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Robust Optimisation of Coordinated Collision Avoidance Manoeuvres in Large Constellations

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This paper presents some preliminary results on the coordinated planning of collision avoidance manoeuvres in two different cases: two or more satellites belonging to the same constellation manoeuvering to avoid the collision with an inert object and two or more satellites belonging to different constellations manoeuvering to avoid a collision. In both cases we consider an uncertainty in the correct calculation of the probability of collision and we plan robust manoeuvres that account for both aleatory and epistemic uncertainty. A Multi-Criteria Decision Making process is then used to select the optimal strategy in both cases
keywords: Mega-constellations, AI, Collision Avoidance Manoeuvre, Robust Optimisation, Uncertainty Quantification,

1. Introduction

The paper explores the case in which multiple satellites in a constellation need to plan and execute a collision avoidance manoeuvre (CAM) in the same time window. The normal approach for a single CAM execution is to avoid a single collision and then implement a minimum number of manoeuvres to recover the orbital position and the overall figure of merit of the constellation (or global performance index).

When multiple satellites need to manoeuvre one can envisage the occurrence of more complicated scenarios: the CAM executed by each satellite needs to avoid more than one collision and avoid collisions with other satellites in the constellation; the uncertainties associated to each of the conjunctions intersect (or satellites need to avoid a cloud of debris); if a number of satellites needs to manoeuvre, a new configuration of the whole constellation, that corresponds to a different value of the performance index, might be more cost effective; members of different constellations need to manoeuvre to avoid a collision and a number of subsequent conjunctions.

In all these scenarios the planning and execution of collision avoidance manoeuvres across the constellations need a level of coordination. More so, if the CAMs are generated autonomously on board each satellite, there is the need to account for post-CAM conjunctions and recovery manoeuvres.

The paper builds upon previous work by the authors on the robust optimisation of CAMs for a single satellite both in the case of single and multiple-encounters.[8,9,12] This body of work led to the development of the CASSANDRA (Computational Agent for Space Situational Awareness aNd Debris Remediation Automation) framework. Within CASSANDRA an Intelligent Decision Support System (IDSS) is coupled with a Robust Bayesian State Estimation[3] and a CAM optimisation modules to make robust decisions on the planing and execution of collision avoidance manoeuvres.[8] The state estimation module computes both the probability of a collision and an associated confidence measure that depends on the uncertainty in the quality of the measurements. The IDSS then makes decision accounting for both the probability of collision and the associated confidence level. If a CAM is scheduled, an optimal solution is computed by solving a robust optimisation problem.

In this paper we propose an extension of this framework under the assumption that some satellites in the constellation need to individually plan and execute and optimal manoeuvre but all the manoeuvres need to be coordinated and need to be optimal in some sense.

The current implementation of CASSANDRA assesses the risk of multiple subsequent conjunctions and the associated confidence interval by merging in a single quantification of uncertainty the uncertainty associated to all the encounters with the same object or with different objects.[8] Either an impulsive or a low-thrust manoeuvre is then planned by solving a min-max problem that reduces the maximum risk of a collision. If it is not possible to reduce this risk below an acceptable level with a single manoeuvre, multiple manoeuvres are implemented, one per conjunction. In this scenario a single satellite is expected to plan all the manoeuvres and all other objects are assumed to be inactive.

In this paper we consider the case in which the two or more satellites need to plan a manoeuvre but the decision has to be coordinated either in the case

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the satellites belong to the same constellation (collaborative scenario) or belong to different constellations (competitive scenario). In the former case the assumption is that the cost of the execution of a manoeuvre affects a global performance index for the whole constellation. This scenario can be formulated as a classic Multi-Criteria Decision Making (MCDM) problem where each manoeuvre is the solution of a min-max problem.[13] On the other hand, if the members of two constellations need to manoeuvre then the problem becomes a game theoretic one in which each satellite needs to minimise its cost and risk without knowing what the other satellites is going to do. This second scenario extends the standard MCDM to incorporate multiple decision makers with conflicting goals.

2. Robust CAM Optimisation

This section presents the approach to calculate an optimal CAM and decide on its execution. The approach was developed in[9] and is based on a linear model proposed in[18] to compute the impulsive manoeuvre to achieve the desired variation of the relative position between satellite (chaser) and debris (target), which experience a close encounter (Fig. 1a).

Given a manoeuvre $\delta\mathbf{v}$, at time t_m, expressed in a spacecraft centred tangential, normal, out-of-plane reference frame, <T,N,H>(Fig. 1c), the corresponding variation of position $\delta\mathbf{x}_b$ at time t_c on the impact plane (or b-plane) of a piece of debris, $<\xi, \eta, \zeta>$, is:

$$\delta\mathbf{x}_b = [\delta\xi \;\; \delta\eta \;\; \delta\zeta]^T = \mathbf{T}\delta\mathbf{v} = \mathbf{B}\mathbf{A}(t_m, t_c)\mathbf{G}\delta\mathbf{v} \quad [1]$$

where matrix $\mathbf{T}$ is the product of three matrices: i) $\mathbf{G}$ relating the instantaneous change in orbital parameters due to the change in velocity at the time of execution of the manoeuvre, ii) $\mathbf{A}(t_m, t_c)$ being the transition matrix between the variation of the Keplerian elements at manoeuvre, t_m, and the variation of the relative position at the time of expected collision, t_c, expressed in a radial, transversal, out-of-plane reference frame, <R,T,H>, iii) and $\mathbf{B}$ the rotation matrix between <R,T,H> and the impact plane reference frames. More details can be found in[9] and.[17]

The direction of the impulse can be defined by the two angles $\phi \in [-\pi, \pi]$ and $\psi \in [-\pi/2, \pi/2]$ (see Fig. 1c). The relation between $[\phi, \psi]$ and the components on the <T,N,H > is:

$$\begin{array}{l} \phi = \arctan\left((\mathbf{v}_1 \times \delta\mathbf{v}) \cdot \mathbf{u}_{1h}, \mathbf{v}_1 \cdot \delta\mathbf{v}\right) \\ \psi = \tan^{-1}\left[\frac{(\delta\mathbf{v}\cdot\mathbf{u}_{1h})\|\delta\mathbf{v}\times\mathbf{u}_{1h}\|}{\delta v^2 - (\delta\mathbf{v}\cdot\mathbf{u}_{1h})^2}\right] \end{array}, \quad [2]$$

with $\mathbf{u}_{1h}$ the normal to the primary object's orbital plane.

Note that the model in Eq. (1) assumes Keplerian motion. The orbital parameters and positions included in the expressions of the matrices refer to the manoeuvrable satellite (the chaser) while the impact plane reference frame $<\hat{\xi}, \hat{\eta}, \hat{\zeta}>$ is centred at the secondary object (the target), and is defined as:

$$\hat{\boldsymbol{\eta}} = \frac{\mathbf{v}_1 - \mathbf{v}_2}{\|\mathbf{v}_1 - \mathbf{v}_2\|}, \quad \hat{\boldsymbol{\xi}} = \frac{\mathbf{v}_2 \times \boldsymbol{\eta}}{\|\mathbf{v}_2 \times \boldsymbol{\eta}\|}, \quad \hat{\boldsymbol{\zeta}} = \hat{\boldsymbol{\xi}} \times \hat{\boldsymbol{\eta}} \quad [3]$$

where $\mathbf{v}_1$ is the velocity vector of the chaser and $\mathbf{v}_2$ the velocity vector of the target (see Fig. 1b).

Taking $\mathbf{r}_{e_0} = [\xi_0, 0, \zeta_0]^T$, the initial unmodified relative position between both objects projected on the impact plane defined above, with combined covariance matrix $\mathbf{\Sigma}$, the optimal CAM can be computed so that the Probability of Collision (PoC) is minimised. The combined covariance matrix is the sum of both object's position covariance matrices projected on the impact plane of the target: $\mathbf{\Sigma} = \mathbf{\Sigma}_{1_{Bp}} + \mathbf{\Sigma}_{2_{Bp}}$.

We assume that the manoeuvre introduces a negligible uncertainty and that it only translates rigidly the uncertain ellipse on the impact plane, defined by the combined covariance matrices, not modifying its shape, size or orientation. This means that the relative position of the modified orbit will present the same covariance matrix $\mathbf{\Sigma}$, with miss distance equal to $\mathbf{r}_e = \mathbf{r}_{e_0} + \delta\mathbf{x}_b$, and $\delta\mathbf{x}_b$ given by Eq. (1). Furthermore, we introduce the short-encounter assumptions defined in:[10] i) rectilinear relative trajectories, ii) no uncertainty in the velocity vector, iii) the uncertainty in the position of the two objects is Gaussian and uncorrelated, iv) and the shape of the two objects is spherical. Under these conditions, the PoC computation can be approximated as the 2D integral:

$$P_C = \frac{1}{2\pi\sqrt{\|\overline{\mathbf{\Sigma}}\|}} \int_{\mathcal{B}((0,0),R)} e^{-\frac{1}{2}\left((\mathbf{b}-\overline{\mathbf{r}}_e)^T \overline{\mathbf{\Sigma}}^{-1} (\mathbf{b}-\overline{\mathbf{r}}_e)\right)} d\xi d\zeta \quad [4]$$

where $\mathbf{b} = [\xi, \zeta]^T$, the two component vector $\overline{\mathbf{r}}_e$ is equal to the first and third components of $\mathbf{r}_e$, and $\overline{\mathbf{\Sigma}}$ is a 2×2 matrix equal to the first and third elements of the first and third rows of $\mathbf{\Sigma}$:

$$\overline{\mathbf{\Sigma}} = \begin{bmatrix} \sigma_\xi^2 & \sigma_{\xi\zeta} \\ \sigma_{\xi\zeta} & \sigma_\zeta^2 \end{bmatrix},$$

Then, in order to minimise the PoC, one needs to solve the following maximisation problem:

$$\begin{array}{l} \max_{\delta\mathbf{v}} \left(\delta\mathbf{v}^T \mathcal{T} \delta\mathbf{v} + 2\mathbf{r}_{e_0}\mathbf{\Sigma}^{-1}\mathbf{Q}\mathbf{T}\delta\mathbf{v}\right) \\ s.t. \\ \|\delta\mathbf{v}\| \leq \delta v_{max} \end{array}, \quad [5]$$

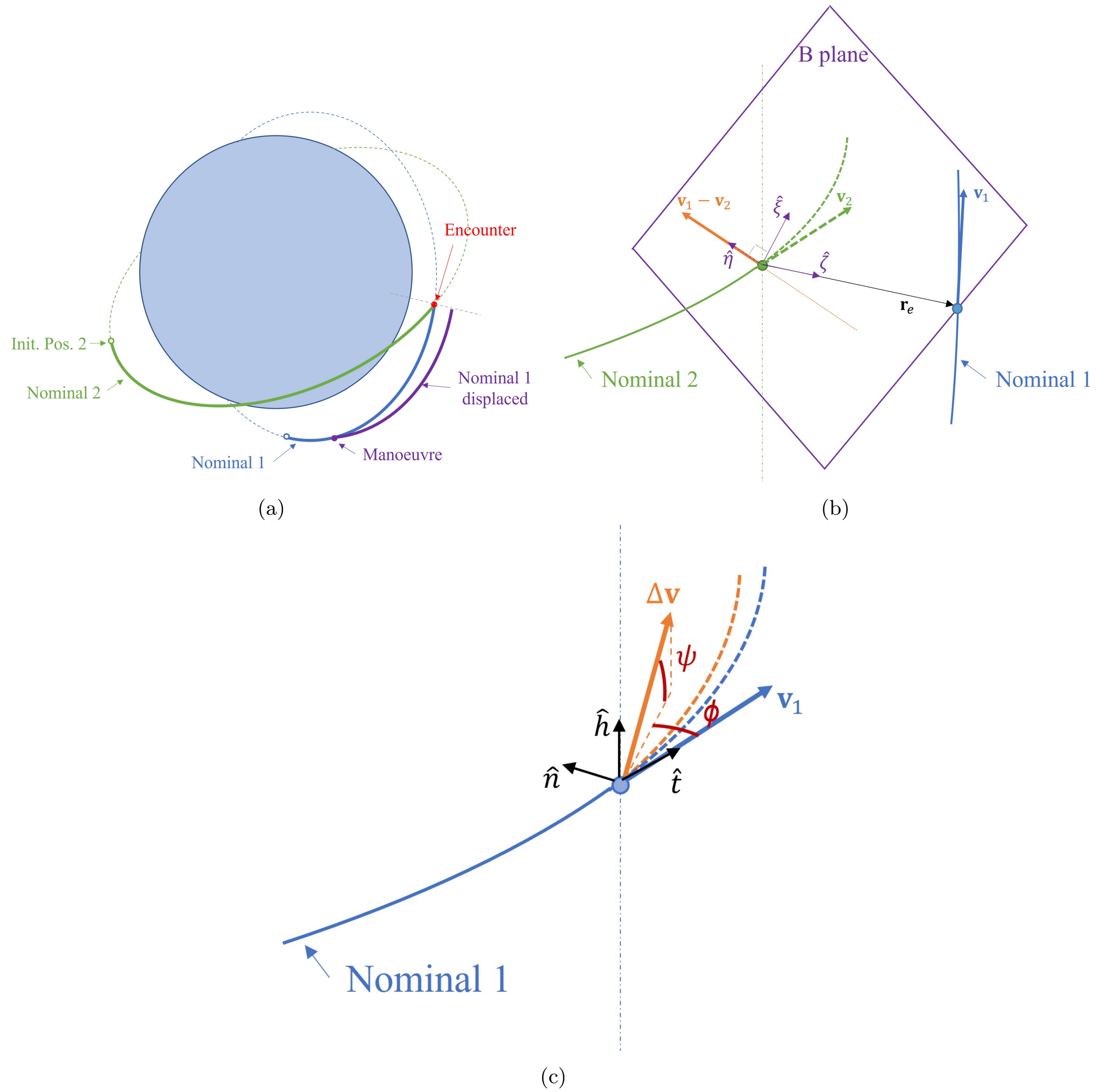


Fig. 1: (a) General configuration of the encounter. Blue: nominal primary satellite orbit, green: nominal secondary satellite orbit, purple: manoeuvre and nominal primary orbit after CAM. Red: encounter position. The encounter and manoeuvre positions are detailed on the subsequent figures. (b) Encounter configuration. Blue: primary satellite (trajectory and orbital velocity, $\mathbf{v}_1$), green: secondary satellite (trajectory and orbital velocity, $\mathbf{v}_2$), black: miss distance ($\mathbf{r}_e$), orange: relative velocity, purple: impact plane and its reference frame $< \hat{\xi}, \hat{\eta}, \hat{\zeta} >$. (c) Manoeuvre position. Blue: primary orbit (trajectory and orbital velocity, $\mathbf{v}_1$), orange: impulsive manoeuvre ($\Delta\mathbf{v}$), red: angles between orbital velocity and impulse (ϕ in plane, ψ out of plane), black: primary object's <T,N,H> reference frame.

where $\mathcal{T} = \mathbf{Q}\mathbf{T}^T\mathbf{\Sigma}^{-1}\mathbf{T}\mathbf{Q}$ and

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In the following, however, we first fix the magnitude of the manoeuvre δv_0 and solve the simplified problem:

$$\begin{array}{l} \max_{\delta\mathbf{v}} \left(\delta\mathbf{v}^T \mathcal{T} \delta\mathbf{v}\right) \\ s.t. \\ \mathbf{r}_e \cdot \mathbf{T}\delta\mathbf{v} > 0 \end{array}, \quad [6]$$

whose solution $\delta\mathbf{v}_{opt}$ is the vector parallel to the eigenvector, $\mathbf{s_1}$, conjugate to the maximum eigenvalue of the matrix $\mathcal{T}$ with magnitude δv_0:

$$\delta\mathbf{v}_{opt} = \delta v_0 \mathbf{s_1} \quad [7]$$

2.1 *Computation of the Probability of Collision under Epistemic Uncertainty*

When the covariance matrix $\mathbf{\Sigma}$ or the miss distance are not precisely known the PoC is computed with a degree of uncertainty. This epistemic uncertainty in the covariance and miss distance can be due to multiple sources of information providing different values of the covariance or can come from a poor knowledge of the measurements or propagation model. As presented in[12] this epistemic uncertainty can be modelled with Dempster-Shafer theory of evidence (DSt).[11]

The idea proposed in,[12] is to use DSt to compute the level of confidence in the correctness of the value of the PoC, given the available evidence on the sources of information. Each component of the covariance, $[\sigma_\xi^2, \sigma_\zeta^2, \sigma_{\xi\zeta}]$, is modelled with one or more intervals and so is the mean value of the relative position $[\mu_\xi, \mu_\zeta]$. A basic probability assumption (bpa) is associated to each interval. Intervals and associated (bpa) can be derived, for example, from a time series of Conjunction Data Messages (CDMs) or directly from the raw observations.

Given the intervals and associated (bpa), we compute the cross product of all the intervals, under the assumption of epistemic independence. Each product of intervals is a Focal Element (FE), and the associated (bpa) is the product of the bpas of the individual intervals. With the focal elements we can compute the *Belief* (lower probability) and *Plausibility* (upper probability) that the PoC associated to a given conjunction event is correct. In the following we call the uncertainty space, U, and the uncertain parameter vector, $\mathbf{u} = [\mu_\xi, \mu_\zeta, \sigma_\xi^2, \sigma_\zeta^2, \sigma_{\xi\zeta}]^T$. We then want to compute the *Belief* and *Plausibility* of $\Phi = \{P_C | P_C \geq \overline{P_{C0}}\}$ where $\overline{P_{C0}}$ is a desirable value. For more information please refer to.[12]

2.2 *The Min-Max CAM Optimisation Problem*

When epistemic uncertainty is considered, the PoC is not defined by a single ellipsoid but by families of ellipsoids corresponding to families of covariances and mean values. Thus, instead of having a single uncertain ellipse on the impact plane one has to consider families of uncertain ellipses each of which has to be displaced by a manoeuvre. This means that an optimal and robust manoeuvre has to displace all the ellipses at once.

Fig. 2 presents this situation where two families of uncertain ellipses (red and green) are shown on the impact plane, against a single ellipse (blue) obtained by combining the information from both sources and considering solely aleatory uncertainty.

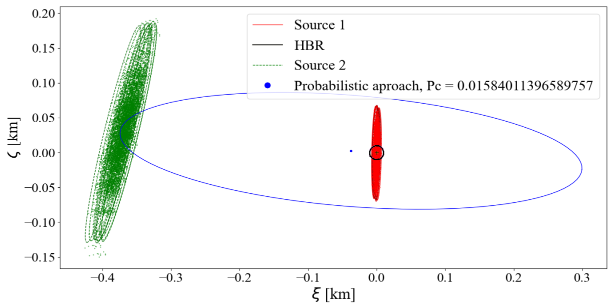


Fig. 2: Encounter geometry accounting for epistemic uncertainty under DSt. Two sources of information are considered. Aleatory and epistemic uncertainty is included. Source 1 (red) is nine times more reliable than Source 2 (green). The maximum and minimum PoC associated to each family of ellipses is: [0.3356, 0.3615] for Source 1 and [0.0, 0.0] for Source 2. The Hard Body Radius (HBR), is shown in black, and equal to 5m. The uncertain ellipse combining information form both sources is shown in blue, with $P_C = 0.00158$.

The presence of families of ellipses means that the optimal CAM has to be able to minimised the PoC corresponding to the worst-case ellipse, which is the uncertain ellipse leading to the highest value of the PoC. Thus one has to solve the the following min-max problem:

$$\begin{array}{l} \min_{\delta\mathbf{v}} \max_{\mathbf{u}\in\Omega_u} P_C \\ s.t. \\ \mathbf{r}_e \cdot \delta\mathbf{v} > 0 \end{array}, \quad [8]$$

which has to be solved over the whole set of FEs.

Similarly to what proposed in Filippi[1] for the solution of general min-max optimisation problems, we propose the following iterative process.

First we compute for each FE the value of the uncertainty vector that gives the highest PoC. We then build the matrix $\mathbf{S} = \mathbf{\Sigma_1^{-1}} + \mathbf{\Sigma_2^{-1}} + ...$ given by the sum of all the worst-case ellipses for all FEs. From $\mathbf{S}$ we compute $\mathcal{T}$ and then use Eq. (7) to compute the manoeuvre.

Since the implementation of a manoeuvre displaces all the ellipses, the process has to be repeated until there is no variation of the PoC.

Up to this point, the optimisation of the manoeuvre assumed a constant magnitude, δv_0. However, this can lead to situations where the correction of the orbit is overestimated, with a reduction of the PoC several orders of magnitude below the minimum safety threshold. In such situation, a smaller impulse could reduce the risk to acceptable levels without an excessive cost of the manoeuvre.

Assuming the maximum capacity of the thruster is defined by δv_{max}, the optimum value of the magnitude will be the minimum one that allows reducing the worst-case scenario PoC below the selected threshold, P_{C0}:

$$\begin{aligned} &\min \ \|\delta \mathbf{v}\| \\ &\text{s.t. } \ P_C < P_{C0} \ , \end{aligned} \qquad [9]$$

Once the optimal direction is computed with Eq. (7), the minimum δv can be simply derived from the solution of problem (9). However, since the magnitude of the impulse affects the deflection of the orbit and, subsequently, the worst-case scenario, the optimal magnitude computation has to be integrated within the min-max optimisation algorithm.

In summary: an outer loop computes the manoeuvre direction at constant magnitude, after computing the optimal direction, $\mathbf{d}_{opt}$ with Eq. (7), the new wort-case ellipse is computed, if $P_C < P_{C0}$ an inner loop reduces the magnitude of the impulse, with a simple bisection method, till $P_C = P_{C0}$, recalculating the worst-case ellipse at each iteration of the inner loop.

2.3 *Multi-encounter CAM Optimisation*

A multiple encounter is a series of successive close conjunctions between one satellite and one or more space objects, whether operational satellites or pieces of space debris. In CASSANDRA we considered the case in which a given CAM has to ensure the reduction of the risk of collision at all conjunctions in a given sequence.

In this case a single manoeuvre is optimised by solving problem Eq. (8) but with Ω_u the space of the epistemic parameters of all the encounters and $P_{C,max} = \max_i P_{C,i}$ the maximum probability of collision among all the encounters. Thus the worst case P_C is optimised over all the families of covariance of all the encounters in a single min-max optimisation loop.

3. MCDM Methodology

In the previous section we presented a methodology to compute robust CAM in the case of single or multiple-encounters. The outcome of this methodology is a number of possible alternative CAMs. Once multiple alternative CAMs are available a decision is required to execute the best CAM with respect to existing operational constraint and other optimally criteria on top of cost and risk. The decision depends on the number of players, or agents, that are responsible to make that decision.

When all agents involved in a decision-making process are known to collaborate, that is, will agree on the decision to be taken to achieve a common goal, the problem can be addressed with Multi-Criteria Decision-Making (MCDM) methods. This is the case when two or more satellites belonging to the same constellation have to manoeuvre within the same time frame maintaining the figure of merit of the constellation.

MCDM is a branch of decision making which provides a compromise solution (in the form of a sorted list) of alternatives evaluated across a set of, usually contradictory, criteria.[15] Hence, there are three type of parameters that should be defined in order to apply the different MCDM methods: the alternatives, the criteria and the weight of the criteria:

- Alternatives are all possibles responses to a given problem built by combining the different answers of all agents.
- Criteria, are quantities used to evaluate the suitability of the alternatives to optimise different aspects related with the problem: risk, cost... They can be beneficial, if they want to be maximised, or non-beneficial, when they want to be minimised.
- Weights, are associated to criteria and are used to assign more importance to certain criteria than others, enabling to address the problem from different perspectives. From a multi-objective point of view weights are used to scalarise the vector of decision criteria and allow a partial ranking of the alternatives.

In this paper, we proposed a method to prioritise the avoidance manoeuvre strategy alternatives for satellites belonging to the same constellation according to a number of criteria.

The different alternatives are defined by the following parameters: the CAM execution position for each satellite, θ_{mi}, and the strategy followed after the CAM after the encounter, s_i: not performing any further action and remaining in the new orbit (strategy 0), execute one manoeuvre to return to the original orbit (strategy 1), or execute two manoeuvres to return to the original position within the original orbit, keeping the relative phase with the other satellites in the same orbital plane.

Once the alternatives are defined, the next step is to define the criteria to evaluate the alternatives. We propose 4 criteria with information of all satellites involved in all events, and other 2 criteria per each encounter. Thus, in the event of two encounters, it is a total of 8 criteria. They are detailed below:

- Total number of manoeuvres, including all satellites. This quantifies the inherent risk of executing a manoeuvring, assuming that the higher the number of manoeuvres, the higher the risk.

- Total cost of the manoeuvre, measured as the sum of all δv, both in the CAM and in the restoring strategy:

$$\delta v_q = \Sigma_i(\delta v_i + \Sigma_k \delta v_{ik}^{-1}), \qquad [10]$$

  where k the recovery strategy, i the encounter, and q the alternative. Note, if Low-Thrust is considering, the manoeuvre cost is measured as the equivalent δv.[13]

- The total manoeuvre cost of each individual satellite: $\delta v_{qi} = \delta v_i + \Sigma_k \delta v_{ik}^{-1}$. This criteria will appear as many times as satellites in the constellation executing at least one manoeuvre. This criteria is included to quantify possible operational constrains as it can be the limitation of manoeuvres in a single satellite due to fuel shortage (i.e at the end of life). It can be tuned according to the necessities by the appropriate weight.

- Total time of the operation, measured as the time between the first manoeuvres executed by any satellite and the last manoeuvre executed by any satellite. This criterion quantifies the cost of the operation, assuming it is directly proportional to the time of operation: the longer the time, the longer the operators have to put specific resources to the operation.

- For each encounter, the reduction of probability of collision due to the CAM, quantified as:[8]

$$\begin{cases} \text{if } P_{Ci} \geq P_{Ci}^* & RPC_{qi} = 0, \\ \text{if } P_{Ci} \leq P_{C0} & RPC_{qi} = 1, \\ \quad \text{else} & RPC_{qi} = \left(\frac{log(P_{Ci}) - log(P_{Ci}^*)}{log(P_{C0}) - log(P_{Ci}^*)} \right)^{16} \end{cases}, \qquad [11]$$

  where P_{Ci}^* is the probability of collision for encounter i if no CAM is executed, and P_{C0} is the constellation probability of collision threshold.

- Constellation's figure of merit. In this paper, this quantity has been modeled based on the total area covered by the constellation assuming sensors mounted on each satellite and pointing to Nadir. The figure of merit is defined as the integral over the considered period of time of the deviation of the total area covered by the constellation with respect the nominal configuration:

$$CFM_q = \int_{t_0}^{t_f} \|A_q(t) - A_0(t)\| \, dt, \qquad [12]$$

  where $A_q(t)$ is the area covered at instant t by the constellation in alternative q, and $A_0(t)$ the area covered by the constellation at the same time if no manoeuvres are executed. Note that the area is made dimensionless by normalising with respect to the Earth's surface. Fig. 3 includes $\|A_q(t) - A_0(t)\|$ for two examples: one where all satellites return to the original position and another where none does.

Having defined the alternatives and criteria, it is possible to build the Alternative-Criteria matrix:

$$\mathbf{T} = \begin{bmatrix} t_{11} & t_{12} & ... & t_{1N} \\ t_{21} & t_{22} & ... & t_{2N} \\ ... & ... & ... & ... \\ t_{M1} & t_{M2} & ... & t_{MN} \end{bmatrix}, \qquad [13]$$

where t_{ij} is the value of alternative i under criteria j.

However, most of the MCDM methods require this matrix to be normalised column-wise and assign all criteria to same categories: beneficial or non-beneficial. There are different normalisation techniques that allows to build a Normalised Alternative-Criteria matrix with all criteria beneficial or non-beneficial. In this paper, we have implemented two of them:[16]

- Linear normalisation, which scales values between 1 and 0, assigning always 1 to the best alternative within each criteria and scaling the rest alternatives accordingly.

$$\begin{cases} \text{if } Beneficial & \bar{t}_{ij} = t_{ij}/\max_i(t_{ij}) \\ \text{if } No\ beneficial & \bar{t}_{ij} = 1 - t_{ij}/\max_i(t_{ij}) \end{cases} \qquad [14]$$

- Vector normalisation, which also scales values between 1 and 0, but uses the magnitude of the "vector" of alternative under each criteria:

$$\begin{cases} \text{if } Beneficial & \bar{t}_{ij} = t_{ij}/\sqrt{\Sigma_{i=1}^{M} t_{ij}^2} \\ \text{if } No\ beneficial & \bar{t}_{ij} = 1 - t_{ij}/\sqrt{\Sigma_{i=1}^{M} t_{ij}^2} \end{cases} \qquad [15]$$

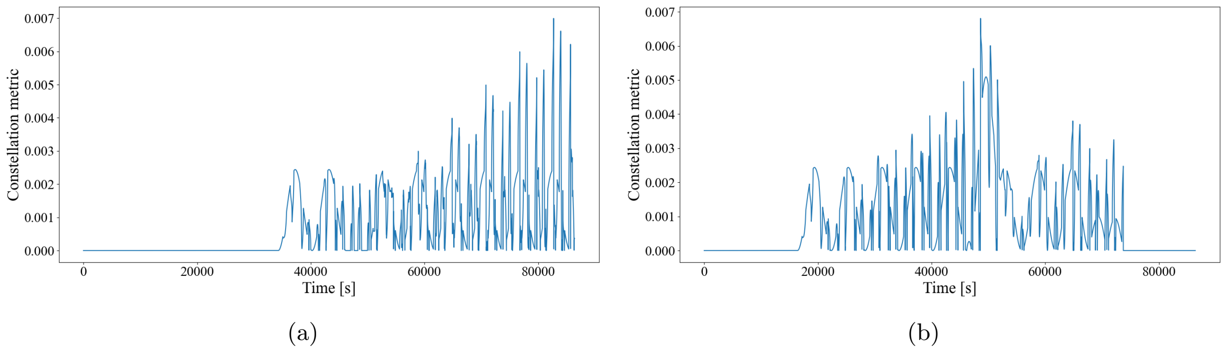


Fig. 3: Absolute difference, as a function of time, of area covered by constellation in alternative one and constellation with no manoeuvres: $\|A_q(t) - A_0(t)\|$. a) No returning manoeuvre for any satellite, b) both satellites returning to position.

Note that different normalisations may lead to different rankings of the alternatives.

The weights are the final parameters to be defined in a MCDM problem. The distribution of weights completely depends on the problem and the specific situation. Then only constraint on the distribution is that the sum of all the weights has to be 1. In the problem treated in this paper, it would be the operator who decides this distribution, assigning a higher relative weight to those criteria that are desired to be more relevant, i.e. if the impact of loosing a satellite is high (small number of satellites in a constellation, little number of back up satellites or huge impact of fragment in orbit), the criterion measuring the encounter risk may be assigned a higher weight; or if one of the satellites has little remaining propellant, the criteria measuring the cost of the manoeuvre can be assigned a higher weight. More about weight selection will be presented in Section 4. Note that changing weights means converging to potentially alternative Pareto optimal solutions. A linear combination of criteria implicitly implies convexity of the Pareto front, and assumption that is not always satisfied.

With all the MCDM parameters defined (normalised Alternative-Criteria matrix and weight distribution), it is possible to obtain a raking of the alternatives. Different methods allows one to rank alternatives.[4] In this paper we have implemented four methods: weighted sum method (WSM),[6] a modified WSM (MWSM),[4] weighted product method (WPM),[14] and TOPSIS (Technique for the Order of Preference by Similarity to the Ideal Solution).[2]

Each combination of normalisation and MCDM methods may lead to a different ranking of the alternatives, which mean, we can have up to 8 different best alternatives. We propose also a method to combine the information provided by each combination. An aspect that is taken into account is the fact that an alternative appearing among the top 3 in all rankings may be a better alternative than one that appears first in one ranking and does not appear again in the top positions in the other rankings. Thus, we count how many times each alternative appears in the g_{top} top values of all the rankings, being the alternatives sorted according to the number of appearances. Note that the number g_{top}, may influence the final combined ranking.

3.1 Game Theoretic MCDM

When the agents are not expected to collaborate, but rather they are competitors, the outcome of the decision may not be the optimal one for every agent. This is the case, for example, when the encounter involves to satellites operated by different operators.

In this situation, the method proposed before would not be adequate to find an optimal strategy for each of the operators. Thus, the problem has to be reformulated as a game theoretic one where each satellite has to optimise their own criteria without necessarily know the action taken by the other satellite. The problem of two manoeuvrable satellites avoiding the same encounter can be formulated as a Multi-Criteria Multi-Decision Makers (MCMDM) problem, where several agents can choose among different alternatives conditional the alternatives of the other agents.[5] While in the collaborative case, optimal decisions were derived from the Alternative-Criteria matrix, in a game theoretic problem, decision are derived from the Game Matrix. From the game matrix one can assess which are the best outcomes of a given strategy. This Matrix quantifies the value of an alternative for one agent, given the alternatives chosen by the other agents. In the following, we summarise the method to obtain this matrix.

As before, the first step is to obtain the alternatives and define the criteria for each agent. For the rest of the paper, without loss of generality, we are going to assume only two agents, that is, two satellites involved in the encounter. For each of the agents, the alternatives are obtained by changing the CAM execution position (and associated magnitude): $\theta_{m1} \in \Theta_{m1}$ and $\theta_{m2} \in \Theta_{m2}$, with $\Theta_{m1} \neq \Theta_{m2}$ in general.

Each agent can define its own criteria. For the scope of this paper, but without loosing generality, both agents will consider the same two criteria: risk reduction and cost of the manoeuvre, defined as before. Although the criteria may be the same, the parameters defining the manoeuvres (magnitude of the impulse) or the risk (PoC threshold) can be different for each agent.

Thus, an Alternative-Criteria matrix as a function of the other agent's answer can be defined for each agent:

$$\mathbf{A} = \begin{bmatrix} a_{111} & \dots & a_{1C1} & a_{112} & \dots & a_{1C2} & \dots \\ a_{211} & \dots & a_{2C1} & a_{212} & \dots & a_{2C2} & \dots \\ \dots & \dots & \dots & \dots & & & \\ a_{A11} & \dots & a_{AC1} & a_{A12} & \dots & a_{AC2} & \dots \end{bmatrix}, \quad [16]$$

where a_{ijk} is the value of alternative i of decision-maker A under criteria j, given decision-maker B chooses its alternative k. A similar matrix, $\mathbf{B}$, is build for the other decision-maker.

In order to build the Game matrix, it is necessary to integrate the criteria of the alternatives, given the other agent's answer.[19] We extend the approach propose in the previous section: each "submatrix" (given the other's agent alternative) is an Alternative-Criteria matrix exactly as the one obtained in the collaborative case. Thus, for each submatrix of each agent, we can apply the method explained in the collaborative case to either evaluate each alternative considering all the criteria or to rank the alternatives (if we want to build the ordinal Game matrix[5]):

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1N} \\ c_{21} & c_{22} & \dots & c_{2N} \\ \dots & \dots & \dots & \dots \\ c_{M1} & c_{M2} & \dots & c_{MN} \end{bmatrix}, \quad [17]$$

where c_{ij} is either the preferred order or the value (weather building or not the ordinal matrix) of the i^{th} alternative of agent A, having integrated all its criteria, with B choosing its alternative j. Similarly, a matrix $\mathbf{D}$ can be obtained for the other decision-makers. Combining the both integrated Alternative-Criteria matrices, $\mathbf{C}$ and $\mathbf{D}$, the Game matrix can be easily obtained:

$$\mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1N} \\ z_{21} & z_{22} & \dots & z_{2N} \\ \dots & \dots & \dots & \dots \\ z_{M1} & z_{M2} & \dots & z_{MN} \end{bmatrix}, \quad [18]$$

where $z_{ij} = (c_{ij}, d_{ij})$

From this matrix, it is possible to apply different stability definitions to find those outcomes (combination of alternatives of each agent) that are optimal for both agents. In this paper, we have implemented the Nash Stability.[7] According to this method, the equilibrium points in the matrix, if any, are those where, given the other agent's alternatives, moving to another alternative implies a loss:

$$z_{ij} \equiv \text{equilibrium point} \iff \begin{cases} c_{nm} = max_i(c_{im}) \\ d_{nm} = max_j(d_{nj}) \end{cases}.$$

Since different rankings can be obtained by different combinations of normalisation techniques (linear or vector normalisation) and MCDM methods (WSM, MWSM, WPM, TOPSIS), the Nash Stability is computed for each of them, with the preferred outcome being the one that appears as an equilibrium point more frequently. The final result is presented with a matrix NE:

$$\mathbf{NE} = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1N} \\ n_{21} & n_{22} & \dots & n_{2N} \\ \dots & \dots & \dots & \dots \\ n_{M1} & n_{M2} & \dots & n_{MN} \end{bmatrix}, \quad [19]$$

where n_{ij} indicates the total number of times the outcome conformed by the i^{th} alternative A and the j^{th} alternative B has been identified as an equilibrium point after combining criteria with the different combinations normalisation techniques and MCDM methods.

4. Case Studies

In this section, we present two examples to show the decision-making process in the collaborative and in the competitive case.

4.1 *Collaborative CAM Planning*

This example shows the case where two satellites belonging to the same constellation faces an encounter with an external object in the same time interval. The satellite are part of a Walker constellations: 60:18/3/1, with semimajor axis (SMA) of 7100 km and circular orbit. In order to compute the constellation metric, each satellite is assumed to be equipped with a 60 deg aperture Nadir-pointing sensor. The piece of debris' orbital parameters at the initial time, $t_0 = 0s$ are:

$$kep_2(t_0) = [6944.26, 0.031016, 141.04, 335.27, 116.15, 47.03], \quad [20]$$

in km and deg. Keplerian motion is assumed for all objects. The time interval of analysis is 1 day ($t_f = 86400s$). The first encounter takes place at $t_1 = 43200s$ after the initial time, and the second encounter, involving a satellite in a different orbital plane, at $t_2 = 64800s$ after t_0.

The satellites of the constellation are assumed to be perfectly known. The state vector of the external object is affected by aleatory uncertainty, expressed in the object's <R,T,H>reference frame at t_0:

$$\mathbf{\Sigma}_{2,rth}(t_0) = \begin{bmatrix} 0.104^2 & 0 & 0 \\ 0 & 0.556^2 & 0 \\ 0 & 0 & 0.139^2 \end{bmatrix} km^2.$$

Assuming two sources (a and b) provides information about the object, there are two components of epistemic uncertainty. It is quantified through an epistemic parameter, given as intervals for the miss distance and covariance, that scale up and down the value of the aleatory uncertainty,[3] meaning the uncertainty ellipse in the impact plane becomes a family of ellipses per source of information. In this example, only the position covariance is affected by epistemic uncertainty:

$$\boldsymbol{\lambda}_{\sigma,a} = [1, 4]$$
$$\boldsymbol{\lambda}_{\sigma,b} = [1/5, 1/2]$$

Thus, the uncertain geometry of the encounters is defined by the following two families of distribution, defined by the intervals in Table 1.

The probability of collision is computed assuming the short-term encounter hypothesis,[10] using Eq. (4), with an $HBR = 10$ m.

For each of the satellites involved in a close encounter, the robust optimal CAM is computed at 6 different positions. Including the alternative of not executing a CAM by one of the satellites, the space of CAM execution position (measured as distance to the encounter) for both objects is $\Theta_m = \{0, p\pi\}$ rads, with $p = 1, 3, .., 11$. The magnitude of the impulse has been set fixed an equal to $dv = 10$ cm/s and the constellation PoC threshold is $P_{c0} = 10^{-6}$.

Three recover strategies has been considered: s_0) no execute any action after the encounter, $dv^{-1} = 0$, s_1) executing one manoeuvre half a revolution after the encounter to return to the original orbit, and s_2) execute one manoeuvre half an orbit after the encounter to enter in a phasing orbit and another manoeuvre after on period to recover the phase in the initial orbital plane, $dv^{-1} = dv_1^{-1} + dv_2^{-1}$.

For each manoeuvre position, there are three returning strategies, except for $\theta_m = 0$ that accepts only one strategy (s_0), which makes 22 options per satellite. Thus, for each first encounter alternative there are 22 options for the second encounter, making a total of 484 alternatives. Each alternative can be identified according to Table 2 or with:

$$N = N_1(M_2 \cdot S_2 + 1) + N_2$$
$$N_i = \begin{cases} \text{if } n_i = 0 \rightarrow N_i = 0 \\ \text{else } N_i = (n_i - 1)S_i + si_j + 1 \end{cases}, \quad [21]$$

where N_i is the number of the alternative, n_i is the argument of θ_{mi} in Θ_{mi} for encounter "i", si_j is the j^{th} returning strategy for encounter "i", S_i is the total number of returning strategies considered for encounter "i", and M_i is the number of CAM execution positions in Θ_{mi}.

Fig. 4 shows the behaviour of the different parameters used in the criteria to evaluate the alternatives as a function of the alternatives.

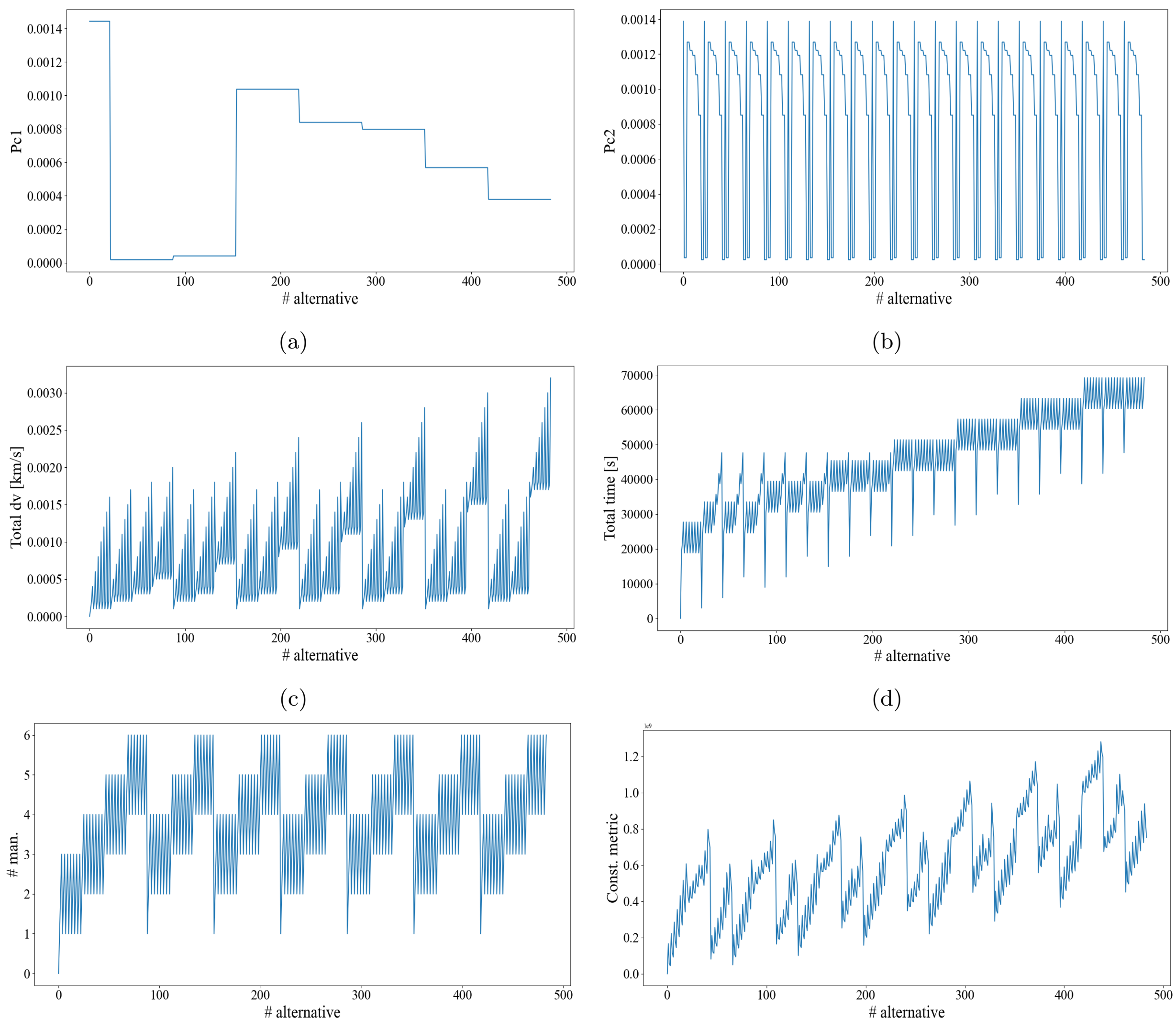
Eight different subscenarios has been studied. Each of those scenarios gives more importance to one or more criteria than the others, by assigning different relative weights Table 4. For each subscenario, each possible ranking has been obtained by combining the results obtained by the combination of two normalisation techniques (linear and vector normalisation) and four MCDM methods (WSM, MWSM, WPM, TOPSIS) with $g_{top} = 5$, as explained in Section 3. Table 3 includes the ranking of *Subscenario 0* for each combination of MCDM method and normalisation technique before and after combination with $g_{top} = 5$. The final ranking with the 5 better alternatives for each subscenario is presented in Table 5.

- *Subscenario 0: Criteria equally weighted*. Alternative most similar to *Subscenario 1*, where PoC is preferred, but with elements rank in the top positions also belonging to other scenarios where other criteria are prioritised. Basically, it is an equilibrium of criteria, not giving as much information as the rest of the subscenarios. The preferred alternatives corresponds to situations where the first satellite smanoevures 0.5 revolution before the encounter ($\theta_{m1} = 0.5$ rad) without returning, $s1 = 0$ and the second satellite at $\theta_{m2} = 6.5$ rad, which the second satellite no returning, $s2 = 0$ (alternative 42) or returning also to the original orbit, $s2 = 1$ (alternative 43). Same alternatives, but with first satellite returning to the original orbit are also well ranked (alternatives 63 and 64).

- *Subscenario 1: Importance given to PoC*. The preferred options are a combination of late ma-

Table 1: Uncertain encounter geometry for both encounters with 2 sources providing information. Upper and lower bound of the components of miss distance and covariance matrix in the impact plane.

	Encounter 1		Encounter 2	
Component	Source 1	Source 2	Source 1	Source 2
μ_ξ (km)	[0.02855,0.08342]	[$1.263\cdot10^{-3}$,0.01529]	[-0.1065,$5.970\cdot10^{-3}$]	[$-7.415\cdot10^{-3}$,$8.664\cdot10^{-3}$]
μ_ζ (km)	[-0.6327,0.7230]	[-0.2317,0.2542]	[-0.7490,0.6741]	[-0.2729,0.2292]
σ_ξ^2 (km^2)	[0.1208,0.4353]	[0.02530,0.05332]	[0.4547,1.2045]	[0.07724,0.1487]
σ_ζ^2 (km^2)	[140.265,505.054]	[29.311,61.202]	[154.274,560.924]	[32.537,68.346]
$\sigma_{\xi\zeta}$ (km^2)	[-14.586,-4.0744]	[-1.8012,-0.8572]	[3.0797,10.613]	[0.6213,1.3313]



(a) (b) (c) (d) (e) (f)

Fig. 4: Value of variables used to compute the MCDM criteria as a function of the alternative. a) Probably of collision of first encounter, b) probability of collision of second encounter, c) Total δv of the alternative, d) total time of the operation, e) total number of manoeuvres, including CAM and returning manoeuvres for both encounters, e) constellation metric.

Table 2: Alternatives in the collaborative case, as function of CAM execution position (θ_{mi}) measured as distance in rad to the encounter, and the returning strategy (si_j).

# Alter.	θ_{m1}	Str. 1	$\theta_{m2}[rad]$	Str. 2
0	0	$s1_0$	0	$s2_0$
1	0	$s1_0$	0.5	$s2_0$
2	0	$s1_0$	0.5	$s2_1$
3	0	$s1_0$	0.5	$s2_2$
...	...	...	...	...
21	0	$s1_0$	6.5	$s2_2$
22	0.5	$s1_0$	0	$s2_0$
23	0.5	$s1_0$	0.5	$s2_0$
24	0.5	$s1_0$	0.5	$s2_1$
25	0.5	$s1_0$	0.5	$s2_2$
...	...	...	...	...
43	0.5	$s1_0$	6.5	$s2_2$
44	0.5	$s1_1$	0	$s2_0$
45	0.5	$s1_1$	0.5	$s2_0$
46	0.5	$s1_1$	0.5	$s2_1$
47	0.5	$s1_1$	0.5	$s2_2$
...	...	...	...	...
65	0.5	$s1_1$	6.5	$s2_2$
...	...	...	...	...
483	6.5	$s1_2$	6.5	$s2_2$

noeuvres in the first encounter and early manoeuvres in the second one: second satellite performing the manoeuvre at $\theta_{m2} = 6.5$ rad, without returning manoeuvre, $s_2 = 0$ or returning to the original orbit, $s_2 = 1$, with satellite A maneuvering late, $\theta_{m1} = 0.5$ rad, without difference among the returning criteria: $s_1 = 0$ first (alternatives 41 and 42), $s_1 = 1$ later (alternatives 63 and 64) and $s_2 = 2$ afterwards (alternatives 85). This combination of CAM execution position provides the smallest PoC for both encounters, minimising other criteria as time of operation, the number of manoeuvres or the total δv required.

- *Subscenario 2: Importance given to manoeuvre cost.* This is the only subscenario presenting a more challenging result to be interpreted. For the first encounter, in general, the CAM of the top alternatives presents a lower δv, which is not the case for the second encounter. Overall, the alternative are within the lower range of total δv (see Fig. 4), but they are not the lowest. This could be due to the (small) influence of other criteria: if some of the other criteria, although with small weight, commonly preferred an alternative with a slightly higher δv, it can explain why not just the alternatives with the lowest total impulse magnitude are selected. In any case, the preferred alternative is no satellite manoeuvring (alternative 0) or only the second one, the closest to the TCA, $\theta_{m1} = 0.5$ rad, without returning strategy, $s_1 = 0$ (alternative 1). The other strategies (41, 42, 64) are associated with alternatives with the first satellite manoeuvring late ($\theta_{m1} = 0.5$ rad) and the other performing the CAM early ($\theta_{m2} = 6.5$ rad).

- *Subscenario 3: Importance given to constellation configuration.* The preferred alternatives are those with: either no manoeuvre by any satellite: alternative 0 (obviously, since there is no disruption of the constellation) or where there is only a CAM in one encounter, with the CAM executed close to the encounter (less disruption of the constellation configuration), with both situations: returning to the original position (no further disruption of constellation) or not returning to the original orbit (but, due to the late CAM, the disruption after the encounter is small): alternatives 2 and 3 when manoeuvres the satellite in the second encounter and alternatives 44 and 66 when the CAM is only performed in the second encounter.

- *Subscenario 4: Importance given to time of operation.* The preferred alternative is not manoeuvring (alternative 0) and, otherwise, only maneuvering for one encounter (the first one), as close to the TCA as possible. For the top alternatives, the time of the operation is restricted form half period (alternatives 22, 44, 66) to a period and a half (alternative 88), essentially. The alternatives where only the executing the CAM for the other encounter are not ranking as high, probably due to a less reduction of probability, bearing in mind this criterion is equally weighed for both encounters, and taking into account that the other parameters remains similar for both approaches.

- *Subscenario 5: Importance given to constellation configuration and time of operation.* The preferred options are those shared by subcenario 3 and 4: only Constellation preference and only time preference, respectively. The manoeuvre is executed only for one encounter (again, the first one), or even for none of the encounter (alternative 0). The CAM is executed close to the encounter ($\theta_{m1} = 0.5$ rad for alternatives

Table 3: Top 5 alternatives in Subscenario 0 for the 8 combinations of normalisation methods and MCDM techniques before combining them into a single ranking with $g_{top} = 5$.

WSM		MWSM		WPM		TOPSIS		Combined
Linear	Vector	Linear	Vector	Linear	Vector	Linear	Vector	
0	22	22	41	41	41	414	65	41
22	41	41	42	42	42	417	87	42
44	42	42	63	63	63	477	351	63
88	63	44	64	64	64	480	417	64
110	64	63	85	85	85	483	483	22

Table 4: Weight distribution through criteria for the different subscenarios.

Subscenario	# man.	δv	δv_1	δv_2	Op. time	RPC_1	RPC_2	Const.
0	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8
1	0.1/6	0.1/6	0.1/6	0.1/6	0.1/6	0.45	0.45	0.1/6
2	0.1/7	0.9	0.1/7	0.1/7	0.1/7	0.1/7	0.1/7	0.1/7
3	0.1/7	0.1/7	0.1/7	0.1/7	0.1/7	0.1/7	0.1/7	0.9
4	0.1/7	0.1/7	0.1/7	0.1/7	0.9	0.1/7	0.1/7	0.1/7
5	0.1/6	0.1/6	0.1/6	0.1/6	0.45	0.1/6	0.1/6	0.45
6	0.1/5	0.1/5	0.1/5	0.1/5	0.1/5	0.3	0.3	0.3

Table 5: Top 5 alternatives in each subscenario.

Subscen. 0	Subscen. 1	Subscen. 2	Subscen. 3	Subscen. 4	Subscen. 5	Subscen. 6
41	85	0	44	0	22	63
42	41	1	66	22	44	64
63	42	41	0	44	66	85
64	63	42	2	88	0	86
22	64	64	3	66	110	65

22. 44, 66 and $\theta_{m1} = 1.5$ rad for alternative 110): less operation time and less disruption of the constellation configuration, and following a returning strategy to the original position.

- *Subscenario 6: Importance given to constellation configuration and PoC.* It is similar to the subscenario 2, where PoC was prioritised, which makes sense, since overall, the PoC criteria still have a bigger relative weight in this subscenario. The preferred alternatives are situations with CAM executing late for first encounter and early for the second one, where the minimum values of PoC are found, as in subscenario 2 (alternatives 63, 64, 85). However, there is a preference to execute a returning manoeuvre to the original orbit (alternatives 86) or to the original position (alternatives 65), due to the higher weight on the constellation criteria.

4.2 *Competitive CAM Planning*

In this example, two manoeuvrable satellites from different constellations are involved in the same encounter to illustrate the competitive case. The initial orbital elements can be found in Table 6. The encounter occurs at $TCA = 43200s$ after t_0.

Table 6: Initial orbital elements satellites involved in the competitive case encounter. Units in km and deg.

Sat.	SMA	ecc.	inc	RAAN	AP	TA
A	7100	0.0	60	0.0	0.0	252.57
B	6944.26	0.031	141.04	335.26	116.15	47.03

Satellite A is assumed to be perfectly known, while satellite B is assumed to be affected by both aleatory and epistemic uncertainty. Note that the choice of which satellite is affected by uncertainty does not change the methodology and the outcome of the decision because both constellations are assumed to be able to compute a single PoC, pre and post manoeuvre.In the case in which the computation of the PoC of one constellations is unknown to the other, the game become stochastic and the approach in this paper has to be further extended.

The aleatory uncertainty is modelled by the covariance matrix at the initial position expressed in the <R,T,H>,

$$\mathbf{\Sigma}_{2,rth}(t_0) = \begin{bmatrix} 0.104^2 & 0 & 0 \\ 0 & 0.556^2 & 0 \\ 0 & 0 & 0.139^2 \end{bmatrix} km^2,$$

while the epistemic uncertainty is modelled with the epistemic parameters $\boldsymbol{\lambda}$, one per each source of information. In this example, two sources are assumed to provide information, with

$$\boldsymbol{\lambda}_{\sigma,a} = [1, 4]$$

$$\boldsymbol{\lambda}_{\sigma,b} = [1/5, 1/2],$$

thus, the uncertain encounter geometry can be defined as in Table 7

Table 7: Uncertain encounter geometry for both encounters with 2 sources providing information. Upper and lower bound of the components of miss distance and covariance matrix in the impact plane.

Component	Source 1	Source 2
μ_ξ (km)	[0.02855,0.08342]	[$1.263 \cdot 10^{-3}$,0.01529]
μ_ζ (km)	[-0.6327,0.7230]	[-0.2317,0.2542]
σ^2_ξ (km^2)	[0.1208,0.4353]	[0.02530,0.05332]
σ^2_ζ (km^2)	[140.265,505.054]	[29.311,61.202]
$\sigma_{\xi\zeta}$ (km^2)	[-14.586,-4.0744]	[-1.8012,-0.8572]

As in the previous example, we assumed an $HBR =$ 10m, computing the PoC with Eq. (4).

The robust optimal CAM is computed at different execution positions for each satellite: $\Theta_{mA} = \{0, 9\pi, 17\pi\}$ rad, $\Theta_{mB} = \{0, 7\pi, 11\pi, 15\pi\}$ rad, measured as distance to the encounter. Two different avoidance conditions have been studied by changing the maximum impulse capacity of each satellite and the PoC threshold of each operator. Table 8 includes the CAM parameters for each case.

For simplicity of analysis no returning strategy is consider. Including returning manoeuvres would only introduce new alternatives and, possibly, new criteria. Other than this, the method would remains the same.

Table 8: PoC threshold and maximum capacity of each satellite involved in the encounter for each of the three Cases considered.

Case	$\delta v_{1,max}$ (km/s)	$P_{C0,1}$	$\delta v_{2,max}$ (km/s)	$P_{C0,2}$
1	$2.5 \cdot 10^{-4}$	10^{-6}	$2.5 \cdot 10^{-4}$	10^{-6}
2	$5 \cdot 10^{-4}$	10^{-5}	$5 \cdot 10^{-4}$	10^{-5}

The criteria for both satellites are the Reduction of PoC, Eq. (10), and the cost of the manoeuvre, Eq. (11). Thus, we have a MCMDM problem with 2 decision-makers with 2 criteria each of them, one selecting among 4 alternatives and the other among 3. In Section 4.2 to Eq. (25), the Alternative-Criteria matrices for each satellite are presented for each of the three CAM approaches in Table 8:

- *Case 1*

$$\mathbf{A_{1,(:,1:4)}} = \begin{bmatrix} 0 & 0 & 0.056 & 0 \\ 0.031 & 2.5 \cdot 10^{-4} & 1 & 1.75 \cdot 10^{-4} \\ 0.156 & 2.5 \cdot 10^{-4} & 1 & 1.675 \cdot 10^{-4} \end{bmatrix},$$

$$\mathbf{A_{1,(:,5:8)}} = \begin{bmatrix} 0.139 & 0 & 0.367 & 0 \\ 1 & 1.875 \cdot 10^{-4} & 1 & 1.925 \cdot 10^{-4} \\ 1 & 1.75 \cdot 10^{-4} & 1 & 1.875 \cdot 10^{-4} \end{bmatrix}, \quad [22]$$

$$\mathbf{B_1} = \begin{bmatrix} 0 & 0 & 0.031 & 0 & 0.156 & 0 \\ 0.056 & 2.5 \cdot 10^{-4} & 1 & 1.875 \cdot 10^{-4} & 1 & 1.925 \cdot 10^{-4} \\ 0.139 & 2.5 \cdot 10^{-4} & 1 & 1.75 \cdot 10^{-4} & 1 & 1.875 \cdot 10^{-4} \\ 0.367 & 2.5 \cdot 10^{-4} & 1 & 1.675 \cdot 10^{-4} & 1 & 1.75 \cdot 10^{-4} \end{bmatrix}. \quad [23]$$

- *Case 2*

$$\mathbf{A_{2,(:,1:4)}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.918 & 1.6 \cdot 10^{-4} & 1 & 9.1 \cdot 10^{-5} \\ 0.982 & 1.7 \cdot 10^{-4} & 1 & 8.2 \cdot 10^{-5} \end{bmatrix},$$

$$\mathbf{A_{2,(:,5:8)}} = \begin{bmatrix} 0.952 & 0 & 0.971 & 0 \\ 1 & 9.4 \cdot 10^{-5} & 1 & 9.6 \cdot 10^{-5} \\ 1 & 8.3 \cdot 10^{-5} & 1 & 8.5 \cdot 10^{-5} \end{bmatrix}, \quad [24]$$

$$\mathbf{B_2} = \begin{bmatrix} 0 & 0 & 0.918 & 0 & 0.982 & 0 \\ 1 & 1.4 \cdot 10^{-4} & 1 & 7.3 \cdot 10^{-5} & 1 & 7.6 \cdot 10^{-5} \\ 0.952 & 1.2 \cdot 10^{-4} & 1 & 6.3 \cdot 10^{-5} & 1 & 6.6 \cdot 10^{-5} \\ 0.971 & 1.0 \cdot 10^{-4} & 1 & 5.5 \cdot 10^{-5} & 1 & 5.8 \cdot 10^{-5} \end{bmatrix}. \quad [25]$$

Five different subscenarios have been presented: i) criteria in both satellites equally weighted, ii) both satellites giving more importance to PoC, iii) both satellites giving more importance to manoeuvre cost, iv) Sat A giving more importance to PoC and Sat B to the cost, v) Sat A giving more importance to cost and Sat B to PoC. For all the subscenarios, the criteria are integrated considering two normalisation: linear and vector, and four MCDM methods: WSM, MWSM, WPM, TOPSIS. An example of the ordinal integrated criteria matrices, **C** and **D**, *Case 1 - Subscenario 0* using Linear normalisation and WPM method to integrate criteria are:

$$\mathbf{C_{1,lin,WPM}} = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad [26]$$

$$\mathbf{D_{1,lin,WPM}} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix}, \qquad [27]$$

and for *Case 2 - Subscenario 0* using Linear normalisation and WPM method:

$$\mathbf{C_{2,lin,WPM}} = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad [28]$$

$$\mathbf{D_{2,lin,WPM}} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}, \qquad [29]$$

where the element c_{ij} indicates the preference of the i^{th} alternative of Sat. A, given the Sat. B chooses alternative j, with the highest values for the most preferred alternatives in each column; and the element c_{kp} the preference of the k^{th} alternative of Sat. B, given the Sat. A chooses alternative p

In the following, the analysis of each subsection is shown, along with the matrices including the number of time an outcome (combination of alternative A, in the rows, and alternative B, in the columns) has been an equilibrium points after computing the Nash Stability for each integration of criteria for the eight combinations of normalisation and MCDM techniques. The best outcome in each subscenario is highlighted in bold.

Case 1

- *Subscenario i.*

$$\mathbf{NE}_i = \begin{bmatrix} \mathbf{4} & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [30]$$

This subscenario tries to optimise both cost and risk for both satellites. The prefer outcome is, surprisingly, that none of the satellite manoeuvres. The explanation can be in the fact that options that minimises the PoC ($RPC = 1$) are at a high cost, and alternatives where only one manoeuvres, which has a zero cost for one of the satellites, does not reduce the risk that much.

- *Subscenario ii.*

$$\mathbf{NE}_{ii} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{8} \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [31]$$

If the PoC is to be minimised, the preferred option, independently of the method, is an outcome where both satellites manoeuvres, reducing the risk below the threshold ($RPC_A = RCP_B = 1$).

- *Subscenario iii.*

$$\mathbf{NE}_{iii} = \begin{bmatrix} \mathbf{7} & 1 & 1 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [32]$$

If the cost is prioritised, the preferred option is, obviously, where no satellites manoeuvres ($\delta v_A = \delta v_B = 0$), even if there is no reduction of risk ($RPC_A = RCP_B = 0$).

- *Subscenario iv.*

$$\mathbf{NE}_{iv} = \begin{bmatrix} \mathbf{4} & 0 & 0 & 0 \\ \mathbf{3} & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [33]$$

When Sat. A prioritises the risk and Sat. B the reduction of the cost, a curios situation is obtained: the most common equilibrium points appears for the outcome where no manoeuvre is executed. The explanation can be that due to the prioritisation of cost reduction of Sat. B, it will prefer alternatives with no manoeuvres for it. However, the reduction of PoC for Sat.A given this outcome is very small in any case. Nevertheless, this is the expected outcome. Other than this, the second alternative, is an outcome where only Sat. A manoeuvres.

- *Subscenario v.*

$$\mathbf{NE}_{v} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{8} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [34]$$

This scenario is the opposite of the previous one: Sat. A search to minimise the cost, while Sat. B to minimise the risk. The preferred option, independently of the method, is that one where Sat. A does not manoeuvre (minimising the cost) and Sat. B manoeuvres the earliest: reducing the risk to the threshold ($RPC_B = 0.367$).

Case 2

This Case presents an important difference with respect the previous one: if only one satellites manoeuvres, the threshold is reached for both satellites, so the outcome where only one manoeuvres will be a more preferred option than in the previous Case.

- *Subscenario i.*

$$\mathbf{NE}_i = \begin{bmatrix} 2 & 0 & 0 & \mathbf{6} \\ 3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \qquad [35]$$

  The preferred options in this case is where only one satellite manoeuvres, executing the CAM at the cheapest position, since in this way, the threshold is reached, and the cost is minimised: one does not spend anything and the other the minimum possible. For the outcome (0,3), being an equilibrium point 6 times, $\delta v_A = 0$ cm/s and $\delta v_B = 10$ cm/s and $RPC_A = RPC_B = 0.971$.

- *Subscenario ii.*

$$\mathbf{NE}_{ii} = \begin{bmatrix} 0 & \mathbf{7} & 0 & 1 \\ 0 & 0 & 0 & 0 \\ \mathbf{7} & 0 & 0 & 0 \end{bmatrix}. \qquad [36]$$

  Similarly, the preferred outcome are those where only one satellite manoeuvres. Since the threshold is reached (or almost) when only satellite manoeuvres, these options are preferred than both satellites manoeuvring. The options is always manoeuvring that reduces the risk the most.

- *Subscenario iii.*

$$\mathbf{NE}_{iii} = \begin{bmatrix} \mathbf{6} & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [37]$$

  As in the previous case, if the cost is to be optimised by both satellites, the preferred option is that no one manoeuvres.

- *Subscenario iv.*

$$\mathbf{NE}_{iv} = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ \mathbf{7} & 0 & 0 & 0 \end{bmatrix}. \qquad [38]$$

  If Sat. A prioritised the risk and Sat. B the cost, the preferred option is that Sat. A manoeuvres and Sat. B does not, with the option where Sat. A reduced the risk the most: $RPC_A = 0.982$.

- *Subscenario v.*

$$\mathbf{NE}_{v} = \begin{bmatrix} 0 & \mathbf{7} & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad [39]$$

  If Sat. A prioritised the cost and Sat. A the risk, the preferred option is that Sat. B manoeuvres the erliest and Sat. A does not, with the options where Sat. B reduced the risk the most: $RPC_B = 1$.

5. Final Remarks

In this paper we have extended the CASSANDRA framework to include coordinated decisions on the planning and execution of collision avoidance manoeuvres. We considered both the collaborative, where satellites belong to the same constellation, and the competitive case, where satellites belong to different constellations. This later case led to an extension of the classic Multi-Criteria Decision Making methodology to incorporate multiple decision makers with conflicting goals.

In the scenarios presented in this paper we found that in the collaborative case the optimal decision for each satellites is always to minimise their individual collision risk and then restore the performance index of the whole constellation.

In the competitive case it was found that in a number of scenarios, game theoretic equilibria occur when only one of the two constellations manoeuvre. This is the case when each manoeuvre can always achieve the minimum safety threshold. On the other hand when operational constraints prevent both constellations to reach the minimum threshold or each operator is putting more weight on manoeuvre cost then equilibria exist where both constellation execute only a partial manoeuvre. In the game defined in this paper both players can compute the total risk of a conjunction with the same approach and, therefore, can predict the risk reduction due to the execution of any manoeuvre. Although in this paper we already considered the case in which the accepted risk differs between two players a more complex case occurs when the risk computed by each player is uncertain. In this case the game become stochastic and requires a further extension of the decision making process.

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