

Impact Analysis of Individual and Network Factors to Household Energy Savings

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Abstract: Reducing energy consumption in residential use sector can largely improve energy savings especially in this post pandemic era when working from home becomes a favoured option for many people. Each home's energy consumption pattern depends on individual user factors and is also influenced by the networks. In this work, four factors that may affect user's adoption decision of energy efficient products are investigated including the personal acceptance level, the influence from the connected neighbours, the overall network adoption rate, and the advertisement influence. The personal acceptance level is further modelled taking account of individual factors on household income, family status, age group and employment status. To enable quantitative analysis, a dynamic network model is established in which each household is taken as a node, and a utility measure is defined for decision making that integrates multiple impact factors described by subsystem models. The relative contribution of each factor towards user's decision making is evaluated by its associated weighting. Two population networks are studied, starting from a small network with 40 homes, followed by a large one with one million nodes. Simulation results from both population networks reveal that, among the four factors considered, the overall network adoption rate is most influential to user's decision on adopting energy efficient products.

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Keywords: Household energy savings; social networks; network model; individual factors; optimisation design.

1. INTRODUCTION

Conserving energy and utilizing energy in an optimal way are structured research and development of global interest (Abrahamse et al., 2005, 2007). One active area is on reduction of residential energy expenditure, which becomes particularly important in this post pandemic era when working from home becomes a choice for many people. To encourage energy savings, governments offer incentives to decrease energy usage in families (Palmer and Cooper, 2013).

The impact of user behaviours has drawn recent research attention in the area of energy innovation. Each individual's decision plays a key role towards improving energy use efficiency. Power companies and engineers are keen to understand how user behaviour may shape energy consumption and how to influence users to reduce energy consumption from social sciences point of view (Bavaresco et al., 2020). To achieve effective energy savings, it is imperative to conduct research on both social and technical scales (Figueiredo et al., 2005; Fischer, 2008). The social scale would detail how consumer behaviour in energy conservation would be affected by information sharing within the social network (Boyd and Ellison, 2007). The technical scale can recommend how such social networks and information shared can be modelled (Ellison et al., 2007; Feng et al., 2013).

While user behaviour affects choices when it comes to energy conservation, information to control or improve energy saving behaviour is a significant parameter that is difficult to obtain. There's a lack of data and models to represent the influence of consumer behaviours on the energy savings outcome. A quantitative research method is useful when the objective is clearly stated, and a hypothesis-based testing is required.

The aim of this work is to develop a mathematical model that can quantify the influence of personal acceptance as well as social networks factors on household energy saving decision. The personal acceptance levels are considered to be dependent on income, age, employment status and family status. The social networks influence are taken from the neighbouring network influence, the network adoption rate and the advertisement influence. All these factors have their impacts on energy efficiency product adoption decision. With a mathematical model in place, it enables examination of each factor's influence on the energy saving target. The model development and the sensitivity analysis are novel contributions of this work.

The remainder of the paper is structured as follows. A mathematical model is developed in Section 2 to quantify the adoption decision of energy efficiency product in relation to the four main factors from individual and networks levels. Case studies including a small population network and a large network are presented in Section 3 with results and analysis provided. Conclusions are given in Section 4.

2. MODEL DEVELOPMENT

2.1 Network Modelling

A social network with N households can be represented by a network with N nodes, in which the connections between nodes are described by edges. The connection state of the whole network is written as $\mathbf{A} = [A_{ij}]$ ($i, j = 1, \dots, N$), A_{ij} is the edge state from node i to node j .

$$A_{ij} = \begin{cases} 1, & \text{when node } i \text{ is connected to node } j \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

In this work, the influence strength between different nodes are taken to be the same. Thus, no weights are added to edges A_{ij} , and $A_{ij} = A_{ji}$. The degree of the i -th node, k_i , can be calculated from A_{ij} by

$$k_i = \sum_{j=1}^N A_{ij}. \quad (2)$$

2.2 Personal Acceptance Level

The personal acceptance level, denoted as L_i , is considered to rely on the following individual factors. The classifications are taken following information in (GfK NOP and Kantar Media Research, 2012).

- (1) State of income, classified into three levels, $a = 1, 2, 3$, representing low (below £17K pa), mid (between £17K pa to £37K pa), and high (above £37K pa) income, respectively.
- (2) State of family situation, divided into two groups, $b = 1, 2$, representing household without and with children, respectively.
- (3) State of age, divided into four groups, $c = 1, 2, 3, 4$, for age ranges of 18 to 34, 35 to 54, 55 to 64, and above 65.
- (4) State of employment status, represented as $d = 1, 2$, for unemployed and employed status, respectively.

Assuming a simple linear model to include the above four individual aspects, the personal acceptance level can be written as

$$L_i = m_1 a_i + m_2 b_i + m_3 c_i + m_4 d_i, \quad (3)$$

where m_1, m_2, m_3 , and m_4 are weighting coefficients for income, family situation, age and employment status, respectively, taking values between -1 to 1. In this work, survey data is used to calculate the four model parameters.

Assume the same set of parameters apply to all nodes,

$$\theta_L = [m_1, m_2, m_3, m_4]^T, \quad (4)$$

The determination of θ_L can be formulated as an optimisation problem, i.e.,

$$\begin{aligned} \theta_L^* &= \arg \min_{\theta_L \in \Theta_L} \sum_{k=1}^{N_L} (m_1 \bar{a}_k + m_2 \bar{b}_k + m_3 \bar{c}_k + m_4 \bar{d}_k - \bar{y}_k)^2 \\ \text{s.t. } & m_1 \bar{a}_k + m_2 \bar{b}_k + m_3 \bar{c}_k + m_4 \bar{d}_k \leq 1 \end{aligned} \quad (5)$$

where k is the index used for acceptance level data, N_L is the total number of data, $\bar{a}_k, \bar{b}_k, \bar{c}_k, \bar{d}_k$ are average values of each individual factor, Θ_L is the searching domain for θ_L , \bar{y}_k are acceptance levels in surveyed data.

2.3 Neighbouring Network Influence

The adoption state of each household is represented as a binary variable, $n_i(t) = 1$ for adopted, and $n_i(t) = 0$ for not adopted, t is the discrete time index. In this work, only the change of $n_i(t)$ from 0 to 1 is considered.

Initially, some households have already adopted the energy efficiency product, they are taken as the seed nodes. These seed nodes will share information to their neighbouring nodes, which may influence the adoption decision of neighbours. When the state of a neighbouring node changes from 0 to 1, this node becomes a new seed node that can further spread the information and influence more nodes.

The influence of neighbouring adoption to node i is quantified as a ratio between the number of adopted neighbours to the degree of the i -th node, that is,

$$s_i(t) = \frac{1}{k_i} \sum_{j=1}^N A_{ij} n_j(t). \quad (6)$$

2.4 Overall Network Adoption Rate

The entire social network has influence on householder's decision making (Bale et al., 2014), which can be taken as indirect influence from the whole network apart from the direct influence from neighbouring nodes. This indirect influence is named as the network trend influence, defined as network adoption rate, $h(t)$, that is calculated by

$$h(t) = \frac{1}{N} \sum_{i=1}^N n_i(t). \quad (7)$$

2.5 Advertisement Influence Model

The use of advertisement is another factor that influence adoption rate significantly (Shapiro, 1983; Du et al., 2023). Following the approach in (West and Harrison, 2006), the advertisement influence is calculated using the Bayesian forecasting method, where the relationship between impressions (number of views), $X(t)$, and the effect of impressions $E(t)$, at time t , can be represented as

$$E(t) = (\beta - \alpha) - (\beta - \alpha - \rho E(t-1)) \cdot \exp(-\gamma X(t)) \quad (8)$$

where α and β are lower and upper thresholds, ρ is a decay factor of advertisement effect over time, γ is a diminishing return factor of advertisement effect. The values of α, β, ρ are between 0 and 1, while γ is larger than 0.

The initial value of $E(t)$ is assumed to be the lower threshold obtained in our earlier work (Du et al., 2023) as no nodes receives advertisement at the initial stage, where $E(0)$ is taken to be 1.697e-2 for the small network, and 3.815e-6 for the large network in Section 3.

Then the advertisement influence, $ad(t)$, can be calculated as the sum of lower threshold and the effect of impressions,

$$\begin{aligned} ad(t) &= \alpha + E(t) \\ &= \beta - (\beta - \alpha - \rho E(t-1)) \cdot \exp(-\gamma X(t)) \end{aligned} \quad (9)$$

2.6 Adoption Utility and Update of Adoption State

Considering the combined contribution from personal acceptance level and influence of neighbouring network, net-

work trend and advertisement to the adoption decision of each node, a *Utility* measure is defined as

$$U_i(t) = \omega_1 \cdot L_i + \omega_2 \cdot s_i(t) + \omega_3 \cdot h(t) + \omega_4 \cdot ad(t) \quad (10)$$

where $\omega_1, \omega_2, \omega_3,$ and ω_4 are the constant weights, and

$$\omega_1 + \omega_2 + \omega_3 + \omega_4 = 1.$$

For node i at time t , when the adoption utility reaches the defined threshold, ε , the adoption decision will occur, i.e., n_i is changed from 0 to 1 at the next time point. The adoption state dynamics is described by

$$n_i(t+1) = \begin{cases} 1, & \text{if } U_i(t) \geq \varepsilon \\ 0, & \text{if } U_i(t) < \varepsilon \end{cases} \quad (11)$$

2.7 Determine Model Parameters and Factor Weightings

In this proposed model, unknown parameters and weighting coefficients are estimated using survey data. A standard least-square method is employed. Denoting

$$\theta_U = [\varepsilon, \alpha, \beta, \rho, \gamma, \omega_1, \omega_2, \omega_3, \omega_4]^T, \quad (12)$$

the values of θ_U is determined by

$$\theta_U^* = \arg \min_{\theta_U \in \Theta_U} \sum_{t=1}^T (h(t, \theta_U) - y(t))^2 \quad (13)$$

s.t. $\omega_1 + \omega_2 + \omega_3 + \omega_4 = 1$

where Θ_U is the searching domain for θ_U , which is a set of positive real numbers in the range between 0 and 1, $y(t)$ is the adoption rate data obtained from survey, $h(t)$ is the adoption rate calculated by the model, T is the total number of samples for data fitting. The configuration of the whole model is shown in Fig. 1.

3. CASE STUDY

Two population networks of different sizes are used for case studies. The first one is a small population network based on a survey of 40 participants in our previous work (Du et al., 2016). The second one is a large population network including one million users in a large trading platform, for which the connection data between nodes are not available, need to be reconstructed through modelling efforts.

3.1 Modelling of Personal Acceptance Level

In order to simulate the personal acceptance impact on adoption behaviour, survey data of Green Deal Segmentation (GfK NOP and Kantar Media Research, 2012) is used to estimate model parameters in (4). The survey data are collected for willingness of taking new energy efficiency product. Participants' income level, age, family situation and employment status are included. The following assumptions are made for the personal factor model.

- (1) The personal acceptance in the survey report is divided into six different levels with corresponding weights of 0, 0.2, 0.4, 0.6, 0.8, 1, representing reject, low acceptance, below average acceptance, above average acceptance, high acceptance, and accept.
- (2) For simplification, averaged values of income level, age, family situation, and employment status are used for each personal acceptance level.

The key results from the survey data are listed in Table 1.

Table 1. Green Deal Segmentation Data

Personal Factors	Proportion of Personal Factors (%)						
	1	2	3	4	5	6	all
Low Income	38	41	30	11	23	32	30
Mid Income	30	40	46	30	39	42	38
High Income	32	19	24	59	38	26	32
No Children	62	76	68	50	74	43	64
Have Children	38	24	32	50	26	57	36
Age group 1	17	23	30	36	35	38	30
Age group 2	46	24	36	50	30	44	36
Age group 3	12	11	12	10	19	10	13
Age group 4	25	42	22	4	16	8	21
Unemployed	45	54	38	19	32	30	38
Employed	55	46	62	81	68	70	62
Segment Proportion	11	24	11	10	24	20	100

The values of θ_L are estimated from (5) with data in Table 1. The optimisation algorithm used in simulation is *fmincon* in Matlab. The initial values of θ_L are taken from the pre-estimation result obtained using a genetic algorithm. The initial settings and the final results for θ_L are given in Table 2, from which it can be seen that, compared to the other three parameters, the coefficient of income, m_1 , has a very low magnitude, which suggests the income state may not affect much of the user decision on adopting energy efficient product. Among the four coefficients, m_3 is the only one that has a negative value, indicating younger people seem to be more willing to adopt new energy product.

Table 2. Personal acceptance model parameters

Coefficients	m_1	m_2	m_3	m_4
Initial values	4.283e-6	0.1347	-0.1287	0.3802
Final results	4.671e-7	0.2830	-0.1385	0.2710

For both the small and the large population networks, when the survey data on (a_i, b_i, c_i, d_i) are given, the calculated parameters, m_1 to m_4 , are used to model the personal acceptance level for each node as in (3).

3.2 Small Population Network

In the survey for the small social network (Du et al., 2016), all 40 participants are PhD students, the values of personal acceptance segments are $a = 1, b = 1, c = 1,$ and $d = 1$. The optimal results of θ_U are calculated using (13) by applying the survey data over 180 days and listed in Table 3. It can be seen that the threshold ε for small population network is 0.1922, and the weight values for social network influence utility, ω_1 to ω_4 , are largely even ranging between 0.2192 to 0.3206.

The simulated adoption rate is compared with the survey data as shown in Fig. 2. The model output matches well with the survey data generally. And it can be seen that the adoption rate converges in the later stage of the experiment.

3.3 Large Population Network

For the large population network, the network degree is assumed to be five, i.e., each node has a degree of five (Du

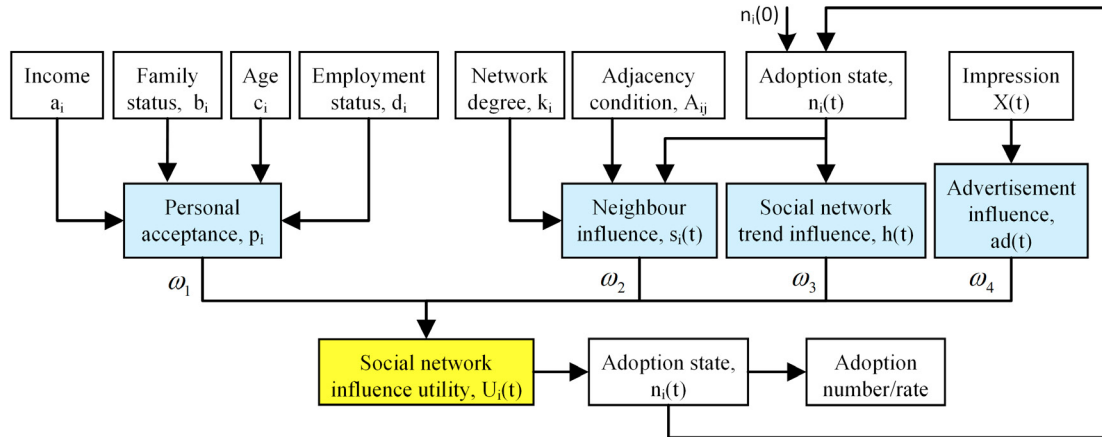


Fig. 1. Model configuration of individual and social networks influence to adoption of energy product

Table 3. Small Network Coefficients

Coefficients	ε	α	β	ρ	γ
θ_U	0.1922	0.3216	0.7588	0.5342	0.7843
	ω_1	ω_2	ω_3	ω_4	
	0.2192	0.2409	0.3206	0.2193	

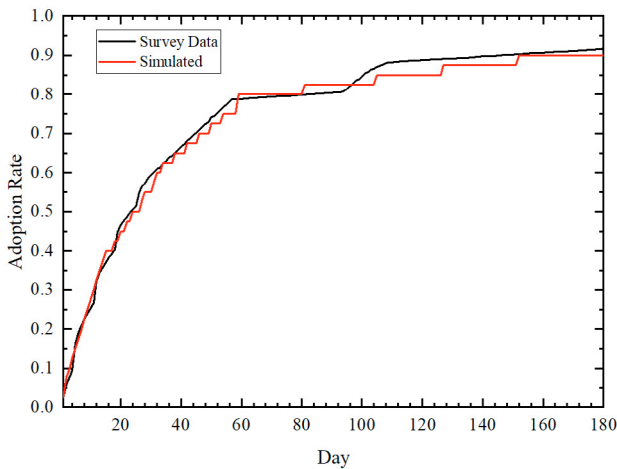


Fig. 2. Adoption rate of small network 180 days

et al., 2023). Since the connection information between nodes are not available, we need to build up the network model by reconstructing the node connections. The W-S small-world network is chosen as it is a well-accepted option for real world network simulation (Chen et al., 2014).

In a W-S small-world network model, different rewiring probability values, β_r , are used to generate the networks representing different properties, from a clustered network ($\beta_r = 0$) to an E-R random network ($\beta_r = 1$), with a step of 0.05 for β_r . Along with generation of the network connections, personal factors, (a_i, b_i, c_i, d_i), are given to each node with the segment proportion values as shown in the last row of Table 1.

Once the network is generated by the W-S small world network, the nodes connection status, A_{ij} , and the personal acceptance level, L_i , for each node are saved for further calculation. The optimal results of θ_U are calculated using (13) with the survey data collected over a period of 28

days, each day makes a sampling point. One set of results are shown in Table 4 for $\beta_r = 0.45$.

Table 4. Large Network Coefficients ($\beta_r = 0.45$)

Coefficients	ε	α	β	ρ	γ
C_U	0.1106	0.1487	0.4233	0.7515	0.3238
	ω_1	ω_2	ω_3	ω_4	
	0.1558	0.1788	0.4304	0.2349	

The modelled results of adoption rate and the survey data are compared in Fig. 3, which shows a close matching. Since the period of 28 days is not long enough to allow the system reach the steady state, only the transient part is shown in Fig. 3.

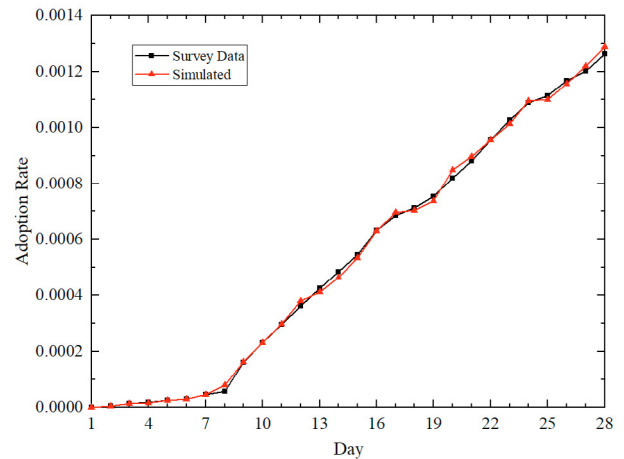


Fig. 3. Adoption rate of large network, $\beta_r = 0.45$

The results under different network conditions, as reflected by values of β_r , are shown in Table 5. It can be seen that the social network trend influence always has the largest weighting (ω_3) among the four factors considered.

Figure 4 shows the variation of the four weights with respect to β_r , from which it can be seen that ω_3 decreases with the increase of rewiring probability. The values of ω_2 and ω_4 seem to be less dependent on the network structure property. The weights of personal acceptance (ω_1) increases when rewiring probability increases. This

Table 5. Large Network Coefficients under different network properties

Rewire Probability	Coefficients of Utilities			
	β_r	ω_1	ω_2	ω_4
0	0.0974	0.1447	0.5333	0.2245
0.05	0.1023	0.1464	0.5205	0.2307
0.10	0.1140	0.1766	0.4830	0.2264
0.15	0.1161	0.1598	0.5052	0.2188
0.20	0.1224	0.1576	0.4863	0.2337
0.25	0.1345	0.1505	0.4715	0.2434
0.30	0.1339	0.1643	0.4792	0.2227
0.35	0.1413	0.1515	0.4571	0.2500
0.40	0.1468	0.1513	0.4546	0.2473
0.45	0.1558	0.1788	0.4305	0.2349
0.50	0.1613	0.1691	0.4276	0.2420
0.55	0.1768	0.1699	0.4126	0.2407
0.60	0.1860	0.2087	0.3914	0.2139
0.65	0.1951	0.2048	0.3704	0.2297
0.70	0.2024	0.2101	0.3567	0.2308
0.75	0.2096	0.2100	0.3404	0.2400
0.80	0.2139	0.2266	0.3314	0.2281
0.85	0.2193	0.2165	0.3186	0.2456
0.90	0.2251	0.1970	0.3051	0.2728
0.95	0.2294	0.2075	0.2959	0.2671
1.00	0.2386	0.2051	0.3393	0.2170

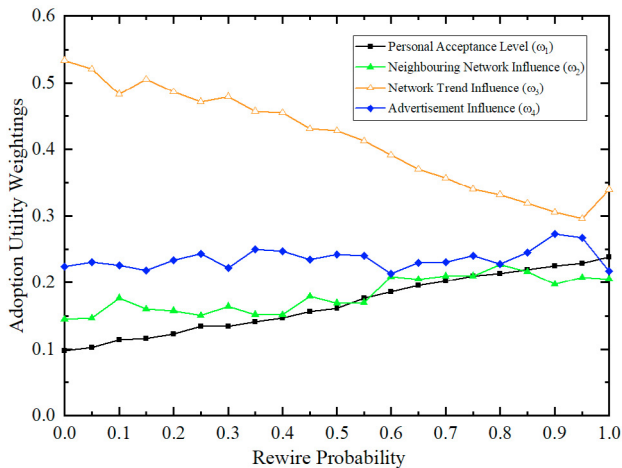


Fig. 4. Network influence utility weights versus rewiring probability of the large network

might due to the fact that a lower rewiring probability corresponds to a network with different clustered groups in which nodes are more influenced by the group adoption rate. However, for a large network that is close to a E-R random network, no such groups exist, the personal acceptance level becomes more important.

3.4 Sensitivity Analysis with Monte Carlo Calculation

The model uncertainty will inevitably affect the design outcome. To understand the impacts from model uncertainties, sensitivity analysis with Monte Carlo calculation are performed for both the small and the large networks. The parameter values in Table 3 and Table 4 are taken as nominal values for the for the two networks. All nine parameters in θ_U are assumed to follow Gaussian distribution, the mean value is the nominal value, the standard deviation is taken at four different levels, $[0.1, 0.2, 0.5, 1]$ of

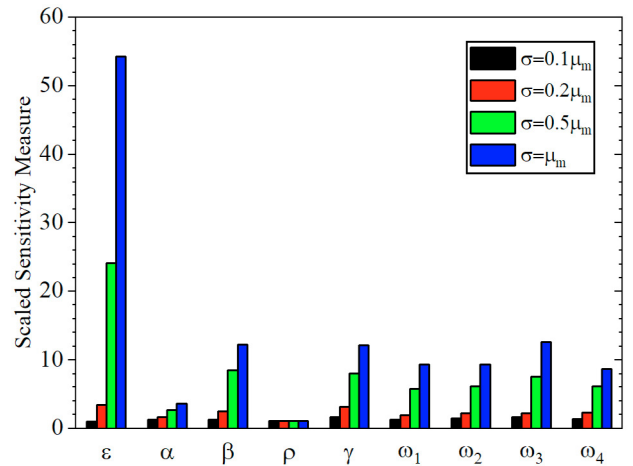


Fig. 5. Sensitivity analysis for small network model

the mean value. The uncertainty range for each parameter is between zero and twice of the nominal value. In each calculation, only one parameter is varied and the other parameters are kept at their nominal values.

Nine points are uniformly sampled within the uncertainty range for each parameter. Taking the k -th parameter in θ_U as an example, the sampled parameters are written as $[\theta_{U,k}(1), \dots, \theta_{U,k}(9)]^T$, ($i = 1, \dots, 9$). The values of the probability density function (pdf) corresponding to the nine sampled parameters are written in vector \mathbf{p}_k . Then for the j -th sampling point in the k -th parameter, the t -th day adoption rate can be calculated as $h(t, \theta_{U,k}(j)) \cdot \mathbf{p}_k(j)$.

To assess the relative impacts from the nine parameters, the following residual function between the calculated adoption rates and the surveyed adoption rates, weighted by the parameter's pdf, is used as the measure for the k -th parameter.

$$J_k = \sum_{t=1}^{28} \sum_{j=1}^9 (h(t, \theta_{U,k}(j)) - y(t))^2 \cdot \mathbf{p}_k(j) \quad (14)$$

For each parameter, taking the standard deviation at four levels, i.e., 10%, 20%, 50% and 100% of the nominal value of, and apply the calculation to all nine parameters in θ_U , the sensitivity analysis results are shown in Fig. 5 for the small network and Fig. 6 for the large network, where the output metric J_k is scaled by the value calculated at the nominal parameter value.

From the sensitivity analysis, it can be observed that among all the parameters, the threshold for the utility in (11), ε , has the strongest influence on the formed output function in (14). This shows the significance of choosing a proper threshold for adoption decision making.

Among the parameters for the advertisement model, (α , β , ρ , γ), the diminishing return factor of advertisement effect (γ) and the upper threshold (β) have larger impact to the model output.

Among the four weighting coefficients, (ω_1 , ω_2 , ω_3 , ω_4), the total network trend influence (ω_3) shows the largest sensitivity, which is consistent with the results shown in Fig. 4, calculated based on the nominal model.

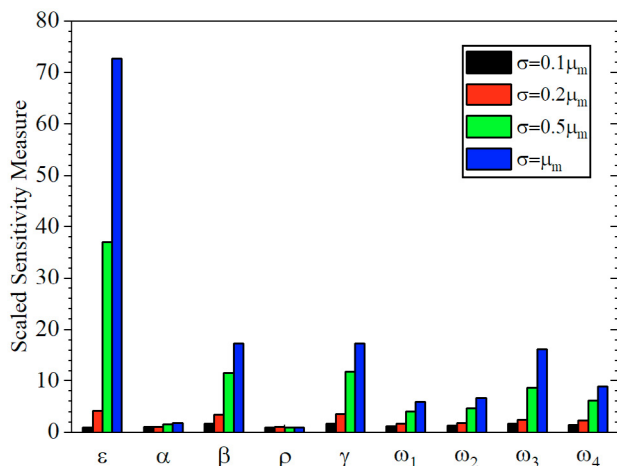


Fig. 6. Sensitivity analysis for large network model

4. CONCLUSIONS

In this study, mathematical models are developed to simulate the dynamics of the adoption rate of energy efficiency product for small and large population networks. A utility measure is defined to support users' decision making. This utility measure is dependent on four factors, the personal acceptance level, the neighbouring network influence, the network trend influence, and the advertisement influence.

A small population network and a large network are simulated in case studies. Based on the developed models, optimal results of the four coefficients are calculated using the survey data, among them the social trend influence has the largest impact on the utility measure. It suggests that the adoption behaviour for an individual is heavily dependent on the adoption rate of the whole social network.

The personal acceptance level is modelled to include four individual states factors, income, family situation, age, employment. The simulation result shows that, except for the income status, the other three factors have clear relevance to the acceptance level.

For the large population network, personal acceptance level has less impact when the network is close to a clustered network, however, when the network approaches a random status, personal acceptance can account for 0.2386 among the four weights, which is close to the result calculated for the small network ($\omega = 0.2388$). According to (Chen et al., 2014), real world social networks are close to W-S small-world with the rewiring probability close to 0.1. Thus, personal acceptance has less impact for large population network compared to small population network.

In this work, we have proposed a model to quantify the influence from individuals and social networks on adoption of energy efficiency products. There are other user behaviour metrics that can be considered for an energy savings program. Also, for a program that lasts for several years, the model parameters are likely to be time-varying. The current model can certainly be improved with the use of more survey data in the future investigation.

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