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Should I open to forecast? Implications from a multi-country unobserved components model with sparse factor stochastic volatility ☆

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ABSTRACT

In this paper, we assess whether and when multi-country studies pay off for forecasting inflation and output growth. Factor stochastic volatility is adopted to allow for cross-country linkages and model economies jointly. We estimate factors and rely on post-processing, rather than expert judgement, to obtain an estimate for the number of factors. This is different from most existing two-step approaches in the factor literature. Our approach is then used to extend the existing unobserved components model, which assumes that 34 economies are independent. The results suggest that allowing for cross-country linkages yields inflation and output growth forecasts that are highly competitive with those obtained from estimating economies independently. Zooming into the forecast performance over time reveals that allowing for cross-country linkages is of particular importance when interest centres on forecasting periods of uncertainty. Another key finding is that the estimated global factors are correlated with the domestic business cycle. We interpret this to mean that part of the variation captured in global factors reflects a global business cycle.

1. Introduction

Since the seminal work by Stock and Watson (2007), the unobserved components (UC) model with stochastic volatility (SV) is commonly used for modelling latent state vectors. These latent state vectors can be interpreted as long-run equilibrium levels, and the UC model has enjoyed great popularity. Surprisingly, to the best of our knowledge, existing literature imposes an assumption of independence across economies. However, studies of global macroeconomic developments argue that national macroeconomic developments depend on international conditions. The dependence holds for both the real business cycle (see Kose, Otrok, & Whiteman, 2003) and inflation (see Ciccarelli & Mojon, 2010).

Investigating whether and when allowing for cross-country linkages pays off for inflation and output forecasting is the key objective of the present paper. Building on recent advances in econometrics, we adopt factor stochastic volatility (FSV) to allow for cross-country linkages and model economies jointly. To avoid omitting some potentially important factors, we adopt shrinkage techniques which use the sparsification on factor loadings and rely on post-processing to obtain an estimate for the number of factors.

From an empirical standpoint it is necessary to investigate how these techniques perform overall and over

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time. We show this by carrying out a thorough forecasting experiment involving 34 economies. The economies considered include 23 advanced economies (AEs) and 11 emerging market economies (EMEs). We include two variables in each economy (quarterly CPI inflation and output growth).

Our results show that the cross-country linkage techniques yield forecasts that are competitive with the ones obtained from estimating economies independently. When the focus is on forecasting periods of uncertainty, we find that these techniques can provide great improvements.

Additionally, we find that the slope of the Phillips curve is lower when using FSV. This seems to be relevant to the debate over the flattening of the Phillips curve. However, we think one needs to interpret this lower value with some care. By checking the correlation between the estimated global inflation factors and the domestic business cycle, we find they are positively correlated. In this sense, we interpret this to mean that part of the variation captured in global inflation factors reflects a global business cycle. Adding these factors can reduce the omitted-variable bias.

This paper is organised as follows. Section 2 reviews the related empirical literature and explains our contributions. In Section 3, we first discuss the UC model for individual economies and then introduce our new model. The details of our new model include FSV and an elaborated account of the sparsification. Section 4 illustrates some full-sample results by fitting our model to 34 economies. Section 5 describes an out-of-sample forecasting exercise. We provide evidence that our proposed model can improve forecasting overall and over time. Finally, Section 6 concludes.

2. Relationship to prior work

To make clear our contributions, we first briefly summarise the most closely related studies of the unobserved components model and global uncertainty. We then detail key differences in our analysis compared to these studies. In broad terms, our work extends the literature by a combination of allowing for cross-country linkages, the use of sparsification, and considering a large number of economies.

The unobserved components model A large body of research has emerged on extending the UC model. One strand of extensions has focused on introducing more indicators into the conditional mean. Another strand has focused on adding bounds on parameters. These extensions overlook the international comovement.

There has been a lot of recent research devoted to introducing suitable indicators into the UC model. These indicators are guided by either economic theory or empirical research. For instance, inspired by the Phillips curve, Stella and Stock (2013) extend the univariate UC model in Stock and Watson (2007) to a bivariate UC model, and assume that it is the inflation gap and unemployment gap that drive the Phillips curve.[1] Based on public commentary that central bankers pay considerable attention to measures of long-run inflation expectations, Chan, Clark, and Koop (2018) develop a bivariate UC model by introducing survey-based long-run forecasts of inflation. To directly address critiques of omitted-variable and omitted-equation bias pointed out by Taylor and Wieland (2016), Zaman (2022) further extends the bivariate UC model to a large-scale UC model and jointly estimates trends of several macroeconomic variables. The observed flattening of the Phillips curve has generated various explanations of this conundrum, and some studies highlight the role played by global factors. Therefore, Kabundi, Poon, and Wu (2021) introduce global factors (global output and oil price) into the bivariate UC model. In this paper, we follow Stella and Stock (2013) to incorporate the Phillips curve into the UC model. One may question the existence of the Phillips curve, but McLeay and Tenreyro (2020) emphasise that the Phillips curve exists and that policymakers are completely aware of its existence. Hasenzagl, Pellegrino, Reichlin, and Ricco (2022) develop a model of inflation dynamics based on the view that the Phillips curve is one of three important components. Stock and Watson (2008) raised the point that the Phillips curve is useful for conditional forecasting. So we expect that the Phillips curve still exists, even though we are observing that it has flattened (e.g., Ball & Mazumder, 2011; Blanchard, Cerutti, & Summers, 2015; Hall et al., 2013).

In parameter-rich models, it is common to use tight priors on coefficients or on error variances (or covariance matrices). And sometimes, directly introducing restrictions on parameters can avoid them moving into undesirable regions. Such restrictions have been explored in many studies. For instance, Chan, Koop, and Potter (2013) bound both the inflation persistence, to avoid the explosive region of the parameter space, and the slope of the Phillips curve, to ensure a slope that is consistent with the economic theory. In this paper, we follow them to restrict the inflation persistence and the slope of the Phillips curve. But there are some differences on the Phillips curve. We acknowledge that using output in log levels gives the usual price Philips curve specification where the level of inflation is linked to the output gap as a measure of excess capacity. However, in this paper, we use output growth, not output in log levels. The reasons are as follows. The trend-cycle decomposition might be sensitive to how the trend is modelled (see Grant & Chan, 2017; Perron & Wada, 2009). For the trend, log output is upward trending, and only a drift is able to generate such a trend. Grant and Chan (2017) further find that this drift for the U.S. is subject to structural breaks, but whether this is true for other countries is an empirical question. The focus of this paper is forecasting, not the estimate of output trend. For the output gap, it is a cycle, and the econometric literature assumes that it is a stationary process with stochastic cyclical behaviour. Prominent researchers have proposed various methods to impose the stationary condition on the AR(2) process. Which method would be suitable in a multi-country study is another empirical

[1] The inflation gap is the deviation of inflation from its trend, and similarly the unemployment gap is the deviation of the unemployment rate from its trend.

question.[2] For instance, it can be computationally efficient to use the method by Grant and Chan (2017), where they directly bound the AR(2) coefficients. But Planas, Rossi, and Fiorentini (2008) stress that putting a prior on AR(2) coefficients makes it difficult to reproduce our knowledge, and the implied distribution for the periodicity and amplitude can be counter-intuitive. Given the amplitude and periodicity of the cyclical movements, they propose to use a trigonometric specification to re-parameterise the AR(2) process, but they exclude a moving-average term. Hasenzagl et al. (2022) brings back the moving-average term. If we use output growth, we do not need a stationary AR(2) process, thus avoiding the need to compare the various methods to impose a stationary condition. Output growth is not a measure of slack, but the use of growth as an alternative is not new. On the Taylor rule, Orphanides (2001) argues that a Taylor rule that reacts to output growth may be more stabilizing than a rule that responds to the output gap. Bullard and Eusepi (2005) develop a rule that responds to the growth gap, rather than output gap. On the Phillips curve, Sbordone (2002) derives a Phillips curve as a function of trend growth. Mattesini and Nisticò (2010) study implications of trend growth on inflation dynamics. Tchatoka, Groshenny, Haque, and Weder (2017) compare the Phillips curve using the output gap and growth. They find that the results remain essentially unchanged when employing output growth, so they concentrate on output growth. Gross and Semmler (2019) find that output growth correlates strongly with the slack measure (one-sided Hodrick–Prescott filter-based output gap) and could be an alternative to assess the empirical link between inflation and real activity.

As to the relationship of our paper to prior studies of the UC model, while our paper shares the two strands of extensions (introducing more indicators and constraining parameters to lie in reasonable intervals), we believe our paper provides further extensions. Firstly, we propose an approach to allow for cross-country linkages in the multi-country UC model. However, previous studies assume that countries are independent of each other. The idea that national macroeconomic developments depend on international conditions is not new. Kose et al. (2003) find that the world common component to expenditure time series of 60 countries explains between one-fourth and one-half of the variance of these series in OECD countries. Ciccarelli and Mojon (2010) provide evidence that a simple average of 22 OECD countries' inflation accounts for almost 70% of the variance of inflation in these countries. So one aim of this paper is to study whether allowing for cross-country linkages will improve forecasts of variables in UC models. Secondly, we allow for cross-country linkages through global factors. These factors are estimated from the model. This is different from studies which use some specific variable to be a proxy of a global factor. In the empirical application, we argue that introducing global factors will help reduce the omitted-variable bias in a single-country UC model.

Global uncertainty Several ways to estimate global uncertainty have been proposed in the literature. Mumtaz and Theodoridis (2017) use a factor-augmented vector autoregression (VAR) model with a common stochastic volatility and a country-specific stochastic volatility. Pfarrhofer (2019) uses a global vector autoregressive specification with FSV in the errors to estimate the impact of global uncertainty on six economies. Cuaresma, Huber, and Onorante (2019) use a large-scale Bayesian VAR with FSV to investigate the macroeconomic consequences of international uncertainty shocks in G7 countries. Carriero, Clark, and Marcellino (2020) measure international macroeconomic uncertainty by featuring the error volatility with a factor structure containing time-varying global components and idiosyncratic components.

As to the relationship of our paper to prior studies on measuring global uncertainty, our paper is closely related to the FSV specification used in Cuaresma et al. (2019) and Pfarrhofer (2019). The contribution of this paper is that we use sparsification to avoid omitting some potentially important factors, whereas prior studies rely on expert judgement (either by subjectively choosing the number or by relying on principal component-based analysis). The sparsification method, proposed by Chakraborty, Bhattacharya, and Mallick (2020), obviates the need to specify a prior on the rank (in this paper, the rank is equivalent to the number of factors), and shrinks the regression matrix towards a low-rank structure. This sparsification method allows us to estimate the factors and use post-processing to obtain an estimate for the number of factors.

Our FSV specification shares with Mumtaz and Theodoridis (2017) the feature of allowing for both common and country-specific stochastic volatility. It is empirically important to allow for stochastic volatility. Ignoring stochastic volatility is expected to exaggerate movements and potentially create transient variations in filtered estimates (see Huber, Pfarrhofer, & Piribauer, 2020; Sims, 2001; Stock, 2001). One difference from Mumtaz and Theodoridis (2017) is that we use sparsification to remove stochastic volatility in a data-based manner. This is important in the heavily parametrised setting. A similar strategy has been explored in Huber et al. (2020). In this paper, we shrink both factor volatilities and idiosyncratic volatilities. This is consistent with Carriero et al. (2020). They find that, in a three-economy macroeconomic data set (U.S.A., euro area, and U.K.), the idiosyncratic component of volatility displays very little time variation. Removing SV in a data-based manner is flexible since it can shrink small time variation to zero while retaining large time variation (e.g., more volatile countries).

As regards the relationship of our paper to Carriero et al. (2020), there are mainly two differences. The first difference is about the number of factors. Carriero et al. (2020) rely on principal component-based analysis, while we use sparsification and rely on post-processing. The second difference is that the model in Carriero et al. (2020) features common factors in both volatilities and in the conditional mean of the VAR. In this paper, we only allow common factors to affect the volatilities of the included variables. The reason that we do not allow common factors to affect the levels is that in the three-economy case, Carriero et al. (2020) find they will suffer

[2] We thank the referee for pointing us in this direction.

from the convergence issue of the Markov chain Monte Carlo (MCMC) sampler if two common factors are both included in the conditional mean. So they include one common factor in the conditional mean. We might have the same issue since we include more factors and our data are shorter.

One other difference between our paper and a number of others in multi-country studies is that we study 34 economies, including 23 advanced economies and 11 emerging market economies, whereas others focus on large economies, small advanced economies, or emerging market economies. As examples, Carriero et al. (2020) focus on large economies, Cross, Kam, and Poon (2018) focus on small advanced economies, Mumtaz and Theodoridis (2017) focus on 11 OECD countries, and Carrière-Swallow and Céspedes (2013) focus on emerging market economies.

3. Sparse factor stochastic volatility for a multi-country UC model

This section begins by detailing the unobserved components model for individual economies, and then introduces factor stochastic volatility to allow for cross-country linkages. We refer to the model as a multi-country UC-FSV model. We then describe sparsification. Finally, we summarise the model.

3.1. Multi-country UC-FSV model specification

We begin with the UC model for output and inflation. In particular, we start from a constant coefficient UC model for inflation, $\pi_{i,t}$, and output growth, $y_{i,t}$, of the form:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = \rho_i(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + \alpha_i(y_{i,t} - \tau^{y}_{i,t}) + \epsilon^{\pi}_{i,t}, \quad (1)$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \epsilon^{y}_{i,t}, \quad (2)$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \epsilon^{\tau\pi}_{i,t}, \quad \epsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i\tau\pi}), \quad (3)$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \epsilon^{\tau y}_{i,t}, \quad \epsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i\tau y}), \quad (4)$$

where i denotes economy i, $i = 1, \ldots, N$. At time t, $\pi_{i,t}$ is the inflation of economy i, and $y_{i,t}$ is the output growth of economy i. $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. These trends are unobserved latent states. In this paper, we refer to them as trend inflation and trend growth.

Eq. (1) is inspired by the Phillips curve. We assume that it is the inflation gap and growth gap that drive the Phillips curve. To ensure stationarity, we bound ρ_i and α_i to be positive and less than one—that is, $0 < \rho_i < 1$ and $0 < \alpha_i < 1$—which also ensures that the Phillips curve has a positive slope. Chan, Koop, and Potter (2016) and Zaman (2022) also bound the coefficients and emphasise the importance of bounding.

Thus, the first equation embodies a Phillips curve, but we are assuming constant coefficients. Many papers have emphasised that the Phillips curve has flattened post-2007 (see Simon, Matheson, & Sandri, 2013) and have proposed allowing for time variation in the coefficients to capture this behaviour (see Zaman, 2022). It seems to be more sensible to start from a UC model with time-varying coefficients. However, using the data in our empirical work (from 1995Q1 to 2018Q1), we considered a model where ρ_i and α_i vary over time, and found that the Bayes factor supports constant coefficients (see Appendix A). Accordingly, the main model does not have time variation in the coefficients.

The second equation implies AR(2) behaviour for the growth gap. The AR(2) assumption is empirically sensible and commonly used. Note that we are assuming constant coefficients in the growth gap equation. This assumption has also been used in Chan et al. (2016), Kabundi et al. (2021), and Zaman (2022). In the broader output literature, Carriero et al. (2020) and Koop, McIntyre, Mitchell, Poon, et al. (2020) also assume constant coefficients.[3]

Eqs. (3) and (4) assume a random walk process for trend inflation and trend growth. This specification is used in Cogley and Sbordone (2008). They find that statistical models with time-varying drifts are able to explain the behaviour of inflation and output growth quite well. For time-varying drift, they assume it follows a random walk.

Thus far, we have specified a UC model for a single economy. In particular, it is a bivariate UC model and incorporates the features from empirical findings (constant coefficients). However, conventional literature would next assume that the errors are independent across economies. It is with this assumption that we part from them.

As discussed above, the assumption of independence across economies might not be plausible when there is significant commonality across economies. To capture such commonality in uncertainty, we assume that, for all economies, the errors in inflation gap equations are driven by common factors and the errors in growth gap equations are driven by common factors. This can be done through factor stochastic volatility (FSV).

To facilitate the FSV specification, at time t, we store all errors in the inflation gap equations in an N-dimensional vector $\boldsymbol{\epsilon}^{\pi}_t$; that is, $\boldsymbol{\epsilon}^{\pi}_t = (\epsilon^{\pi}_{1,t}, \ldots, \epsilon^{\pi}_{N,t})'$. $\epsilon^{\pi}_{i,t}$ is the error for economy i. Similarly, we store all errors in the growth gap equations in an N-dimensional vector $\boldsymbol{\epsilon}^{y}_t$; that is, $\boldsymbol{\epsilon}^{y}_t = (\epsilon^{y}_{1,t}, \ldots, \epsilon^{y}_{N,t})'$. $\epsilon^{y}_{i,t}$ is the error for economy i. Through FSV, $\boldsymbol{\epsilon}^{\pi}_t$ can be decomposed as:

$$\boldsymbol{\epsilon}^{\pi}_t = L_{\pi}\boldsymbol{f}_t + \boldsymbol{u}^{\pi}_t \quad (5)$$

$$\begin{pmatrix} \boldsymbol{u}^{\pi}_t \\ \boldsymbol{f}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_\pi} \end{pmatrix}, \begin{pmatrix} \Sigma^{\pi}_t & 0_{r_\pi} \\ 0_N & \Omega^{\pi}_t \end{pmatrix} \right) \quad (6)$$

and $\boldsymbol{\epsilon}^{y}_t$ can be decomposed as:

$$\boldsymbol{\epsilon}^{y}_t = L_{y}\boldsymbol{g}_t + \boldsymbol{u}^{y}_t \quad (7)$$

$$\begin{pmatrix} \boldsymbol{u}^{y}_t \\ \boldsymbol{g}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_y} \end{pmatrix}, \begin{pmatrix} \Sigma^{y}_t & 0_{r_y} \\ 0_N & \Omega^{y}_t \end{pmatrix} \right) \quad (8)$$

where $\boldsymbol{f}_t = (f_{1,t}, \ldots, f_{r_\pi,t})'$ is an r_π-dimensional vector of latent factors, and L_π is the associated $N \times r_\pi$ loading matrix. Similarly, $\boldsymbol{g}_t = (g_{1,t}, \ldots, g_{r_y,t})'$ is an r_y-dimensional vector of latent factors, and L_y is the associated $N \times r_y$ loading matrix. Furthermore, we follow Chan (2022) to

3 Since we use output growth, we do not bound the constant coefficients in the growth gap equation.

assume that the factor loading matrices L_π and L_y are both a lower triangular matrix with ones on the main diagonal and $r_\pi \leq (N-1)/2$, $r_y \leq (N-1)/2$.[4] Let $n_{l,\pi}$ denote the number of free elements in L_π, then $n_{l,\pi} = N \times r_\pi - \frac{(1+r_\pi)r_\pi}{2}$. Let $n_{l,y}$ denote the number of free elements in L_y, then $n_{l,y} = N \times r_y - \frac{(1+r_y)r_y}{2}$.

We assume that inflation gap and growth gap equations across economies are driven by different factors, $\boldsymbol{f}_t$ and $\boldsymbol{g}_t$. This assumption is for reasons of parsimony. If the interest is in understanding the underlying causal relationship between output and inflation across countries, then the dependence assumption across the factors would make sense. One possible method is to assume that both $\boldsymbol{f}_t$ and $\boldsymbol{g}_t$ follow a VAR process.[5]

Based on preliminary empirical work that shows that errors in inflation gap equations exhibit stochastic volatility, we assume that the disturbances $\boldsymbol{u}_t^\pi$ exhibit stochastic volatility. This is why the error variance of $\boldsymbol{u}_t^\pi$ is Σ_t^π. Regarding growth gap equations, with the exception of Mertens (2014) and Zaman (2022), previous literature assumes that the errors remain homoscedastic; that is, $\boldsymbol{u}_t^y$ are homoscedastic. However, we assume that the disturbances $\boldsymbol{u}_t^y$ exhibit stochastic volatility. This is why the error variance of $\boldsymbol{u}_t^y$ is Σ_t^y. Such a specification will capture time variation in output variance unique to that economy. It has been used in Carriero et al. (2020) and Cesa-Bianchi, Pesaran, and Rebucci (2020). If the error is homoscedastic, our specification of the log volatility can (nearly) remove SV in a data-based manner (through the horseshoe prior).

For the latent factors $\boldsymbol{f}_t$ and $\boldsymbol{g}_t$, we assume that they exhibit stochastic volatility. This is why the error variance of $\boldsymbol{f}_t$ is Ω_t^π and the error variance of $\boldsymbol{g}_t$ is Ω_t^y.

The resulting time-varying variance matrix is $\Sigma_t^\pi = \text{diag}(e^{h_{1,t}^\pi}, \ldots, e^{h_{N,t}^\pi})$,

$\Sigma_t^y = \text{diag}(e^{h_{1,t}^y}, \ldots, e^{h_{N,t}^y})$, $\Omega_t^\pi = \text{diag}(e^{h_{1,t}^f}, \ldots, e^{h_{r_\pi,t}^f})$, and

$\Omega_t^y = \text{diag}(e^{h_{1,t}^g}, \ldots, e^{h_{r_y,t}^g})$.

We use $\exp(\boldsymbol{h}_t^\pi/2)$ to measure the **idiosyncratic inflation uncertainty**, $\exp(\boldsymbol{h}_t^y/2)$ to measure the **idiosyncratic growth uncertainty**, $\exp(\boldsymbol{h}_t^f/2)$ to measure the **global inflation uncertainty**, and $\exp(\boldsymbol{h}_t^g/2)$ to measure the **global growth uncertainty**. To facilitate the expression, we store the four types of log volatilities in an N_h-dimensional vector $\boldsymbol{h}_t = (\boldsymbol{h}_t^\pi, \boldsymbol{h}_t^y, \boldsymbol{h}_t^f, \boldsymbol{h}_t^g)$ where $N_h = 2N + r_\pi + r_y$. We summarise the definitions and descriptions of uncertainty in Table 1.

[4] Chan, Koop, and Yu (2021) show that we do not need a lower triangular loading matrix when the factors are heteroscedastic. If we stick to a lower triangular matrix, the prior will be order-dependent. In our empirical application, we experimented with different orderings of economies. We found that our conclusion did not change. So we simply followed the ordering from the database where we downloaded the data. In Appendix G, we report some results on the number of factors when the factor loading matrices are full.

[5] We leave this flavour to the future.

3.2. Sparsification

One of our contributions is the use of sparsification. We use sparsification to estimate the factor loadings and rely on post-processing to obtain an estimate for the number of factors. We also use sparsification to remove stochastic volatility in a data-based manner. In this subsection, we first talk about the number of factors and then about removing stochastic volatility.

To facilitate the discussion, note that a generic horseshoe prior takes the form:

$$\beta_j \mid \lambda_j^\beta, \tau^\beta \sim \mathcal{N}\left(0, \lambda_j^\beta \tau^\beta\right), \tag{9}$$

$$\lambda_j^\beta \sim \mathcal{C}^+(0,1), \tag{10}$$

$$\tau^\beta \sim \mathcal{C}^+(0,1), \tag{11}$$

where $\mathcal{C}^+(\cdot,\cdot)$ denotes the half-Cauchy distribution, λ_j^β is the local shrinkage parameter, and τ^β is the global shrinkage parameter. In Appendix H, we show that the horseshoe prior can take a hierarchical form using inverse-gamma hyper-priors, and in the estimation we use the inverse-gamma representation.

The number of factors This is done through the prior on the factor loading matrix. To avoid specifying a prior on the number of factors, Chakraborty et al. (2020) consider a potentially full-rank matrix and shrink out the redundant columns. Then they post-process the posterior draws to get the posterior estimate of the rank of the matrix (in this paper, the rank of a matrix is the number of factors). We follow their method. Theoretically, one can use a full matrix. In our case, this means setting the number of factors to the number of economies N. However, we do not do this.[6] Thanks to the bulk of empirical studies, some guidance on the number of factors is available. We choose a slightly higher number.[7]

To assign shrinkage priors on factor loading matrices (L_π and L_y), we use the horseshoe prior and specify a column-specific global shrinkage parameter and an element-specific local parameter. For instance, for L_π, let $L_{\pi,j}$ denote the jth column of factor loading matrix L_π. Then the prior on the ith element in $L_{\pi,j}$ is the horseshoe prior with a column-specific global shrinkage parameter ($L_{\pi,j}$) and an element-specific local parameter ($L_{\pi,ij}$).

The final step is to post-process the posterior draws. We threshold the singular values of the factor loading matrix and estimate the rank as the number of non-zero thresholded singular values. We refer our readers to Chakraborty et al. (2020) for more details.

To remove stochastic volatility To allow the data to decide whether there is time variation in their log volatility, we model the evolution of the log volatility as a random

[6] In fact, we experimented with this but found that the computation became a burden and that the forecast performance did not improve much.

[7] For instance, Carriero et al. (2020) find that there is one global factor driving the 19-country GDP. What we did was to set the number of global output factors to two.

Table 1
Definitions and descriptions of uncertainty $\exp(\boldsymbol{h}_t/2)$.

Definition	Description of uncertainty $\exp(\boldsymbol{h}_t/2)$
Idiosyncratic inflation uncertainty	$\exp(\boldsymbol{h}_t^{\pi}/2)$, the standard deviation of $\boldsymbol{u}_t^{\pi}$
Idiosyncratic growth uncertainty	$\exp(\boldsymbol{h}_t^{y}/2)$, the standard deviation of $\boldsymbol{u}_t^{y}$
Global inflation uncertainty	$\exp(\boldsymbol{h}_t^{f}/2)$, the standard deviation of $\boldsymbol{f}_t$
Global growth uncertainty	$\exp(\boldsymbol{h}_t^{g}/2)$, the standard deviation of $\boldsymbol{g}_t$
Global inflation factor	$\boldsymbol{f}_t$
Global growth factor	$\boldsymbol{g}_t$

walk. This random walk is in non-centred parameterisation. Then we use a global–local shrinkage prior (the horseshoe prior) to control time variation. More specifically, for each $j = 1, \ldots, N_h$, the evolution of the log volatility is modelled as:

$$h_{j,t} = h_{j,0} + \omega_j^h \tilde{h}_{j,t} \tag{12}$$
$$\tilde{h}_{j,t} = \tilde{h}_{j,t-1} + +\epsilon_{j,t}^h, \quad \epsilon_{j,t}^h \sim \mathcal{N}(0,\ 1)$$

The non-centred parameterisation decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$, and a time-varying part $\omega_j^h \tilde{h}_{j,t}$. The time-varying part has a constant coefficient ω_j^h, which controls the time variation. If the error is homoscedastic, then we expect that ω_j^h may be zero (or close to zero). If the error is heteroscedastic, then we expect that ω_j^h is different from zero. This case is exactly the advantage of global–local shrinkage priors. Many papers have documented that global–local shrinkage priors can cope with the case where a matrix is characterised by zero and non-zero elements (e.g., Kastner & Huber, 2020; Polson & Scott. Shrink globally, 2010). So we use the empirically successful global–local shrinkage prior, namely the horseshoe prior, for ω_j^h.

If the error (factor) really is homoscedastic, the horseshoe prior will shrink ω_j^h to (nearly) zero and automatically remove (or nearly remove) the SV from the error (factor). The horseshoe prior has a global shrinkage parameter. It will push all elements (ω_j^h) towards zero. We assume that there is a single global shrinkage parameter. This is a restricted version of the horseshoe prior in Feldkircher, Huber, Koop, and Pfarrhofer (2021). They specify that the global shrinkage parameter differs across economies and across equations within a given economy. However, we notice that such a flexible prior is used for the coefficients in their panel VARs. Our horseshoe prior is for the time-varying part of log volatility. Since the log volatilities all represent the uncertainty, we expect that they have a single global shrinkage parameter. To capture the differences across factors, economies, and equations, we rely on the local shrinkage parameter.

For ω_j^h, we consider the horseshoe prior. For the time-invariant part of log volatility, $h_{j,0}$, we also consider the horseshoe prior. Such priors might be too strong on log volatility, so in Appendix G, we consider a normal prior with zero mean and variance one on $h_{j,0}$.

To complete the priors, we assume that the constant coefficients and initial states (ρ_i, α_i, $\varphi_{i,j}$, $\tau_{i,1}^{\pi}$, $\tau_{i,1}^{y}$) follow a normal distribution with zero mean and variance ten.[8] The error variances ($\sigma_{\tau\pi}^2$ and $\sigma_{\tau y}^2$) are assumed to follow an inverse-gamma distribution $\mathcal{IG}(10,\ 0.18)$.

3.3. Summarizing the model

To summarise the model including all economies:

$$\begin{aligned}
\boldsymbol{\pi}_t - \boldsymbol{\tau}_t^{\pi} &= P(\boldsymbol{\pi}_{t-1} - \boldsymbol{\tau}_{t-1}^{\pi}) + A(\boldsymbol{y}_t - \boldsymbol{\tau}_t^{y}) + L_{\pi}\boldsymbol{f}_t + \boldsymbol{u}_t^{\pi}, \\
&\quad \boldsymbol{f}_t \sim \mathcal{N}(0,\ \Omega_t^{\pi}),\ \boldsymbol{u}_t^{\pi} \sim \mathcal{N}(0,\ \Sigma_t^{\pi}) \\
\boldsymbol{y}_t - \boldsymbol{\tau}_t^{y} &= \Phi_1(\boldsymbol{y}_{t-1} - \boldsymbol{\tau}_{t-1}^{y}) + \Phi_2(\boldsymbol{y}_{t-2} - \boldsymbol{\tau}_{t-2}^{y}) \\
&\quad + L_y\boldsymbol{g}_t + \boldsymbol{u}_t^{y},\ \boldsymbol{g}_t \sim \mathcal{N}(0,\ \Omega_t^{y}),\ \boldsymbol{u}_t^{y} \sim \mathcal{N}(0,\ \Sigma_t^{y}) \\
\tau_{i,t}^{\pi} &= \tau_{i,t-1}^{\pi} + \epsilon_{i,t}^{\tau\pi},\quad \epsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{i\tau\pi}^2),\ i = 1, \ldots, N \\
\tau_{i,t}^{y} &= \tau_{i,t-1}^{y} + \epsilon_{i,t}^{\tau y},\quad \epsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{i\tau y}^2) \\
h_{j,t} &= h_{j,0} + \omega_j^h \tilde{h}_{j,t} \\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon_{j,t}^h,\quad \epsilon_{j,t}^h \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, N_h
\end{aligned} \tag{13}$$

where $\boldsymbol{\pi}_t = (\pi_{1,t}, \ldots, \pi_{N,t})'$ is an $N \times 1$ vector, $\boldsymbol{\tau}_t^{\pi} = (\tau_{1,t}^{\pi}, \ldots, \tau_{N,t}^{\pi})'$ is an $N \times 1$ vector, $P = \mathrm{diag}(\rho_1, \ldots, \rho_N)$ is an $N \times N$ matrix, $A = \mathrm{diag}(\alpha_1, \ldots, \alpha_N)$ is an $N \times N$ matrix, $\boldsymbol{y}_t = (y_{1,t}, \ldots, y_{N,t})'$ is an $N \times 1$ vector, $\boldsymbol{\tau}_t^{y} = (\tau_{1,t}^{y}, \ldots, \tau_{N,t}^{y})'$ is an $N \times 1$ vector, $\Phi_1 = \mathrm{diag}(\phi_{1,1}, \ldots, \phi_{N,1})$ is an $N \times N$ matrix, and $\Phi_2 = \mathrm{diag}(\phi_{1,2}, \ldots, \phi_{N,2})$ is an $N \times N$ matrix.

In what follows, we use "the multi-country UC-FSV" for the model defined through Eq. (13). Many models can be written as a restricted version of the multi-country UC-FSV model. These restrictions can help us to investigate some aspects of our model. The restricted models, along with their acronyms, are as follows:

(1) UC-FSV-$r_y = 0$: this is the restricted version of the UC-FSV where there are no common factors in growth gap equations; that is, $r_y = 0$. And errors in growth gap equations are allowed to exhibit stochastic volatility.

(2) UC-FSV-$r_y, r_\pi = 0$: this is the restricted version of UC-FSV where there are no common factors in inflation gap and growth gap equations; that is, $r_\pi = 0$, $r_y = 0$. Errors in inflation gap and growth gap equations are allowed to exhibit stochastic volatility.

(3) UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: this is the restricted version of UC-FSV where there are no common factors in inflation gap and growth gap equations—that is, $r_\pi = 0$, $r_y = 0$—and errors in growth gap equations u_t^y are homoscedastic, while errors in inflation gap equations u_t^{π}

8 Remember that we bound ρ_i and α_i to be positive and less than one.

exhibit stochastic volatility. This is the model that is used in Chan et al. (2016) and Stella and Stock (2013).[9]

4. Full-sample results

4.1. Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 economies: 23 advanced economies (AEs)[10] and 11 emerging market economies (EMEs).[11] They span the period from 1995Q1 to 2018Q1. The choice of countries and the sample size were based on data availability. The series include the headline consumer price index (CPI), representing domestic headline inflation, and real gross domestic product (GDP), which reflects domestic demand. Real GDP data were obtained from Haver Analytics. We transformed the data to annualised growth rates as: $400\log(z_t/z_{t-1})$. And because the growth gap equation follows an AR(2) process, our estimation starts from 1995Q4. We set $r_\pi = 5$ and $r_y = 2$. That is, we include five factors in inflation gap equations and two factors in growth gap equations. The posterior results are based on 100,000 draws after a burn-in period of 20,000.

4.2. Overview of empirical results

We divide our full-sample results into three sub-sections. The first sub-section is the estimate of global inflation uncertainty $\exp(\boldsymbol{h}_t^f/2)$ and global growth uncertainty $\exp(\boldsymbol{h}_t^g/2)$. Then we report the correlation between global inflation factors and domestic business cycles.

The second sub-section is the sparsification. We use sparsification to avoid omitting some potentially important factors and to remove stochastic volatility in a data-based manner. We report the posterior estimates of the number of factors. The evidence of removing stochastic volatility is provided in Appendix F.

The third sub-section is the Bayesian model comparison. We compare the multi-country UC-FSV with the alternative models (UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$, and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) described in Section 3.3.

4.3. Estimates of global uncertainty

Although the multi-country UC-FSV estimates of global uncertainty reflect contemporaneous effects of global factors on (the volatility of) macroeconomic data, the effect is also directly related to the loadings on global factors. These loadings are reported in Appendix B. We report the posterior mean of the five factors' loadings (recall that we set $r_\pi = 5$), but only the 16% and 84% quantiles of the first factor's loadings for brevity. Most of the economies have sizable loadings on the first global inflation factor, and the quantiles (except for Russia and Brazil) do not include zero. Then we report the loadings on the global growth factor. The quantiles of the first global growth factor for all economies do not include zero. This provides strong evidence of significant commonality of output growth in the 34 economies. Carriero et al. (2020) obtain similar result in their case of a 19-country GDP data set.

Fig. 1 displays the posterior estimates of global uncertainty obtained from the multi-country UC-FSV using the full sample. The left panel is the estimate of global inflation uncertainty, and the right panel is global growth uncertainty. In both figures, the solid lines represent the posterior means of the first uncertainty, while the dotted lines are the associated 16% and 84% quantiles. The dashed lines represent the posterior means of the remaining uncertainties. For instance, with regard to global inflation uncertainty, we set $r_\pi = 5$. So we obtain the posterior estimates of the five global inflation uncertainties from MCMC, including their posterior means and quantiles. Then, in Fig. 1(a), we plot the posterior means and quantiles of the first global inflation uncertainty (see the solid lines and dotted lines), but for brevity, we only plot the posterior means of the remaining uncertainties (the second, third, fourth, and fifth uncertainties) using dashed lines.

As indicated in Fig. 1(a), we only observe evident and meaningful time variation in the first global inflation uncertainty. The estimated global inflation uncertainty shows significant increases around some of the political and economic events that Bloom (2009) highlights as periods of uncertainty, including 9/11, the Enron scandal, the second Gulf war, and the Global Financial Crisis period. These spikes associated with the global factor are documented in Kastner and Huber (2020) using U.S. macroeconomic data. Since our data come from 34 economies, the consistency (between the estimates in Kastner and Huber (2020) and in our study) indicates that global macroeconomic uncertainty is closely related to uncertainty in the U.S., which might not seem surprising given that the international economy is tied to the U.S. economy. One spike that is not documented in Kastner and Huber (2020) is that volatility increases from 2013 onward. This may indicate that such an increase is driven by economies other than the U.S. In addition, at the end of our sample (2018Q1), global inflation uncertainty still exists and continues to influence all economies under consideration. This is supported by a related study, namely Forbes (2019). They add commodity price volatility to explain inflation and find that commodity price volatility plays a large role in CPI inflation.

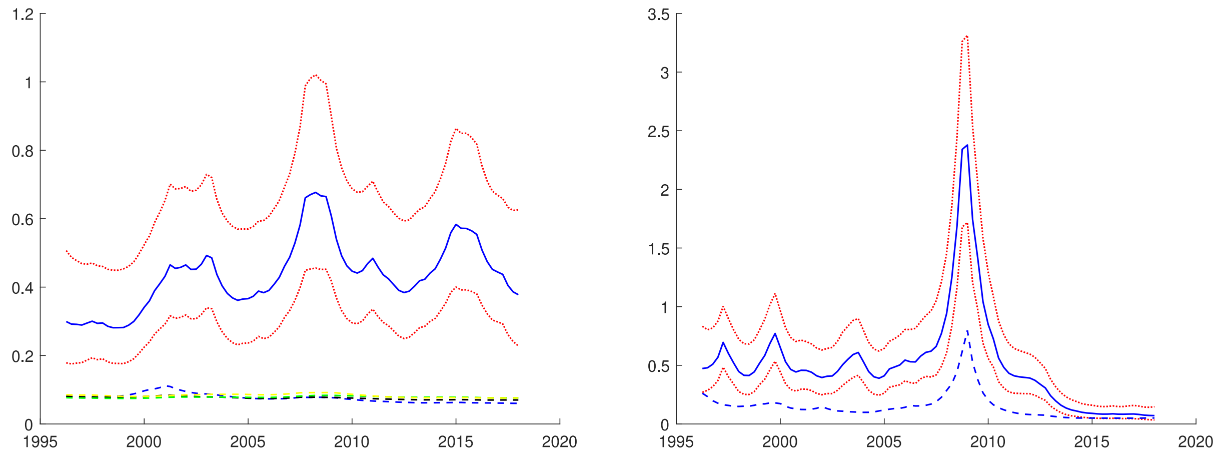
However, we find a different story with regard to the time variation in global growth uncertainty from Fig. 1(b). First, the two global growth uncertainties both increase during the Global Financial Crisis (GFC) of 2008, but except for this, we do not observe other meaningful time variation from the second global growth uncertainty.[12] Before the GFC, there was global growth uncertainty but it

[9] The coefficients in this paper are restricted to be constant.

[10] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, the Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, the U.K., and the U.S.A.

[11] Bolivia, Brazil, China, Hungary, Indonesia, Mexico, the Philippines, Russia, South Africa, Thailand, and Turkey.

[12] This is the first reason for including only two factors in the growth gap equation.



(a) Global inflation uncertainty (b) global growth uncertainty

Fig. 1. Posterior estimates for global inflation uncertainty exp($\boldsymbol{h}_t^f/2$) and global growth uncertainty exp($\boldsymbol{h}_t^g/2$) under the multi-country UC-FSV. The solid lines represent the posterior means of the first global uncertainty, while the dotted lines are the associated 16% and 84% percentiles. The dashed lines represent the posterior means of the remaining uncertainties.

Table 2
Posterior estimates of correlation between global inflation factors and domestic business cycles.

	Correlation
Mean	0.22
16% quantile	0.19
84% quantile	0.25

did not show much time variation. Then, during the GFC, such uncertainty increased substantially. In the aftermath of the GFC, it decreased sharply and, importantly, in 2015 global inflation uncertainty reached a very low level.[13] These features are documented in Carriero et al. (2020) in their 19-country GDP data set.

Next, Table 2 reports the correlation between global inflation factors $\boldsymbol{f}_t$ and domestic business cycle $(y_{i,t} - \tau_{i,t}^y)$ on average. We check this correlation mainly because we get a lower α when adding FSV (see Table 13 in Appendix C). Recall that α is the slope of the Phillips curve. This may give us an impression that allowing for cross-country linkages will flatten the Phillips curve. But we think we need more care when interpreting this lower value. So we checked the correlation between two variables: global inflation factors, $f_{1,t}$,[14] and domestic business cycles, $(y_{i,t} - \tau_{i,t}^y)$. Then we took the average across economies.

The results show that the estimated global inflation factor is positively correlated with domestic business cycles. We interpret this to mean that part of the variation captured in the global inflation factor reflects a global business cycle. Introducing factors could reduce the omitted-variable bias.

[13] This is the second reason for including only two factors in the growth gap equation.

[14] $f_{1,t}$ is the global inflation factor with the highest variation. Since other factors are quite flat and do not have meaningful interpretations, we do not consider them.

4.4. Sparsification: Number of factors

To obtain the posterior estimate of the number of factors, we post-process the posterior draws by thresholding the singular values of the factor loading matrix, and then estimating the rank as the number of non-zero thresholded singular values. One choice of the threshold is proposed in Chakraborty et al. (2020). Using their choice, we get the result in Table 3. We have inflation gap equations and growth gap equations. The second column is singular values for inflation gap equations, and the third column is singular values for growth gap equations. The first row reports the threshold. The number of factors is determined as the number of singular values larger than the threshold. In growth gap equations, we find that there is one singular value (119.82) that is larger than the threshold (= 108.46). This means that there is one global factor in the growth gap equations. This finding is consistent with prior studies.

In the inflation gap equations, we find that there is no singular value that is larger than the threshold (= 118.47). This means that there is no global factor in the inflation equation.[15] Even if in the full-sample result we do not find strong evidence of global factors in inflation gap equations, in the out-of-sample forecasting exercise we find that the FSV specification does improve the forecast of inflation (although the improvement of forecasting inflation is smaller than the improvement of forecasting growth; see Section 5).

[15] In Appendix G, we provide additional results that provide robustness checks on the estimate of the number of factors. The two dimensions to assess the robustness are (1) the identification constraint on the factor loading matrices L_π and L_y, and (2) the shrinkage on the time-invariant part of log volatility $h_{j,0}$. Our conclusion remains the same. Another method for choosing the number of factors is by the model's forecast performance, for instance, a table reporting the sum of one-step-ahead log predictive likelihoods, as in Table 4, but with different numbers of factors.

Table 3
Posterior number of factors.

Singular values (descending)	Inflation equation threshold = 118.47	Output equation threshold = 108.46
First	49.08	119.82
Second	23.33	45.85
Third	5.17	20.62

Table 4
Sum of one-step-ahead log predictive likelihoods.

Model	Against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0
UC-FSV-$r_y, r_\pi = 0$	520.37
UC-FSV-$r_y = 0$	658.57
UC-FSV	883.34

4.5. Bayesian model comparison

As discussed above, the computation of marginal likelihoods can be a challenge when there are a large number of states. Therefore, we use an approximation of the marginal likelihood (see Cross, Hou, & Poon, 2020; Geweke, 2001; Geweke & Amisano, 2010). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 economies can be viewed as an approximation to the marginal likelihood, and therefore provide a direct measure of in-sample fit. We compare four competing models: the multi-country UC-FSV, UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$, and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Before computing the sums of one-step-ahead joint log predictive likelihoods, we need to define some basics. Let $\hat{y}_{t+k}^{(i,j)}$ denote, at time t, the k-step-ahead forecast of the jth variable in the ith economy, and let $y_{t+k}^{(i,j)}$ denote the actual value. In our empirical work, $i = 1, \ldots, N$ with $N = 34$ and $j = 1, 2$, where $j = 1$ denotes inflation and $j = 2$ denotes growth. $\mathrm{Y}_{1:t}^{(i,j)}$ stores the data up to time t, so $\hat{y}_{t+k}^{(i,j)} = \mathrm{E}\,(y_{t+k}^{(i,j)} \mid \mathrm{Y}_{1:t}^{(i,j)})$. Then we compute the k-step-ahead log predictive likelihoods (LPLs) at time t of the ith economy and the jth variable:

$$\mathrm{LPL}_{t,i,j,k} = \log p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)}), \;\; t = T_0, \ldots, T-k$$

Then the sums of one-step-ahead joint log predictive likelihoods are computed:

$$\mathrm{LPL}_{\cdot,\cdot,\cdot,1} = \sum_{t=T_0}^{T-1} \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\hat{y}_{t+1}^{(i,j)} = y_{t+1}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of the 34 economies in Table 4.

In Table 4, the results are presented relative to the forecast performance of the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so that a positive number indicates that a model is forecasting better than the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.[16] The results show that the multi-country UC-FSV provides the best fit compared to all other models. In addition, since we find that UC-FSV-$r_y, r_\pi = 0$ provides higher model fit than UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, we view this as more evidence in support of allowing for idiosyncratic stochastic volatility in growth gap equations.

[16] Note that we only take the sum, and not the average. That may be why the number seems so large. For instance, the sum of LPLs under UC-FSV is 895.02. Their average over time is 14.67. If we take the average across economies, it is 0.43.

5. Out-of-sample forecasting results

Since our modifications are about uncertainty, we focus on the density forecast. We use data from 1995Q4 to 2002Q4 as an initial estimation period, and we use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 4, 8, 12, 16$ quarters. So our forecast evaluation period begins in 2003Q1. We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output growth, and jointly forecasting inflation and output growth. For each part, we discuss the results in three dimensions. The first dimension is the aggregate forecasting performance over time and across economies (the aggregate LPL, by summing all economies and all time periods). Since we observe international macroeconomic uncertainty, it is natural to expect that considering such uncertainty will provide more accurate forecasts in economic recessions. Thus, the second dimension is about forecasting performance over time. (We can study how the sums of LPLs change over time by summing all economies at time t.) After providing evidence that our multi-country UC-FSV can produce more accurate forecast in economic recessions, we further study whether such good forecast performance is driven by particular economies. Hence, the third dimension is about the forecasting performance at the level of the particular economy. All results are presented relative to the forecast under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so a positive number indicates that a model is forecasting better than the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.1. Forecasting inflation

We first report the aggregate forecasting performance for inflation over time and over economies in Table 5. It is calculated by summing the LPLs for the N economies over T_0 to $T - k$ (and recall that $j = 1$ denotes inflation):

$$\mathrm{LPL}_{\cdot,\cdot,1,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \mathrm{Y}_{1:t}^{(i,1)})$$

The results show that the model with cross-country linkages in inflation (UC-FSV-$r_y = 0$ and UC-FSV) provides more accurate forecasts for inflation than the model without cross-country linkages (UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) at all horizons.

The forecasting results of inflation in Table 5 demonstrate the benefits of allowing for cross-country linkages, which is done by considering global inflation uncertainty in our paper. It is natural to expect that the good forecasting results may largely arise from periods of uncertainty.

Table 5
Sum of k-step-ahead log predictive likelihoods for 34-country inflation.

Model	$k=1$	$k=4$	$k=8$	$k=12$	$k=16$
UC-FSV-$r_y, r_\pi=0, \omega_y^h=0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi=0$	−4.27	71.83	127.82	138.85	185.26
UC-FSV-$r_y=0$	98.92	**265.09**	286.02	350.53	333.11
UC-FSV	**101.63**	257.39	**294.76**	**379.19**	**356.89**

To investigate this point, we calculate the sums of LPLs over time. As in Feldkircher et al. (2021), a common method is to sum the LPLs for the N economies at time t:

$$\mathrm{LPL}_{t,\cdot,1,k} = \sum_{i=1}^{n} \log p(\hat{y}^{(i,1)}_{t+k} = y^{(i,1)}_{t+k} | \mathrm{Y}^{(i,1)}_{1:t})$$

For instance, suppose we are at the time point of 2007Q4. Then $k=1$ means we are forecasting the data in 2008Q1, and $k=4$ means we are forecasting the data in 2008Q4. So this method helps to answer (at time t) which model can provide the most accurate forecast in the future.

However, recall that global inflation uncertainty shows significant increases around 2008 and 2015 (see Fig. 1(a), and because our forecast starts from 2003Q1, we omit the increase in 2001). Such global inflation uncertainty drives strong co-movement across economies. So a more interesting study is to investigate whether this global inflation uncertainty can improve the forecast performance during periods of uncertainty. For instance, suppose that we want to know which model can provide the most accurate forecast of 2008Q1. Different forecast horizons will provide the forecast made at a different time t. If $k=1$, then this means the forecast is made at 2007Q4 (one step ago). If $k=4$, then this means the forecast is made at 2007Q1 (four steps ago). Overall, the difference is the x-axis. Suppose that we are at time t. In Feldkircher et al. (2021), the x-axis is t and represents when we make the forecast, but in our paper, the x-axis is $t+k$ and represents when to forecast. This is how we produce Fig. 2. About the starting time, since we make the first forecast at 2002Q4, if $k=1$, the time to forecast (at 2002Q4) is 2003Q1, so in Fig. 2, the x-axis (time to forecast) starts from 2003Q1 when $k=1$. If $k=4$, the time to forecast (at 2002Q4) is 2003Q4, so in Fig. 2, the x-axis (time to forecast) starts from 2003Q4 when $k=4$. Similarly, if $k=16$, the time to forecast (at 2002Q4) is 2006Q4, so in Fig. 2, the x-axis (time to forecast) starts from 2006Q4 when $k=16$.

We plot the results (against UC-FSV-$r_y, r_\pi=0, \omega_y^h=0$) in Fig. 2 (but for brevity, we only plot the results of UC-FSV). To forecast inflation during periods of uncertainty (like 2008), we find good overall forecast performance for UC-FSV at all horizons, particularly at long horizons. This indicates the importance of taking into account cross-country linkages for improving forecasts of inflation, especially to forecast periods of uncertainty. To forecast more stable periods, it does not hurt to take into account cross-country linkages.

The sums of LPLs over time in Fig. 2 are for the 34 economies. Someone may question whether the good forecasting result is driven by particular economies. To investigate this point, we present the forecasting results for individual economies. The LPL of inflation for economy i at time t is calculated as follows:

$$\mathrm{LPL}_{t,i,1,k} = \log p(\hat{y}^{(i,1)}_{t+k} = y^{(i,1)}_{t+k} | \mathrm{Y}^{(i,1)}_{1:t})$$

We plot the results (against UC-FSV-$r_y, r_\pi=0, \omega_y^h=0$) in Fig. 3. Here, the period of uncertainty that we plot is 2008Q4, so the time to forecast is 2008Q4 ($t+k=$ 2008Q4). If $k=1$, then the time we make the forecast is 2008Q3, and we find good overall forecast performance for most economies, with more pronounced gains in advanced economies. (The first 23 economies are AEs, and the following 11 economies are EMEs.) A similar pattern is found if $k=16$. The time we make the forecast is 2004Q4, and we also find good overall forecast performance for most economies. We also find significant gains in Spain and the U.S.A. The gain is not so significant when $k=1$ compared to when $k=16$. In Fig. 3, we only plot the shortest horizon, $k=1$, and the longest horizon, $k=16$. For middle horizons ($k=4,8,12$), we found good forecasting results across most economies and did not find that a particular economy was important for driving the good forecasting results. Overall, we find good forecast performance for UC-FSV for most economies, where this performance is not driven by particular economies.

5.2. Forecasting output growth

With regard to output growth, we report the sums of LPLs of output over time and over economies in Table 6. This is calculated by summing the LPLs for the N economies over T_0 to $T-k$ (and recall that $j=2$ denotes output growth):

$$\mathrm{LPL}_{\cdot,\cdot,2,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \mathrm{Y}^{(i,2)}_{1:t})$$

The results show that the model that allows for both idiosyncratic stochastic volatility and cross-country linkages in growth gaps provides the most accurate forecast for output growth at all horizons.

Similar to the analysis of inflation, the second dimension of discussion for output growth is the sum of LPLs over time (by summing all economies at time t), which can be calculated by:

$$\mathrm{LPL}_{t,\cdot,2,k} = \sum_{i=1}^{n} \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \mathrm{Y}^{(i,2)}_{1:t})$$

We plot the results (against UC-FSV-$r_y, r_\pi=0, \omega_y^h=0$) in Fig. 4. To forecast output growth during periods of uncertainty (like 2008), we find good overall forecast performance for UC-FSV at all horizons. This indicates the importance of taking into account cross-country

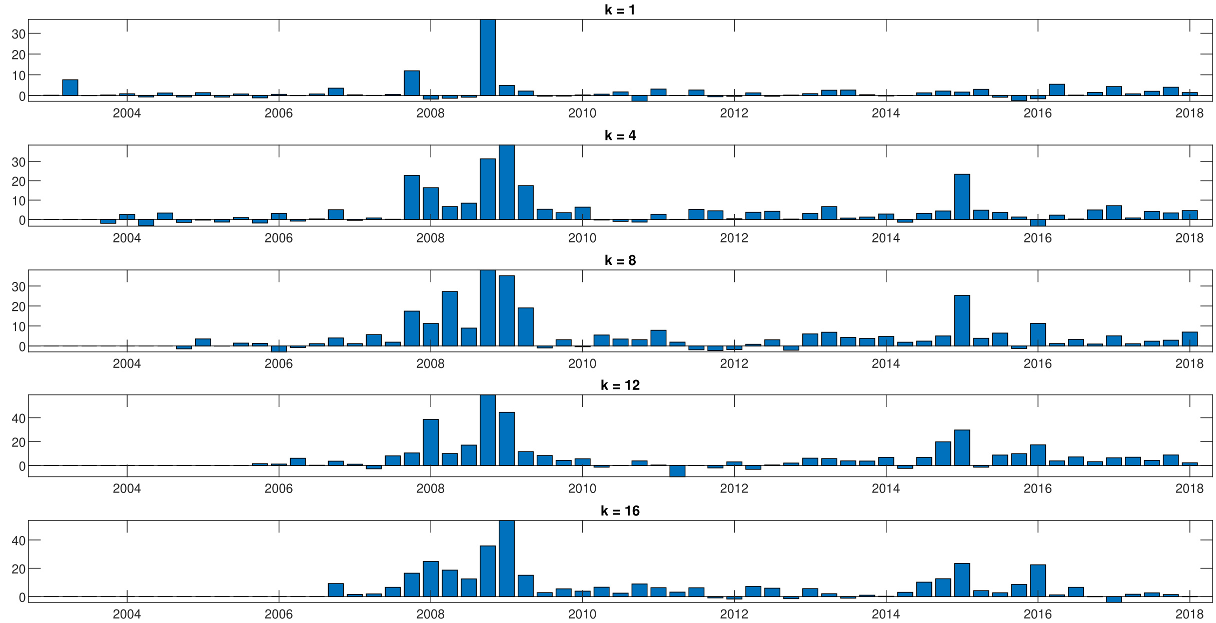


Fig. 2. Sums of k-step-ahead LPLs of inflation for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The x-axis is $t + k$ and represents when to forecast.

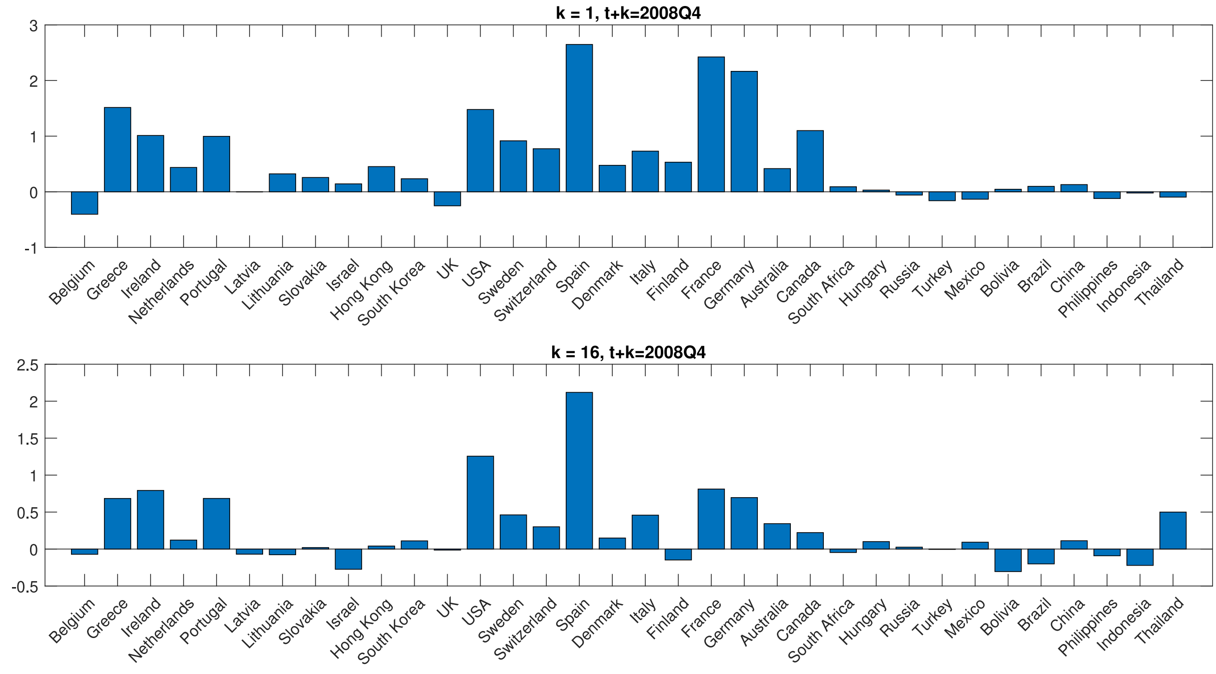


Fig. 3. Sums of k-step-ahead LPLs of inflation for economy i under UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Table 6
Sum of k-step-ahead log predictive likelihoods for 34-economy output growth.

Model	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	577.02	694.98	811.01	797.25	684.98
UC-FSV-$r_y = 0$	566.81	668.04	852.04	772.90	680.99
UC-FSV	**762.93**	**1194.99**	**1211.17**	**1208.10**	**1052.36**

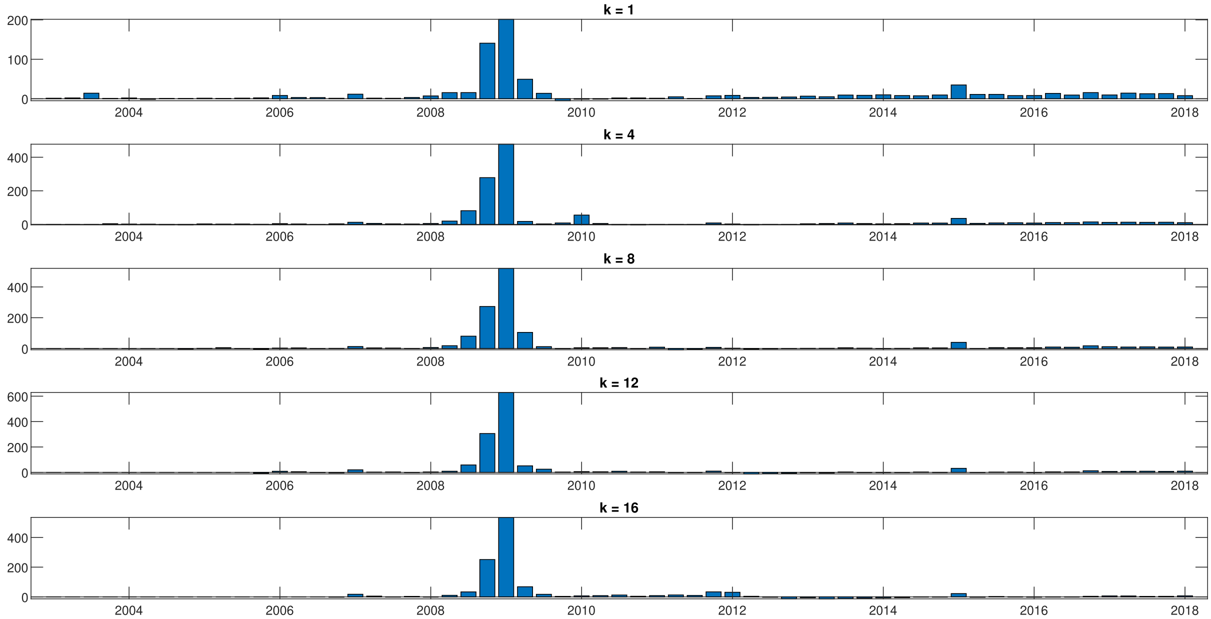


Fig. 4. Sums of k-step-ahead LPLs of output for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The x-axis is $t+k$ and represents when to forecast.

linkages for improving forecasts of output growth, especially to forecast periods of uncertainty. To forecast more stable periods, it does not hurt to take into account cross-country linkages.

To investigate whether the good forecast performance is driven by particular economies, we calculate the sum of LPLs of output growth for economy i at time t:

$$\text{LPL}_{t,i,2,k} = \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \text{Y}^{(i,2)}_{1:t})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Fig. 5. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 16$, we both find good overall forecast performance for UC-FSV for all economies. The highest gain is found for Hungary, followed by Sweden. However, unlike the conclusion in the case of forecasting inflation, where more pronounced gains were found in AEs, we found significant gains in both AEs and EMEs. This implies that allowing for idiosyncratic stochastic volatility and cross-country linkages in output growth is important for both AEs and EMEs.

5.3. Jointly forecasting inflation and output growth

With regard to the joint predictive density for inflation and output growth, we first report the sums of joint LPLs over time and over economies in Table 7. This is calculated by summing the LPLs for the N economies over T_0 to $T-k$ (and for all j, recalling that $j = 1$ denotes inflation and $j = 2$ denotes output):

$$\text{LPL}_{\cdot,\cdot,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\hat{y}^{(i,j)}_{t+k} = y^{(i,j)}_{t+k} | \text{Y}^{(i,j)}_{1:t})$$

The results show that the model that allows for idiosyncratic stochastic volatility in output growth and

Table 7
Sum of k-step-ahead joint log predictive likelihoods for 34-economy inflation and output growth.

Model	$k=4$	$k=8$	$k=12$	$k=16$
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	679.42	751.62	794.16	615.81
UC-FSV-$r_y = 0$	898.62	1084.28	1084.13	1148.35
UC-FSV	**1513.05**	**1545.20**	**1824.70**	**1672.17**

cross-country linkages in both inflation and output growth (UC-FSV) provides the most accurate joint forecasts of inflation and output growth at all horizons.[17]

Next, we study the time variation in forecast performance to see whether the benefits arise from forecasts during periods of uncertainty. So the second dimension of discussion for the joint predictive density for inflation and output growth is the sum of joint LPLs over time (by summing all j and all economies at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,\cdot,k} = \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\hat{y}^{(i,j)}_{t+k} = y^{(i,j)}_{t+k} | \text{Y}^{(i,j)}_{1:t})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Fig. 6. A pattern similar to that for inflation and output growth was found. To jointly forecast inflation and output growth during periods of uncertainty (like 2008),

[17] We do not report the horizon $k = 1$ since this has been reported in Table 4. We refer the reader to Table 4 to see the sum of one-step-ahead joint log predictive likelihoods for 34-economy inflation and output growth.

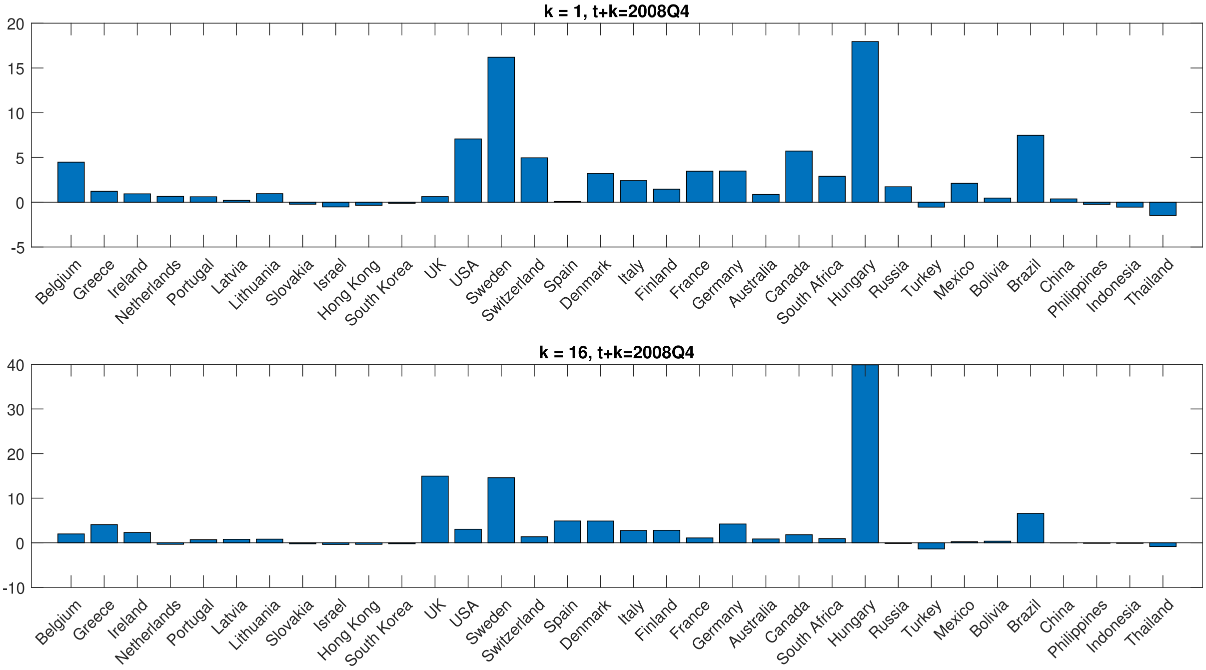


Fig. 5. Sums of k-step-ahead LPLs of output in economy i for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

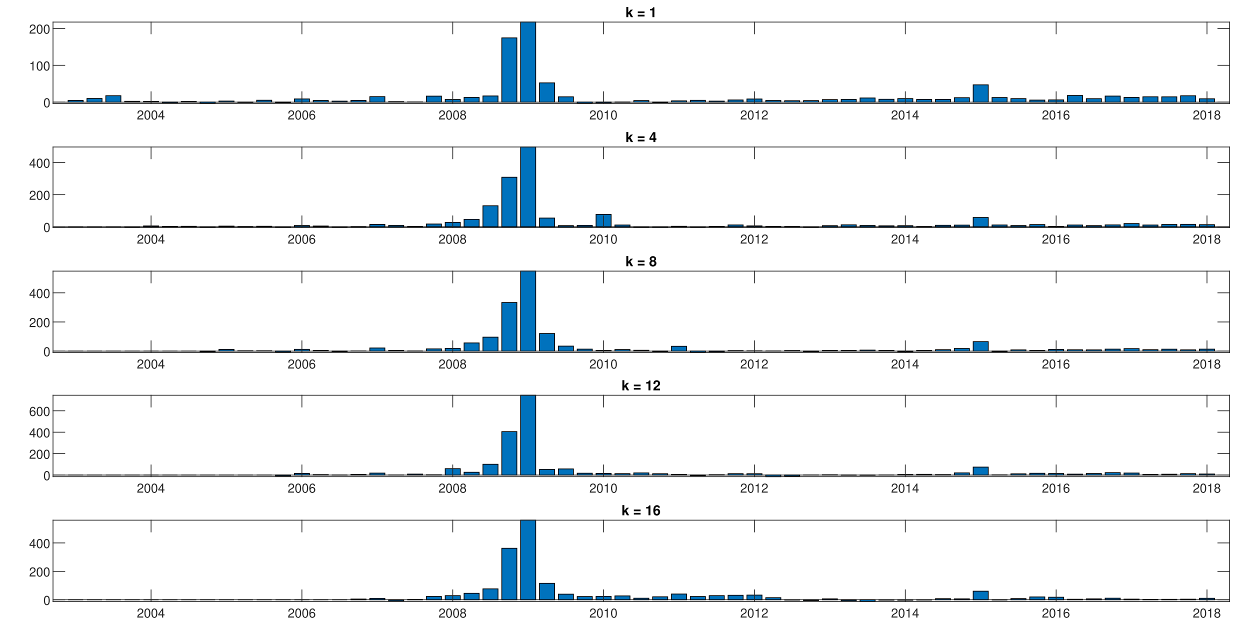


Fig. 6. Sums of k-step-ahead joint LPLs for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The x-axis is $t + k$ and represents when to forecast.

we find good overall forecast performance under UC-FSV at all horizons. This indicates the importance of taking into account cross-country linkages (in inflation and output growth) for improving forecasts of inflation and output growth, especially during periods of uncertainty.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular economies, so the third dimension of discussion for the joint predictive density for inflation and output growth is sum of joint LPLs at the economy level (by summing all j for economy i), which can be calculated by:

$$\mathrm{LPL}_{t,i,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{j=1}^{2} \log p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

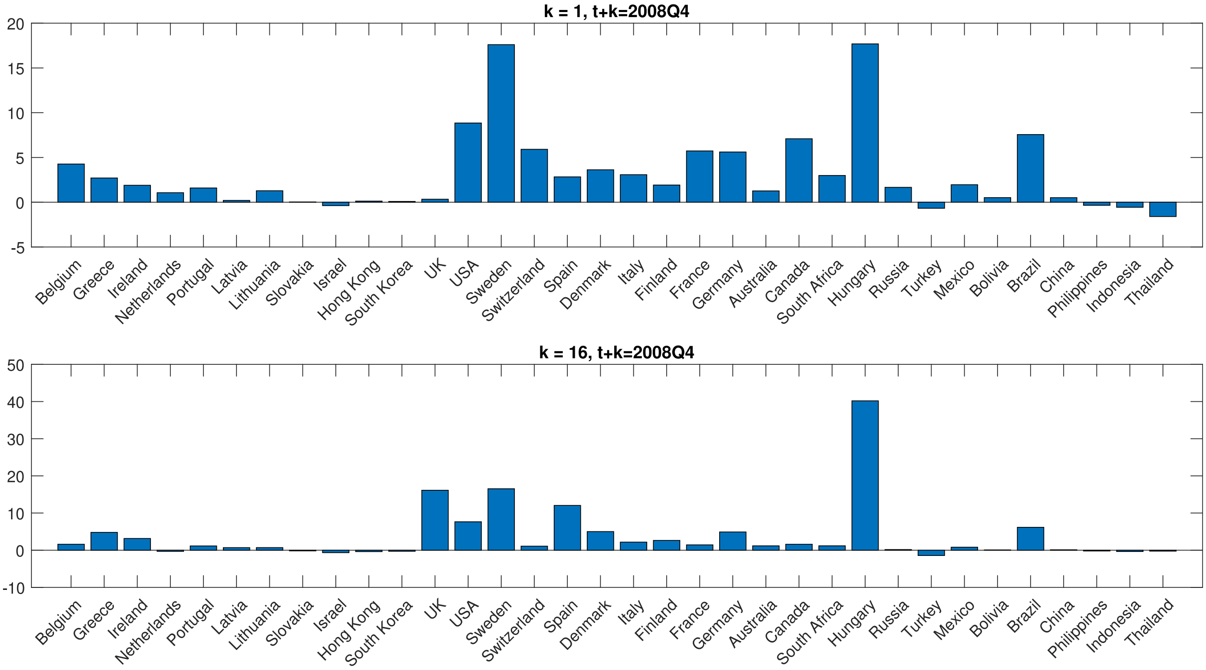


Fig. 7. Sums of k-step-ahead joint LPLs in country i for UC-FSV relative to UC-FSV-r_y, $r_\pi = 0, \omega_y^h = 0$.

We plot the results (against UC-FSV-r_y, $r_\pi = 0, \omega_y^h = 0$) in Fig. 7. A pattern similar to that for output was found. (This is sensible since the gains in output are much larger than the gains in inflation; see Fig. 3 and Fig. 5.) We found good overall forecast performance for UC-FSV for all economies.

6. Conclusion

This paper developed a multi-country unobserved components model that allows for cross-country linkages and models economies jointly. The important feature is realised through factor stochastic volatility. The factor stochastic volatility specification enables us to study the commonality in international macroeconomic uncertainty (global uncertainty). Another important feature of our model is the use of sparsification. We use sparsification to estimate factor loadings and rely on post-processing to obtain an estimate for the number of factors. We also use sparsification to remove stochastic volatility in a data-based manner. Recent research has been devoted to speeding up computation, and one prominent development is equation-by-equation estimation. The factor stochastic volatility specification also enables us to estimate this high-dimensional model equation-by-equation.

In an empirical application, we first presented evidence of global uncertainty that coincided with major economic events. Part of the variation captured in the global inflation factor reflected a global business cycle. Finally, we provided a detailed forecasting exercise to evaluate the merits of our model. We found that our model can provide more accurate density forecasts, especially if the aim is to forecast periods of uncertainty. And such good forecast performance is for most economies, and not driven by particular economies.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: For the purpose of open access, the author has applied a Creative Commons Attribution (CC BY) licence to any Author Accepted Manuscript version arising from this submission.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ijforecast.2023.07.005.

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