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Polynomial Power Method: An Extension of the Standard Power Method to Para-Hermitian Matrices

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ABSTRACT

This paper expands the concept of the power method to polynomial para-Hermitian matrices in order to extract the principal analytic eigenpair. The proposed technique involves repeatedly multiplying the para-Hermitian matrix by a polynomial vector, followed by an appropriate normalization of the resulting product in each iteration, under the assumption that the principal analytic eigenvalue spectrally majorises the remaining eigenvalues. To restrain the growth in polynomial order of the product vector, truncation is performed after normalization in each iteration. The effectiveness of this proposed method has been verified through simulation results on an ensemble of randomly generated para-Hermitian matrices, demonstrating superior performance compared to existing algorithms.

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Figures and tables

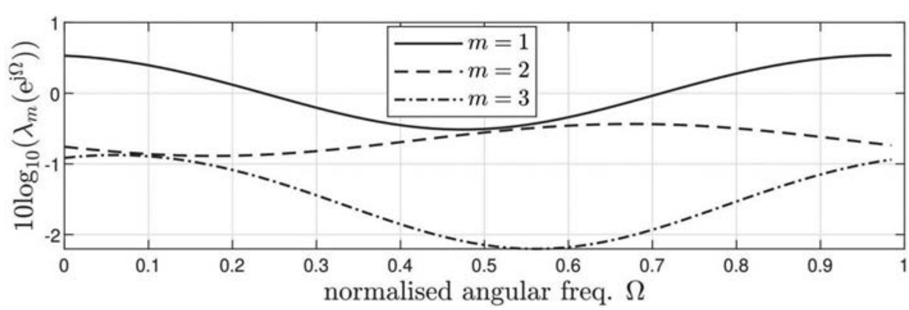

Fig. 1. Example of analytic, spectrally majorised eigenvalues $\lambda_1(z) = \frac{6+j}{100}z + 1.01 + \frac{6-j}{100}z^{-1}$, $\lambda_2(z) = \frac{-1+2j}{100}z + 0.86 + \frac{-1-2j}{100}z^{-1}$, and $\lambda_3(z) = \frac{5-2j}{100}z + 0.71 + \frac{5+2j}{100}z^{-1}$ of a para-Hermitian matrix $R(z) : C \rightarrow C^{3\times 3}$ when evaluated on the unit circle for $z = e^{j\Omega}$. For the decomposition of such polynomial matrices [1], an analytic eigenvalue decomposition (EVD) exists [2–4], which can be approximated by a number of different algorithms [5–12]. For many applications in the communications [13,14], beamforming and angle of arrival estimation [15–18], speech enhancement [19,20], and signal detection [21–24], a reduced analytic eigenvalue decomposition suffices; hence this contribution investigates the extension of the power method [25] to the polynomial domain [26,27].

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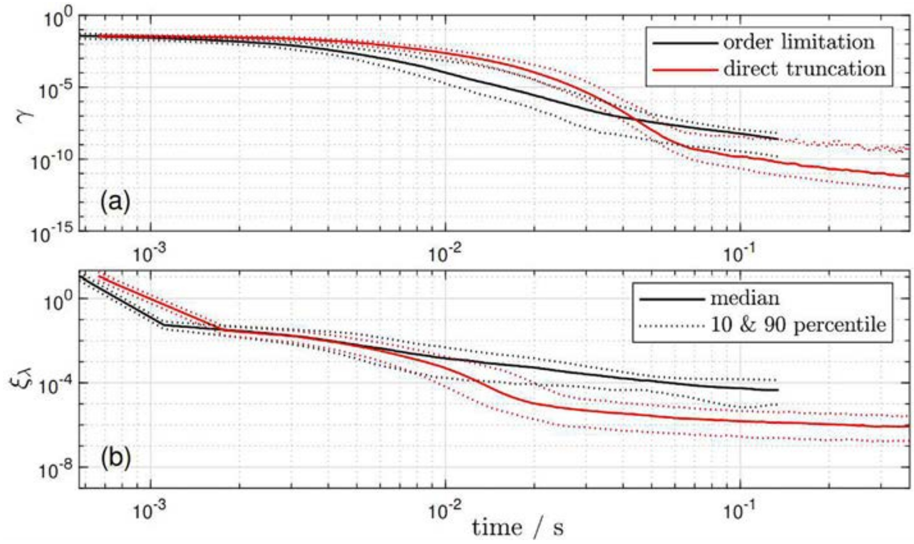

Fig. 2. Comparison of two different truncation methods – order limitation via [28] in the DFT domain [29] and order truncation as described in [30,31] – applied to the normalized product vector (a) illustrating γ, the overall deviation between of the normalized product vector of two consecutive iterations and (b) showing ξ_λ, the normalized error between the ground truth and the estimated dominant eigenvalues.

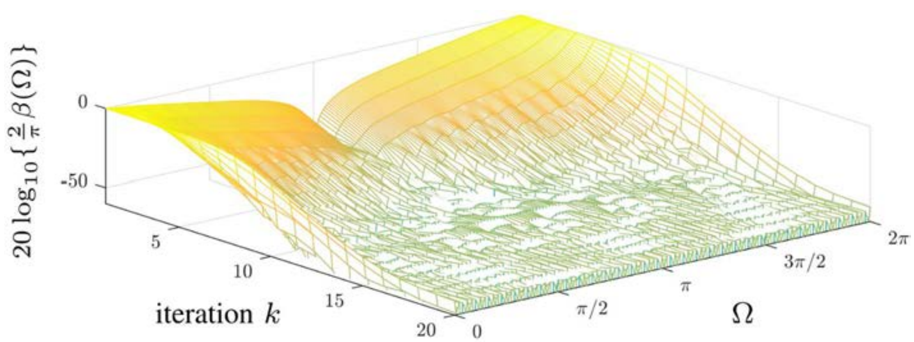

Fig. 3. Example of convergence of the polynomial power method for the metric $\beta(\Omega) = \angle\{v^{(k)}(e^{j\Omega})q_1(e^{j\Omega})\}$ with $v_0(z) = \frac{(1-z^{-1})}{\sqrt{2}}q_1(z) + \frac{(1+z^{-1})}{\sqrt{2}}q_2(z)$ where $q_{m=1,2}(e^{j\Omega})$ are analytic eigenvectors. The initial vector has a singularity of $c_1(e^{j\Omega})$ for $\Omega = 0$. The normalization is performed with a regularization parameter $\varepsilon = 0.1$ to avoid division by zero, and to bias the result, thus iteratively filling in the spectral null at $\Omega = 0$.

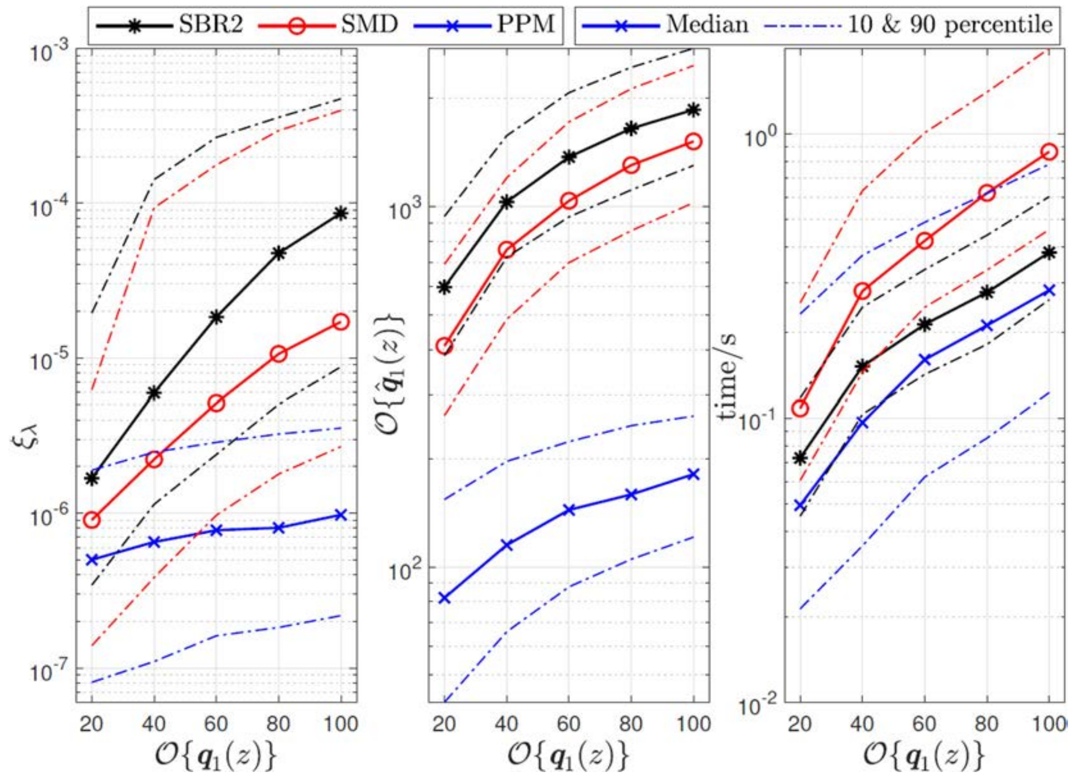

Fig. 4. Comparison on an ensemble of para-Hermitian matrices between the polynomial power method with the second order sequential best rotation (SBR2) [5,6] and the sequential matrix diagonalisation (SMD) [7] algorithms on the basis of metric (a) ξ_λ, the normalized error between the estimated and ground truth dominant eigenvalue, (b) the polynomial order of the estimated dominant eigenvector denoted with $O\{q_1(z)\}$, and (c) execution time. Estimated space-time covariance matrices are spectrally majorised [32] with probability one [33], such that the polynomial EVD targeted by the SBR2 and SMD algorithms is equivalent to the analytic decomposition targeted by the proposed method.

CRediT authorship contribution statement

Faizan A. Khattak: Formal analysis, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Ian K. Proudler:** Conceptualization, Supervision, Writing – review & editing. **Stephan Weiss:** Conceptualization, Software, Supervision, Validation, Writing – review & editing.

Data availability

No data was used for the research described in the article.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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