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Paraunitary Approximation of Matrices of Analytic Functions - the Polynomial Procrustes Problem

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Abstract: The best least squares approximation of a matrix, typically e.g. characterising gain factors in narrowband problems, by a unitary one is addressed by the Procrustes problem. Here, we extend this idea to the case of matrices of analytic functions, and characterise a broadband equivalent to the narrowband approach which we term the polynomial Procrustes problem. Its solution relies on an analytic singular value decomposition, and for the case of spectrally majorised, distinct singular values, we demonstrate the application of a suitable algorithm to three problems via simulations: (i) time delay estimation, (ii) paraunitary matrix completion, and (iii) general paraunitary approximations.

Figures and tables

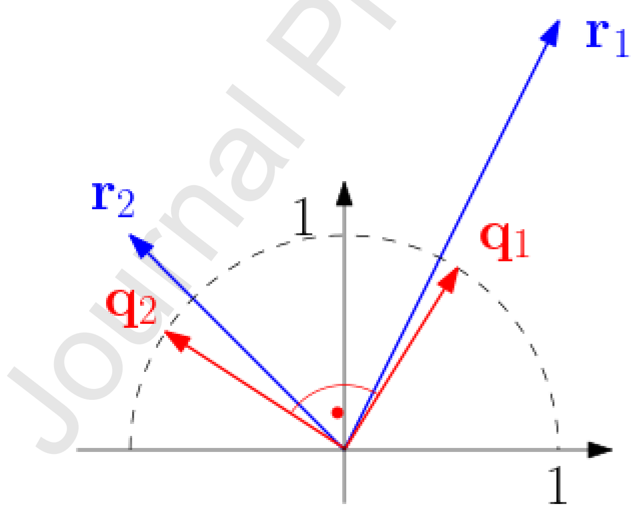

Fig. 1: Example of the Procrustes solution [1] for a 2x2 matrix with column vectors $r_1$and r_2 (in blue), finding the closest unitary matrix whose columns form an orthonormal basis with basis vectors q_1 and q_2 (in red). We are looking for an extension of this problem to the case of polynomial or generally analytic matrices [2-4] in order to admit e.g. the approximation of matrices of transfer functions by paraunitary systems representing lossless filter banks, as desired in e.g. [5-7].

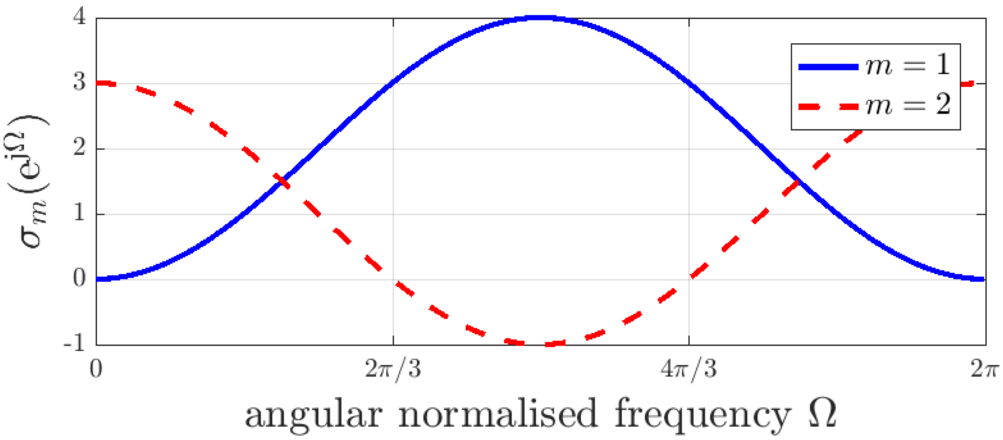

Fig. 2: Example for the analytic singular values $\sigma_m(z)$, $m = 1{,}2$, of a 2x2 matrix of analytic functions when evaluate on the unit circle; note that analyticity for the 2nd singular value requires the singular value to become negative [8-11]. Various algorithms can help to address such singular value decompositions: analytic solutions for the similar EVD case are obtainable in [12-15]; under some circumstances, analytic solutions are free of intersections [16] such that a number of other polynomial matrix SVD algorithms such as in [17-19] suffice.

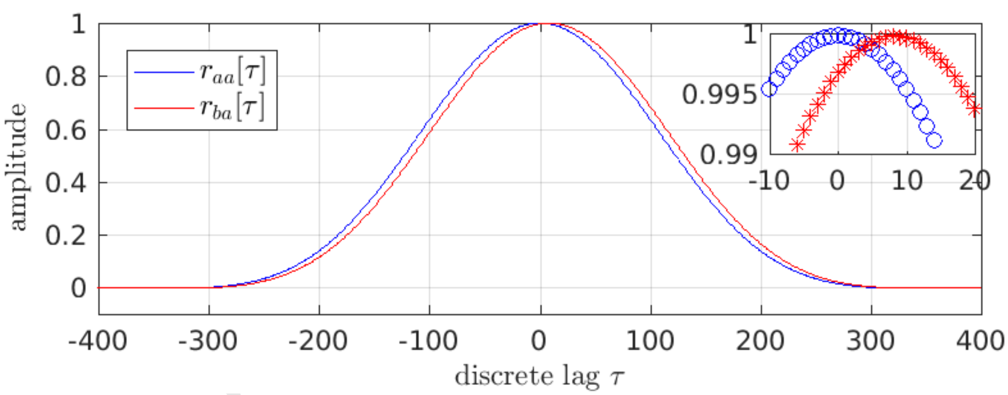

Fig. 3: Autocorrelation $r_{aa}[\tau]$a lowpass signal $a[n]$; for a signal $b[n]$, which is shifted by a fractional delay of 7.3 sampling periods [20], determining this fractional delay from the cross-correlation sequence $r_{ba}[\tau]$ is difficult, since its peak is ill-defined.

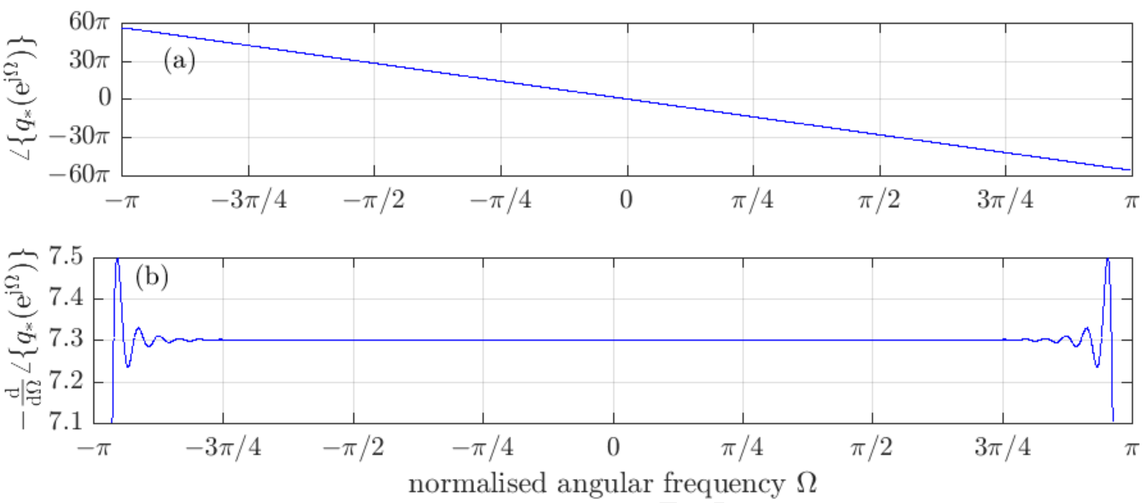

Fig. 4: Polynomial Procrustes solution applied to the cross-correlation function $r_{ba}[\tau]$ in Fig. 3 yields an allpass whose (a) phase response and (b) group delay permit to extract the fractional delay akin to approaches in [21-24].

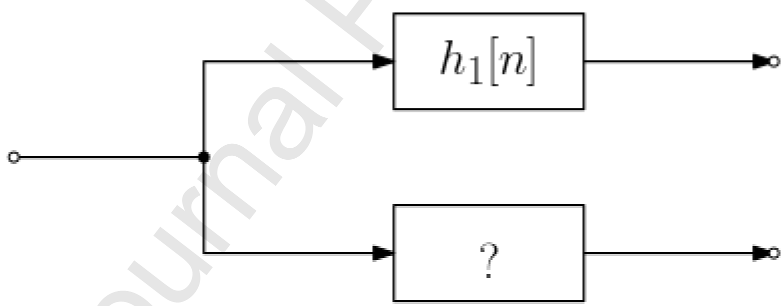

Fig. 5: Paraunitary matrix completion problem: finding the orthogonal complement to a lowpass filter $h_1[n]$.

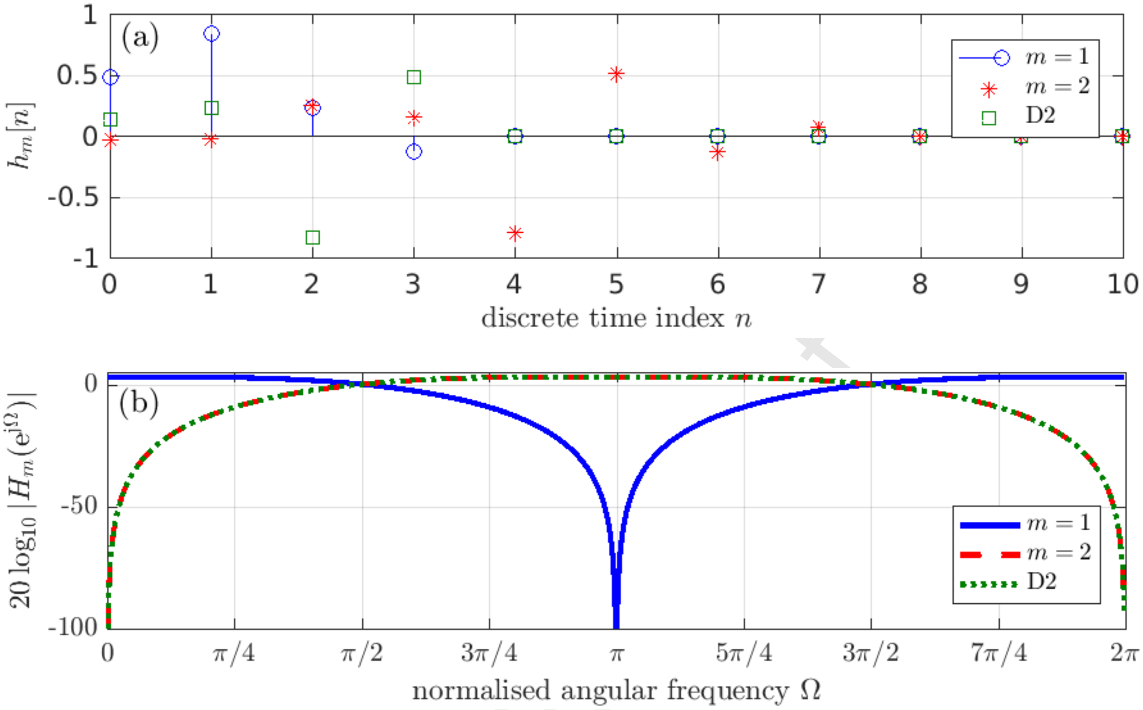

Fig. 6: (a) Lowpass filter $h_1[n]$ for a Daubechies D2 wavelet [25] of length 4 (in blue), the identified solution ($m = 2$, in red), and the known orthogonal complement of the D2 wavelet of length 4 (in green): an allpass ambiguity leads to a Procrustes solution that does not possess maximum compactness; (b) corresponding magnitude responses, with the given lowpass response (in blue), and the identified solution (red) as well as the Daubechies D2 highpass filter (green); phase ambiguity to the solution means that the identified response and the ideal D2 filter possess the same magnitude responses.

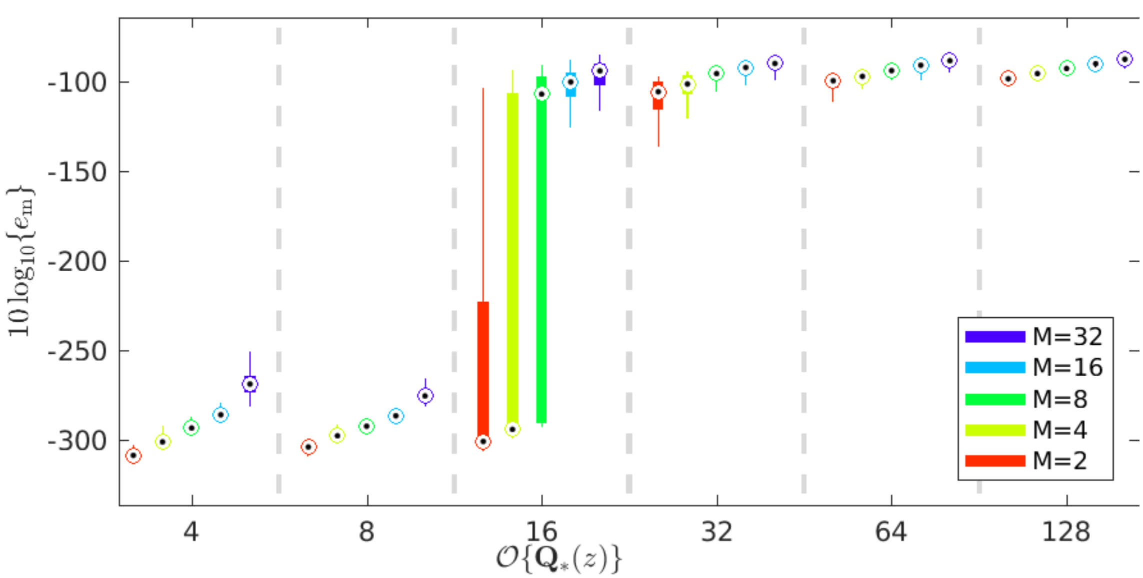

Fig. 7: Ensemble test generated over a number of systems generated by random innovation filter [26], where the mismatch between the ground truth and the identified Procrustes solution is evaluated for different dimensions M and for different orders of the ground truth solution; for small orders, the mismatch is close to machine accuracy; for higher orders, a preset truncation parameter of 1e-10 limits the accuracy.

CRediT author statement

Stephan Weiss: Formal Analysis, Software, Writing – Original Draft; **Sebastian J. Schlecht:** Conceptualisation, Formal Analysis, Software, Writing – Reviewing and Editing; **Orchisama Das:** Formal Analysis, Writing – Reviewing and Editing; **Enzo De Sena:** Conceptualisation, Writing – Reviewing and Editing.

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Declaration of interests

X☐ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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