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Modelling The Impulsive Breakdown Characteristics of Sub-mm to mm Spheroidal Voids

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***Abstract*—Solid insulation is widely used in power and pulsed power systems. In the past, great attention has been paid to gas-filled voids found in solid dielectrics within the context of partial discharges for power equipment, such as high voltage cables. However, it is equally important to consider the effect of such voids on the degradation of solid insulation within pulsed power systems. Discharge activity within gas-filled voids, or across cavities at solid-solid interfaces, can potentially threaten equipment longevity and durability. Therefore, the transient ionization processes initiated inside these voids, particularly under fast-rising impulsive energization, are of great importance. Yet, the additional complexities of transient stress compared to steady-state conditions typically render classical steady-state breakdown models inapplicable, and necessitates a different approach for the impulsive regime. In this work, an approach for the breakdown voltage and time-to-breakdown estimation of spheroidal air-filled voids, subjected to fast-rising high voltage impulses has been developed. The proposed model links the time-dependent electric field developed within a spheroidal void to gas ionization and breakdown criteria, allowing the estimation of the formative time-to-breakdown and breakdown voltage of the void. This study investigated the effects of *dV/dt* on estimated void breakdown parameters, and the approach may be of use for the further characterization of solid insulation under impulsive energization, aiding in the future development of novel insulation technologies for current and future pulsed power systems.**

***Index Terms*—solid dielectrics, gas voids, electrical breakdown, gas discharge, pulsed power systems, electron avalanche**

I. Introduction

As the complexity of power and pulsed power systems increases, so too does the requirement for correspondingly advanced and novel high voltage (HV) insulation technology. Modern HV systems may exert significant transient electrical stress on their insulating components, and coupled with the growing requirement for systems to be highly performant yet compact, the further development of electrical insulation solutions has become a necessity. A comprehensive understanding of factors which influence insulation performance under fast-rising stresses would benefit a wide range of industries that may utilize pulsed power technologies, including the power, defence, biomedical, and aerospace sectors.

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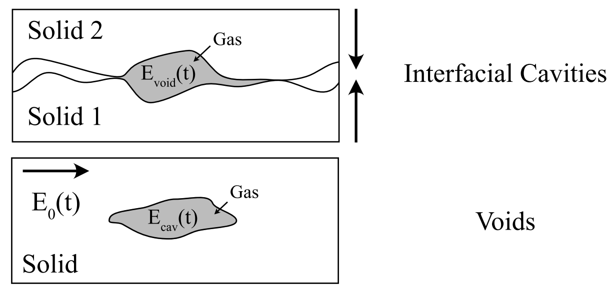

Fig. 1. Diagram depicting (top) void-like gaps formed by cavities at solid-solid dielectric interfaces due to surface roughness, and (bottom) sealed void defects formed inside solid dielectrics.

In this work, analysis has been conducted on gaseous voids of spheroidal geometry embedded within an external dielectric bulk, with a particular focus on the initiation of gas discharge within said voids. An analytical approach based on simplified electron transport has been employed to estimate the void breakdown strength, but which also incorporates the effects of time-dependent applied voltages and voltage rate-of-rise, *dV/dt*. The developed model can be considered an extension to the classical Meek-Raether criterion, but with the explicit consideration of *dV/dt*, and the redefinition of the avalanche-to-streamer transition to be based upon the electric field, rather than an approximate critical charge density (or often, a value for the ionization integral, K). It is remarked that the present work assumes streamer transition is possible within the gas-filled cavities across solid-gas interfaces. Under certain conditions (relating the the rate of voltage rise, gas pressure, overvoltage, etc.), the critical distance can be shown to be shorter than a typical void or cavity length for short mm and sub-mm gaps. Treatment of the breakdown regime where the length of the void is less than the critical transition distance is left as a topic for future work. Likewise, voids with dimensions and internal pressure nearing the Paschen limit ($\lesssim$15 μm at 1 atm.) were also outside the scope of this study.

II. Model Formulation

A. Spheroidal Voids

Voids within solid insulation (or void-like features formed from contacting cavities at solid-solid dielectric interfaces,

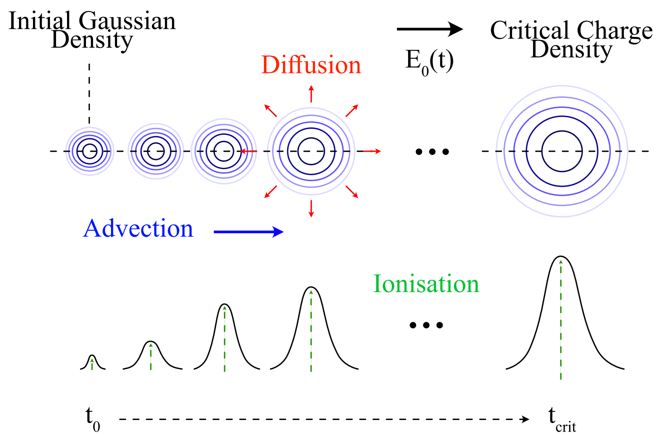

Fig. 2. Graphical depiction of an evolving initial Gaussian electron density, under the action of simplified advection, diffusion, and reaction. t_{crit} is the critical time: the moment of avalanche-to-streamer transition.

e.g., as illustrated in Fig. 1) can lead to a significant reduction in the net dielectric strength of a solid insulation system [1], and therefore formed the subject of the present study. Specifically, voids with dimensions on the order of hundreds of micrometers to several millimeters, stressed with voltages rising at several kV/ns were of interest. In the authors' previous work [2], closed-form mathematical expressions for the time-dependent electric field developed inside spherical and spheroidal voids were shown to be obtainable. These models allowed the estimation of the electric field magnitude under any arbitrary time-varying external field, taking into consideration dielectric relaxation and the effects of Maxwell-Wagner polarization due to the interfacial regions between the gas void and dielectric bulk.

For the purposes of this study, the approach from [2] was once against used, assuming a double-exponential form for the external field, emulating impulsive voltage signals with defined rise and fall characteristics. For brevity, the field expressions will not be repeated here, and the reader should refer to [2] for details. It follows, therefore, that the link between the estimated intra-void electric field and the initiation of ionization processes within the cavity (and eventually to void breakdown) must be established.

B. Intra-void Avalanche-To-Streamer Transition

For the development of pulsed power systems, importance lies not only with the breakdown voltage, but also the time to breakdown. Estimating these parameters under the action of fast-rising impulses necessitates the consideration of the time-varying aspects of gaseous discharge, and may often prohibit breakdown criteria (or empirical scaling relationships) developed under standard AC or DC steady-state conditions from providing accurate predictions.

This work draws inspiration from results arising from the hydrodynamic modelling of streamer discharges and electron transport. Following an approach originally developed by Montijn and Ebert [3], an avalanche-to-streamer transition model based on the simplified transport of a Gaussian electron cloud has been developed, and has been applied to the pulsed breakdown of gas-filled gaps, which includes the subject of the present work: gaseous voids. In the model description that follows, Fig. 2 provides a graphical illustration of the modelled avalanche development process. It is remarked that the inclusion of statistical time for the appearance of an initial electron is not within the scope of this work, and therefore the analysis begins by assuming that a localised electron seed already exists in the gap, where the electron density n_e is Gaussian-distributed. Thus, in spherically-symmetric coordinates:

$$n_e(r, t=0) = n_0(t=0) \exp\left(-\frac{r^2}{2\sigma_0^2}\right), \tag{1}$$

where $n_0(t)$ is the peak magnitude of the seed at time t, σ_0 is the initial spread of the seed, and r is the radial coordinate. The spatiotemporal evolution of the initial electron cloud was modelled by considering a constant diffusion coefficient D, electron mobility $\mu_e(t)$, and the effective ionization coefficient $\bar{\alpha}(t) = \alpha(t) - \eta(t)$, where α and η are the ionization and attachment coefficients, respectively. Note that the electron mobility and effective ionization coefficient were assumed to be time-dependent parameters due to their dependence on the electric field, which itself is also time-dependent. The diffusion coefficient, however, was considered constant as a simplification. Though, it is remarked that for fast impulses as considered in this work, diffusion is unlikely to be significant. Then, according to Fick's law of diffusion, the Gaussian seed will grow like:

$$n_e(r, t) = n_0(t) \frac{\sqrt{2\sigma_0^2}^3}{\lambda_2^3} \exp\left(-\frac{r^2}{\lambda_1^2}\right), \tag{2}$$

where λ_1 and λ_2 are given by:

$$\begin{aligned} \lambda_1 &= \sqrt{2\sigma_0^2 + 4Dt}, \\ \lambda_2 &= \sqrt{2\sigma_0^2 + 4\pi Dt}. \end{aligned} \tag{3}$$

The term λ_2^3 found in the denominator of (2) therefore determines the reduction in the Gaussian magnitude as a result of electron diffusion. The growth of the electron cloud is determined by impact ionization, from which the time-dependent peak electron concentration $n_0(t)$ is given by the solution to the differential equation:

$$\frac{\partial n_0(t)}{\partial t} = \bar{\alpha}(t)\mu_e(t)n_0(t)|E(t)|, \tag{4}$$

where $E(t)$ is the electric field. The electron growth caused by impact ionization must therefore overcome diffusive reduction and attachment before net growth of the electron cloud becomes possible [3]. It was further assumed that in the initial stages proceeding voltage application, space charge effects would be negligible and therefore the entire electron cloud would be subject to drift under action of only the external

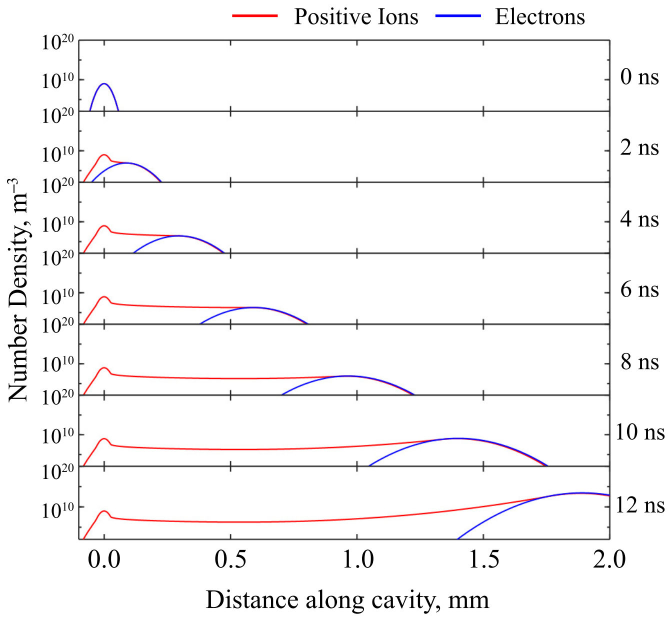

Fig. 3. Spatiotemporal evolution of the electron cloud (and trail of positive ions) as estimated by the present model, up to 12 ns, and across a spheroidal void with a semi-major axis of 1 mm, from one side to the other.

field, such that the position of the Gaussian would be translated according to the electron drift velocity:

$$r = \sqrt{[x - s(t)]^2 + y^2}, \qquad \frac{\partial s(t)}{\partial t} = \mu_e(t) E(t), \tag{5}$$

where x and y are the Cartesian coordinates, and $s(t)$ is the linear displacement of the electron cloud. The initial Gaussian distribution is therefore maintained throughout its transport, where only its peak concentration, location, and spread changes with time. Consequently, the electron induced electric field has the analytical solution:

$$E_e(r,t) = \frac{e n_0(t)}{4\pi\varepsilon_0} \frac{\lambda_1^3 \sqrt{2\pi\sigma_0^2}^3}{\lambda_2^3} \times \left[\frac{1}{r^2}\operatorname{erf}\left(\frac{r}{\lambda_1}\right) - \frac{2}{\lambda_1\sqrt{\pi}}\exp\left(-\frac{r^2}{\lambda_1^2}\right)\right] \tag{6}$$

where e is the elementary charge, ε_0 is the vacuum permittivity, and erf is the error function. In the present work, the field arising from the positive ions has been neglected. As the avalanche leaves a trail of positive space charge in its wake, the computation of the positive ion field is nontrivial. Therefore, only the electron field has been considered for the transition criterion, which is taken to be the moment that the peak of the electron field becomes equal to the external field in magnitude, or:

$$\max_{r,t} E_e(r, t_{br}) = E(t_{br}), \tag{7}$$

and t_{br} would therefore be the estimated formative time to breakdown, with $E(t_{br})$ being the corresponding breakdown

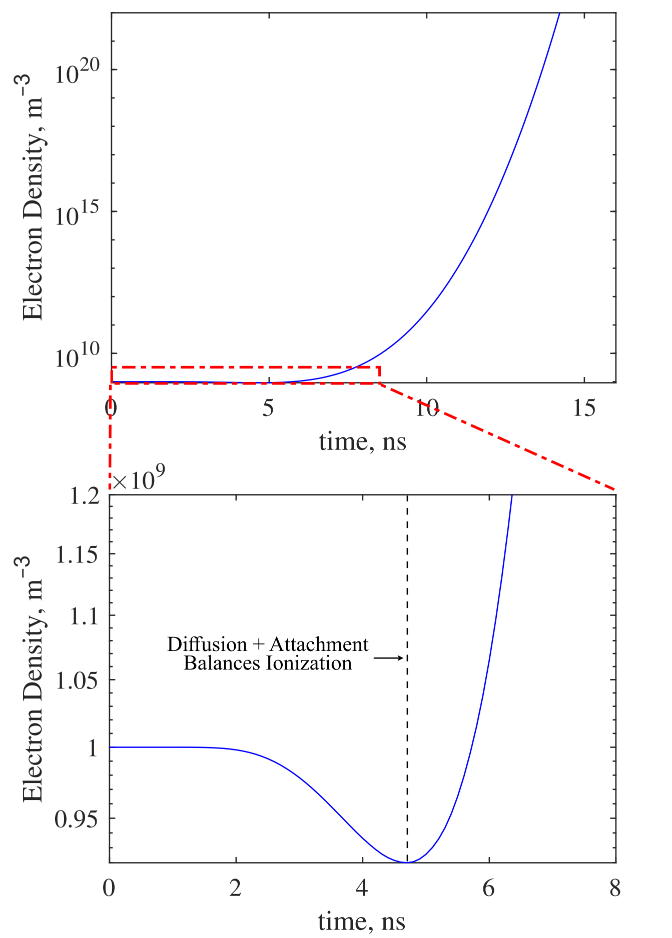

Fig. 4. (top) Rapid growth of the peak electron density as estimated by the present model up to the point of predicted streamer transition. (bottom) Initial reduction of electron density before ionization balances attachment and diffusion.

field strength, and $U_0(t_{br})$ the breakdown voltage. It is remarked that the positive ion density $n_+(r,t)$ can be solved numerically (neglecting ion diffusion and advection, both of which would be negligible under the timescales considered here), following:

$$\frac{\partial n_+(r,t)}{\partial t} = \max\left(\frac{\partial n_e(r,t)}{\partial t}, 0\right). \tag{8}$$

The $\max$ function is used here to ensure that only the positive parts of $\partial n_e/\partial t$ are taken, as under the assumptions of the present model, the only source of positive ions would be due to electron-ion pair production resulting from impact ionization. Since electron-ion recombination was not considered, negative components of $\partial n_e/\partial t$ can only be a result of attachment (to form negative ions, which were also not considered) or diffusive reduction, both of which would not have any impact on the positive ion density. Neither equation (8) nor its corresponding electric field has an analytical solution, but it is nonetheless useful for the purposes of visualisation.

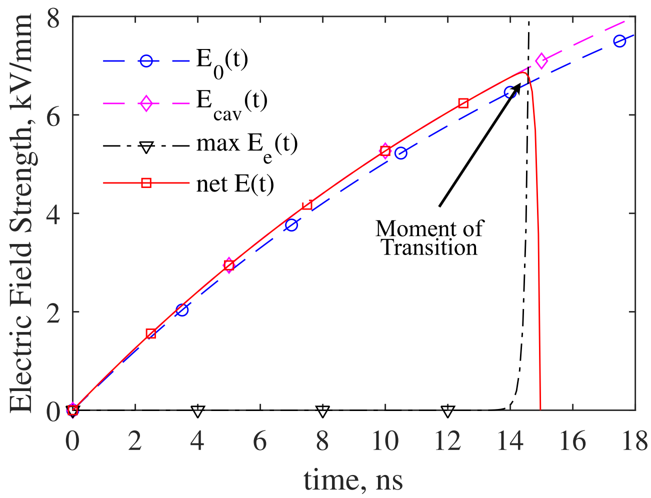

Fig. 5. Plot of the external field, intra-void field, maximum space-charge induced field, and the net intra-void field over time. Labelled is the point of avalanche-to-streamer transition when the field is rapidly screened by the growing space charge field.

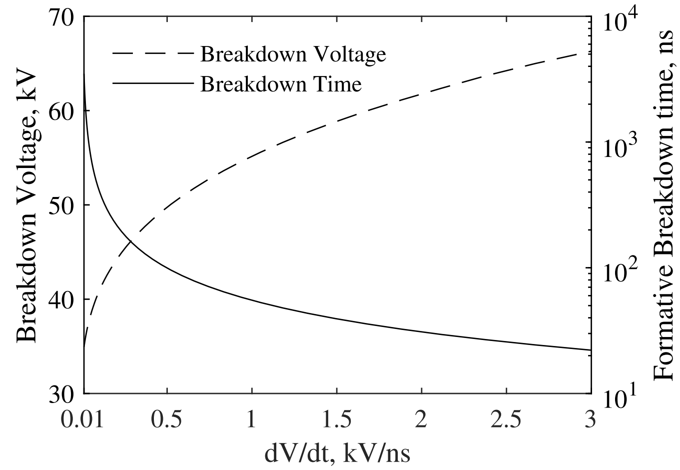

Fig. 6. Estimated breakdown voltage and formative time to breakdown for a simple 10 mm plane-plane gap, for *dV/dt* between 0.01 kV/ns and 3 kV/ns.

III. Avalanche Development

Equations (2) and (8) were solved for a gas void encased in a bulk solid with ε_b = 2.5, peak applied voltage of U_0 = 100 kV, and r_{maj} = 1 mm, r_{min} = 250 μm, which are the semi-major and minor radii, respectively. These dimensions were informed by estimated cavity dimensions determined from experimental surface characterization of rough surface contact. The double-exponential voltage was configured to produce an impulse with 10-90% rise-time of 50 ns, and a full width at half maximum (FWHM) of 1 μs. Transport parameters were interpolated from tabulated data [4], [5], assuming that the voids were filled with air. Fig. 3 shows the development of the electron avalanche, indicated by the initial Gaussian density which changes in peak magnitude and spread over time. In the initial stages (up to around 5 ns) the total number of electrons decreases, as attachment and diffusion dominate over ionization. However, this rapidly changes with the rising electric field in the void, as the propagation and development of the electron avalanche begins soon after, with the electron production quickly becoming exponential. This is shown in detail in Fig. 4, where the bottom panel shows the reduction of the electron density from the initial 10^9 m^{-3} peak magnitude.

Fig. 5 shows the external field, the field in the void, the maximum electron-induced electric field, and the net electric field. Labelled is also the moment of avalanche-to-streamer transition (in this case, the estimated moment of void breakdown), marked by the rapid shielding of the intra-void field by the exponential increase of the space charge induced electric field.

IV. Effect of The Rate of Voltage Rise

Using the present approach, the time-dependent model allows the effects of the voltage rise of rise to be investigated. To do so, ramp voltages rising with *dV/dt* between 0.01 kV/ns and 3 kV/ns were investigated in a simple 10 mm gap (in absence of any dielectric, so as to remove any possible relaxation effects and isolate the effects of *dV/dt*), assuming a uniform field. Estimations of breakdown voltages and formative time-to-breakdowns by the present model were recorded, and have been plotted against *dV/dt* in Fig. 6. An increase of the predicted breakdown voltage was recorded with increasing *dV/dt*, aligning with typical experimental trends observed for fast pulsed breakdown. Correspondingly, a reduction in the predicted breakdown time was estimated for faster rising voltages - an expected trend considering that breakdown will always occur on the 'rising slope' of a ramp voltage, if the ramp voltage were to be considered the rising slope of an impulse. It is remarked that the model is also capable of estimating breakdown on the falling edge of an impulse, should an impulse with defined rise and fall characteristics be used instead of a continuously rising ramp.

V. Conclusions

In this work, an analytical approach to the modelling of the avalanche to streamer transition, originally developed in [3], has been applied to the fast-rising breakdown of spheroidal dielectric voids of sub-mm and mm dimensions. By considering the dynamics of a Gaussian-distributed electron cloud, the spatiotemporal evolution of the space charge induced electric field up to the moment of external field distortion (avalanche-to-streamer transition) has been estimated. In combination with the authors' previously-developed analytical model for the time-dependent electric field inside a spheroidal void [2], the process of estimating the void breakdown strength under a 50/1000 ns impulse has been demonstrated. Furthermore, an investigation of the predicted dependency between breakdown voltage and formative breakdown time on *dV/dt* was conducted for a simple gas gap without any dielectric. The present approach estimated a nonlinear increase in the breakdown voltage, and corresponding decrease in the breakdown time, with increasing *dV/dt* from 0.01 kV/ns up to 3 kV/ns. Resulting predictions provided by this approach may be beneficial for the design of novel and composite insulation systems for power and pulsed power applications.

However, to explore the full range and capability of the model for pulsed breakdown, a number of aspects continue to be under development. Namely, the breakdown regime under conditions where the void length does not exceed the critical distance necessary for streamer transition, and where a Townsend-like mechanism may dominate, possibly involving the effects of secondary electron emission, should be studied further. Additionally, closed-form approximations to the full numerical model are also under development, with the objective to produce a convenient mathematical description which has the potential to be used as design curves for equipment involving fast pulsed gas breakdown.

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