This is a peer-reviewed, accepted author manuscript of the conference paper: Zhao, X., Huang, Y., & Wong, T. C. (2023). General decay projective synchronization of drive-response reaction-diffusion memristive neural networks. In 2023 IEEE 13th International Conference on CYBER Technology in Automation, Control, and Intelligent Systems (CYBER) (pp. 1253-1257). https://doi.org/10.1109/cyber59472.2023.10256521

General Decay Projective Synchronization of Drive-response Reaction-diffusion Memristive Neural Networks

Xin Zhao, Yanli Huang* and Tse Chiu Wong

Abstract— **This paper studies the general decay projective synchronization (GDPS) of a class of drive-response reaction-diffusion memristive neural networks (RDMNNs). Firstly, a suitable controller is designed, which does not ask for any knowledge about the activation functions. Then we investigate the GDPS of drive-response RDMNNs by constructing a suitable Lyapunov functional, and an adequate condition for guaranteeing the GDPS of this type of network is inferred. Lastly, the validity of the result is demonstrated by one numerical example with simulation results.**

I. Introduction

How to simulate synapses in the human brain and make artificial neural networks (NNs) more similar to the human brain in information processing has always been a hot topic in the research of artificial NNs. The emergence of memristive neural networks (MNNs) based on memristor with the same functional components provides a good solution to the above problem. In practical terms,the memristor has the advantages of high integration, low power consumption as well as synaptic plasticity. As a consequence, a large number of papers have been published in recent years to witness the development of MNNs [1-4]. MNNs based on memristors have gradually become a hot spot for scientists. In [5], the authors proposed a physical layer security strategy of space division multiplex orthogonal frequency division multiplexing system based on MNNs.

As is well known, incomplete uniform electromagnetic field can bring about the diffusion phenomena of electrons in circuit simulation experiments of MNNs. In this situation, the state trajectory of electrons needs to be represented by partial differential equations with respect to time and space variables. In recent years, synchronization has always been a frequently discussed topic on drive-response reaction-diffusion memristive neural networks (RDMNNs). Wu et al. [6] studied the pinning synchronization of stochastic neutral RDMNNs by designing an appropriating Lyapunov-Krasovskii functional. Cao et al. [7] discussed the global exponential anti-synchronization issue for an array of delayed MNNs with reaction-diffusion terms and leakage term.

By and large, the types of synchronization can be divided into generalized synchronization, impulsive synchronization, lag synchronization, complete synchronization and projective synchronization [8-12]. Due to the unpredictability of scaling constants, projection synchronization has outstanding advantages over other types of synchronization, which can extra promote communication security. The article [13] investigated the combinatorial projection synchronization issue of fractional order complex dynamic networks with external interference and time-varying delay coupling. By making use of a suitable controller, He et al. [14] explored the projection scaling factor to a general constant matrix and researched the global matrix projection synchronization for the delayed fractional-order NNs. In view of the different synchronization speeds and times, it was difficult to gain the convergence rate or time of the NNs. for some realities. Fortunately, the general decay projective synchronization (GDPS) can get over this issue. The GDPS of a class of memristive competitive NNs with time delay was discussed in [15]. However, according to our knowledge, there is no result on the GDPS of drive-response RDMNNs. With inspiration from above, this work explores the GDPS of drive-response RDMNNs.

The rest of this paper is outlined as follows. Some notations, several relevant definitions and essential lemmas are introduced in Section II. Section III is devoted to establishing the GDPS criterion of the considered derive response RDMNNs. In Section IV, a numerical example is given to verify the feasibility of the derived synchronization result. We end this

Yanli Huang is the corresponding author and with School of of Computer Science and Technology, Tiangong University, Tianjin 300387, China. huangyanli@tiangong.edu.cn. Xin Zhao is with School of of Computer Science and Technology, Tiangong University, Tianjin 300387, China. Tse Chiu Wong is with Department of Design Manufacturing and Engineering Management (DMEM), Faculty of Engineering, University of Strathclyde, 75 Montrose Street, Glasgow, Scotland G1 1XJ, UK.

paper in Section V with summary of our work.

II. Preliminaries

A. Notations

$\mathbb{R} = (-\infty, +\infty)$ delegates the set of real numbers. $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{n\times n}$ denotes the $n\times n$ dimensional real matrix space. We define $\Omega = \{s = (s_1,\ s_2,\ \cdots,\ s_j)^T \big| |s_h| < \alpha_h,\ h = 1,\ 2,\ \cdots,\ j\} \subset \mathbb{R}^j$ is an open bounded domain with smooth boundary $\partial\Omega$.

B. Lemma and definition

Lemma 1. *Let* $\Omega = \{s = (s_1,\ s_2,\ \cdots,\ s_j)^T \big| |s_h| < \alpha_h,\ h = 1,\ 2,\ \cdots,\ j\} \subset \mathbb{R}^j$ *be a cube and* $\xi(s) \in C^1(\Omega)$ *be a real valued function satisfying* $\xi(s)|_{\partial\Omega} = 0$*, then:* $\int_\Omega \xi^2(s)ds \leqslant \alpha_h^2 \int_\Omega \left(\frac{\partial\xi}{\partial s_h}\right)^2 ds$.

Definition 1. *If there exists a function* $\psi(\varpi) : \mathbb{R}^+ \to (0, +\infty)$ *which satisfies the following conditions:*
(1) it is nondecreasing and differentiable;
(2) $\psi(+\infty) = +\infty$ *and* $\psi(0) = 1$*;*
(3) $\overline{\psi}(\varpi) := \frac{\dot{\psi}(\varpi)}{\psi(\varpi)}$ *is decreasing;*
(4) for any $u, v \geqslant 0,\ \psi(u+v) \leqslant \psi(u)\psi(v)$*;*
then the function $\psi(\varpi)$ *is* ψ*-type function.*

III. GDPS of Drive-response RDMNNs

In this part, we will consider the network model of RDMNN as below:

$$\begin{aligned}\frac{\partial\xi_q(s,\varpi)}{\partial\varpi} &= -c_q\xi_q(s,\varpi) + I_q + p_q\Delta\xi_q(s,\varpi) \\ &\quad + \sum_{b=1}^{n} e_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi)), \qquad (1)\end{aligned}$$

where $q = 1,2,\cdots,n$, $\xi_q(s,\varpi)$ is the state of the qth neuron in space s and at time ϖ; $\Delta = \sum_{h=1}^{j} \frac{\partial^2}{\partial s_h^2}$; $g_b(\xi_b(s,\varpi))$ stands for the activation function of the bth neuron; I_q is the external input; $c_q > 0$ is the self-inhibition rate of neuron; $p_q > 0$ is the diffusion coefficient; $(e_{qb}(\xi_q(s,\varpi)))_{n\times n}$ denotes the connection weight of memristor synapses, where $e_{qb}(\xi_q(s,\varpi))$ is defined as:

$$e_{qb}(\xi_q(s,\varpi)) = \frac{\mathscr{F}_{qb}}{\mathscr{O}_q} \times \text{sign}_{qb},$$

$$\text{sign}_{qb} = \begin{cases} 1, & q \neq b, \\ -1, & q = b, \end{cases}$$

where $b, q = 1,\ 2,\ \cdots,\ n$, $\mathscr{F}_{qb}$ denotes the memductances of memristors $\mathscr{J}_{qb}$, $\mathscr{J}_{qb}$ represents the memristor between $g_b(\cdot)$ and $\xi_q(s,\varpi)$. Based on the voltage-current features of memristor, and we define the memristive connection weight as below:

$$e_{qb}(\xi_q(s,\varpi)) = \begin{cases} \hat{e}_{qb}, & |\xi_q(s,\varpi)| \leqslant V_q, \\ \check{e}_{qb}, & |\xi_q(s,\varpi)| > V_q, \end{cases}$$

where the switching jumps $V_q > 0$, $\hat{e}_{qb}$ and $\check{e}_{qb}$ are constants, $\tilde{e}_{qb} = \max\{|\check{e}_{qb}|, |\hat{e}_{qb}|\}$, $q,\ b = 1,\ 2,\ \cdots,\ n$.

For the system (1), $\xi_q(s,0) = \phi_q(s,\varpi)$, $\xi_q(s,\varpi) = 0$ if $(s,\varpi) \in \partial\Omega \times [0,+\infty)$, where the function $\phi_q(s,\varpi)$ is continuous and bounded on Ω.

We regard network (1) as the drive system, then the corresponding response system is constructed as

$$\begin{aligned}\frac{\partial\hat{\xi}_q(s,\varpi)}{\partial\varpi} &= -c_q\hat{\xi}_q(s,\varpi) + I_q + p_q\Delta\hat{\xi}_q(s,\varpi) \\ &\quad + \sum_{b=1}^{n} e_{qb}(\hat{\xi}_q(s,\varpi))g_b(\hat{\xi}_b(s,\varpi)) + l_q(s,\varpi), \qquad (2)\end{aligned}$$

where $q = 1,\ 2,\ \cdots,\ n$, $\hat{\xi}_q(s,\varpi)$ is the state of the q-th node; $l_q(s,\varpi)$ denotes the controller to be designed; c_q, p_q, $\triangle$, I_q, $e_{qb}(\cdot)$, $g_b(\cdot)$ are defined similarly to the network (1).

The initial condition and Dirichlet boundary condition of system (2) are: $\hat{\xi}_q(s,0) = \varphi_q(s,\varpi)$, $\hat{\xi}_q(s,\varpi) = 0$ if $(s,\varpi) \in \partial\Omega \times [0,+\infty)$, where the function $\varphi_q(s,\varpi)$ is continuous and bounded on Ω.

Assumption 1. *There are several positive numbers* χ_b *and* $\tilde{\chi}_b$*, for* $b = 1,\ 2,\ \cdots,\ n$*, such that:* $|g_b(z_1) - g_b(z_2)| \leqslant \chi_b|z_1 - z_2|, |g_b(z)| \leqslant \tilde{\chi}_b$*, for* $\forall z,\ z_1,\ z_2 \in \mathbb{R}$.

Assumption 2. *There is a positive constant* κ *and a function* $r(\varpi) \in C(\mathbb{R},\ \mathbb{R}^+)$ *such that:*

$$\sup_{\varpi\in[0,\infty)} \int_0^{\varpi} \psi^{\kappa}(s)r(s)ds < \infty, \overline{\psi}(\varpi) \leqslant 1,$$

in which $\psi(\varpi)$*,* $\overline{\psi}(\varpi)$ *are given in Definition 1.*

By letting $\mu_q(s,\varpi) = \hat{\xi}_q(s,\varpi) - m_q\xi_q(s,\varpi)$ be the error vector, m_q is a proportional constant, then we get

$$\begin{aligned}\frac{\partial\mu_q(s,\varpi)}{\partial\varpi} &= \sum_{b=1}^{n} e_{qb}(\hat{\xi}_q(s,\varpi))f_b(\mu_b(s,\varpi)) + R_q \\ &\quad + l_q(s,\varpi) + p_q\Delta\mu_q(s,\varpi) - c_q\mu_q(s,\varpi), (3)\end{aligned}$$

where $f_b(\mu_q(s,\varpi)) = g_b(\hat{\xi}_b(s,\varpi)) - g_b(m_q\xi_b(s,\varpi))$, $R_q = (1-m_q)I_q + \sum_{b=1}^{n}[e_{qb}(\hat{\xi}_q(s,\varpi))g_b(m_q\xi_b(s,\varpi)) - m_q e_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi))]$.

The controller $l_q(s,\varpi)$ is designed:

$$\begin{aligned}l_q(s,\varpi) &= -\eta_q \frac{||\mu(\cdot,\varpi)||^2\mu_q(s,\varpi)}{||\mu(\cdot,\varpi)||^2 + r(\varpi)} \\ &\quad -\varsigma_q \text{sign}(\mu_q(s,\varpi)), \qquad (4)\end{aligned}$$

where $q=1,\ 2,\ \cdots,\ n$, $\mathbb{R}\ni\eta_q>0$, $\mathbb{R}\ni\varsigma_q>0$, and

$$\mathrm{sign}(\mu_q(s,\varpi))=\begin{cases}1, & \mu_q(s,\varpi)\geqslant 0,\\ -1, & \mu_q(s,\varpi)<0.\end{cases}$$

For convenience, we define $\eta=\max\limits_{1\leqslant q\leqslant n}\{\eta_q\}$.

Definition 2. *If there is a real positive number κ, such that:*

$$\limsup_{\varpi\to\infty}\frac{\log||\mu(\cdot,\varpi)||}{\log\psi(\varpi)}\leqslant-\kappa,$$

where $\mu(s,\varpi)=(\mu_1(s,\varpi),\ \mu_2(s,\varpi),\ \cdots,\ \mu_n(s,\varpi))^T$, $\psi(\varpi)$ is a ψ-type function, then the error system (3) is ψ-type stable, or the corresponding drive-response systems (1) and (2) achieve GDPS, in which κ is the convergence rate when $\mu(s,\varpi)\to 0$.

Lemma 2. *Under the condition of Assumption 2, if there are two constants $0<\gamma_1\in\mathbb{R},\ \ 0<\gamma_2\in\mathbb{R}$ and a differential function $V(\varpi,\mu_q(\varpi)):\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^+$, then:*

$$(\gamma_1||\mu_q(\varpi)||)^2\leqslant V(\varpi,\mu_q(\varpi)),$$

$$\dot V(\varpi,\mu_q(\varpi))|_{(3)}+\kappa V(\varpi,\mu_q(\varpi))\leqslant\gamma_2 r(\varpi),$$

where κ and $r(\varpi)$ are defined in Assumption 2, and $\mu_q(\varpi)$ is a solution of the error network (3), then the system (3) is ψ-type stable, which means that the drive-response systems (1) and (2) achieve GDPS. Besides, $\frac{\kappa}{2}$ is the rate of convergence.

Theorem 1. *If Assumptions 1, 2 and the following conditions hold:*

$$\Xi_1^{(q)}=\frac{1}{2}\sum_{b=1}^{n}(\tilde e_{qb}^2+\chi_b^2)-\sum_{h=1}^{j}\frac{p_q}{\alpha_h^2}+\frac{\kappa}{2}-(\eta_q+c_q)<0,\quad (5)$$

$$\Xi_2^{(q)}=\sum_{b=1}^{n}\tilde e_{qb}\tilde\chi_b(1+|m_q|)-\varsigma_q+|1-m_q|I_q\leq 0,\quad (6)$$

where $q=1,2,\cdots,n$, then, under the action of controller (4), the system (3) is ψ-type stable. Moreover, the drive-response systems (1) and (2) match up to GDPS when the rate of convergence is $\frac{\kappa}{2}$.

Proof. Define the following Lyapunov functional for network (3):

$$V(\varpi)\ =\ \frac{1}{2}\int u_q^2(s,\varpi)ds.\qquad (7)$$

Taking the derivative of $V(\varpi)$, we obtain

$$\begin{aligned}
\dot V(\varpi)&=\sum_{q=1}^{n}\int_\Omega\mu_q(s,\varpi)\frac{\partial\mu_q(s,\varpi)}{\partial\varpi}ds\\
&=\sum_{q=1}^{n}\int_\Omega\mu_q(s,\varpi)\{p_q\Delta\mu_q(s,\varpi)-c_q\mu_q(s,\varpi)\\
&\quad+\sum_{b=1}^{n}e_{qb}(\hat\xi_q(s,\varpi))f_b(\mu_b(s,\varpi))+(1-m_q)I_q\\
&\quad+\sum_{b=1}^{n}[e_{qb}(\hat\xi_q(s,\varpi))g_b(m_q\xi_b(s,\varpi))\\
&\quad-m_qe_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi))]\\
&\quad-\eta_q\frac{||\mu(\cdot,\varpi)||^2\mu_q(s,\varpi)}{||\mu(\cdot,\varpi)||^2+r(\varpi)}-\varsigma_q\mathrm{sign}(\mu_q(s,\varpi))\}ds\\
&\quad+\sum_{q=1}^{n}\eta_q\int_\Omega\mu_q^2(s,\varpi)ds-\sum_{q=1}^{n}\eta_q\int_\Omega\mu_q^2(s,\varpi)ds\\
&=\sum_{q=1}^{n}\int_\Omega\mu_q(s,\varpi)\{p_q\Delta\mu_q(s,\varpi)-c_q\mu_q(s,\varpi)\\
&\quad+\sum_{b=1}^{n}e_{qb}(\hat\xi_q(s,\varpi))f_b(\mu_b(s,\varpi))+(1-m_q)I_q\\
&\quad+\sum_{b=1}^{n}[e_{qb}(\hat\xi_q(s,\varpi))g_b(m_q\xi_b(s,\varpi))\\
&\quad-m_qe_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi))]\\
&\quad-\eta_q\mu_q(s,\varpi)-\eta_q\frac{r(\varpi)\mu_q(s,\varpi)}{||\mu(\cdot,\varpi)||^2+r(\varpi)}\\
&\quad-\varsigma_q\mathrm{sign}(\mu_q(s,\varpi))\}ds\\
&\leqslant\sum_{q=1}^{n}\int_\Omega\mu_q(s,\varpi)\{p_q\Delta\mu_q(s,\varpi)-c_q\mu_q(s,\varpi)\\
&\quad+\sum_{b=1}^{n}e_{qb}(\hat\xi_q(s,\varpi))f_b(\mu_b(s,\varpi))+(1-m_q)I_q\\
&\quad+\sum_{b=1}^{n}[e_{qb}(\hat\xi_q(s,\varpi))g_b(m_q\xi_b(s,\varpi))\\
&\quad-m_qe_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi))]-\eta_q\mu_q(s,\varpi)\}ds\\
&\quad-\sum_{q=1}^{n}\varsigma_q\int_\Omega|\mu_q(s,\varpi)|ds+\eta\frac{||\mu(\cdot,\varpi)||^2r(\varpi)}{||\mu(\cdot,\varpi)||^2+r(\varpi)}.(8)
\end{aligned}$$

In view of Green formula and boundary conditions, we know

$$\begin{aligned}
&\sum_{q=1}^{n}\int_\Omega\mu_q(s,\varpi)p_q\Delta\mu_q(s,\varpi)ds\\
&=-\sum_{q=1}^{n}\sum_{h=1}^{j}p_q\int_\Omega\left(\frac{\partial\mu_q(s,\varpi)}{\partial s_h}\right)^2ds\\
&\leqslant-\sum_{q=1}^{n}\sum_{h=1}^{j}\frac{p_q}{\alpha_h^2}\int_\Omega\mu_q^2(s,\varpi)ds.\qquad (9)
\end{aligned}$$

According to Assumption 1 and the inequality $|\ell-\hbar|\leqslant|\ell|+|\hbar|$, we can obtain

$$\begin{aligned}&\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}[e_{qb}(\hat{\xi}_q(s,\varpi))g_b(m_q\xi_b(s,\varpi))\\&\qquad-m_qe_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi))]\mu_q(s,\varpi)ds\\&\leqslant\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}(\tilde{e}_{qb}\tilde{\chi}_b+|m_q|\tilde{e}_{qb}\tilde{\chi}_b)|\mu_q(s,\varpi)|ds\\&=\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}\tilde{e}_{qb}\tilde{\chi}_b(1+|m_q|)|\mu_q(s,\varpi)|ds.\end{aligned}\tag{10}$$

By using Assumption 1, we know

$$\begin{aligned}&\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}\mu_q(s,\varpi)e_{qb}(\hat{\xi}_q(s,\varpi))f_b(\mu_b(s,\varpi))\\&\leqslant\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}|\mu_q(s,\varpi)|\tilde{e}_{qb}|f_b(\mu_b(s,\varpi))|ds\\&\leqslant\frac{1}{2}\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}(\mu_q^2(s,\varpi)\tilde{e}_{qb}^2+\chi_b^2\mu_b^2(s,\varpi))ds\\&=\frac{1}{2}\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}\mu_q(s,\varpi)\tilde{e}_{qb}^2\mu_q(s,\varpi)ds\\&\quad+\frac{1}{2}\sum_{b=1}^{n}\int_{\Omega}\mu_b(s,\varpi)\chi_b^2\mu_b(s,\varpi)ds.\end{aligned}\tag{11}$$

Substituting (9)-(11) into (8), one gets

$$\begin{aligned}\dot{V}(\varpi)\leqslant&\sum_{q=1}^{n}\int_{\Omega}\mu_q(s,\varpi)[-\eta_q\mu_q(s,\varpi)-c_q\mu_q(s,\varpi)]ds\\&+\eta\frac{||\mu(\cdot,\varpi)||^2r(\varpi)}{||\mu(\cdot,\varpi)||^2+r(\varpi)}+\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}[\tilde{e}_{qb}\tilde{\chi}_b(1\\&+|m_q|)-\varsigma_q+|1-m_q|I_q]|\mu_q(s,\varpi)|ds\\&+\frac{1}{2}\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}\mu_q(s,\varpi)\tilde{e}_{qb}^2\mu_q(s,\varpi)ds\\&+\frac{1}{2}\sum_{b=1}^{n}\int_{\Omega}\mu_b(s,\varpi)\chi_b^2\mu_b(s,\varpi)ds\\&-\sum_{q=1}^{n}\sum_{h=1}^{j}\frac{p_q}{\alpha_h^2}\int_{\Omega}\mu_q^2(s,\varpi)ds\\\leqslant&\sum_{q=1}^{n}\int_{\Omega}\mu_q(s,\varpi)[\frac{1}{2}\sum_{b=1}^{n}(\tilde{e}_{qb}^2+\chi_b^2)-\sum_{i=h}^{j}\frac{p_q}{\alpha_h^2}\\&-(\eta_q+c_q)]\mu_q(s,\varpi)ds+\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}[\tilde{e}_{qb}\tilde{\chi}_b(1\\&+|m_q|)-\varsigma_q+|1-m_q|I_q]|\mu_q(s,\varpi)|ds\\&+\eta r(\varpi).\end{aligned}\tag{12}$$

Therefore, it follows from (7) and (12) that

$$\begin{aligned}\dot{V}(\varpi)+\kappa V(\varpi)\leqslant&\sum_{q=1}^{n}\int_{\Omega}\mu(s,\varpi)[\frac{1}{2}\sum_{b=1}^{n}(\tilde{e}_{qb}^2+\chi_b^2)+\frac{\kappa}{2}\\&-\sum_{h=1}^{j}\frac{p_q}{\alpha_h^2}-(\eta_q+c_q)]\mu(s,\varpi)ds+\sum_{q=1}^{n}\sum_{b=1}^{n}\int_{\Omega}[\tilde{e}_{qb}\tilde{\chi}_b(1\\&+|m_q|)-\varsigma_q+|1-m_q|I_q]|\mu_q(s,\varpi)|ds+\eta r(\varpi).\end{aligned}$$

Based on (5) and (6), we get the final inequality: $\dot{V}(\varpi)+\kappa V(\varpi)\leqslant\eta r(\varpi)$. We let $\gamma_1=\frac{1}{\sqrt{2}}$, $\gamma_2=\eta$, it follows that the system (3) is ψ-type stable under the action of controller (4), which means that the relevant drive-response systems (1) and (2) attain GDPS and the rate of convergence is $\frac{\kappa}{2}$ in the meantime. □

IV. Examples

In this section, our goal is to verify the effectiveness of the GDPS criterion for the network under consideration derived above by an example with simulation results.

Example 1. *Consider the following network model of drive-response RDMNNs:*

$$\begin{cases}\frac{\partial\xi_q(s,\varpi)}{\partial\varpi}=-c_q\xi_q(s,\varpi)+I_q+p_q\Delta\xi_q(s,\varpi)\\\qquad+\sum_{b=1}^{2}e_{qb}(\xi_q(s,\varpi))g_b(\xi_b(s,\varpi)),\\\frac{\partial\hat{\xi}_q(s,\varpi)}{\partial\varpi}=-c_q\hat{\xi}_q(s,\varpi)+I_q+p_q\Delta\hat{\xi}_q(s,\varpi)\\+\sum_{b=1}^{2}e_{qb}(\hat{\xi}_q(s,\varpi))g_b(\hat{\xi}_b(s,\varpi))+l_q(s,\varpi),\end{cases}\tag{13}$$

where $q=1,\ 2,\ I_1=I_2=0,\ \Omega=\{s|-1<s<1\},\ g_b(h)=\frac{|h+1|-|h-1|}{8}$, *in which* $b=1,2,\ c_1=0.1,\ c_2=0.2,\ p_1=2.3,\ p_2=2.4$. *Moreover,* $e_{qb}(\varepsilon)$ *are chosen as following:*

$$e_{11}(\varepsilon)=\begin{cases}-0.21, & |\varepsilon|\leqslant1.5,\\ \ \ 0.26, & |\varepsilon|>1.5,\end{cases}$$
$$e_{12}(\varepsilon)=\begin{cases}-0.35, & |\varepsilon|\leqslant1.5,\\ -0.29, & |\varepsilon|>1.5,\end{cases}$$
$$e_{21}(\varepsilon)=\begin{cases}-0.28, & |\varepsilon|\leqslant1.5,\\ -0.24, & |\varepsilon|>1.5,\end{cases}$$
$$e_{22}(\varepsilon)=\begin{cases}0.22, & |\varepsilon|\leqslant1.5,\\ 0.21, & |\varepsilon|>1.5,\end{cases}$$

Select the appropriate parameters for controller (4)*:* $\eta_1=2.6,\ \eta_2=2.3,\ \varsigma_1=1.1,\ \varsigma_2=1.4$. *By computation, we take* $m_1=-2,\ m_2=-3,\ \chi_b=\tilde{\chi}_b=0.25$.

Let $\kappa=0.3$, *so the convergence rate is* $\frac{\kappa}{2}=0.15$. *We can get* $\Xi_1^{(q)}=-9.4665$ *and* $\Xi_2^{(q)}=-0.5575$ *by virtue of exploiting the MATLAB.*

According to Theorem 1, the drive-response system (13) realize GDPS under the action of the controller (4). The simulation result is displayed in Fig.1.

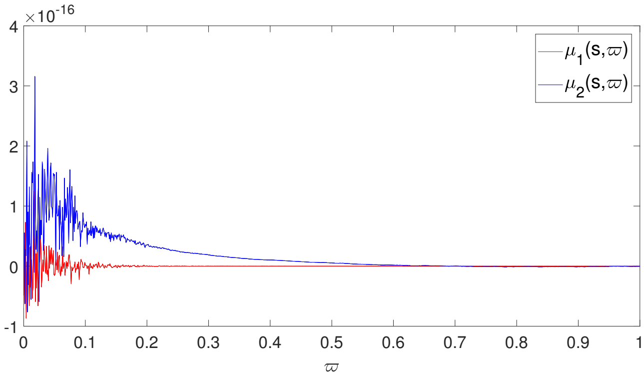

Fig. 1. $u_q(s,\varpi)$, $q=1,2$.

V. Conclusions

This study has concerned with the GDPS of drive-response RDMNNs. After introducing the network model of drive-response RDMNNs, the GDPS concept based on ψ-type function and ψ-type stability has been presented firstly. Then, a sufficient condition for attaining GDPS of the considered drive-response system has been inferred in view of a suitable Lyapunov function and an elaborate controller. Eventually, a numerical example has been given to demonstrate the validity of the obtained conclusion.

References

[1] Q. Lai, C. Lai, Kuate. PDK et al., “Chaos in a simplest cyclic memristive neural network,” International Journal of Bifurcation and Chaos, vol. 32, no. 07, pp. 2250042, 2022.
[2] C. Zhou, C. H. Wang et al., “Cluster output synchronization for memristive neural networks,” Information Sciences, vol. 589, pp. 459-477, 2022.
[3] S. Zhu, H. B. Bao, “Event-triggered synchronization of coupled memristive neural networks,” Applied Mathematics and Computation, vol. 415, pp. 126715, 2022.
[4] A. Lin, J. Cheng, J. H. Park et al., “Fault detection filtering of nonhomogeneous Markov switching memristive neural networks with output quantization,” International Journal of Bifurcation and Chaos, vol. 632, pp. 715-729, 2023.
[5] Q. Zhong, B. Liu, J. X. Ren et al., “Secure SDM-OFDM signal transmission based on memristive neural network,” Optics Communications, vol. 515, pp. 128030, 2022.
[6] X. Wu, S. T. Liu, H. Y. Wang, “Pinning synchronization of stochastic neutral memristive neural networks with reaction-diffusion terms,” Neural networks, vol. 157, pp. 1-10, 2023.
[7] Y. Y. Cao, W. J. Jiang, J. H. Wang, “Anti-synchronization of delayed memristive neural networks with leakage term and reaction-diffusion terms,” Knowledge-based Systems, vol. 233, pp. 107539, 2021.
[8] J. P. Yang, H. L. Li, J. K. Yang, L. Zhang et al., “Quasi-synchronization and complete synchronization of fractional-order fuzzy BAM neural networks via nonlinear control,” Neural Processing Letters, vol. 54, no. 4, pp. 3303-3319, 2022.
[9] Y. D. Ji, W. Wu, “Reliable mode-dependent memorized projective lag synchronization for semi-Markovian jumping coupled networks,” Proceedings of the Institution of Mechanical Engineers Part I-Journal of Systems and Control Engineering, vol. 235, no. 8, pp. 1367-1374, 2021.
[10] D. Ding, Z. Tang, Y. Wang, Z. C. Ji, “Pinning impulsive synchronization of complex networks with multiple sizes of delays via adaptive impulsive intervals,” Circuits Systems and Signal Processing, vol. 40, no. 9, pp. 4259-4278, 2021.
[11] J. Liu, G. R. Chen, X. Zhao “Generalized synchronization and parameters identification of different-dimensional chaotic systems in the complex field,” Fractals-Complex Geometry Patterns and Scaling in Nature and Society, vol. 29, no. 04, pp. 1-13, 2021.
[12] Z. X. Ding, C. Chen, S. P. Wen et al., “Lag projective synchronization of nonidentical fractional delayed memristive neural networks,” Neurocomputing, vol. 469, pp. 138-150, 2022.
[13] J. Fang, D. Wei, N. N. Yin et al., “Combination projection synchronization of fractional-order complex dynamic networks with time-varying delay couplings and disturbances,” Communications In Theoretical Physics, vol. 74, no. 11, pp. 16-26, 2022.
[14] J. M. He, T. F. Lei, F. Q. Chen, “Global matrix projective synchronization of delayed fractional-order neural networks,” Soft Computing, DOI: 10.1007/s00500-023-07834-5, 2023.
[15] M. Sader, F. Y. Wang, Z. X. Liu, Z. Q. Chen, “General decay projective synchronization of memristive competitive neural networks via nonlinear controller,” International Journal of Nonlinear Sciences and Numerical Simulation, vol. 23, no. 6, pp. 867-878, 2022.