(Preprint) AAS 22-641

ESCAPE STRATEGIES FROM THE CAPTURE INTO 1:1 RESONANCE USING LOW-THRUST PROPULSION

Wail Boumchita* and Jinglang Feng†

Vesta is the second largest celestial object of the main asteroid belt and it was visited and studied by the DAWN mission in 2011. The spacecraft used solar-electric propulsion that generates continuous low-thrust. As the spacecraft slowly descends from high altitude mission orbit (HAMO) to low altitude mission orbit (LAMO), it crosses the 1:1 ground-track resonance, with the risk of being captured by this resonance and being trapped at this altitude. The objective of this paper is to analyze different escape strategies from the 1:1 resonance with Vesta based on the change in the low-thrust magnitude. Firstly, the dynamics is modelled considering the irregular gravitational field up to the fourth order and degree and the thrust constant in magnitude and opposite to the velocity direction of the spacecraft. Then, a reference case of capture into 1:1 resonance is considered and the effects of the resonance on the spacecraft's trajectory are described. A Monte Carlo analysis is performed to study the probability of escape from 1:1 resonance as a function of the thrust magnitude. The analysis reveals that there are regions in which the escape from the 1:1 resonance requires more thrust with respect to other ones. Additionally, some cases require to increase the thrust magnitude over the operational limit, making the escape impossible by only increasing the thrust magnitude. This paper contributes to increase the awareness on the risk of resonance capture and the strategies to escape for designing future space missions to asteroids.

INTRODUCTION

In 2011, the spacecraft Dawn successfully approached the asteroid Vesta.[1] The DAWN mission was one of the first missions to use low-thrust propulsion during both the cruise phase and the approach phase to an asteroid. It demonstrates the possibility of relying on low-thrust propulsion for the majority of the mission duration[2].[3] As the spacecraft slowly approaches the asteroid, there is a possibility that it is captured by the 1:1 ground-track resonance and being permanently trapped into it.[4] The spacecraft at each revolution encounters the same gravitational configuration, the effect of which accumulates over the revolutions and change noticeably the orbit eccentricity and inclination. Small variations in the initial state of the spacecraft can make a difference in whether the spacecraft manages to cross the resonance and reach lower altitudes from if the spacecraft remains trapped in the resonance despite the continuous thrusting.

Tricarico and Sykes[4] and Delsate[5] studied in depth the phenomenon of capture into 1:1 resonance. The former brought the awareness that the mission was potentially in danger due to the

*wail.boumchita@strath.ac.uk, Ph.D. student, Department of Mechanical and Aerospace Engineering, University of Strathclyde, Glasgow, G1 1XJ, UK
†jinglang.feng@strath.ac.uk, Lecturer, Department of Mechanical and Aerospace Engineering, University of Strathclyde, Glasgow, G1 1XJ, UK

1:1 resonance, estimating its probability of capture. The latter refined the previous study by updating the estimate of the probability of capture into 1:1 resonance, giving more information on the phenomenon and providing a general methodology to search and analytically locate the main resonances. Delsate concluded his study by considering the possibility of escaping from the resonance and highlighting how this depends on the phase of the resonance angle.

For this research, we extend the previous investigation and advance the knowledge of resonance escape by analyzing the possibility of escaping from the 1:1 resonance around Vesta by increasing only the thrust magnitude. These analysis contribute to the mission design phase so that the hazards of the resonance capture and the action needed to escape from it are taken in consideration. In fact, resonance capture interferes with the spacecraft's trajectory and, for missions that require precision, the resonance capture prevents the mission requirements from being met. In the worst case, the accumulation of gravitational perturbations could lead the spacecraft to crash into the asteroid's surface.

This work is organized as follows: the equations of motion and the Hamiltonian dynamical model are defined. The phase-space of the model is obtained and it provides important insights on the phenomenon of resonance. Then, the analysis is focused on the effects of the 1:1 resonance on the descent trajectory of Dawn, in particular, on how the accumulation of the gravitational perturbation affect the evolution of the semi-major axis (SMA), eccentricity and inclination. Subsequently, the study concentrates on the Monte Carlo simulations providing insights on the best strategies when performing a maneuver to escape the resonance. The final section concludes the study.

DYNAMICAL MODELING

The model considered is the perturbed two-body problem with perturbations from the irregular gravitational field of Vesta and the low-thrust to which the spacecraft is subject. Tricarico[4] has proved that with a spherical harmonic expansion up to the 8^{th} order, the dynamics is dominated by the 3^{rd} and 4^{th} order terms. Therefore, the gravitational field truncated to the fourth order and degree is used. The low-thrust is constant in magnitude and it always in the opposite direction of Dawn's velocity.

Kaula[6] defined the gravitational potential of a central body as a function of the spherical harmonics. The characteristics of the asteroid including the shape and volume density variations are taken into account through the Stokes coefficient. The gravitational field V of degree n and order m in spherical harmonics expansion is given in spherical coordinated (r,λ,ϕ) as

$$V = \frac{\mu}{r} + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \frac{\mu}{r} \left(\frac{R_e}{r} \right)^n P_{nm}(\sin\phi)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda) \tag{1}$$

where r is the distance of the spacecraft from Vesta, μ is Vesta's gravitational constant, R_e is the reference radius of Vesta, n and m are respectively the degree and order of the spherical harmonic expansion considered, C_{nm} and S_{nm} are the unnormalized Stokes coefficients.

The equations of motion are defined in cartesian coordinates that describe the absolute spacecraft's motion in the asteroid's centered inertial frame as

$$\ddot{\vec{x}} = \vec{\nabla} V - \frac{T}{m(t)}\hat{\vec{v}} \tag{2}$$

$$\dot{m} = -\frac{T}{I_{sp} g_0} \tag{3}$$

where $\ddot{\vec{x}} = [a_x, a_y, a_z]$ is the acceleration vector along the cartesian coordinates, V represents the potential expanded in spherical harmonics as a function of the cartesian coordinates (x, y, z), T is the thrust magnitude, m is the spacecraft's mass and $\hat{v}$ is the spacecraft's velocity unit vector. The rate of change of the spacecraft's mass over time is a function of I_{sp} and g_0 which represent the specific impulse and Earth's gravitational acceleration respectively.

More information can be extrapolated by using the Hamiltonian formalism. The gravitational field previously defined in Eq. 1 can be expressed as a function of the orbital parameters as

$$V = \frac{\mu}{r} + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\sum_{p=0}^{n}\sum_{q=-\infty}^{\infty} \frac{\mu R_e^n}{a^{n+1}} F_{nmp}(i) G_{npq}(e) S_{nmpq}(\omega, M, \Omega, \theta) \tag{4}$$

where $F_{nmp}(i)$ is a function of the inclination, $G_{npq}(e)$ is a function of the eccentricity, ω is the argument of periapsis, M is the mean anomaly, Ω is the longitude of the ascending node, θ is the sidereal time, n, m, p, q are all integers and

$$S_{nmpq} = \begin{cases} C_{nm}\cos\Psi_{nmpq} + S_{nm}\sin\Psi_{nmpq}, & \text{if } n-m \text{ is even} \\ -S_{nm}\cos\Psi_{nmpq} + C_{nm}\sin\Psi_{nmpq}, & \text{if } n-m \text{ is odd} \end{cases} \tag{5}$$

where Ψ_{nmpq} is the Kaula's phase angle that is defined as

$$\Psi_{nmpq} = (n-2p)\omega + (n-2p+q)M + m(\Omega - \theta) \tag{6}$$

Resonances arise when the time derivative of Kaula's phase angle $\dot{\Psi}_{nmpq}$ is approximately zero. Physically, we define that the spacecraft Dawn is in ground-track resonance with Vesta when the frequencies of Dawn's revolution and of Vesta's rotation are linearly dependent on a set of integers. In particular

$$\dot{\Psi}_{nmpq} = k_1\dot{\theta} - k_2 n = 0 \tag{7}$$

where $\dot{\theta}$ is the angular velocity of Vesta and n is the angular velocity of Dawn.

The Hamiltonian that describes the motion of the spacecraft around an asteroid with an irregular gravitational field is expressed using Delaunay variables as

$$\mathcal{H} = -\frac{\mu^2}{2L^2} + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\sum_{p=0}^{n}\sum_{q=-\infty}^{\infty} R_e^n \frac{\mu^{n+2}}{L^{2n+2}} F_{nmp}(i) G_{npq}(e) S_{nmpq}(\omega, M, \Omega, \theta) + \dot{\theta}\Lambda \tag{8}$$

where Λ is the conjugated momentum of the sidereal time θ and the term $\dot{\theta}\Lambda$ accounts for the asteroid's rotation. The dynamics around the 1:1 ground-track resonance is mostly influenced by the term of second degree and order, J_{22}. For this reason, the Hamiltonian considered in Eq. 11 was limited to the second order and degree. For a polar circular orbit, the Hamiltonian is

$$\mathcal{H}_{1:1} = -\frac{\mu^2}{2L^2} + \frac{1}{4}R_e^2\frac{\mu^4}{L^6}C_{20} + \frac{3}{4}R_e^2\frac{\mu^4}{L^6}\sqrt{C_{22}+S_{22}}\cos\left[\Psi_{2200} + \arctan\left(-\frac{S_{22}}{C_{22}}\right)\right] + \dot{\theta}\Lambda \quad (9)$$

We introduce the resonance angle $\sigma = \lambda - \theta$ in the case of 1:1 resonance. where $\lambda = \Omega + \omega + M$. However, to maintain a set of canonical variables, the symplectic transformation used by**?** has to be adopted. This defines the new set of canonical variables

$$\sigma \quad , \quad L' = L \quad , \quad \theta' = \theta \quad , \quad \Lambda' = \Lambda + L$$

The Hamiltonian expressed as a function of this new set is thus transformed as

$$\mathcal{H}_{1:1} = -\frac{\mu^2}{2L^2} - \frac{1}{4}R_e^2\frac{\mu^4}{L^6}C_{20} - \frac{3}{4}R_e^2\frac{\mu^4}{L^6}\sqrt{C_{22}+S_{22}}\cos\left[2\sigma + \arctan\left(-\frac{S_{22}}{C_{22}}\right)\right] - \dot{\theta}L \quad (10)$$

in which we dropped the prime sign for simplicity and dropped the constant $\dot{\theta}\Lambda'$ term since the expression is no more explicitly dependent on θ. For Vesta, $S_{22} = 0$ so the Hamiltonian is simplified as

$$\mathcal{H}_{1:1} = -\frac{\mu^2}{2L^2} - \frac{1}{4}R_e^2\frac{\mu^4}{L^6}C_{20} - \frac{3}{4}R_e^2\frac{\mu^4}{L^6}C_{22}\cos 2\sigma - \dot{\theta}L \quad (11)$$

CAPTURE INTO 1:1 RESONANCE

The 1:1 ground-track resonance is one of the major resonances that Dawn goes through during its slow descent from HAMO to LAMO. In particular, it is the resonance with greater amplitude. The location of the 1:1 resonance depends on the initial inclination and eccentricity of the descent trajectory and the condition to locate it is

$$\frac{\partial \mathcal{H}}{\partial L} = 0 \quad (12)$$

evaluated at the equilibrium point. Equilibrium points can be determined from

$$\frac{\partial \mathcal{H}}{\partial \sigma} = 0 \quad (13)$$

Vesta has four points of equilibrium, two stable and two unstable. In particular

$$\sigma_{st}^{1,2} = 0, \pi \quad , \quad \sigma_{un}^{1,2} = \frac{\pi}{2}, \frac{3\pi}{2} \quad (14)$$

The initial conditions are reported in Table 1.

Table 1: Dawn's nominal initial conditions at HAMO[4]

Mass	m	1000 kg
Thrust magnitude	T	20 mN
Specific impulse	I_{sp}	3000 s
Semi-major axis	a_0	1000 km
Eccentricity	e_0	0
Inclination	i_0	90°
Longitude of the ascending node	Ω_0	0°
Argument of periapsis	ω_0	0°
True anomaly	θ_0	30°

From Eq. 12, it can be determined that the location of the 1:1 resonance is 550 km. Therefore, in case that the spacecraft is captured into 1:1 resonance around Vesta the radial distance oscillates around the resonance location as in Figure 1. The integration of the dynamics of the spacecraft is stopped at 400 km.

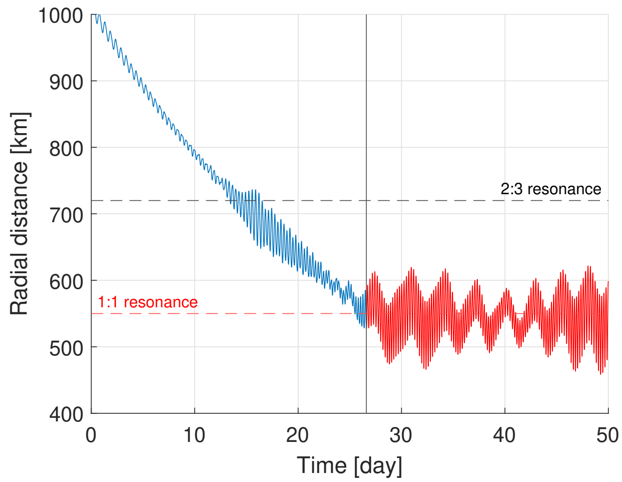

Figure 1: The radial distance evolution with respect to time in the case of the spacecraft being captured into 1:1 resonance with Vesta. The vertical line indicates the time epoch of the resonance capture.

Resonance capture can also be illustrated in phase-space. We define the resonance region as the region of the phase-space included within the separatrices as in Figure 2. An averaging process is applied to filter high frequency oscillations.

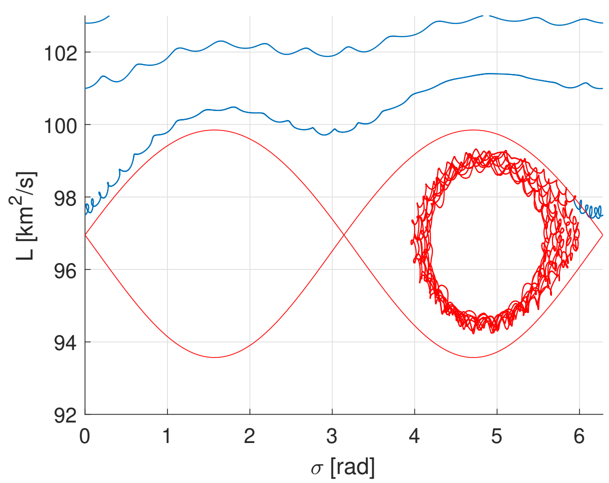

Figure 2: Representation in phase-space of the last part of the descent.

The y-axis in Figure 2 represents the momentum L from 92 km^2/s to 103 km^2/s, which corresponds respectively to a SMA interval from 483.65 km to 606.23 km. When the spacecraft is trapped into resonance, its trajectory in phase-space rotates around the stable equilibrium point.

The resonance amplifies the gravitational perturbations on the motion of the spacecraft and acts on the orbital parameters of the trajectory. The consequences of such an accumulation of perturbations causes the excessive need for corrective maneuvers or, in the worst case, deviate the spacecraft from its nominal trajectory, causing it to impact on the surface of the asteroid. In particular, the eccentricity fluctuates with great amplitude bringing it to assume values from 0 to 0.16 in the first month within the resonance as in the right plot of Figure 3. As in Figure 1, the changes to which the eccentricity is subjected leads the orbital radius to oscillate more or less according to the value that the eccentricity assumes, the larger is the eccentricity the bigger the oscillations.

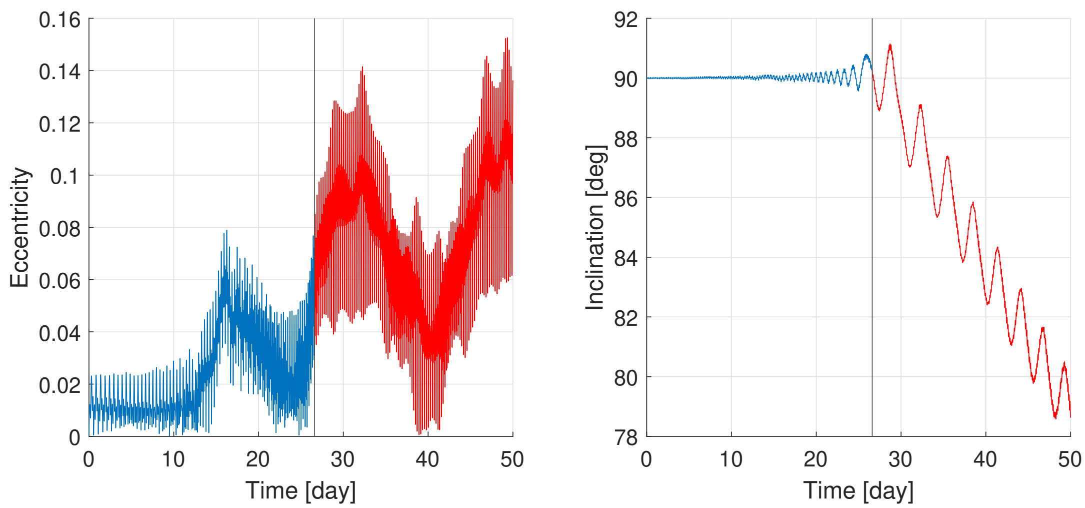

Figure 3: The eccentricity (on the left) and the inclination (on the right) evolution with respect to time in the case of the spacecraft being captured into 1:1 resonance with Vesta. The vertical line indicates the time epoch of the resonance capture.

Additionally, the inclination oscillates and the average value of the oscillations decreases linearly. In one month within the 1:1 resonance, the inclination decreases by 12° as in the left plot of Figure 3.

ESCAPE FROM 1:1 RESONANCE

To limit the effects of the 1:1 resonance, it is necessary to study different strategies to escape from the resonance. The escape is sensitive with respect to the time epoch when the thrust is increased. In Figure 4, it is noticed that the successful escape from the resonance is closely related to the initial conditions. In particular, when the thrust is increased to 30 mN from 30.9 days after the spacecraft started its descent, the spacecraft escapes as indicated by the red curve. But when the thrust is increased by the same amount from day 30.8, the spacecraft fails to escape as indicated by the blue curve.

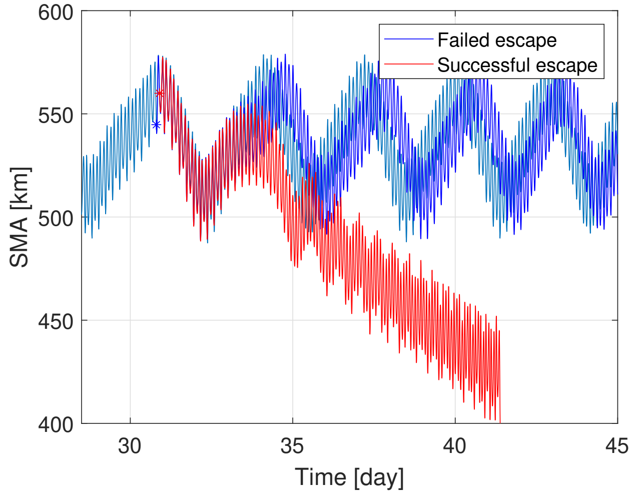

Figure 4: Sensibility of the resonance escape on the firing time. In both cases the thrust magnitude is increased from 20 mN to 30 mN. The initial condition for the successful escape and its SMA evolution are highlighted in red, while the failed case are in blue.

In the previous section, the study on resonance capture was performed with a constant thrust of 20 mN and the maximum thrust that the spacecraft can generate is equal to 92 mN. Initially, a time interval between 28.5 and 32.3 days is considered as in Figure 5.

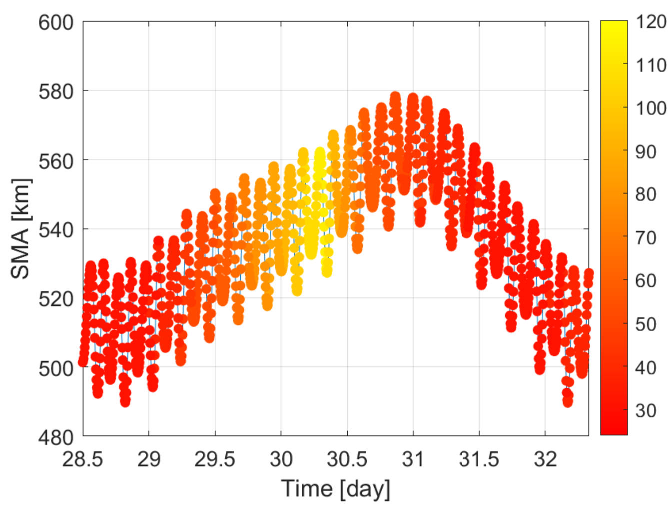

Figure 5: Minimum thrust required to escape the 1:1 resonance for different initial conditions.

The thrust considered in the Monte Carlo simulations ranges from 24 mN to 120 mN and a total of 1000 different initial conditions are considered. For each initial condition, the minimum thrust required to escape the resonance is obtained. There are regions in which escape from the resonance is more difficult than in other regions. In Figure 5, the higher the thrust required to escape the brighter the color. In these regions the thrust required for certain initial conditions to escape the resonance exceeds the maximum thrust magnitude that Dawn's engines can provide, making it practically impossible to escape from resonance by only increasing the thrust magnitude. In the phase-space, the initial conditions near the lower separatrix require less thrust than that near the upper separatrix as shown in Figure 6.

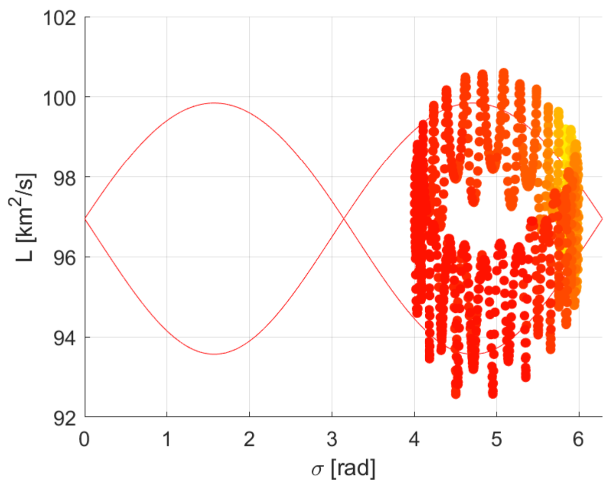

Figure 6: Representation in phase-space of the minimum thrust required to escape the 1:1 resonance for different initial conditions.

Figure 7 indicates the number of cases that the spacecraft successfully escapes from resonance as the thrust magnitude varies. The number of escape cases evolves with an approximate logarithmic trend: increasing the thrust up to 50 mN, the probability of escape from the resonance is 80%; the remaining 20% requires a larger thrust magnitude. In particular, by increasing the thrust up to the maximum thrust magnitude that Dawn's engine can generate the probability of escape from the resonance reaches about 95%. The remaining 5% requires a thrust magnitude that the spacecraft cannot provide.

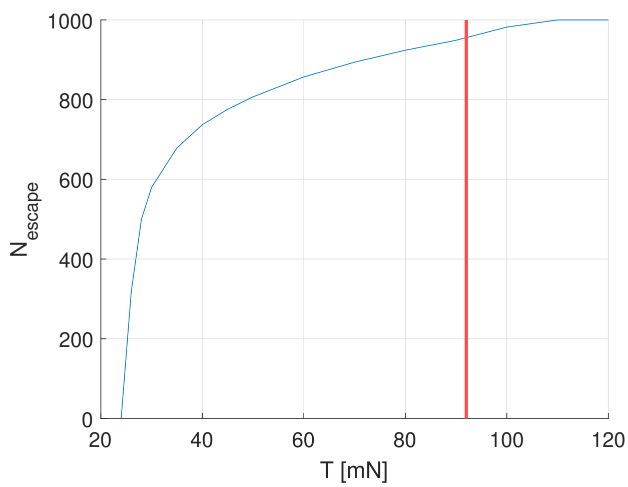

Figure 7: Number of successful escape cases for different thrust magnitudes. The red line indicates the limit of thrust magnitude that Dawn's engine can generate

An additional risk to the spacecraft and its ability to escape the 1:1 resonance is noticeable in the long term. Considering Figure 8 in which the red points correspond to the cases of successful escape while the blue points are the cases in which the escape fails, we notice that with the same thrust magnitude the cases of successful escape decrease if this thrust maneuver is performed late.

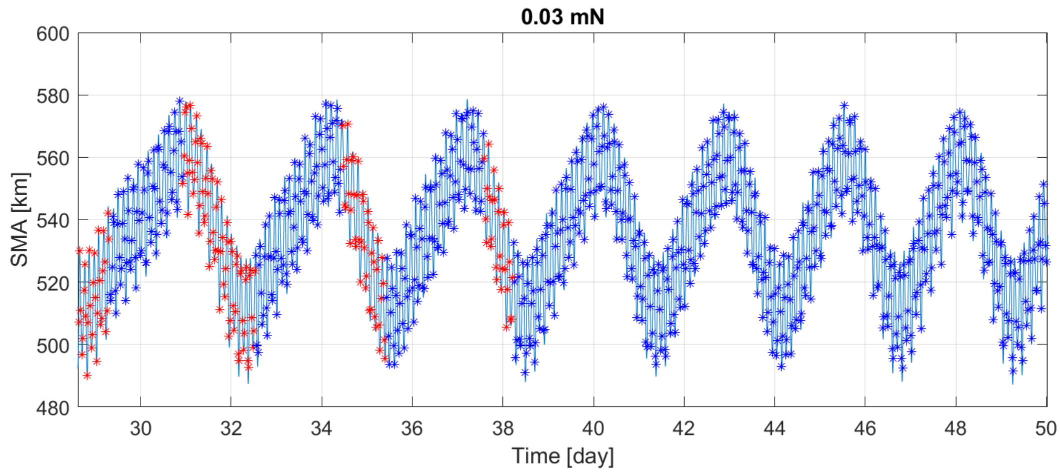

Figure 8: Long term evolution of the escape cases increasing the thrust magnitude to 30 mN. The initial conditions related to the successful cases (red points) diminishes as the maneuver is postponed.

In fact, as the spacecraft postpones the maneuvers for the escape from the resonance, the probability that the spacecraft successfully escapes from the resonance decreases with the same thrust magnitude. In Figure 9, the minimum thrust magnitude required to escape the resonance for different initial conditions is demonstrated.

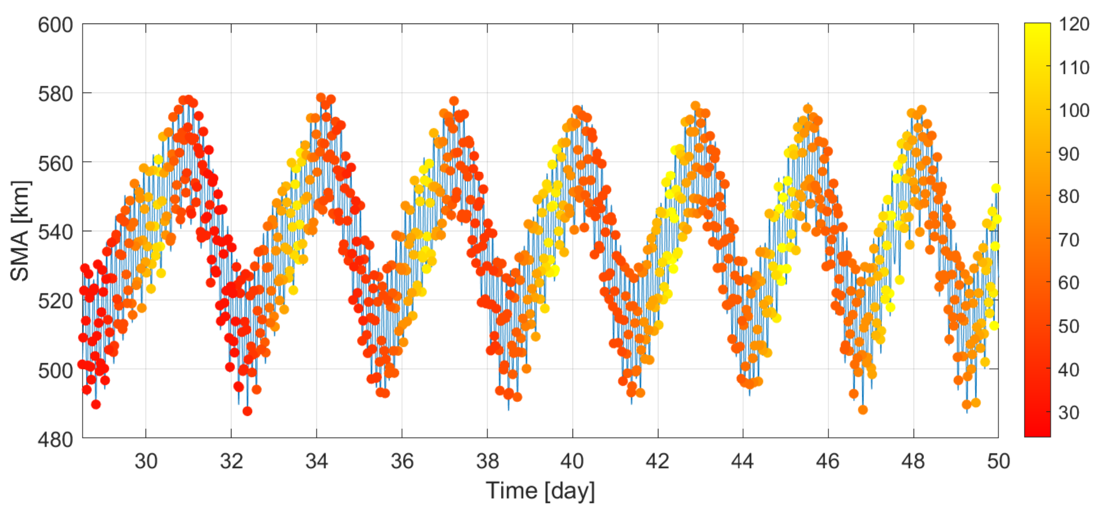

Figure 9: Long term evolution of the thrust required to escape for different initial conditions.

If the spacecraft is trapped for a long time, the escape from the resonance will be more difficult. The brighter region, which corresponds to the interval of initial conditions that require a high thrust magnitude to escape, grows as the maneuver is postponed. For the same value of SMA, the thrust required to escape from the resonance for day 28 is equal to 24 mN, while if the maneuver is postponed until day 49, the required thrust increases up to 80 mN.

The 1:1 resonance can be exploited to maneuver the spacecraft, saving on mission cost. One of the effects of the resonance is to decrease the inclination of the trajectory as shown in Figure 10.

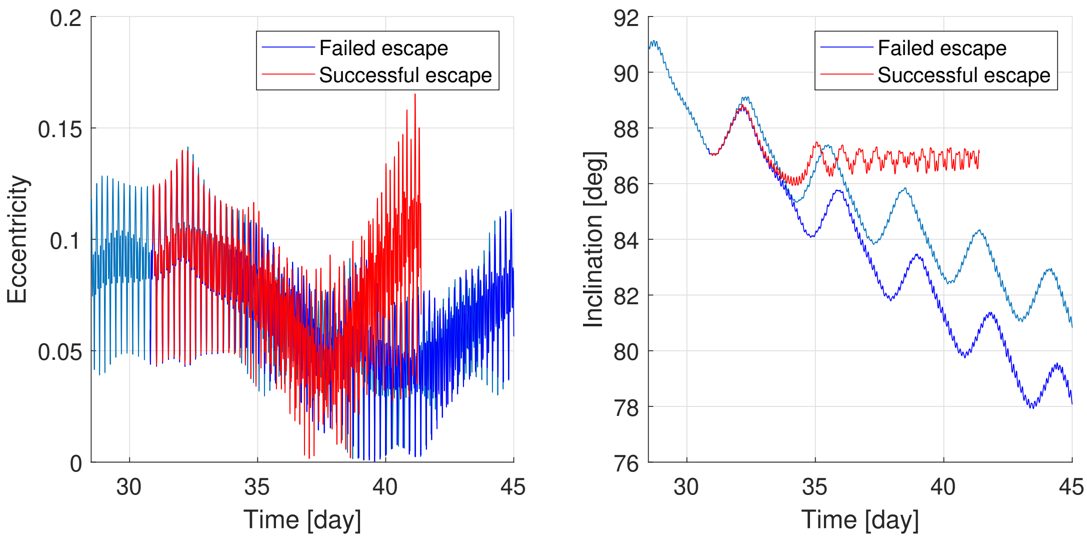

Figure 10: The eccentricity (on the left) and the inclination (on the right) evolution with respect to time after two escape maneuvers are performed at day 30.8 and 30.9. The red line represents the parameters evolution for the successful escape case, while the blue line represents the failed escape case.

Once escaped from the resonance, the inclination stops decreasing and remains constant except for small oscillations. This can be used to perform a plane change maneuver, saving on fuel cost. On the other hand, if the spacecraft increases the magnitude thrust and fails to escape from the resonance, the effects of the gravitational perturbations on the evolution of the inclination become more severe by increasing the speed with which the inclination decreases. The escape maneuver is performed in day 30.9, when the inclination reached 87°. After a transition period of 4 days, characterized by large oscillations, the inclination value asymptotically oscillates around 87°. The eccentricity is more sensible to other resonances rather that the outcome of the escape maneuver. It is evident that if the escape fails, the evolution of the eccentricity remains unchanged from the eccentricity evolution with a thrust magnitude of 20 mN; while if the escape is successful the eccentricity does not undergo notable variations from the 20 mN case until it is influenced by other resonances.[5] In fact, the eccentricity begins to deviate before day 40, when the spacecraft reaches the altitude of the 4:3 ground-track resonance.

CONCLUSION

In this paper, we extend the previous research on escape from 1:1 ground-track resonance. The escape from the resonance is a necessary maneuver to guarantee the success and long lifetime of the mission. The escape from the resonance is not guaranteed even by increasing the thrust to the maximum value. It is also related to the time epoch when the thrust maneuver is performed. In fact, there are intervals of initial conditions for which the escape from the resonance is impossible and an ad-hoc analysis for the mission considered must be conducted. In general, it is found that it is counterproductive to postpone the escape maneuver. For the same value of SMA, the more the maneuver is postponed, the greater is the thrust required to escape from the resonance. Finally, it is shown how the resonance can be exploited to perform a plane change maneuver, saving on mission costs.

ACKNOWLEDGEMENTS

This work is funded by ESA OSIP with project title ”Resonance Capture of Low-Thrust Spacecraft Around a Small Body” and by the John Anderson Research Award Studentship.

APPENDIX

The unnormalized gravitational coefficient for Vesta are reported in Table 2.

Table 2: Unnormalized gravitational coefficient for Vesta[7]

Coefficients	Vesta
C00	1.0
C20	-6.872555 $\times 10^{-2}$
C21	0.0
S21	0.0
C22	3.079667 $\times 10^{-3}$
S22	0.0
C30	6.286305$\times 10^{-3}$
C31	-7.982112 $\times 10^{-4}$
S31	1.825409 $\times 10^{-4}$
C32	-3.162892 $\times 10^{-4}$
S32	5.943231 $\times 10^{-5}$
C33	2.565757$\times 10^{-5}$
S33	7.264998$\times 10^{-5}$
C40	9.6 $\times 10^{-3}$
C41	6.394125$\times 10^{-4}$
S41	-1.347130$\times 10^{-4}$
C42	-3.152856 $\times 10^{-5}$
S42	6.551679 $\times 10^{-5}$
C43	-3.113571$\times 10^{-5}$
S43	-2.689264$\times 10^{-6}$
C44	3.190457 $\times 10^{-6}$
S44	5.514632 $\times 10^{-6}$

REFERENCES

[1] C. Russell, F. Capaccioni, A. Coradini, M. De Sanctis, W. Feldmann, R. Jaumann, H. Keller, T. Mccord, L. Mcfadden, S. Mottola, C. Pieters, T. Prettyman, C. Raymond, M. Sykes, D. Smtih, and M. Zuber, “Dawn Mission to Vesta and Ceres,” *Earth, Moon and Planets*, Vol. 101, 2007, pp. 65–91, 10.1007/s11038-007-9151-9.

[2] N. Wallace, O. Sutherland, J. Bolter, H. Gray, A. Altay, F. Striedter, F. Budnik, S. Manganelli, E. Montagnon, and C. Steiger, "BepiColombo - Solar Electric Propulsion System Operations for the Transit to Mercury," 2019.

[3] K. Nishiyama, S. Hosoda, K. Ueno, R. Tsukizaki, and H. Kuninaka, "Development and Testing of the Hayabusa 2 Ion Engine System," *The Japan Society for Aeronautical and Space Sciences*, Vol. 14, 2016, pp. 131–140, 10.2322/tastj.14.Pb131.

[4] P. Tricarico and M. Sykes, "The Dynamical Environment of Dawn at Vesta," *Planetary and Space Science*, Vol. 58, 2010, pp. 1516–1525, 10.1016/j.pss.2010.07.017.

[5] N. Delsate, "Analytical and numerical study of the ground-track resonances of Dawn orbiting Vesta," *Planetary and Space Science*, Vol. 59, 2012, 10.1016/j.pss.2011.04.013.

[6] W. M. Kaula, *Theory of Satellite Geodesy: Applications of Satellites to Geodesy*. 1966.

[7] J. Feng, R. Noomen, X. Hou, P. Visser, and J. Yuan, "1:1 Ground-track resonance in a uniformly rotating 4th degree and order gravitational field," *Celestial Mechanics and Dynamical Astronomy*, Vol. 127, 2017, 10.1007/s10569-016-9717-9.