Flexibility service design for mitigating voltage unbalance in the distribution network

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***Abstract*— It is expected that unbalanced voltages will increase in future distribution systems as the level of single-phase distributed energy resources (DERs) rises significantly, and this paper presents a methodology to address such problems. The approach is based on a three-phase optimal power flow (OPF) method seeking to mitigate the voltage unbalance factor (VUF) by using demand-side flexibility provided by DERs. A novel flexibility service proposed here allows Distribution System Operators (DSOs) to mitigate VUF through local flexibility resources. Considering the differences in preferences among DSOs, we compare and analyze three different flexibility service design options to provide a reference for DSOs to develop their strategies. The proposed convex relaxation model was tested using an IEEE network and the results showed that various flexibility service schemes could lead to differences in the allocation of flexibility resources and hence changes in the system state. Furthermore, it was found that the activation of flexibility resources might further increase the unbalance of the system if the voltage unbalance constraint was ignored.**

Keywords—Distributed energy resource (DER), local flexibility market (LFM), convex relaxation, demand-side flexibility, voltage unbalance factor (VUF).

I. Introduction

In recent years, there is a significant increase in distributed energy resources (DERs) connections in the distribution systems, but their intermittent and uncertain nature brings additional operational challenges for Distribution Network Operators (DNOs) [1]. To cope with the rapid changes in distribution networks, the DNOs are transitioning to Distribution System Operators (DSOs) [2]. Flexibility is the key factor during this transition, it allows the DSOs to manage the distribution network constraints in a cost-efficient manner, thus avoiding or delaying network reinforcement.

In general, flexibility is defined as the ability to adjust the generation/consumption pattern according to an external signal for facilitating services [3]. Transmission System Operators (TSOs) can use flexibility for system balancing and planning [4], while for other stakeholders, such as DSOs, local congestion management and voltage support are more commonly addressed issues [5-6].

The Local Flexibility Market (LFM) provides a competitive platform for DSOs (buyers) and aggregators (sellers) to trade flexibility services locally. According to the current LFM framework [7], the attributes of flexibility services are including the following aspects: a) the direction of the flexibility activation b) the rate of the change of flexibility c) flexibility duration d) starting time e) flexibility purpose and f) location in the network.

However, most flexibility service designs ignore the fact that they do not consider the phase information of flexibility resources, although these resources are located at medium and low voltage levels and are already allowed to participate in LFM, e.g., in the UK, the minimum capacity permitted to enter the market is 10 kW [8]. Ideally, at the planning stage, the loads on the distribution system should be evenly distributed across three phases, however, it is often not the case. In addition, DERs are often connected to a single phase, e.g., Electric Vehicle (EV) charging points or rooftop photovoltaic (PV) panels, thus leading to significant phase unbalances. This results in the poor power quality, such as voltage unbalance, which may increase beyond the permissible ranges of distribution grid code and engineering recommendations [9]. The potential risk in ignoring phase connection information in LFM is that the utilized flexibility services could lead to new, or aggravated existing, problems of a voltage unbalance. Based on the above facts, this paper suggests that phase connection information for flexibility resources should be considered in the LFM when designed flexibility services and associate market clearing process in order to address the voltage unbalance issues.

Traditional solutions for mitigating voltage unbalance include reconnecting loads, manually changing the reconfiguration of link boxes and relying on, often costly, equipment such as phase switches, tap changers and STATCOMs [10], however, the above solutions may not be suitable for future distribution systems scenarios where there are a significant amount of DERs integrated into the network. Thus, there is an increasing need to consider a cost-effective way for DSOs to manage flexibility resources per phase and in this work, we propose a market-based solution for DSOs to cope with the voltage unbalance issue while respecting other network constraints. This requires a three-phase network representation, as well as a three-phase OPF tool to clear LFM and allocate local flexibility resources. In the current LFM designs, the network constraints are often either disregarded [11-12] or represented by DC approximation power flow [13], complex AC power flow formulation [14], convex-relaxed AC power flow [15] and linearized power flow equation [16]. Neglecting the physics network in LFM facilitates fast problem-solving, but cannot guarantee the feasibility of the optimal solution. The DC approximation approach is successfully used for the transmission system but is not appropriate for the distribution systems. The AC OPF can be computationally challenging in practical applications and therefore the convex relaxation and linearization-based OPF may be more suitable for application for LFM. Thus, methodology presented in [17] adopted a semidefinite programming (SDP) for voltage regulation in distribution systems, but neglected the voltage unbalance, while [18] investigated the different dispatch models of PV inverters at the unbalanced distribution system also omitting voltage unbalance. In [16], the author adopted linearized network constraints for LFM design, but using a single-phase model. In this paper, we adopt a convex relaxation power flow model based on [25], with the main advantage of this relaxation approach is that it can at least provide a lower bound of the original problem, unlike the approximations approach. To verify the accuracy of the method, a corresponding error analysis was performed, and the results confirmed the reliability of the approach with potential for LFM application.

The main purpose of this paper is to develop a novel local flexibility service used for mitigating voltage unbalance while considering other network constraints. In addition, it is critical to understand the possible options when DSOs are seeking to develop this new type of flexibility service. In [19-20], the goal of mitigating the voltage unbalance is achieved by modifying the original objective function, while in our previous work [14], we included a new set of constraints for limiting the voltage unbalance factor (VUF) at each of the three-phase buses. Furthermore, voltage unbalance can be mitigated by balancing the net load of each phase. There is a lack of research comparing various voltage unbalance solutions, however, it is essential to understand the differences between these strategies for DSOs to develop specific flexibility services.

The main contributions of this paper include: (i) proposing a new local flexibility service design for DSOs to mitigate the voltage unbalance and proposing and (ii) comparing three different voltage unbalance strategies to provide an effective reference for DSOs to make a choice on which approach to use.

II. VOLTAGE UNBALANCE MANAGEMENT STRATEGIES

For the task of designing a new flexibility service, the first step is to select an appropriate metric, after which the DSO should consider potential solutions for achieving this target, as well as the differences between the various strategies. In the case of voltage unbalance management, different strategies can lead to deviations in the allocation of flexibility resources and thus to different system states, which may require DSOs to make trade-offs according to their respective preferences. In this section, we first illustrate the voltage unbalance metrics and then describe the different models in more detail.

A. Metrics for Measuring the Voltage Unbalance

The balanced voltage means the three-phase voltages has equal magnitudes and a 120° difference between phases. However, in practice, there is the unbalance current flow along the distribution feeder leading to an offset between each of the phase voltages. There are three different definitions of voltage unbalance from IEC, IEEE and NEMA [21-23], respectively. The definition of voltage unbalance given by the IEC as VUF, which consists of negative and positive sequence voltage components, as shown in equation (1).

$$VUF = \frac{|V_{neg}|}{|V_{pos}|} \tag{1}$$

In addition, NEMA and IEEE have adopted the line-to-line and line-to-ground voltage magnitude to evaluate the voltage unbalance, respectively. A detailed summary and comparison of different voltage unbalance metrics could be found in [24]. In [26], the author recommends utilizing the voltage magnitude-based approach to identify the voltage unbalance level as it is easily obtained from metered data. However, VUF is commonly used in practice, and it can be estimated based on three-phase power flow techniques, hence we use VUF as the metric for the proposed flexibility service design.

B. Voltage Unbalance Management Models

Now, we present three potential models for managing voltage unbalance, which could provide a valuable reference for DSOs when designing the corresponding services.

To comply with the VUF requirements in [9], in model 1 the regulation of the VUF is modelled by introducing a new set of hard constraints in the optimization process. For model 2, we assume that the DSO is willing to reduce the overall level of VUF as much as possible, which is achieved by modifying the original objective function. In model 3, we mitigate the voltage unbalance level by rebalancing the load distribution across each phase. The detailed mathematical formulations representing the various models are shown in the following subsections.

The following notation is used in this part to represent the voltage unbalance management models. VUF_{max} is the upper limitation of VUF. $V_{neg}, V_{pos}, A_{neg}$ and A_{pos} are the negative (positive) sequence voltage components and the symmetrical voltage transformation vectors, respectively. $\boldsymbol{v}$ is the phase voltage column vector of a three-phase bus and $\boldsymbol{V} = \boldsymbol{v} * \boldsymbol{v}^T$. $\boldsymbol{p}^{a,sub}$, $\boldsymbol{p}^{b,sub}$ and $\boldsymbol{p}^{c,sub}$ are represented for the power exchange at the substation bus in different phases.

1) VUF Regulation Model

To keep the VUF below its maximum value, the constraints are introduced and modeled as:

$$\frac{|V_{neg}|}{|V_{pos}|} \leq VUF_{max} \tag{2}$$

where $V_{neg} = A_{neg} * \boldsymbol{v}$ and $V_{pos} = A_{pos} * \boldsymbol{v}$.

However, the fractional equation above is a non-convex form and we need to convert it by squaring both sides simultaneously, so that:

$$A_{neg} * \boldsymbol{V} * A_{neg}^H - VUF_{max}^2 * \left(A_{pos} * \boldsymbol{V} * A_{pos}^H\right) \leq 0 \tag{3}$$

2) VUF Minimization Model

In this model, we assume that the DSOs are willing to minimize the VUF and achieve this by modifying the objective function. The standard definition of VUF is a non-convex expression that cannot be directly added to the objective function, thus we propose an alternative method to minimize the total voltage unbalance level. Recalling the definition, VUF is determined by the positive and negative sequence voltage components. However, in practice, the lower negative sequence voltage contributes to a lower VUF level, so the task of minimizing the VUF can be approximated by minimizing the negative sequence voltage instead, as shown in equation (4). A similar approach can be found in [28].

$$Min\, A_{neg} * \boldsymbol{V} * A_{neg}^H \tag{4}$$

3) Power Balance Model

In this approach, instead of using the VUF as the flexibility service metric, we seek to balance the power exchange between the different phases on the substation buses. The mathematical formulation for this model is:

$$Min|\boldsymbol{p}^{a,sub} - \boldsymbol{p}^{b,sub}|+|\boldsymbol{p}^{a,sub} - \boldsymbol{p}^{c,sub}|+|\boldsymbol{p}^{b,sub} - \boldsymbol{p}^{c,sub}| \tag{5}$$

III. FLEXIBILITY SERVICE DESIGN

In this section, we discuss the necessary attributes to standardize the proposed flexibility services and, at the same time, the roles of the main participants in the flexibility market are described.

A. Explanation of Flexibility Service Attributes

As a commodity in the market, flexibility services need to be clearly defined to facilitate trading by market participants. In general, existing flexibility services requirements contain at least the type of service, start and end times, the amount of flexibility, location, rate of change and power type [7]. Specifically for voltage unbalance services, it is necessary to include information on the phase in which the flexibility resource is located. It is a reasonable requirement, as flexibility located in medium and low-voltage levels is generally connected to the network in a single-phase configuration. We only consider flexibility services from active power, i.e., assuming the flexibility resources operate in a fixed reactive power mode. Solving voltage unbalance problems through reactive power flexibility services is beyond the scope of this paper.

B. The Roles of Market Participants

The participants in LFM and their objectives depend on the scope of the market design and in this paper, we focus on the behavior of DSOs, aggregators, and independent market operators. Other potential stakeholders, such as TSOs and Balancing Responsible Parties (BRPs), are not covered here.

DSOs are responsible for the safe operation of the distribution system and the cost-effective delivery of electricity to customers. They can procure flexibility in the LFM for different purposes [6]. In this work, DSOs are purchasers of flexibility services and when there is a problem with the network, they submit distribution network data together with a request for flexibility to the market operator for market clearing purposes.

Aggregators are providers of flexibility services and participate in LFM on behalf of small groups of DER owners. When they receive a signal from the LFM, they need to submit a bid to the market operator containing available flexibility in both upward (increased generation or reduced consumption) and downward (increased consumption or reduced generation) direction. In this work, the Market Time Units (MTUs) is 1 hour, and this assumption could be easily modified according to the different LFM designs. It is worth noting here, not all MTUs will be activated, only for the duration when network constraints are violated. To avoid ambiguity, we adopt Activated Market Time Units (AMTUs) to refer to the time slot when flexibility services are needed.

Independent market operators run the LFM and execute market clearing. They offer DSOs a market tool to cope with distribution network problems and give demand-side customers a platform to make a profit by selling flexibility services.

IV. PROBLEM FORMULATION

We present here a formulation of the BFM-based convex relaxation three-phase OPF [25]. Assume that set N denotes all the buses of the distribution system, where bus 0 stands for the substation, and define the remaining bus set as $N^+ = N\backslash\{0\}$. The buses with flexibility resources connected are denoted as $N_{flex} \subset N$. The E denotes the set of distribution lines $i \rightarrow j$ connected between adjacent buses. Let $\Phi^i = \{a, b, c\}$ and $\Phi^{ij} = \{a, b, c\}$ denote the set of phases $\emptyset$ of bus $i \in N$ and line $i \rightarrow j \in E$, respectively. If the bus or line only contains one or two phases, then the missing phase should be removed from the set, e.g., if a bus only contains phase a and phase b then $\Phi^i = \{a, b\}$. In this paper, the upper-case (lower-case) boldface letters stand for matrices (vectors). The $\boldsymbol{v}_i = [V_i^{\emptyset}]^T$ is the complex voltage vector at bus $i \in N$, phase $\emptyset \in \Phi^i$. Let $\boldsymbol{i}_{ij} = [I_{ij}^{\emptyset}]^T$ denotes the complex current vector of the line $i \rightarrow j \in E$, phase $\emptyset \in \Phi^{ij}$. The decision variables used for the proposed problem are in matrix form which are written as equations (6)-(8).

$$\boldsymbol{V}_i = \boldsymbol{v}_i * (\boldsymbol{v}_i)^H, \ i \in N \tag{6}$$

$$\boldsymbol{I}_{ij} = \boldsymbol{i}_{ij} * (\boldsymbol{i}_{ij})^H, \ i \rightarrow j \in E \tag{7}$$

$$\boldsymbol{S}_{ij} = \boldsymbol{v}_i * (\boldsymbol{I}_{ij})^H, \ i \rightarrow j \in E \tag{8}$$

where $\boldsymbol{V}_i \in \mathbb{H}^{|\Phi_i| * |\Phi_i|}$, $\boldsymbol{I}_{ij} \in \mathbb{H}^{|\Phi_{ij}| * |\Phi_{ij}|}$ and $\boldsymbol{S}_{ij} \in \mathbb{C}^{|\Phi_{ij}| * |\Phi_{ij}|}$. In addition, the set of $\mathbb{H}$ and $\mathbb{C}$ denotes the Hermitian matrix and complex number, respectively.

The $\boldsymbol{p}_i^{fix} = [P_i^{\emptyset,fix}]^T$ and $\boldsymbol{q}_i^{fix} = [Q_i^{\emptyset,fix}]^T$ is the fixed active and reactive power at bus $i \in N$, phase $\emptyset \in \Phi^i$ respectively. They could either come from the day-ahead energy market or be forecasted by DSOs. Let $\boldsymbol{p}_i^{up} = [P_i^{\emptyset,up}]^T$ and $\boldsymbol{p}_i^{down} = [P_i^{\emptyset,down}]^T$ denote the upward and downward flexibility at bus $i \in N_{flex}, \emptyset \in \Phi^i$. The $\boldsymbol{p}_0^{sub} = [P_0^{\emptyset,sub}]^T$ and $\boldsymbol{q}_0^{sub} = [Q_0^{\emptyset,sub}]^T$ represent the active and reactive power exchange at the substation bus, respectively.

A. Unbalanced Distribution Network Model

The three-phase lines are represented by the π model, and we neglect the branch shunt admittance due to its small value compared to the series impedance. The diagonal and off-diagonal entries in $\boldsymbol{Z}_{ij}$ are the self-impedance and mutual impedance, respectively. The formulation of line impedance $\boldsymbol{Z}_{ij}$ is defined as:

$$\boldsymbol{Z}_{ij} = \begin{bmatrix} z_{ij}^{aa} & z_{ij}^{ab} & z_{ij}^{ac} \\ z_{ij}^{ba} & z_{ij}^{bb} & z_{ij}^{bc} \\ z_{ij}^{ca} & z_{ij}^{cb} & z_{ij}^{cc} \end{bmatrix} \tag{9}$$

Noted that the impedance matrix is in a complex form and is usually written as $\boldsymbol{Z}_{ij} = \boldsymbol{R}_{ij} + j\boldsymbol{X}_{ij}$, where the $\boldsymbol{R}_{ij}$ and $\boldsymbol{X}_{ij}$ is the resistance and reactance matrix, respectively.

The voltage regulators are important equipment for maintaining the voltage within the safety range. We assume that three single-phase voltage regulators are installed at a three-phase bus and the voltage at the primary and secondary side of the regulator is linked by the tap changer ratios R as equation (10):

$$\boldsymbol{v}_{sec} = R * \boldsymbol{v}_{pri} \tag{10}$$

where R= [$r_a \, r_b \, r_c$], r_{abc} =1+0.00625*Tap and Tap is integer representing the tap changer positions (constant value in this work) for each phase.

B. Objective Function

The original cost function to be minimized is defined as:

$$f(\boldsymbol{I}_{ij}, \boldsymbol{p}_i^{up}, \boldsymbol{p}_i^{down}, \boldsymbol{p}_o^{sub}) = \omega_l l(\boldsymbol{I}_{ij}) + \omega_g g(\boldsymbol{p}_0^{sub}) + \omega_\gamma \gamma(\boldsymbol{p}_i^{up}, \boldsymbol{p}_i^{down}) \tag{11}$$

where function $l(\boldsymbol{I}_{ij})$ denotes the line losses, $g(\boldsymbol{p}_0^{sub})$ is the cost of buying power from the main grid, and $\gamma(\boldsymbol{p}_i^{up}, \boldsymbol{p}_i^{down})$ is the upward and downward flexibility bids. The above multi-objective problem could be solved by the weighted sum method [27]. Also, ω_l, ω_g and ω_γ denote the weighting coefficients, which are positive values. In this paper, we have assumed that the weighting coefficients related to system status share a higher value than the cost component, e.g., $\omega_l > \omega_g = \omega_\gamma$, although the order could be adjusted depending on DSOs' preference.

The line losses component of the objective function is:

$$l(\boldsymbol{I}_{ij}) = \sum_{\emptyset \in \Phi^{ij}} \sum_{(i,j) \in E} r_{ij}^{\emptyset} {I_{ij}^{\emptyset}}^2 \tag{12}$$

where the ${I_{ij}^{\emptyset}}^2$ is the current magnitude square of the line $i \rightarrow j$.

The cost of the buying power from the main grid could be written as:

$$g(\boldsymbol{p}_0^{sub}) = \sum_{\emptyset \in \Phi^i} k_0^{\emptyset,sub} P_0^{\emptyset,sub} \tag{13}$$

where k_0^{sub} is the price of buying energy from the upper grid.

The bids for upward and downward flexibility services are shown in equation (14):

$$\gamma(\boldsymbol{p}_i^{up},\boldsymbol{p}_i^{down}) = \sum_{i\in N_{flex}}\sum_{\emptyset\in\Phi^i} k_i^{\emptyset,down}P_i^{\emptyset,down} + \sum_{i\in N_{flex}}\sum_{\emptyset\in\Phi^i} k_i^{\emptyset,up}P_i^{\emptyset,up} \quad (14)$$

where $k_i^{\emptyset,up}$ and $k_i^{\emptyset,down}$ are the bid coefficients for upward and downward flexibility services, respectively, depending on the behavior and bids of the aggregators.

C. Flexibility Constraints

The headroom of upward and downward flexibility resources is limited by $\overline{P_i^{\emptyset,up}}$ and $\overline{P_i^{\emptyset,down}}$, i.e.:

$$0 \le P_i^{\emptyset,up} \le \overline{P_i^{\emptyset,up}} \;,\; i \in N_{flex}, \emptyset \in \Phi^i \quad (15)$$

$$0 \le P_i^{\emptyset,down} \le \overline{P_i^{\emptyset,down}} \;,\; i \in N_{flex}, \emptyset \in \Phi^i \quad (16)$$

D. Network Constraints

The power flow constraint is defined as:

$$\sum_{i:\mathrm{i}\to\mathrm{j}} diag(\boldsymbol{S}_{ij} - \boldsymbol{Z}_{ij}\boldsymbol{I}_{ij}) + \boldsymbol{s}_j = \sum_{k:j\to\mathrm{k}} diag(\boldsymbol{S}_{jk}),\; j \in N \quad (17)$$

where $\boldsymbol{s}_j$ is the net apparent power injection, composed of $\boldsymbol{p}_j$ and $\boldsymbol{q}_j$ defined as:

$$\boldsymbol{p}_j = \boldsymbol{p}_j^{sub} + \boldsymbol{p}_j^{fix} + \boldsymbol{p}_j^{up} - \boldsymbol{p}_j^{down}, j \in N \quad (18)$$

$$\boldsymbol{q}_j = \boldsymbol{q}_j^{sub} + \boldsymbol{q}_j^{fix} \;, j \in N \quad (19)$$

Using Ohm's law voltage magnitude is:

$$\boldsymbol{V}_j = \boldsymbol{V}_i - (\boldsymbol{S}_{ij}\boldsymbol{Z}_{ij}^H + \boldsymbol{S}_{ij}^H\boldsymbol{Z}_{ij}) + \boldsymbol{Z}_{ij}\boldsymbol{I}_{ij}\boldsymbol{Z}_{ij}^H,\; i \to j \in E \quad (20)$$

upper and lower bounds on voltages magnitudes are defined as:

$$|\underline{V_i^\emptyset}|^2 \le diag(\boldsymbol{V}_i) \le |\overline{V_i^\emptyset}|^2 \;, i \in N^+, \emptyset \in \Phi^i \quad (21)$$

while the thermal limits of the distribution line are:

$$diag(\boldsymbol{S}_{ij}) \le \overline{S_{ij}^\emptyset},\; i \to j \in E, \emptyset \in \Phi^{ij} \quad (22)$$

E. Substation Model

The substation is the slack bus that provides a reference voltage as:

$$V_0^\emptyset = [1\angle 0, 1\angle 120, 1\angle -120]^T, \emptyset \in \Phi^i \quad (23)$$

In addition, assuming the substation services as generators, the lower and upper power exchange limitations are:

$$\underline{P_0^{\emptyset,sub}} \le P_0^{\emptyset,sub} \le \overline{P_0^{\emptyset,sub}}, \emptyset \in \Phi^i \quad (24)$$

F. Convex Relaxation Constraints

Proposed models are presented in matrix forms, and to ensure the solutions can be recovered to the original variables it is necessary to link the original variables to the new variables as:

$$\begin{bmatrix} \boldsymbol{V}_i & \boldsymbol{S}_{ij} \\ \boldsymbol{S}_{ij}{}^H & \boldsymbol{I}_{ij} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{i}_{ij} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{i}_{ij} \end{bmatrix}^H \quad (25)$$

where the equation (25) could be replaced by a rank and semidefinite constraint. Neglecting the non-linear rank constraint yields a convex relaxation optimization model which can be solved quickly using readily available solvers.

G. Flexibility Service Models

In this subsection, using the above objective function (11) and constraints (15)-(25), we define four different flexibility service models to be tested in this paper.

1) Benchmark Model

The benchmark model ignores the voltage unbalance limitation and is formulated as:

$$min(11)$$
$$s.t.\ (15) - (25)$$

2) VUF Regulation Model

The VUF regulation model is defined as managing the VUF value at each of the three-phase buses and is given by:

$$min\ (11)$$
$$s.t.\ (3), (15) - (25)$$

3) VUF Minimization Model

In this model, we assume the DSO is interested to reduce the VUF as low as possible, which can be expressed as achieved by the following formulation:

$$min\ ((4) + (11))$$
$$s.t.\ (15) - (25)$$

4) Phase Power Balance Model

Voltage unbalance mitigation is achieved by balancing the load on each phase, rather than introducing constraints on the VUF. This problem is formulated as:

$$min\ ((5) + (11))$$
$$s.t.\ (15) - (25)$$

V. Numerical Case Studies

Now, the above defined models will be used to investigate the proposed LFM designs. Firstly, we introduce the case study network, including its modifications and assumptions made. Secondly, the input data are defined. Next, we describe the work of DSOs before they sent the flexibility request to the market. Finally, we implement and compare different flexibility service models and carry out an error analysis to identify the accuracy of the proposed methods.

A. Network Modification and Assumption

A modified 13-bus IEEE test network shown in Figure 1 is used in the case studies. We assume the switch between buses 671 and 692 is closed, and the distribution transformer is modelled as a suitable line. The tap changer position, Tap, of the regulator for improving under-voltage situations during the high loads period is set to values of [5 5 6]. The distributed load is modelled as two individual loads located at the two ends of the line, with the assumption that all flexibility resources are single-phase connected at buses 671 and 675.

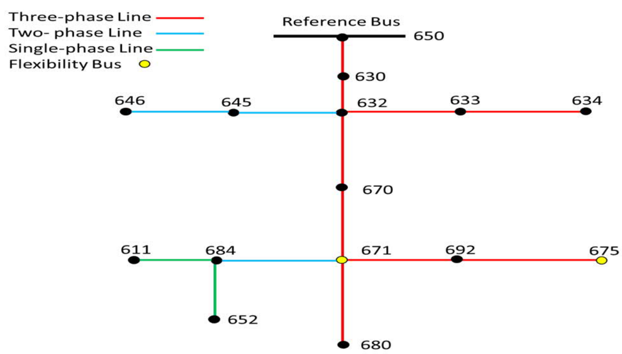

Fig. 1. Modified IEEE-13 bus test network

B. Input Data Description

The load profile of the network can be obtained from the day-ahead energy market or DSO forecasts. Figure 2 shows the net power exchange on the substation bus during the day. Due to the contribution of renewable energy generators such as PVs, light loads, as seen by the network, occur at midday, while heavy loads occur at night. In general, distribution networks cover a relatively concentrated geographical area, thus we assume a consistent pattern of net load variation over time for each phase.

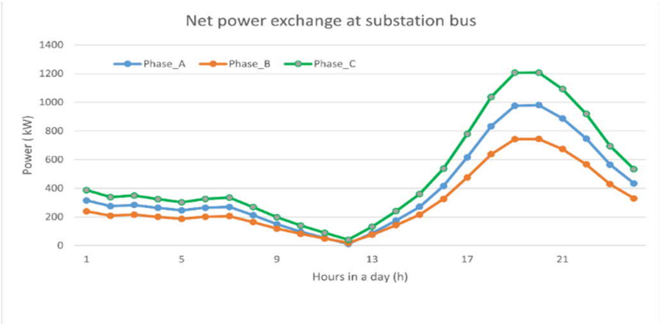

Fig. 2. Net load profiles at the substation bus during a day

Table 1 shows the available upward and downward flexibility, bids and location information for the case studies. In addition, assume this information is available before the market-clearing process and submitted by aggregators.

TABLE I. AVAILABLE FLEXIBILITY, BIDS AND LOCATIONS

Bus	Upward/Downward (kW)			Flexibility Bids (£/kWh)		
	Ph- A	*Ph-B*	*Ph-C*	*Ph- A*	*Ph-B*	*Ph-C*
671	30/30	20/20	20/20	0.2/0.2	0.37/0.37	0.8/0.8
675	30/30	20/20	40/40	0.3/0.3	0.38/0.38	0.9/0.9

C. Network Issues Identification

In the proposed LFM design, the role of the DSO is to identify the violations of network constraints and send corresponding requests for flexibility services to the market operator. The network status is checked by the OpenDSS software, which has a three-phase power flow function. In this paper, the permitted voltage magnitude is between 0.95 p.u and 1.05 p.u with a maximum VUF of 2%, which is used to determine whether a flexibility service needs to be triggered. Figure 3 shows the voltage magnitude and VUF values of the unbalanced network during a day. Based on the results of power flow, we can see that over-voltage events occur in phase C at times 9-14h, while under-voltage events occur at 19-21h, together with voltage unbalance problems. At 18h, the voltage magnitude of the phases is within the permissible range, however, the VUF exceeds the maximum value and therefore requireed the activation of flexibility resources to improve the operation of the network.

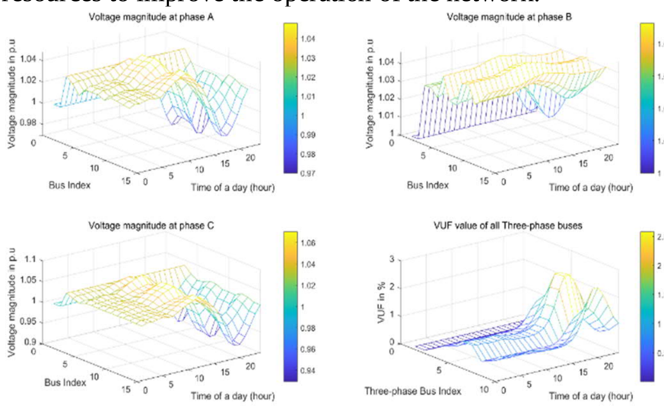

Fig. 3. Voltage magnitude and VUF values of a day

D. Results of Flexibility Market Clearing

From the above results, the flexibility services are required in the periods from 9-14h and 18-21h. For consistency with the content below, we have rearranged the chronological order from 0-10 to represent all AMTUs.

The network state after the market-clearing process is shown in Figure 4. For all models, the voltage magnitude is within the permissible range, which means the over and under-voltage problems are successfully addressed. However, for the benchmark model, the VUF still occurs, and is even higher than before optimization. This is because the market clearing process does not consider the voltage unbalance in that model. Figure 5 presents the maximum VUF at all AMTUs for different models. Therefore, the solution of the benchmark model may be rejected due to the failed VUF results starting from AMTU time 6. Models 2,3, and 4 have a similar trend but they successfully keep VUF within the limitations. Note that model 4 has provided lower VUF then 2 and 3, thus demonstrating, thus demonstrating the feasibility of mitigating voltage unbalance by balancing the load on each phase. VUF has a greater value during high load periods due to the large differences in load between phases from periods 6 to 10.

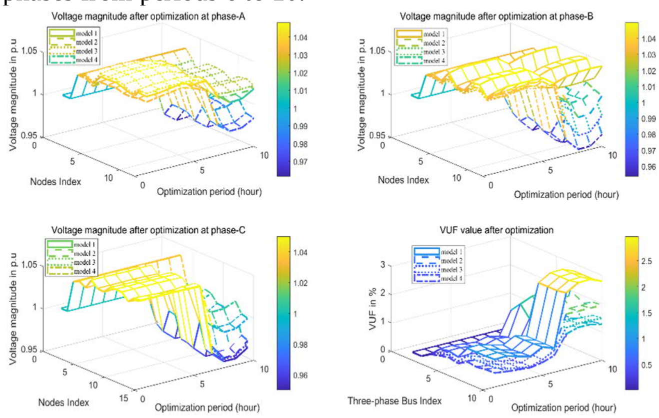

Fig. 4. Voltage magnitude and VUF values after the optimization

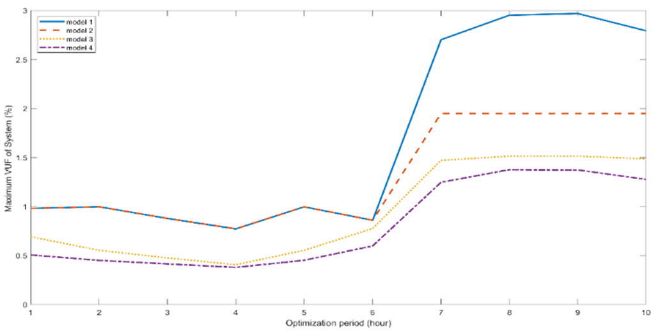

Fig. 5. Maximum VUF in the network of different models

Figure 6 shows the results of flexibility resources activation in different models, and the positive and negative signs represent the upward and downward flexibility, respectively. For the benchmark model, downward flexibility is required to reduce the voltage magnitude of phase C from time 1 to 6 at light loads periods, and vice versa stands for upward flexibility. From periods 1 to 6, the situation for model 2 is similar as for model 1, as the VUF constraint is not activated, as can also be seen in Figure 5. However, from periods 6 to 10, model 2 requires more flexibility resources to bring the VUF back within the permissible range by using upward flexibility in the high-load phases A and C and downward flexibility in the light-load phase B.

The difference between models 2 and 3 is mainly during periods 1 to 6, which can be explained by fromulation as VUF is part of the objective function of model 3, therefore during light load periods, model 3 will still attempt to reduce the VUF of the system by suppressing the tendency for load differences, e.g., by limiting the downward flexibility of phase C at bus 675, which may lead to an overcompensation problem. In model 4, instead of using metrics related to voltage unbalance, an effort is made to minimise the load differences between the phases. Thus, phase B, which has the lightest load during the considered time, will need to activate downward flexibility to facilitate the balancing of the phases, and, likewise, phase C, which is at a heavy load, will activate upward flexibility to achieve the same goal.

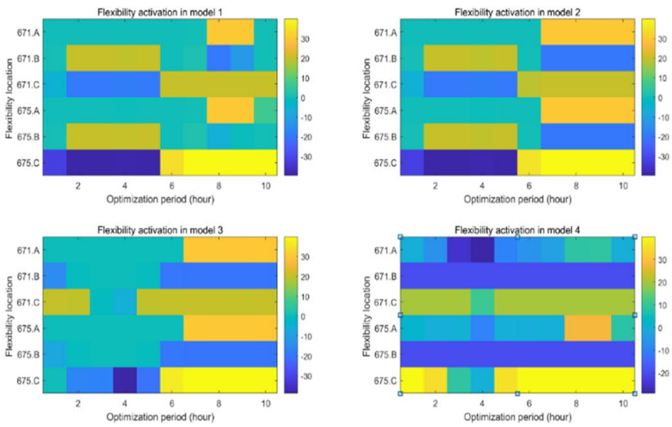

Fig. 6. The results of flexibility resources activation in different models

E. Error Analysis

The maximum voltage magnitude difference for all phases of the entire AMTUs are presented in Figure 7. The maximum voltage error is below $3.5e^{-5}$ for all time slots, thus demonstrating the accuracy of the proposed modes.

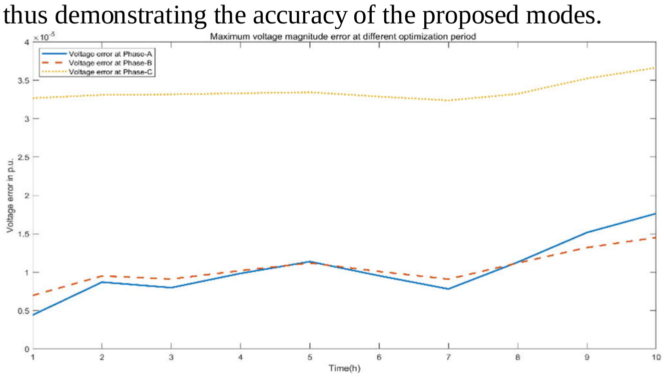

Fig. 7. Maximum voltage error results compared with OpenDSS

VI. Conclusions

The results show that voltage unbalance needs to be considered when designing flexibility services in the LFM. In addition, DSOs can choose between different options according to their respective preferences, but this is the questions which requires their additional attention and anlysis.

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