# Voluntary overbooking in commercial airline reservations

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This paper studies a voluntary overbooking model under rational expectation equilibrium to promote cooperation between customers and airlines, maintain goodwill of customers, and maximize the expected total returns to airlines. A decision tree analysis is constructed for both customers and airlines. Sensitivity analysis is conducted in both realistic and simulated no-show random variables for validation. The findings suggest considerable mutual benefits associated with a 'voluntary overbooking' policy that emphasizes mutual cooperation between passengers and commercial airlines. The main underlying assumption of the paper is that customers are willing to provide valuations to airlines seeking volunteers for overbooking. The originality of the proposed

This is a peer-reviewed, accepted author manuscript of the following article: Dalalah, D., Ojiako, U., & Chipulu, M. (2020). Voluntary overbooking in commercial airline reservations. *Journal of Air Transport Management*, *86*, Article 101835. https://doi.org/10.1016/j.jairtraman.2020.101835

model is the incorporation of elements of the Rational expectations hypothesis into classical overbooking models gleaned from the literature.

**Keywords**: Logistics; Modelling; Overbooking; Commercial airlines; Reservations

## 1. Introduction

### *1.1 Articulation*

The importance of revenue management in general and overbooking in particular to the commercial airline industry cannot be over-emphasized. This is particularly the case in light of the intense competitive within the industry (and ensuring narrow operating and profit margins). While the practice of overbooking plays a critical role in the income of commercial airlines, its practice is susceptible to high-profile cases of service failure that can lead to serious reputational damage. For example, on 9 April 2017, a video of David Dao Duy Anh, being violently dragged off from United Airlines Express Flight 3411 went viral on social media, sparking global outrage. Thus, in addition to the United Airlines Express Flight 3411 incident, other high profile service failures due to overbooking have attracted the attention of scholars. For example, in July 2017, it was reported that United Airlines had forced a passenger to hold her two-year-old child on her lap over the duration of a four-hour flight. This was after reselling the child's seat just before departure to a standby passenger (Zhang, 2017).

The United Express Flight 3411 incident was a direct result of a well-recognized (at least among scholars) practice of 'overbooking', which commercial airlines engage in. Generally, in order to safeguard against 'last minute' passenger cancellations and no-shows (which will lead to loss of revenue coming from refunds and unused seats), most commercial airlines engage in a number of practices. *First*, airlines sometimes engage in '*Discount allocation'*. In effect, this policy involves product segmentation and differentiation (Botimer, 1996). In other words, on a standard flight, airlines may offer different classes of product – usually ‘business class’, ‘premium economy class’ and ‘economy class’. These classes tend to be differentiated based on easily identifiable product and service offerings – seating position, size of seats, services – such as check-in services and meals and premium drinks. Discount allocation can also entail offering seats to customers (usually, those who book and confirm early) at discounted prices. *Second,* commercial airlines can also engage in ‘*Destination control’*. This policy is highly dependent on the use of airport hubs and involves commercial airlines exploiting the diversity of the destinations they fly to in order to board as many customers as possible. One approach that airlines are utilizing to exploit destination control is through their membership of various sky alliances (Iatrou and Alamdari, 2005). *Third* (finally), commercial airlines can also engage in ‘*Overbooking’.* This is the practice of ‘selling’ more tickets for seats than are actually available on a specific flight.

An outline review of the literature suggests that ‘Overbooking’ is a popular form of revenue management (Rose, 2016; Yeoman, 2016; Balaiyan et al., 2019). It is also a revenue management practice of significant economic importance to airlines. For example, Zaki (2000) and Bailey (2007) claim that revenue management contributes approximately 10% additional income to commercial airlines.

Originating in the airline industry (see McGill and van Ryzin, 1999), revenue management serves as a means for inventory management of perishable items such as airline seats. Of particular importance is the need to maximize earned revenue from the sale of airline seats within a specific period of time (in this case, the scheduled flight departure time). In the context of our study, perishable and non-perishable products are differentiated by the fact that where products (such as airline seats) are presumed perishable, it is assumed that their economic value is permanently lost when un-occupied during a scheduled flight.

Revenue management is particularly important to the commercial airline industry because it is an industry not only characterized by intense competition (Douglas, 2019), but also one that generally operates within very narrow profit margins. Balaiyan et al. (2019) for example quote a profit margin of approximately 6.8% for commercial airlines. Commercial airlines therefore practice overbooking because they cannot predict whether allotted seats will be occupied by a passenger until the flight departs. ‘No shows’ is an interesting area, generally, estimated to be between 10% and 15% of all reservations (Suzuki, 2006). It does have a major financial implication for commercial airlines. Although cancellations and ‘no shows’ are more likely to emanate from business than from leisure passengers (Walczak et al., 2012), commercial airlines are generally reluctant to penalize customers for ‘no shows’ because these tend to be business customers who pay full fares (Schubmehl et al., 2002).

### *1.2 The question of goodwill*

The practice of overbooking primarily assumes that, for a combination of factors, not all passengers will eventually turn up to board a pre-booked flight. Contributing factors could be that the passengers may have cancelled their reservation almost immediately before the flight was scheduled for boarding or departure, or that the customers (passengers) arrived late and were not able to board, or did not turn up at all (the ‘no shows’), or were denied boarding for a number of reasons. These reasons could be for example where the customer (passenger) did not have the appropriate form of identification or for other security or health reasons.

Nevertheless, as in the case of United Airlines Express Flight 3411, sometimes a particular flight may experience most (if not all) passengers with a valid ticket turning up to board. This creates a situation that there are more passengers ready to board than are available seats on a particular flight. When this happens, commercial airlines will usually explore a number of options to determine those customers who will fly. For example, the commercial airline may choose to ask for passengers to volunteer not to board (volunteer-denied boarding), or the airline may deny a passenger the opportunity (or perhaps, the right) to board their flight. This is even when those specific customers have a perfectly valid ticket or boarding pass. Alternatively, as in the case of United Airlines Express Flight 3411, passengers may be denied the opportunity to fly (even when they have a perfectly valid boarding card) and, as in the case of David Dao, this may well be when the customer has already boarded their flight and been seated. This form of denied boarding is referred to as *involuntary denied boarding*. It is also the form of denied boarding likely to bring reputational risks (and loss of goodwill) to commercial airlines (Guo et al., 2016; Ma et al., 2019).

As expected, following the backlash over the violent nature of the attempt by United Airlines Express Flight 3411 to enforce its involuntary denied boarding policy, the company announced a series of initiatives to address its overbooking policies (Mutzabaugh, 2017) and facilitate regaining customer trust and goodwill. These initiatives included *firstly* an actual reduction of the volume of overbooking that the company engaged in. *Secondly*, the company increased customer compensation for voluntary denied boarding to $US10,000 and further announced that customers may be able to negotiate their own compensation to allow for voluntary denied boarding. *Thirdly*, the company created an automated system to solicit for volunteers prepared to voluntarily be denied boarding. This particular initiative was in response to accusations that the David Dao incident was triggered by how he had been ‘selected’ for denied flight. Some will indeed claim that the rate of involuntary denied boarding has sharply declined following the United Express Flight 3411 incident (Zumbach, 2018) and it may no longer be a major problem for airlines. Data obtained from the United States Department of Transportation (2019) for example, suggest that between January and March 2018, the average rate of reported involuntary denied boarding (per 10,000) passengers was 0.15%, compared to 0.32% for the months between January and March 2019. However, this argument must take into consideration studies by Chipulu et al. (2016) who found that individuals will undertake a number of positive steps and activities when they perceived that service operations entities have violated specific ethical norms and expectations. Thus, it is very possible that the United Express Flight 3411 incident was perceived as a major ethical violation by specific customers. The literature further tells us that customers are no longer passive in terms of their engagement with service providers (Chipulu et al., 2016, 2018).

### *1.3 The problem statement*

Commercial airlines utilize overbooking as a revenue management policy tool. However, as shown in the United Airlines Express Flight 3411, if not carefully implemented, its use can serve as a source of reputational risk concerning significant loss of customer goodwill costs (Guo et al., 2016; Ma et al., 2019). Pizam (2017) suggests that commercial airlines are legally permitted to invoke involuntary denied boarding either by (i) discretion, in effect, randomly, (ii) by relying on loyalty program membership – where non-members of its loyalty program are involuntary denied boarding, or (iii) selecting customers based on their fare status – here, customers with discounted and then cheaper economy tickets are involuntary denied boarding in order to cater for those with full priced or premium priced tickets.

The main premise of our study therefore is that while the new incentives announced by the commercial airlines industry following the United Express Flight 3411 incident may help alleviate the overbooking problem (and go a long way towards rectifying the associated reputational damage and loss of customer goodwill), there are no guarantees that these initiatives will prevent another such incident from occurring in the near future.

The view that there are no guarantees that the different initiatives announced by commercial airlines to hedge against involuntary denied boarding will work is a sentiment a number of airlines and commentators share. In fact, some commercial airlines such as Southwest Airlines have ceased the practice of overbooking (from May 2018). Conversely, commentators such as Lori Aratani (2018) opined that no substantial progress has been made to address the overbooking problem experienced by commercial airlines.

### *1.4 The research aim*

Driven by the notion that revenue management is particularly important to the commercial airline and also that overbooking is not only a popular form of revenue management, but also one of significant economic importance to airlines, it is safe to opine that it is unlikely that other airlines will follow the example of Southwest Airlines and cease practicing overbooking. With this in mind, interest turns to how the practice of overbooking can be undertaken in a manner whereby (i) the inevitable involuntary denied boarding does not occur (in effect, we experience a wholly *voluntary* denied boarding policy) and (ii) customer goodwill is maintained. We draw upon the marketing notion of the customer as co-producer (see for example, Wikström, 1996) which we extend to the *Rational expectations hypothesis* (see Lucas, 1972, 1973, 1975) to suggest that an overbooking policy based on voluntary denied boarding that ensures the maintenance of customer goodwill is feasible where there is full co-operation (underpinned by full information disclosure) between commercial airlines and their customers. Increased *voluntary* denied boarding will also certainly lead to fewer passengers being violently dragged off commercial airlines. It will also certainly lead to fewer perceptions of ethical violation by other and potential passengers, and certainly protect the revenues of commercial airlines.

With this in mind, *we aim to develop a new 'voluntary overbooking' customer -focused model that espouses cooperation between passengers and commercial airlines*. The model will represent the reaching/establishment of a rational expectation equilibrium (REE) between commercial airlines and their customers (passengers). We argue that such a model serves as the best means of preventing the loss of valuable customer goodwill. The proposed model will take into consideration a number of factors from both the airline customers' perspective and that of the commercial airline. For example, from the customers' perspective, the model will consider the bargain-hunting behavior of airline customers as relates to the prospect of cheaper flight tickets, money-back guarantees, and refunds. Another factor we consider is the customer belief of being seated in a booked flight; for example, a passenger has to weigh a choice between overbooking on his preferred flight (if the flight is booked) at a discounted or lower price against buying a seat on another flight at a regular price. On the other hand, for the commercial airline, the model will have to take into consideration the number of tickets the airline will allocate for overbooking, the price for such tickets, and their refund value (for customers who accept overbooking and may be denied boarding upon their prior knowledge of the possibility of this act). It is important to highlight at this juncture that our study has focused exclusively on passengers operations and that our model will not include cargo.

The rest of the paper proceeds in the following manner. In the section that follows (section 2), a literature review of overbooking revenue management and overbooking models is presented. In section 3, we present the voluntary overbooking model. The Rational Expectations Equilibrium (REE) is illustrated in section 4 while data validation is undertaken in section 5. Finally, section 6 concludes.

## 2. Literature Review

### *2.1 Overbooking and revenue management*

According to Klein et al. (2019), revenue management originated from research on commercial air transport studies. For a number of years, logistics research in air transport focused more on cost reduction than on revenue generation (and its management) (see Ballou, 2006). With its focus on demand management (in effect, the need to balance customer requirements with supply chain capabilities – see Croxton et al. 2002; Bish et al., 2004), revenue management sought to optimize the selling of commercial airline seats within a specific time. The importance of revenue management to the commercial airline industry stems from the reality that commercial airlines have operated their services almost solely on the basis that the customer is obliged to book for a service prior to its use (in effect, customers must make advanced bookings for flights). Furthermore, seats on commercial airlines were ‘perishable’ as they were not only characterized by low variable costs and high fixed costs, but also that unused seats attracted a zero residual value (Guo et al., 2013, 2016). Unused airline seats thus created significant unrecoverable revenue to commercial airlines.

Overbookings as a revenue management topic (Ivanov and Zhechev, 2012) has also been the subject of much debate from as far back as 1958 with Beckman’s (1958) seminal paper and was of much interest among scholars such as Simon (1970). Our review of these early literatures shows their particular interest in reducing the possibility of involuntary denied boarding. Among the various options and approaches that these studies investigated were the use of auctions and also the use of different ticket/fare classes for booking reservations. Drawing from Harewood (2006), the ability to effectively utilize market segmentation allows commercial airlines to – in effect – sell airline tickets to different segments of air travelers at competing prices. However, revenue management plays an important role in commercial airlines operations in that it provides the necessary tools to effectively balance changing demand to supply in a short period of time. Commercial airlines' revenue is considerably impacted by fixed and variable costs. For example, in the case of fixed costs, airlines incur significant costs associated with expenditures such as fuel and oil, maintenance, and hangar rentals (McGill and Van Ryzin, 1999). These fixed expenditures are generally present regardless of the number of flights being operated. To thus ensure that overall costs are under control and do not escalate (and profit margins are increased), commercial airlines expend considerable effort to reduce variable costs, such as those relating to customers (passengers).

Generally, commercial airlines face either of two demand and supply challenges emanating from their low variable cost and high fixed cost. When the demand for airline tickets exceeds the number of airline seats available, commercial airlines will have to determine which customers to sell tickets to (generally, at pre-announced prices). Conversely, when commercial airlines have more seats available than there is demand for (when supply is more than demand), noting the perishable nature of tickets, airlines will generally be prepared to sell tickets at discounted prices (as long as it is above its variable cost).

Scientific methods have been successfully used to improve revenue management in the commercial airline industry (Singhaseni et al., 2013; Rose, 2016; Yeoman, 2016). One of those improved areas is in seat pricing and allocation (see Singhaseni et al., 2013; Lan et al., 2015; Hjorth et al., 2018; Ko, 2019). Commercial airlines will reserve airline seats in excess of their existing capacity (overbooking) as a way to counter the loss of revenue from ‘no shows’, late cancellations, and early and/or late departures. Passengers may also have to disembark a flight for a number of reasons – for example, when a passenger becomes unwell shortly after boarding. Generally speaking, there is substantial research available that has not only highlighted the overbooking problem but has also specifically sought to identify means of making the process more effective and efficient (Post and Spann, 2012; Lan et al., 2015). There are also numerous studies available that have examined commercial airlines overbooking from a *customer satisfaction* perspective (Guo et al., 2016; Pizam, 2017; Ma et al., 2019), from the perspective of *ethics* (Sorell, 1994) and also from a *legal* perspective (Crespo-Almendros and Del Barrio-García 2016; Pizam, 2017). Some studies tend to focus more on the operations perspective, addressing the practice from both operations management and operations research perspectives. For example, such studies examine the optimization of booking levels (Lardeux et al., 2019), strategies for seat allocations (Lan et al., 2015; Büsing et al., 2019), and various inventory management practices (Rose, 2016).

### *2.2 Overbooking models and customers*

A good body of research exists on overbooking policy in commercial airlines. However, most of this research assumes passenger unawareness of their overbooking policy (Suzuki, 2002). A general review of the overbooking literature (see Bergemann and Said, 2011) suggests that overbooking models come in two types; those that are *dynamic* and those that are *static*. The static model ignores the changing nature of reservation requests and customer cancellations. On the other hand, the dynamic model accounts for such temporal changes. In most static models, the demand distribution is assumed to be binomial (Talluri and van Ryzin, 2004). Other static models assume that the random show-up rate can be modeled using a parametric distribution such as the uniform or Beta distributions (Luo et al., 2009). Researchers also argue that a normal distribution of show-ups, while quite common in practice, is not necessarily the best choice (Popescu et al., 2006).

Restricting the available fare options to a single fare type means commercial airlines potentially lose their ability to attract different classes of passengers. However, in dynamic pricing schemes, the system status is accounted for and different passenger classes are considered (Schütz and Kolisch, 2013). In early studies undertaken by Simon (1994), auction mechanisms were observed to be acceptable responses to customers. Generally, customers for boarding denial were selected on the basis of their willingness to give up their seats (in effect, they were mainly volunteers). However, it was unclear as to whether those volunteers took such decisions before or after they became aware of their values. In response, a number of scholars (see Deb, 2010; Möller and Watanabe, 2010) sought to develop decisional models that provided commercial airlines with the tools necessary to determine which customers should be subject to boarding denial based on the nature of their prior information and valuation. Some research papers assumed that customers face no uncertainty about their future valuations; examples include Gershkov and Moldovanu (2013), Hinnosaar (2015) and Board and Skrzypacz (2016). Further examples include work by Pai and Vohra (2013) and Mierendorff (2016). The treatment of both dynamic arrivals and valuation uncertainty builds on recent work of Garrett (2016, 2017).

While overbooking appears to provide benefits to both commercial airlines and customers (see Schubmehl et al., 2002), it presents customer service-related challenges which commercial airlines (particularly in light of the United Airlines Express Flight 3411 incident) need to be aware of (see Guo et al., 2016; Pizam, 2017; Ma et al., 2019).

More specifically, Wangenheim and Bayón (2007) explored how customers tend to respond to specific overbooking experiences from the perspective of three overbooking recovery strategies: (i) denied service (in other words, involuntary denied boarding by airlines); (ii) downgrading (when an alternative service, but of poorer grade or quality is offered), or (iii) upgrading (when an alternative service, but of superior grade or quality is offered). Their study found that customers who experienced negative consequences of overbooking (such as involuntary denied boarding or downgrading) generally will seek to eventually lower the nature of any future transactions they will have with the specific airline. In particular, they found that the impact of such negative experiences was particularly more pronounced among higher-value customers (who are generally business travelers). Along the same lines as Wangenheim and Bayón (2007), Lindenmeier and Tscheulin (2008) found that involuntary denied boarding induced major customer dissatisfaction. In fact, they suggested that the rate of this dissatisfaction countered any potential gain from overbooking as a form of revenue management. Wittman (2014) also found that involuntary denied boarding attracted increased customer complaints. More recently, studies have found that customers' engagement (particularly repeat customers) may be significantly driven by how customers assess the way and ethical manner within which organizations (are perceived to) engage with other customers (Chipulu et al., 2016, 2018; Beck et al., 2018; Ma et al., 2019). Interestingly enough, seasonal variation has also been found to play an important role in airfare and capacity management. For example, Merkert and Webber (2018) found stronger seasonal variation in the average airfare than in the seat factor. The implications of their findings being that airlines were to be better placed allowing higher seat factor and average airfares at times of seasonal highs as against seasonal lows.

We do know from Zhang and Chen (2013) that while different customers hold different expectations relating to overbooking recovery strategies, theories of aversion to loss (see Kahneman and Tversky, 1979; Dalalah et al., 2015) suggest that the prospect of losing will have a greater influence on how an individual judges their options than the prospect of an equivalent gain would have. Guo et al. (2016) examined the prospect of involuntary denied boarding by airlines by seeking to establish the level of monetary compensation that will convince a customer to volunteer for denied boarding. They opined that involuntary denied boarding was deemed a terrible experience by most customers. Pizam (2017) outlined lessons learnt from the United Airlines Express Flight 3411 incident, noting for example that it was essential that one proposed approach to address the current controversies with overbooking was not to implement the policy as a means of increasing aircraft occupancy for seats which had been booked with non-refundable tickets.

## 3. The Model

### *3.1 Starting assumptions*

The model that we propose maintains some underlying assumptions. Generally, in risk-neutral revenue management works of scholarship, we observe that the anticipated revenue is generally maximized in the absence of considering other parameters (Terciyanlı and Avșar, 2019). In this study, we *first* assume that, over time, individual passengers will become more familiar with ascertaining the value of their airline tickets and will be willing to provide those valuations to airlines seeking volunteers for voluntarily overbook. *Second*, we assume that the airlines will fully commit to their terms and conditions at the time of sale. *Third*, we also assume that the airlines will have the expertise to interrogate data in their possession and utilize analytics to arrive at informed decisions on ‘no shows’ as a means of encouraging customers to willingly engage in voluntarily overbooking. We posit that both passengers and commercial airlines are likely to benefit from such a policy in that passengers develop more awareness of their boarding/seating chances while commercial airlines develop more understanding of passengers’ reserved prices. Matching the two will help attain an equilibrium of beliefs of which optimal quantities and prices will become computable over time. By informing customers purchasing flight tickets that they can make a reservation at a cheaper price (but that there is the possibility of overbooking/denied boarding), the decision is left to the passenger whether or not to commit to the terms of overbooking. We acknowledge that in real life applications, those responsible for revenue management will need to take into consideration numerous parameters in order to capture the essence of revenue behavior under different policies.

Consider a customer who is exploring flights to a specific destination and all the flights are being offered at the same regular seat price. Among these flights, the customer favors a specific one (referred to as ‘preferred flight’). If the customer’s preferred flight is not yet booked, he or she can proceed to purchase a seat at its regular price. However, if the preferred flight is completely booked, the passenger is offered to overbook his or her preferred flight at a lower price without a confirmed boarding. Alternatively, the passenger can purchase a ticket at the regular price with a confirmed seat in another (likely undesirable) flight heading that still has empty seats. However, a customer who accepts the overbooking deal, ‘money-back’ and refunds are guaranteed. Thus, unless the customer’s valuation of his preferred flight is the same as the valuation of other flight or flights with different departure times, there is a probability (inspired by the cheap price and potential seat availability) that the customer will accept the overbooking offer. Hence, overbooking becomes a ‘win-win’ gamble from the point of view of the customer. This is the policy we refer to as ‘voluntary overbooking’. In this policy, it is both the price (expected to be lower) and the quantity (number) of tickets available to customers that changes the revenue position. Hence, the key decision variables in this policy are (i) ticket prices and (ii) the quantity of tickets available for purchase.

### *3.2 Overbooking in economy class*

The model is set within the economy class of an airline. Economy-class customers are considered in this model for two reasons. First, as earlier highlighted, they are the least likely to be late cancellations and ‘no shows’ (see Walczak et al., 2012). Second, we seek to avoid the impact of the ‘adjustable-curtain’ strategy whereby some airlines adjust the size of the business class compartment to accommodate more higher-paying customers and thereby subject economy-class passengers (who are flying on the lower price tickets), to involuntary denied boarding (see Ringbom and Shy, 2002).

Thus, let the number of economy-class seats of some flight be $s$, where each seat is sold by the airline company for a regular price $p_2$. The ticket prices do not change as long as there are vacant seats. Once the seats are sold out and to make overbooking more attractive, the seat price drops to $p_1$ and the overbooking option becomes available to customers. Those who buy an overbooked seat will have a probabilistic chance of boarding while they wait and expect ‘no shows’. If there is a prevailing philosophy of cooperation between passengers and commercial airlines, then both will be willing to share information such as flight valuations and no-show statistics.

Two questions of interest arise. First, why would any customer be inclined to purchase a ticket under overbooking terms? The answer indeed depends on the valuations of the decision problem, customer beliefs and urgency to travel. We opine that as long as there is a belief a customer may not show up, there will be other customers who will accept the policy of overbooking (provided that the past average of no shows is announced for the flight). Here, low averages will imply low overbookings while high averages will suggest likely high overbookings (inducing customers to overbook). Overbooking can be of great help for customers who have experienced interruptions during transit trips, those travelling for emergencies or those who have made a late plan to travel on a specific day (for example, for national holidays such as Thanksgiving or for Spring break). Besides, the customer would have had to value the various incentives available, such as the prospect of a cheaper flight ticket, money-back guarantees, refunds, assessment of the probability that there will be 'no shows' and also allocate a valuation against the preferred flight. A fixed cost of $C$ per flight and a variable cost of $c$ per passenger are incurred by the airline. All seats can be sold for $p_2$ as long as there are available seats on the passenger's preferred flight, which is the same price as the seat on other flights (most likely, later flights) departing to the same destination. Suppose that the customer values the preferred flight seat by $v_1$ and that of other or later departing flight by $v_2$ where $v_2 \leq v_1$ (i.e., the passenger favors to travel on his preferred flight rather than other flights, where, the earlier the flight, the higher the chance that the flight is fully booked). Passengers of valid tickets, if not seated, will get their money back in addition to a refund of $r$. For correct settings of the model, the inequalities $c \leq p_1 \leq p_2 \leq v_2 \leq v_1$ must hold, which simply state that the cost must be less than the prices and the prices must be less than the passenger flight valuations. The second question of interest then asks why would an airline guarantee 'money back' and possibly consider paying extra to a customer denied boarding who signed up to its overbooking policy? The answer resides in the airline's interest in maintaining its overbooking policy, but doing so in a manner that will secure profit and customer goodwill.

Let the number of overbooked seats be $Q$ and the number of no-shows a random infinitesimal variable $X \geq 0$ of the probability density function $f(x)$ and a cumulative function $F(x)$. By assumption, $f(x)$ exhibits an increasing failure rate, i.e., $f(x)/(1-F(x))$ is increasing in $x$. The goal is to determine the optimal overbooking quantity (number) and the best prices for which overbooked seats are sold. However, since the passenger is a component of the decision-making in this policy, the temporal behavior of the passengers is considered via a Rational Expectation Equilibrium (REE).

### *3.3 The passenger decision problem*

In a prevailing environment of cooperation between passengers and commercial airlines, we posit that there will be three main underlying assumptions governing the decision problems that passengers face. *First*, we assume their willingness to declare their seat valuation to the airline. *Second*, anticipated passengers who voluntarily select to overbook acknowledge that there is a possibility that they will be subjected to involuntary denied boarding. *Third*, we assume that information on historical 'no show' is available to these passengers.

The decision problem faced by these passengers if they find their preferred flight fully booked is that they have a choice between overbooking on their preferred flight at a price $p_1$ or buying a seat on another (most likely later) departing flight at a price of $p_2$. In the model settings, the airline commits to seat all regular-price customers (in effect, customers holding 'economy-class' tickets), while those on the overbook list may be denied boarding, but with their complete knowledge of this possibility. Voluntary overbooking reduces the potential of damaging customer goodwill (Wirtz et al., 2003). Our argument is that there is an equilibrium point that makes overbooking on a preferred flight as attractive as a confirmed booking on other flights characterized with undesirable times/dates/attributes. The equilibrium point should also maximize the airline revenues and should break the tie in the passenger decision problem.

On a certain flight, when 'no shows' occur – that is the boarding gates are closed – if $(Q \leq x)$, then $(x - Q)$ seats could have been sold and those passengers would have been seated. On the other hand, if $(Q > x)$, then $(Q - x)$ passengers will be transferred to another flight with refunds. Accordingly, the net wealth of each decision and the states of nature are shown in Figure 1 (below). The "Not-seated" bottom branch means money-back plus refund – that is $-p_1 + p_1 + r$.

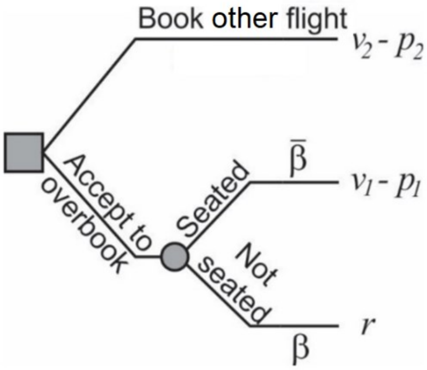

**Figure 1**: The decision problem of the passenger

Customers who are strategic (those who will entertain the decision problem) would expect lower overbooking price $p_1$. However, the passengers' decision depend on their valuations and risk outlook. If the passengers have a belief (i.e., probability) $\bar{\beta}$ of their chances of being seated, then, to prefer overbooking over another/later departing flight, the following inequality must hold:

$$v_2 - p_2 \leq \bar{\beta}(v_1 - p_1) + \beta r \tag{1}$$

where, $\bar{\beta} + \beta = 1$. Note that, if the passenger prefers to overbook, this means that his expected utility of this option is higher than that of a confirmed booking. On contrary, if the passenger prefers confirmed booking, his utility in this case would be higher for this option. Indeed, if the utility is to be considered, our model will not violate the Expected Utility Theory anyway. In this model, both the passenger and airline build their own risk outlook (i.e., probabilities/prices). As the passengers form a probability of their chances of being seated, they can reserve a seat price $p_p$ that is not exposed to the airlines. Similarly, the airlines form an outlook on the passenger's reserved price $p_A$.

Since $p_1 \leq p_2$ and $v_2 \leq v_1$, summing the two inequalities and rearranging the variables result in $v_2 - p_2 \leq v_1 - p_1$. As airlines will not pay back more than the maximum valuation of the passenger in the case of denied boarding, the refund is bounded by the difference between the passenger valuation and the airline price of an empty seat, that is, $r \leq v_2 - p_2$. Since $r \leq v_2 - p_2 \leq v_1 - p_1$, there is always a probability that overbooking on a preferred flight (which entails a chance that the customer can get the seat at a lower price $p_1$) is preferred over the act of waiting for a confirmed booking on undesirable/other flight.

We adopt the following sequence of events. First, the airline develops a belief $p_A$ over the customers' reserved prices then optimally chooses the price $p_1$ and the quantity $Q$ according to the proposed formulas for those quantities in this model. On the other side, the customer privately establishes a belief/probability $\bar{\beta}$ over the chances of being seated and then forms the reservation price $p_p$. Next, 'no shows' are realized (that is, they occur), and then boarding/denials take place.

Finally, all those who are not seated are refunded an amount of $r$, and then their flights are rescheduled upon their approval.

Based on the customer outlook of $\bar{\beta}$, the expected surplus at an overbooking price of $p_1$ is $\max\{v_2 - p_2, \bar{\beta}(v_1 - p_1) + \beta r\}$. The first term is the surplus from the purchase of a seat at its regular price $p_2$. The second term is the expected surplus if the passenger decides to overbook with a probability $\bar{\beta}$ that they will end up with $v_1 - p_1$ and a probability of $\beta$ with $r$. Because the passenger will pick the more attractive option between overbooking on the preferred departing flight or booking on another undesirable flight, they will overbook at a price $p_1$ if and only if $v_2 - p_2 \leq \bar{\beta}(v_1 - p_1) + \beta(r)$. In other words, given the passenger's expectation $\bar{\beta}$, the maximum passenger's reserved price for overbooking is

$$p_p \leq \frac{v_1 - v_2 + p_2 - \beta(v_1 - r)}{1 - \beta}\,. \tag{2}$$

As a remark, if the passengers' outlook probability of being seated is high ($1 - \beta = \bar{\beta}\uparrow$), they will expect a higher overbooking price $p_1$ and thus, is likely to reserve higher $p_p$. Similarly, if the passengers' probability of being seated is low ($\bar{\beta}\downarrow$), their reserved price $p_p$ will be low. If the passengers' assessment is that they are unlikely to be seated, that is, $\beta$=0, their reserved price will be the same as the regular price $p_2$ plus the difference in valuation of other/subsequent flights. This can be easily verified as $\lim_{\beta\to 0} p_p = v_1 - v_2 + p_2$ and $\lim_{\beta\to 1} p_p = \infty$. The norm in such decisions is that the passenger valuation of the preferred flight ($v_1$) is higher than the passenger's valuation of other flight ($v_2$). If the customer valuation of both flight seats is the same – that is $v_1 = v_2$, then for any positive probability of being seated, the passenger's reserved price will be less than $p_2$. The passengers will reserve exactly as much as $p_2$ if their belief that they have an excellent chance of being seated – i.e., $\bar{\beta} \to 1$. If the announced price $p_1$ is higher than the passenger's reserved price $p_p$, the passenger will not agree to overbook. The price $p_1$ is the tipping point in terms of the decision to either overbook on a preferred flight or book (confirmed) on another-but-undesired flight. The lower the benchmark price, the greater the passenger's preference for overbooking, as shown in Figure 2, below.

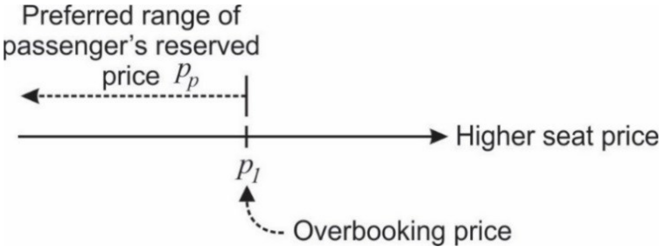

**Figure 2**: The range of the passenger's reserved price compared to the actual price

Underlying the above is the assumption that not only will similar passengers share the same outlook $\bar{\beta}$ and the same reservation price $p_p$, but also that all passengers are risk-neutral and hence will not discount future payoffs. In effect, the passenger's valuation of the difference between any two prices will be the same regardless of their financial status (wealth).

### *3.4 The airline decision problem*

For the commercial airline, the decision problem consists of identifying three quantities. These are (i) the overbooking quantity (in other words, the number of tickets available for overbooking $Q$), (ii), the overbooking price (in other words, the price of tickets available for overbooking) $p_1$ and (iii) the refund (in other words, the value of the refund offered when involuntary denied boarding occurs) $r$. The airline does not know the passenger's reserved price $p_p$ so they estimate what that price is likely to be. Suppose that the airline forecasts $p_A$ as the passenger reserved price.

Accordingly, the airline will prefer to have $p_1$ (actual money needed for purchasing an overbooked seat) higher than their expectation ($p_A$) in order to increase revenues. Selling overbooked tickets at a price that is lower than $p_A$ yields less revenue anyway. Figure 3 (below) shows the preferred range of $p_A$, that is $p_A \geq p1$.

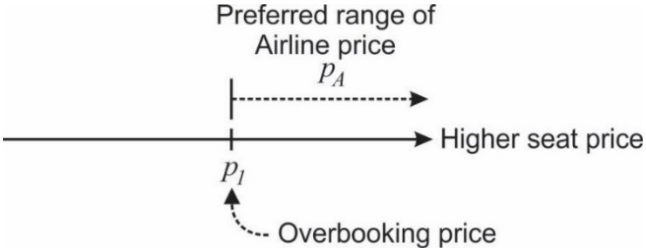

**Figure 3**: The range of airline preferred price compared to actual price

The potential revenues and the net wealth are shown in Figure 4 (below).

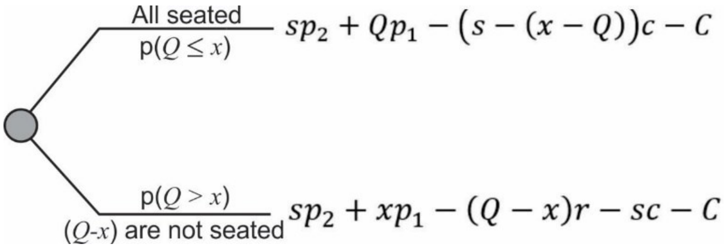

**Figure 4:** The potential revenues from the airline's point of view

If the aircraft is underutilized, the net wealth will be equivalent to $sp_2 + Qp_1 - \left(s - (x - Q)\right)c - C$, which is detailed as: $s$ sold seats at a price of $p_2$, the overbooked quantity at a price of $p_1$, $s - (x - Q)$ boarded passengers at a cost of $c$, and the fixed cost $C$ per flight. When the number of 'no shows' is less than the quantity $Q$, the total net wealth is given by $sp_2 + xp_1 - (Q - x)r - sc - C$, which includes the revenue of regular seats at a price of $p_2$, the overbooked quantity (number) at a price of $p_1$, the refund for denied boardings, the variable cost of boarding passengers at $c$, and the fixed cost $C$ per flight. Since the number of 'no shows' is stochastic of a probability distribution $f(x)$, given the number of flight seats, the flight fixed cost, the cost per customer and the refund for unseated passengers, the expected net wealth can be found by

$$\pi(Q, p_1) = \int_0^Q (sp_2 + xp_1 - (Q - x)r - sc - C) f(x) dx$$

$$+ \int_Q^\infty \big(sp_2 + Qp_1 - (s - (x - Q))c - C\big) f(x) dx \,, \tag{3}$$

where $\pi(Q, p_1)$ is the expected revenue as a function of the allocated quantity and the price $p_1$. If the airline expects that all passengers have a reservation price $p_A$, the airline will choose the price $p_1 = p_A$ and the quantity $Q(p_1) = \arg\max_Q \pi(Q, p_1)$, where $\pi(Q, p_1)$ is given in equation 3. Note that, given the belief $p_A$, the airline is essentially facing a variant of the newsvendor problem of a fixed price $p_1$, where

$$\pi(Q, p_1) = sp_2 - sc - C + \int_0^Q xp_1 f(x) dx - \int_0^Q r(Q - x)\, f(x) dx$$

$$+ \int_Q^\infty Qp_1 f(x) dx + \int_Q^\infty c(x - Q)\, f(x) dx \quad . \tag{4}$$

Using Leibniz differentiation rule, the first derivative of the above expression results in

$$\frac{\partial}{\partial Q}\big(\pi(Q, p_1)\big) = p_1 \int_Q^\infty f(x)\, dx - r \int_0^Q f(x)\, dx - c \int_Q^\infty f(x)\, dx. \tag{5}$$

Alternatively, this can be expressed as

$$\frac{\partial}{\partial Q}\big(\pi(Q, p_1)\big) = -rF_Q + p_1 \bar{F}_Q - c\bar{F}_Q, \tag{6}$$

where $F_Q = \int_0^Q f(x)\,dx$, $\bar{F}_Q = \int_Q^\infty f(x)\,dx$ and $F_Q = 1 - \bar{F}_Q$. By equating the above to "0" we simply get the maximizer. However, to verify the uniqueness of the maximizer, the second derivative has to be tested using the same differentiation rule applied above:

$$\frac{\partial^2}{\partial^2 Q}\left(\pi(Q, p_1)\right) = -rf(Q) - (p_1 - c)f(Q)\,. \tag{7}$$

Clearly, $\frac{\partial^2}{\partial^2 Q}\left(\pi(Q, p_1)\right) < 0$ since the charged price $p_1$ must be higher than the cost $c$. Hence, using equation 6, the solution of the optimal quantity can be expressed as:

$$F_Q = \frac{p_1 - c}{p_1 + r - c}\,. \tag{8}$$

The above overbooking formula is indeed a variant of the newsvendor problem customized to commercial airlines. For a given price $p_1$, higher optimal quantities will result when the refund is low (with the opposite applying for lower optimal quantities). In other words, the airlines should offer higher quantities (numbers) of tickets for overbooking in the case of low refund, and lower (less) quantities of tickets for overbooking when the refund is high. Cost component per passenger is generally insignificant when overbooking is under consideration since in airline operations it is the fixed costs that generate the major operating expenses.  When the company raises the price of its tickets $p_1$, it is supposed to increase the quantity of those tickets available for overbooking; however, this may contradict with the passenger's interest in obtaining low-priced tickets. In the long run, both the passenger and the airline will come to Rational Expectation Equilibrium (REE) over such quantities.

## 4. Rational Expectations Equilibrium (REE)

### *4.1 Overview*

The traditional perspective of economic theory is that individuals in a market place have a tendency to make optimal decisions based on their interpretation of information, preference parameters and market prices. The main premise of the *Rational expectations hypothesis* (see Lucas, 1972, 1973, 1975) is that any available information is utilized by individuals at an optimal level. More specifically, drawing from Frydman (1982), Lucas's perception of the *Rational expectations hypothesis* is that individual develop their forecasts by reducing their expectations of forecasted errors that are dependent on information at their disposal.

In a marketplace, when both parties who form opinions over the relevant quantities (numbers) of tickets available for overbooking are anonymous to each other, an equilibrium will be attained over the long run where all parties align their outlook to agree on expectations (thus creating an equilibrium). In effect, airlines will form an opinion $p_A$ of the passenger's likely reserved price, while the passenger forms an opinion $\bar{\beta}$ on the chance of being seated, which in turn articulates the passenger's belief of their reserved price $p_p$. However, whatever belief the passenger maintains is likely to be dependent on the distribution $f$ and the offered quantity $Q$. The passengers prefer lower prices, while the airlines prefer higher prices of their seats as shown in Figure 5. However, this is a supply-demand issue that affects the equilibrium price. If the airlines offer a low number of overbooking seats, the prices will rise to higher values. Conversely, if the number of reserved overbooking seat is high, the availability will make the prices low. An equilibrium will result between the passenger and the airline when $p_1 = p_A = p_p$. Figure 5 (below) shows that $p_1$ is a compromise value for both $p_p$ and $p_A$ when the Rational Equilibrium of Expectations (REE) is attained over the long run.

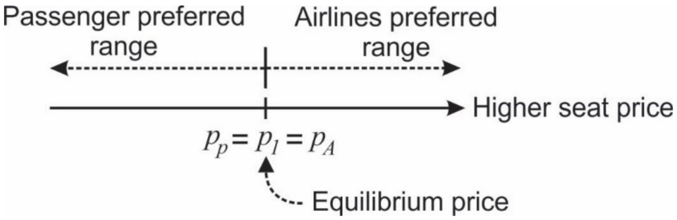

**Figure 5**: Rational Expectation Equilibrium (REE)

*4.2 Definition*

A rational equilibrium of the quantities $(p_1, Q, r, \beta, p_A)$ should satisfy the following:

(i) $p_p = \frac{v_1 - v_2 + p_2 - \beta(v_1 - r)}{1-\beta}$,

(ii) $p_1 = p_A$,

(iii) $Q = \arg\max_{Q} \pi(Q, p_1)$,

(iv) $\beta = F_Q$, alternatively, $\bar{\beta} = \bar{F}_Q$ and

(v) $p_A = p_p$.

The first three conditions assert that both passengers and airlines will choose the rationally appropriate utility-maximizing actions. The remaining two conditions require that the expectations should be consistent with the outcomes. For instance, in (iv) the expected belief of $\bar{\beta}$ concurs with the observed chance of being seated if the passenger chooses to overbook. For the passengers to accept overbooking on a preferred flight as attractive in the equilibrium state, the airline company must price the overbooked tickets at the customer's reserve price $p_p$. However, if the passenger decides to make a confirmed booking on another flight, and under the assumption of infinitesimal numbers, the remaining number of passengers is still *x*; hence, this individual will not be seated if

$(Q > x)$. On the other hand, if $(Q \leq x)$, the passengers will be seated and they obtain the ticket at a cheap price $p_1$. Therefore, when a passenger decides to overbook on his preferred-but-booked flight, he will get the seat with a probability of $\bar{F}(Q)$ which must be consistent with the value of $\bar{\beta}$ as revealed in (iv). In this situation, we are implicitly assuming efficient rationing where customers who decide to overbook get the seat at a cheaper price. This is reasonable because passengers who are eager to travel on a specific preferred flight are those who usually agree to overbook and are likely to purchase overbooked flight tickets. As for the last condition in (v), the airline must correctly anticipate the passenger's reservation price.

Aside from the above discussion, the equilibrium can alternatively be modeled as a game where the airline sets a price and a quantity of tickets available for overbooking, while the passengers choose their reservation prices and hence will consume the offered quantity accordingly. Indeed, this situation will essentially result in a Nash equilibrium; however, REE is quite intuitive and will drive explicit deliberations among all parties. The conditions for REE can be reduced to two equations in $p_1$ and $Q$, where $p_1 = \frac{v_1 - v_2 + p_2 - \beta(v_1 - r)}{1 - \beta}$ and $Q = \arg\max_{Q} \pi(Q, p_1)$, which specifically characterize the Rational Equilibrium (REE).

### *4.3 Proposition*

In the REE, passengers weighing whether to overbook on a preferred-but-booked flight or purchase a ticket on another undesired flight will agree to overbook on the preferred flight when the airline's price of overbooking tickets is given by

$$p_1 = \frac{r(v_1 - c)}{v_2 - p2} - (r - c), \tag{9}$$

which results from equations 1 and 8 at REE conditions (i to v) – i.e., when $p_p = p_A = p_1$ and by substituting the value of the belief; that is $\beta = F$. The above two new equations (equations 8 and 9) demonstrate the REE for any problem setting such that $\Delta v \leq \Delta p$ and any $r$. However, when $r \geq v_2 - p_2$, the expected value of the overbooking choice will always be higher than booking on another flight, but we are interested in the refund value that will keep booking net wealth between $r$ and $v_1 - p_1$. Under the assumption that the passengers have the quantity $p_1$, they still need $\Delta p$ to book on another flight, i.e., $r \geq \Delta p$, alternatively, this is, $p_2 - r \leq p_1$. By plugging the equality part of this expression in equation 9 and solving for $r$, leads to

$$r = \frac{(v_2-p_2)(p_2-c)}{v_1-c}. \tag{10}$$

Substituting $r$ in equation 9 results in

$$p_1 = p_2 - \frac{(v_2-p_2)(p_2-c)}{v_1-c} . \tag{11}$$

Plugging $r$ and $p_1$ from equations 10 and 11 into equation 8 leads to

$$F_Q = 1 - \frac{v_2-p_2}{v_1-c} . \tag{12}$$

The new expressions in equations 10, 11 and 12 introduce the best refund, overbooking price, and optimal quantity of tickets available for overbooking as a function of the passenger's valuations, the cost, and the airline's regular price. To compute the three quantities both the passengers and the airlines have to cooperate in a strategic manner. Clearly, passengers will have higher valuation ($v_1$) of their preferred flight. However, higher values of $v_1$ entails increasing the quantity $Q$ (that is, offering more overbooking seats). From the expectation point of view, higher valuation $v_1$

means the passengers' willingness to accept overbooking gets higher as the expected surplus increases for the overbooking option. Similarly, with the two valuations close to each other, the optimal quantity will be low, meaning that the airline should not offer many extra seats as long as passengers do not discriminate in their valuations of other flights heading to the same destination. Likewise, high regular price means offering more seats as the passengers will reconsider the cheap price $p_1$when $p_2$is high (the price of tickets available as part of overbooking policy is always less than or equal to the regular price of a ticket). As for the refund, clearly, a low refund value should be considered if the valuation of the preferred flight is high. Conversely, higher refunds result when the valuation of the other flight/flights is low. Note that the optimal quantity in equation 8 can be rewritten as $F_Q = \frac{p_1 - c}{p_2 - c}$ at the equilibrium state.

*4.4 Comparison with non-strategic passengers*

While we had construed strategic passengers as those embracing a prevailing philosophy of cooperation between passengers and commercial airlines, who are willing to share information such as flight valuations and no-show statistics with airlines, we also consider that they are commercial airline customers who may not necessarily share such an outlook – who we refer to as '*non-strategic passengers'*. If customers are not strategic and they face the decision problem of overbooking, it is likely that they will be keen to pay their valuation $v_1$ since they are eager to travel on the preferred flight and therefore the airline has the opportunity to impose charges $v_1$ and the optimal quantity, which will be $F_{Q_0} = \frac{v_1 - c}{v_1 + r - c}$.

Increasing the refund will decrease the optimal quantity of tickets available for overbooking for non-strategic customers. However, when strategic customers are considered, changing the refund will result in new overbooking price $p_1$. Indeed, the decision-making capability of strategic passengers is likely to serve as a reason for airlines to lower their ticket prices below $v_1$ and $v_2$. On the other hand, the equilibrium quantity $Q$ will be lower than the optimal quantity $Q_0$ with non-strategic passengers. Note that when the airline company sells at $v_1$, we have

$$F_Q = \frac{p_1 - c}{p_1 + r - c} < \frac{v_1 - c}{v_1 + r - c} = F_{Q_0} . \qquad (13)$$

That is, $Q \leq Q_0$. This behavior of generating scarcity in offered quantities is adopted in some business such as auctions where increased number of bidders induces aggressive bidding behavior (Vulcano et. al., 2002).

### *4.5 Sensitivity of the model parameters*

As earlier alluded to, fixed costs account for the biggest proportion of expenditures in the airline industry. Consequently, they impose a burden on revenue management in that only a narrow margin is left to address variable costs. Fixed costs do not appear in the optimal REE expressions shown in equations 10, 11 and 12. Conversely, the variable cost seems to have little effect on the optimal values. Figure 6 depicts $F(Q)$ as a function of the cost for different valuations. Note that low $F(Q)$ entails low optimal quantities, and vice versa. The norm in real cases is to have dissimilar valuations of $v_1$ and $v_2$. Clearly, when the gap between the valuations increases, the variable cost will have less effect on the resulting optimal quantity. For instance, when $v_1$=100, $v_2$=60, and by increasing the cost (50 times higher), the effect will be less than 10%. However, the effect will increase when $v_2$ approaches $v_1$ since the difference between $p_1$ and $p_2$ will be the main incentive to overbook. In general, higher cost means fewer seats offered in the overbooking list.

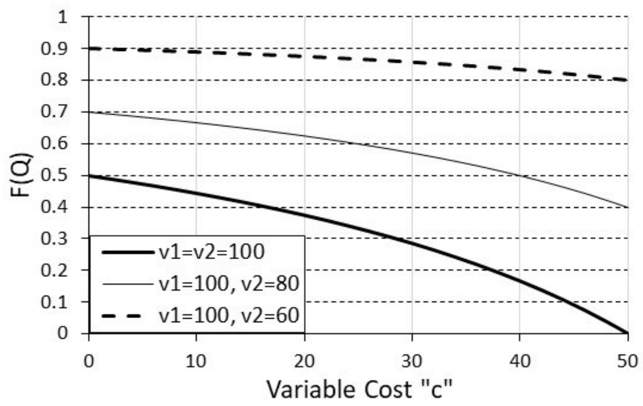

**Figure 6:** The effect of the cost on the optimal quantity for different flight valuations

The effect of the passenger valuation on the optimal quantity can be easily understood from equation 12 where higher values of preferred flight ($v_1$) necessitates offering a higher number of tickets available for overbooking. Higher valuation means greater needs to travel on the preferred flight, and thus a greater willingness to overbook. In this situation, a commercial airline should exploit the opportunity and increase the number of tickets available for overbooking to increase revenues. In contrast, higher valuation of the subsequent flight ($v_2$) will force the airline to offer fewer tickets available for overbooking due to passengers increasingly favoring other departing flights. A proportional relationship exists between the regular price $p_2$ and $F(Q)$ as given by equation 12 – that is, higher regular prices lead to higher quantities.

Due to REE, the overbooking price $p_1$ and the offered overbooking quantity are linked in their outlook (in effect, if commercial airlines offer more seats on overbooking policy, the prices of those tickets will be lower). A higher overbooking price $p_1$ will increase $F(Q)$; thus, a higher number of tickets available for overbooking should be offered. This is a reasonable result as higher overbooking prices induce less willingness to overbook. Conversely upon equilibrium, higher refunds are always accompanied with lower quantities. Figure 7 (below) presents the effect of the refund on $F(Q)$ for suggested prices.

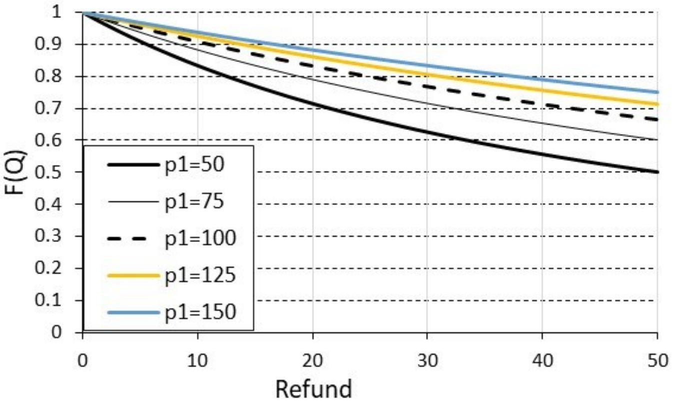

**Figure 7**: The effect of refund on the optimal quantity

The overbooking price increases as the passenger valuation $v_1$ increases. Certainly, the desire to travel on a preferred flight attracts a higher valuation and thus a commercial airline can be expected to assign higher prices to those flights. Conversely, higher valuation of $v_2$ (the undesired flight) makes overbooking unattractive; therefore, lower overbooking price will result in the equilibrium state. In addition, due to REE, the overbooking price will increase as the regular price increases. Figure 8 shows that high $p_1$ values result when $p_2$ is increased; however, the price of overbooking will always be less than the regular seat price. A tie will result only when the cost is the same as $p_2$, where in this case it will not be possible to generate a profit. The same applies when the regular price is as high as the valuation of the passenger, where in this case the other-but-undesired flight is not an attractive option (i.e., $v_2 - p_2 = 0$). We would like to point out, though, that regular prices will be less than $p_1$ only when $p_2 \leq c - \frac{1}{2}(v_1 - v_2)$. Under any circumstances the cost must be less than $p_2$, otherwise no profit will be collected.

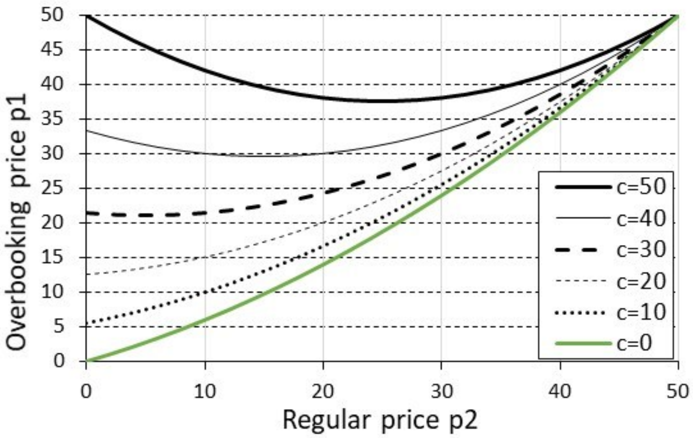

**Figure 8:** Overbooking price vs. regular seat price at REE for $v_1$=100 and $v_2$=50

To build better insight of the resulting equilibrium Table 2 summarizes the model behavior by increasing the valuations, the regular price, and the cost. Conversely, opposite behavior can be noticed at each parameter.

**Table 2**: Sensitivity analysis of REE values. "-" means the result depends on other factors

| Valuation of preferred flight ($v_1$) | Valuation of next flight ($v_2$) | Regular seat price ($p_2$) | Cost per passenger ($c$) | Refund ($r$) | Overbooking equilibrium price ($p_1$) | Optimal quantity ($Q$) |
|---|---|---|---|---|---|---|
| ↑ | | | | ↓ | ↑ | ↑ |
| | ↑ | | | ↑ | ↓ | ↓ |
| | | ↑ | | - | ↑ | ↑ |
| | | | ↑ | - | - | ↓ |

## 5. Validation

For validation, we conduct two different case analyses: (i) a real-life case assessment from an operating commercial airline and (ii) Monte Carlo simulation of a daily flight.

### *5.1 Economy-class overbooking*

To test the validity of the proposed model, we chose real data derived from Kuwait Airways Flight number KU560. This is a daily flight operating from the State of Kuwait to the Hashemite Kingdom of Jordan. In normal travel seasons (between the beginning of September and the end of May), Flight KU560 is operated using an Airbus A330-200 which accommodates 36 business-class and 290 economy-class seats. Table 3 shows the number of show-ups for a two-month time period in 2016 while Figure 9 shows the frequency of ‘no shows’.

**Table 3**: Flight KU561, passengers’ show-up data for two successive months (2016)

| | | | | | |
|---|---|---|---|---|---|
| 290 | 288 | 288 | 282 | 290 | 290 |
| 285 | 285 | 281 | 290 | 290 | 290 |
| 284 | 280 | 290 | 286 | 287 | 289 |
| 290 | 282 | 289 | 284 | 290 | 290 |
| 290 | 286 | 289 | 290 | 288 | 290 |
| 288 | 289 | 289 | 290 | 284 | 288 |
| 290 | 289 | 286 | 290 | 286 | 289 |
| 284 | 284 | 290 | 289 | 285 | 285 |
| 289 | 282 | 288 | 284 | 290 | 290 |
| 286 | 290 | 287 | 289 | 284 | 289 |
| | | | | | 290 |

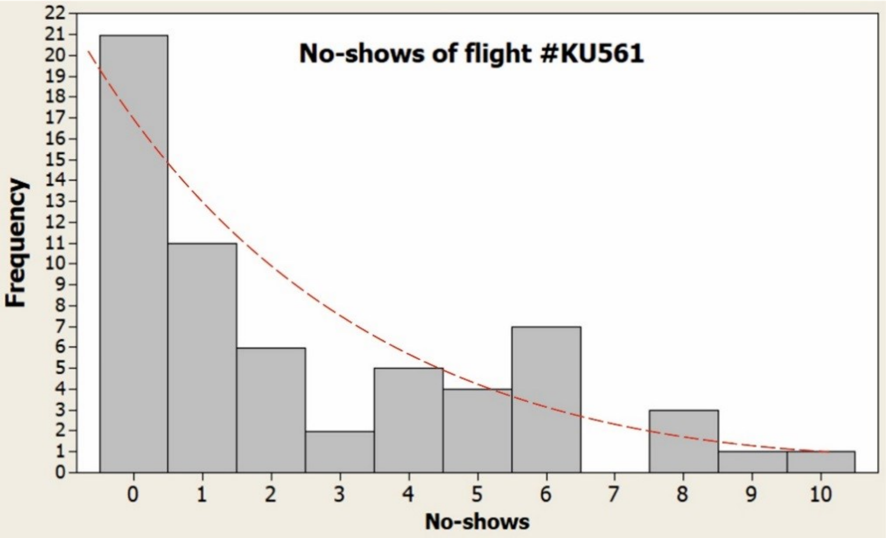

**Figure 9**: Histogram of no-shows and the best fit curve for Flight #KU561

Table 4 summarizes the results of this flight along with the optimal quantity of seats available for overbooking, seat prices and value of refunds. Note that the fixed cost is assumed to be zero as it does not affect the optimal quantities and prices.

**Table 4**: Flight #KU561 observed data and the model results

| Flight data | Value |
|---|---|
| Seats | 290 |
| Fixed cost | 0 |
| Variable cost | 5KD=\$16.64 |
| Distribution | Expo(2.5246) |
| $p_2$ | 70KD=\$233.00 |
| $v_1$ | 77KD=\$256.25 |
| $v_2$ | 77KD=\$256.25 |
| Average observed revenue | 18685.9KD=\$62,186.7 |
| Revenue at full capacity | 18850KD=\$62,732.8 |
| Results: | |
| $F(Q)$ | 0.9028 |
| $Q$ | 5.8842 ≈ 6 |
| Refund *r* | 6.319KD =\$21.03 |
| $p_1$ | 63.681KD =\$212 |
| Revenue of our model | 18987KD =\$63,188.7 |

| | |
|---|---|
| Expected number of overbooked but boarded passengers | 1.7331 |
| Expected number of voluntary denials | 0.7917 |

Kuwait Airways generally offers refundable tickets and does not incorporate overbooking into its general booking settings in order to allow for free online seat reservation. The average ticket price during the two months is around 70KDs (Kuwaiti Dinar) which is equivalent to $US233 at the time of this study (July 2019). If a boarding denial occurs, the airline operates a money-back guarantee plus additional $US200 compensation. We fit the data to an exponential distribution of the parameter $\lambda = \frac{1}{2.5246} = 0.3961$. For this flight, we show that the airline can achieve higher profits if it implements the voluntary overbooking policy while maintaining customer goodwill. To accommodate a scenario where all customers with a valid ticket turn up for boarding, we will make overbooking even less attractive to passengers by assuming that their valuations for the preferred and other flights are the same – i.e., $v_1 = v_2$. By so doing, we relax some of the assumptions for the benefit of the passenger, where the prices will be the sole incentive for passengers to overbook in this case. Suppose that the customers value the utility of the ticket by 10% more than the announced price, then, accordingly, $v_1 = v_2$=77KDs = $US256.25.

The resulting optimal price $p_1$ is $US212 and the resulting refund is $US21.03. Table 4 shows that when Flight KU561 operates a flight on full capacity ("0" no-shows), a total revenue of 290×(70-5)=18850KD=$US62,732.80 will be collected, while the observed average revenue falls behind due to flying with empty seats in this flight (18685.9KDs=$US62,186.7). In contrast, the proposed model can achieve a revenue of 18987KDs ($US63, 188.70) if voluntary overbooking is employed. Note that, on average, 1.7331 passengers will manage to board from the overbooked list with a service level of 90% (in effect, 90% of the customers who possess overbooked ticket will be able to board the flight). Although the average no-shows is 2.5246, the optimal overbooking quantity is around six seats.

If Kuwait Airways was to employ overbooking with regular passengers (in effect, customers holding 'economy-class' tickets), these passengers are willing to pay $v_1$, where $p_1$=$v_1$=77KD. Given the equilibrium refund *r*=6.319KD, larger quantities in case of non-strategic customers will be found. This is because $F_Q = \frac{p_1 - c}{p_1 + r - c} = 0.9028 < F_{Q_0} = \frac{v_1 - c}{v_1 + r - c} = 0.9193$. Indeed, strategic passengers (as earlier stated, these are passengers willing to share information such as flight valuations and no-show statistics with airlines), the airline company tends to lower its overbooking quantity. This behavior of generating scarcity in offered quantities is adopted in auctions (Vulcano et al., 2002).

### *5.2 Simulated flight*

Due to difficulty in accessing censored airlines data, we use simulated flight scenarios for further validations. Here, random 'no shows' are generated via Monte Carlo simulation for a daily flight over five years. Boeing 737-800 aircraft (a series that is heavily used by American Airlines) is considered for this flight with a capacity of 114 standard economy seats. The selected distribution is voluntarily simple; that is, *Norm*(μ=5, σ=3). The valuations of the passengers for subsequent flights are $v_1$ =$US300 and $v_2$ =$US250. The regular standard seat price is $US200 and the variable cost is set to $US20. A total of 5×365 =1825 flights are simulated and the histogram of the resulting no-shows is depicted in Figure 10. The results are shown in Table 5.

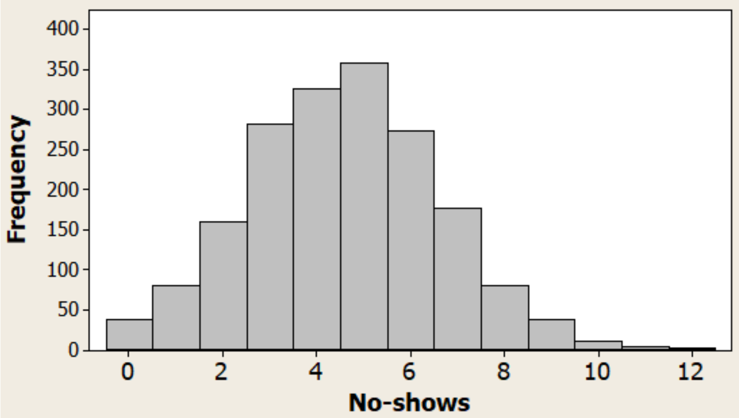

**Figure 10**: No-shows histogram of a daily flight over five years

**Table 5**: Simulated flight data and results

| Flight data | Value |
|---|---|
| Seats | 114 |
| Fixed cost | 0 |
| Variable cost | $20 |
| Distribution: | Norm(5,2) |
| $p_2$ | $200 |
| $v_1$ | $300 |
| $v_2$ | $250 |
| Average observed revenue | $19696.44 |
| Revenue at full capacity | $20520 |
| Results: | |
| $F(Q)$ | 0.8214 |
| $Q$ | 8.6138≈9 |
| Refund *r* | $32.1429 |
| $p_1$ | $167.8571 |
| Revenue of our model | $21,228 |
| Expected number of overbooked and boarded passengers | 4.7784 |
| Expected number of denials | 0.2219 |

By incorporating voluntary overbooking policy, our model achieves (21,228-19696.44)/ 19696.44 = 7.8% increase in the revenue. Disregarding voluntary overbooking results in a loss of $US1531.56 per flight. The equilibrium overbooking price is $US167.85. A refund of $US32.14 in addition to the money paid by the passenger is reimbursed upon boarding denial. That said, we posit that voluntary overbooking is indeed a mutually beneficial policy for both passengers and airline operators. With voluntary overbooking employed, we attain an equilibrium in passenger-airlines risk outlook, revenues are maximized and involuntary boarding denial is eliminated. However, as in other revenue management approaches, voluntary overbooking may be associated with a number of advantages and disadvantages. For airlines, when voluntary overbooking forms part of revenue management strategy, it implies that involuntary boarding denials will not occur and the airlines are able to maintain customer goodwill and in the process, protect profit margins. From the travellers' point of view, voluntary overbooking comes along with the incentives of cheap priced tickets and the knowledge (of average no-shows statistics), that they do still have a chance to travel on their preferred flights. The reality however is that it is debateable what precise percentage of travellers will be willing to take the risk and accept voluntary overbooking as a travel strategy. However, we opine that this number is more of a question of achieving an equilibrium between the supply (number of no-shows) and demand (passengers under overbooking seats). If the average number of no shows is small, then only limited number of passengers will accept such deals meaning that airlines will offer fewer overbooked seats. Similarly, if the number of no shows is high, it is more than likely that a higher number of seats under the overbooking conditions will be reserved. However, while cargo and seasonality were not considered in this study, if the number of passengers denied boarding is high, a lower number of passengers are likely to accept this deal. This will result in even cheaper ticket prices and low number of offered seats under the overbooking policy.

## 6. Conclusions

This paper studied a voluntary overbooking model under rational expectation equilibrium to promote cooperation between customers and airlines, maintain goodwill of customers, and maximize the expected total returns to airlines. Decision tree analysis were constructed for both customers and airlines. Sensitivity analysis were conducted and simulation on the no-show random variables of the model was used for validation, the results of which illustrate mutual benefits associated with the voluntary overbooking equilibrium model.

Our objective in this study was to develop and test a model for optimized voluntary overbooking in commercial airline reservations. We argue that such a model will serve to hedge against the potential loss of customer goodwill, which is a major source of reputational risk associated with involuntary denied boarding. Underpinning this voluntary overbooking model is the *Rational expectations hypothesis* and the argument that a particular equilibrium point exists that will present confirmed booking for a customer on another flight as attractive as overbooking on the preferred flight (which attracts higher risk of involuntary denied boarding). However, we do point out that this policy requires cooperation between passengers and commercial airlines, which is only plausible where there is full information disclosure between the two parties.

To test for validity, the proposed model was subjected to two tests, first, a real-life case assessment from an operating commercial airline and second, Monte Carlo simulation of a daily flight. The decision to utilize different test scenarios was driven by our observations that, within the literature, there appears to be very limited exploration of how incentive parameters such as optimal quantity of seats available for overbooking, seat prices and value of refunds interact with customer bargain-hunting behaviour. Following tests, we found that where a passenger is assessing whether to overbook on his preferred flight or purchase a ticket for another flight, the decision to purchase a ticket for the latter departing flight and therefore expose oneself to voluntary denied boarding was dependent on the appeal of available incentives in the offers from commercial airlines to their customers. To enable commercial airlines to optimize the price of their overbooked seats, we assumed that passengers willing to assess whether to overbook on a preferred flight or purchase a ticket for another undesired flight will be prepared to provide their personal valuations of the seats to the airline. In our model, the passengers and the airline company form conflicting beliefs on their expectations which, by implicit bargaining, will approach an equilibrium of expectations. This equilibrium of expectations is used to establish an optimization model that maximizes the expected revenues. The main contribution of our study is that we introduced new means of representing equilibrium to the literature on overbooking policies. They can be easily implemented. Indeed, an added attraction is that our approach demonstrates higher revenues compared to the revenues associated with existing involuntary denied boarding policies.

As expected, the study was not without limitations. For example, the model was developed on the basis of a number of assumptions. From the customer perspective, the model assumed that customers will be willing to provide valuations to airlines seeking volunteers for voluntarily overbook. However, we do know that this is not always the case. Indeed, as our model was developed from data collected from a daily operating flight we do not expect that the model is applicable to overbooking problems experienced on last flights of the day. In addition, since the model was developed using data obtained from a mid-size wide-body jetliner, its applicability to overbooking problems on small aircraft is also questionable. Furthermore, as the United Airlines Express Flight 3411 incident showed us, not all customers will be willing to provide valuations to airlines seeking volunteers for voluntarily overbook. In fact, the United Airlines Express Flight 3411 incident arose as three customers randomly selected for involuntary flight denial had agreed to the policy. The incident with David Dao only occurred after he had refused the range of incentives on offer. These limitations call for future research on not only overbookings on last flights of the day and flights on small aircraft, but also, more specifically, the use of incentives to manage overbookings on these specific flights. The model also assumed that customers will be faced with only two decision types –a choice between overbooking on a preferred-but-booked flight at a low price and buying a seat on another flight at higher price.

On the side of the commercial airline, our model had assumed a decision problem consisting of the number of tickets available for overbooking, the price of tickets available for overbooking, and the value of the refund offered when involuntary denied boarding occurs. No monetary value had been allocated to the risk of reputational damage, although this was considered a key attribute of the model. The model had also assumed that airlines will fully commit to their terms and conditions at the time of sale. However, the report on the child who was de-seated by United Airlines suggests that commercial airlines may not always fully commit to the terms and conditions of carriage at the time of sale. Finally, the model assumed full information transparency, particularly from the commercial airline operators.

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