled using the following two inequalities, the first one for offers and the second one for bids $(g \in \mathcal{G}_A)$. The inequalities ensure that following an instruction, the all-or-nothing unit continues to ramp until it reaches its full capacity:

$$\left(x_{g,t+1}^{\mathsf{U}} - x_{g,t}^{\mathsf{U}} - 1 \right) M + \frac{\mathsf{S}_g^{\mathsf{UB}}}{\mathsf{R}_g^{\mathsf{U}}} \le \sum_{k=t+1}^{t + \frac{\mathsf{S}_g^{\mathsf{UB}}}{\mathsf{R}_g^{\mathsf{U}}} + 1} x_{g,t}^{\mathsf{U}},$$
 (13a)

$$\left(x_{g,t+1}^{D} - x_{g,t}^{D} - 1\right)M + \frac{S_g^{LB}}{R_g^{D}} \le \sum_{k=t}^{t + \frac{S_g^{LB}}{R_g^{D}}} x_{g,t}^{D}.$$
 (13b)

The motivation behind introducing this requirement is to ensure that both BM and non-BM units that provide services such as Short Term Operating Reserve (STOR) and Fast Reserve (FR) could be included into Bulk Dispatch problems. Besides, pumped storage units are all-or-nothing when operating in the pumping mode.

I. Closed instructions

The mathematical program is described over a time horizon \mathcal{T} . The optimal solution may be to keep some of the instructions open, meaning that at the end of the time horizon

TABLE II
RAMP RATE, MINIMUM FLAT TOP, NOTICE-TO-OFFER, NOTICE-TO-BID,
AND BID/OFFER PRICE VALUES USED.

	RU/RD (MW/min)	FT (mins)	NTO/NTB (mins)	bid/offer (£/MWh)
Fast & expensive unit (FTEX)	7	20	0	70/50
Slow & cheap unit (SLCH)	4	5		35/20

the dispatch is not equal to the CCL/FPN of a BMU. Control Room may desire to close instructions at the end of the optimisation window, in particular, when the price information for the next settlement period is not yet revealed. This requirement can be achieved by the following constraints ($q \in \mathcal{G}_c$):

$$p_{a,T} = P_{a,T},\tag{14}$$

where \mathcal{G}_c is the set of all units for which closed instructions are required, and T is the last time point of the optimisation window.

J. Overall formulation

The objective of the optimisation problem is to minimise the total cost of redispatch actions to meet a given energy requirement. The overall formulation is then given as follows:

$$\min f(p_g), \tag{15a}$$

subject to

$$(1) - (14),$$
 $(15b)$

where $f(p_g)$ is a cost curve of unit g represented by a piecewise linear profile (for BMUs such cost curves are derived from their bid and offer data, also known as BOD, and are convex profiles). The overall formulation is a mixed-integer linear programming problem. We have implemented the above model using Pyomo [15] and Gurobi optimiser v9.1.2 [16] to solve bulk dispatch problem instances created using both synthetic and historical operational data based test cases.

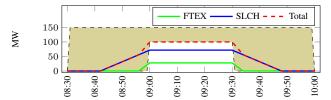
IV. NUMERICAL ILLUSTRATION

To demonstrate the impact of functional constraints presented in Section III, we start with a simple example of using two BMUs. Later in this section we extend our results to cases with up to 500 units.

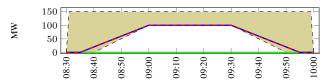
Let us consider two BMUs—a fast and expensive unit (FTEX) with a 7 MW/min ramp rate and a slow and cheap (SLCH) unit with a 4 MW/min ramp rate. The costly FTEX unit is twice as expensive as the more economical SLCH. Table II provides key parameters used in the optimisation problem. In the subsequent subsections, we analyse how different constraints impact the optimal solution.

A. The shape of a requirement

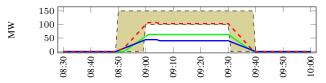
The energy requirement is described by upper and lower bounds on total active power redispatch. Figure 3a presents a scenario in which additional 100 MW are required between 09:00 and 09:30. This requirement can be expressed as a



(a) Optimal solution of meeting 100 MW of requirement from 0900 to 0930. Both units are dispatched to meet the requirement.



(b) A user's requirement of 100 MW from 0900 to 0930 is defined with a ramp rate of 5 MW/min. The optimal solution is 3% more expensive than the solution in Fig 3a.



(c) A user's requirement of 100 MW from 0900 to 0930 is defined with an upper bound of 0 MW from 0830 to 0850. The optimal solution is 17% more expensive than the solution in Fig 3a.

Fig. 3. The impact of changing energy requirement on the optimal redispatch.

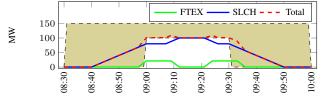
lower bound of 100 MW over 30 minutes and an upper bound of 150 MW from 08:30 to 10:00, allowing units to adapt. Figure 3b shows a gradual power increase by applying a 5MW/min ramp. In this case the optimal solution delivers 20% more energy and the overall cost is approximately 3% more than in the previous scenario. If no extra energy is required at the beginning and end, as in Figure 3c, the power/energy profile can be represented accordingly. In this scenario, the volume delivered is 9% less, however, the cost of meeting the requirement is 17% more expensive than in the first scenario. Despite these differences, all three scenarios ensure delivery of at least 100 MW between 09:00 and 09:30, showcasing the flexibility of using two bounds in defining extra power (or power reduction) requirements.

B. Minimum flat top times (MFTTs)

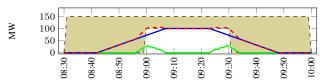
The minimum flat top length requirement specifies how long a given BMU must maintain a specific MW level. Figure 3a presents a solution with FTEX with a 20-minute flat-top requirement. In Figure 4a, reducing the MFTT requirement to ten minutes results in decreased energy delivered by FTEX and 1% decrease in the overall cost. Figure 4b goes further, reducing the MFTT to 1-minute, leading FTEX to support SLCH in meeting maximum requirement and 12% reduction in the cost of meeting the energy requirement.

C. Maximum delivery time and volume

Maximum delivery volume is a constraint on a BMU that defines the largest amount of energy the unit can provide



(a) MFTT requirement changed from 20 mins to 10 mins. The optimal solution is 1% better than the solution in Fig 3a.



(b) MFTT requirement changed from 20 mins to 1 min. The optimal solution is 12% better than the solution in Fig 3a.

Fig. 4. Impact of the MFTT requirement on the optimal solution. The MFTT for FTEX unit is modified from 20 mins to 10 mins and 1 min, respectively.

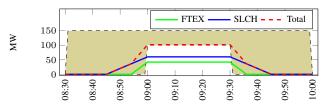


Fig. 5. The impact of reducing the maximum delivery volume of SLCH from 36 MWh to 21 MWh. FTEX is redispatch to a higher level as compared to the optimal solution in Fig 3a to accommodate this constraint.

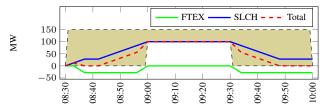
within a given period. In Figure 3a, SLCH delivers energy of 36 MWh from 09:00 to 09:30. When constrained to deliver only 21 MWh over the same half-hour interval, the optimal solution is shown in Figure 5. Note that this change redispatches the FTEX BMU to a higher level, increasing the solution cost by 36% compared to Figure 3a solution.

D. Energy arbitrage and closed instructions

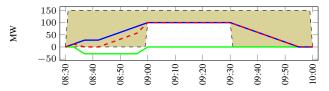
BMUs submit price information in advance, and the optimisation model may identify opportunities for cost-effective energy arbitrage. When FTEX's bid price is set to $100~\pounds/\text{MWh}$, indicating willingness to pay the system operator for reducing its own power output, the optimal solution in Figure 6a shows FTEX accepting bids. However, this solution leaves instructions open at the end of the time horizon, which may not be ideal due to price uncertainty in the next time horizon and as the balancing window moves in time. Figure 6b presents a solution when a constraint is enforced to close instructions at the end of the time horizon.

E. Testing the model on larger number of BMUs

The primary aim of the proposed optimisation model is to support decision making for redispatching a large fleet of BMUs. As the number of BMUs increases, the cost of meeting power/energy requirement decreases. Figure 7 illustrates a solution using five BMUs, including 3 SLCH and 2 FTEX units, achieving a 22% cost reduction compared to using only FTEX and SLCH.



(a) Optimal solution by modifying the cost curve of FTEX. With this modification FTEX accepts bids



(b) Optimal solution when a constraint is added to close all instructions at the end of the optimisation window.

Fig. 6. The impact of changing price on the optimal solution. With an incentive on accepting bids, an optimal solution may involve energy arbitrage that may leave instructions open at the end of time horizon.

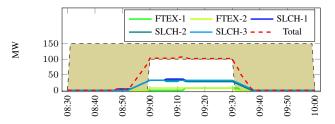


Fig. 7. Using 5 units (2 FTEX and 3 SLCH) to meet the requirement of 100 MW from 09:00 to 09:30. Using these five units results in a 22% reduction in costs of meeting the power requirement as compared to using two units in Fig 3a.

In Figure 8, we present solver times for identifying an initial feasible solution with an increased number of BMUs. While this is an ongoing area of research, our findings indicate that the model can handle hundreds of units while meeting the functional requirements. The reported results are for finding a feasible solution. For application of this model, optimality of a feasible solution is an important consideration. Our initial findings indicates the first feasible solution on average arrives with an optimality gap of 60%, which reduces to 20% within seconds. There are cases which take a long time (in the order of minutes) to reduce the optimality gap to under 10%. We plan to report the findings on run-times in greater detail in a subsequent paper.

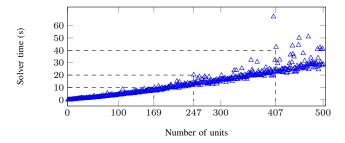


Fig. 8. Solver run times for finding a feasible solution for up to 500 BMUs.

V. REVIEW OF FUNCTIONAL REQUIREMENTS

This paper introduces a subset of requirements for power redispatch, acknowledging the broader set of requirements not covered here. The authors acknowledge limitations stemming from this reduced scope and plan to publish a comprehensive set of requirements and corresponding mathematical formulation separately.

In the model presented, all requirements are applied to units' target level profiles $p_{g,t}$. This means that if units' preoptimisation profiles violate a requirement, the model rectifies the inconsistency without cost incentives. For example, if unit g's minimum flat top time F_g is 5 minutes and all flats that exist in its pre-optimisation profile $P_{g,t}$ are of smaller lengths, optimisation will adjust g's redispatch, either by removing such flats or, indeed, extending them with the aim of ensuring durations of at least five minutes for such extended flats. In practice, inconsistencies in pre-optimisation profiles should be allowed, with requirement constraints applying only during redispatch.

We have assumed single ramp rates in modelling unit's target level dynamics, simplifying the model and deviating from real-world scenarios where ramp rates vary with power output. We have assumed convexity of cost curves, though this assumption may not always hold true. For example, if unit's CCL moves below its CL (due to redeclaration of unit's maximum export limit), there should be zero cost associated with not moving the unit. However, a given redispatch of the unit down (i.e. an instruction to reduce its power) must incur the cost of redispatch from CL to CCL plus the cost associated with the actual redispatch away from CCL. This creates a cost curve with a discontinuity at zero MW (in the cost-versus-MW redispatch space). Another example that would lead to such situation is the case of renewable generation units whose outputs may be following the forecasts and not units' FPNs. Furthermore, in cases of equal merit order among units, a mechanism for tie-breaking is necessary.

Optimisation over finite optimisation intervals means that the application of requirements on such windows should be consistent with units' outputs outside of them. For example, if we find a unit to be in a non-zero MW state at the beginning of the optimisation window, we need to know how long that state lasted for in order to correctly apply the MNZT constraint. Similarly, starting the optimisation window with constant power is not in itself informative on whether such constant power belongs to a flat in CCL or can potentially become a flat due to redispatch; hence, additional information would be required for completeness and in order to know whether an interval of constant power could potentially become a flat and what kind of a flat that could be.

VI. DISCUSSION AND CONCLUSIONS

The paper presents findings of a collaborative research project between the University of Strathclyde and National Grid ESO. We have demonstrated the capability to model a range of Balancing Mechanism (BM) constraints into a

mathematical programming framework. As mentioned in Section I, the model introduced here has since been significantly expanded to accommodate a broader range of needs to support the bulk dispatch capability of NGESO's Open Balancing Platform [13], [14]. The paper serves as an introduction to the host of requirements elicited during the course of the project and the associated MILP model. Future publications will explore these requirements and modelling intricacies in greater detail, along with their resulting performance.

The ever increasing number of electricity assets participating in balancing prompts development of decision support tools that would allow Control Room to efficiently dispatch them in order to maintain demand and generation balance. This requirement has been outlined in [17]. The proposed model directly addresses the pressing need for power balancing. It is important to note that while this formulation covers market constraints, it does not currently encompass transmission constraints, voltage, frequency, or reserve-related constraints. Ongoing research focuses on identifying which additional constraints can be integrated into the model to effectively manage the added complexity.

Our optimisation model hinges on accurate power requirement information, typically provided by experienced Control Room engineers. An ongoing innovation project is addressing the challenge of constructing more precise and adaptive energy requirement profiles by developing predictive models of system frequency.

Another key area of ongoing research centres on the computational performance of the model. The time required to find a feasible solution and the optimality of that solution are critical factors. The trade-offs between these aspects depend on the specific operational needs. For instance, during periods of rapid frequency decline necessitating additional MWs, a high-quality solution that may not be optimal might be acceptable. However, in less time-sensitive scenarios, users may be willing, and indeed prefer to wait longer to obtain the best possible solution (or a range of suboptimal solutions). Research in this area explores methods for warm-starting the optimisation model to enhance efficiency.

Furthermore, our work is confronted with complexities stemming from input data. While BMUs submit ramp rates, the CCL/FPN profiles do not always align with these submitted rates. According to current BM rules, this discrepancy is permissible, and an optimisation model should not attempt to rectify CCL/FPN profiles that deviate from prescribed min flat top times or ramp rates. We intend to address such challenges in our forthcoming publications.

In conclusion, this paper represents a significant step towards addressing the evolving needs of Great Britain's electricity system operator, particularly in the context of Balancing Mechanism market environment and constraints. Our ongoing research will continue to refine and expand upon this model, offering more comprehensive solutions to the complex challenges associated with energy balancing and market operations.

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