Structuring ultracold atoms with light in an optical cavity

Grant W. Henderson[a], Gordon R. M. Robb[a], Gian-Luca Oppo[a], and Alison M. Yao[a]

[a]SUPA & Department of Physics, University of Strathclyde, Glasgow, Scotland G4 0NG, United Kingdom

ABSTRACT

We numerically model the propagation of light through a Kerr medium and through a far-red-detuned Bose Einstein condensate (BEC) inside an optical cavity using Lugiato-Lefever and Gross-Pitaevskii equations. We demonstrate the formation of light-atomic ring lattices with rotation speeds and direction that can be controlled by the orbital angular momentum of the light. In the BEC, we show the possibility of moving from a lattice to a ring-shaped atomic circuit by changing the atomic scattering length, and explore the potential generation of rotating BEC cavity solitons. Our results may be of interest as slow light pulses with fully controllable speed and structure for use in optical quantum memories, for particle manipulation and trapping, and for the novel realization of highly controllable, tightly confined, rotating atomic lattices.

Keywords: Bose-Einstein Condensate, Orbital Angular Momentum, Lugiato-Lefever Equation, Gross-Pitaevskii Equation, Pattern Formation, Coupled Atom-Light Solitons

1. INTRODUCTION

Pattern formation is ubiquitous in nonlinear dynamical systems, the most famous example being Turing patterns in reaction-diffusion systems.[1] In optical cavities, spontaneous spatial pattern formation derives from the interplay of nonlinearity, cavity detuning and a spatial coupling, such as diffraction. Such systems are very well described by the Lugiato-Lefever equation.[2]

In this paper we consider the effect of two different types of media providing the non-linearity, a self-focussing Kerr medium and a Bose-Einstein Condensate (BEC), and on using structured light to pump optical cavity. In both cases, we demonstrate the formation of rotating Turing structures when the pump carries optical angular momentum (OAM),[3] with a rotation speed determined entirely by the OAM of the input pump, m, and the radius R of the ring structure according to $\omega = 2m/R^2$. We show that the BEC has additional control parameters/degrees of freedom that affect the pattern formation, but that these do not affect the rotation speed.

Spatial optical structures rotating on a transverse ring can be considered as slow light pulses with fully controllable speed and structure for use in optical quantum memories and delay lines. Equivalent atomic structures can form rotating atomic lattices without the requirement for sophisticated static magnetic or optical trapping potentials. Moreover, by using structured light to control the atomic rotation we can produce atomic lattices that are highly controllable and dynamically reconfigurable: we can change the speed and direction of rotation simply by changing the size of the pump beam or switching the sign of its OAM. Pumps consisting of superpositions of OAM modes can also produce concentric counter-rotating atomic Turing patterns, an atomic peppermill, with independent control over the speed and direction of each ring.

2. PATTERN FORMATION IN KERR OPTICAL CAVITIES

A ring cavity containing a Kerr nonlinear medium is well-described by the Lugiato-Lefever equation (LLE) in two transverse dimensions:[2]

$$\partial_t E = P - (1 + i\theta)E + i\eta|E|^2 E + i\nabla^2 E \tag{1}$$

where E is the intracavity field, P is the amplitude of the input pump, θ is the detuning between the input pump and the closest cavity resonance, η is proportional to the Kerr coefficient of the nonlinear material, and the

Further author information: (Send correspondence to A.M.Y.)
A.M.Y.: E-mail: alison.yao@strath.ac.uk

This is a peer-reviewed, accepted author manuscript of the following paper: Henderson, G. W., Robb, G. R. M., Oppo, G-L., & Yao, A. M. (2023). Structuring ultracold atoms with light in an optical cavity. In D. L. Andrews, E. J. Galvez, & H. Rubinsztein-Dunlop (Eds.), Proceedings of SPIE Complex Light and Optical Forces XVII (Vol. 12436). SPIE. https://doi.org/10.1117/12.2655449

transverse Laplacian term ∇^2 describes diffraction. The time scale has been normalised by the mean lifetime of photons in the unidirectional ring cavity $\tau = 2L/(cT)$ where L is the cavity length, T the (intensity) transmission coefficient of the cavity mirrors and c the speed of light in vacuum. As we will be considering structured pumps, the transverse spatial scale $(x,\ y)$ has been normalised by $a = L/\left(Tz_R\right)^{1/2}$, where $z_R = kw_0^2$ is the Rayleigh range of the optical field.

We have recently shown that when such cavities are pumped by optical fields carrying OAM they can form rotating Turing patterns on rings of fixed radius.[4] To analyse this we consider pumps with transverse amplitudes described by Laguerre-Gaussian modes with radial index $p = 0$:[5]

$$LG_0^m(r,\phi) = \sqrt{\frac{2}{\pi|m|!}}\frac{1}{w_0}\left(\frac{r\sqrt{2}}{w_0}\right)^{|m|}\exp\left(\frac{-r^2}{w_0^2}\right)e^{im\varphi} = P_m e^{im\varphi}, \tag{2}$$

where m is the OAM, w_0 is the beam waist and P_m is a complex amplitude independent of φ. It is then more natural to consider polar coordinates (R,φ) and, as R can be considered to be constant for a particular LG mode $(m>0)$, we can write the LLE Eq. (1) in one angular transverse dimension:

$$\partial_t E = P - (1+i\theta)E + i\beta|E|^2E + \frac{i}{R^2}\frac{\partial^2 E}{\partial\varphi^2}\ . \tag{3}$$

We start by considering solutions of the form:

$$E(\varphi,t) = F(\varphi,t)e^{im\varphi} \tag{4}$$

which satisfy the equation:

$$\frac{\partial F}{\partial t} + \frac{2m}{R^2}\frac{\partial F}{\partial\varphi} = P_m - \left[1 + i\left(\theta + \frac{m^2}{R^2}\right)\right]F + i\beta|F|^2F + \frac{i}{R^2}\frac{\partial^2 F}{\partial\varphi^2}\ . \tag{5}$$

From Eq. (5) it is clear that there exist rotating solutions of the form $F(\varphi - \omega\, t)$ where the angular velocity ω, found by setting the l.h.s. of Eq (5) to zero:

$$\omega = \frac{2m}{R^2}\ , \tag{6}$$

depends on the OAM m and radius R of the optical pump.

Such dynamic optical pattern formation may have potential applications in particle manipulation by using the rotating peak intensities to dipole trap atoms, molecules and small particles. More recently, it has been proposed that rotating Turing patterns may be used to induce circular transport of cold atoms using opto-mechanic nonlinearities instead of Kerr.[6]

3. PATTERN FORMATION IN BEC-LOADED OPTICAL CAVITIES

Here, we extend the work in[4,6] to ultracold atoms, or Bose-Einstein Condensates (BECs), with the aim of producing controllable atomic lattices with potential applications in matter-wave interferometry[7] and in atom-tronics.[8,9] We follow the approach in section (2), replacing the Kerr medium with a pancake-shaped BEC, as shown in Fig. (1). In this case the Lugiato-Lefever equation Eq. (1) becomes

$$\partial_t F = P - (1+i\theta)\,F + i\nabla_\perp^2 F - is|\psi|^2 F, \tag{7}$$

where the temporal evolution of the BEC atoms, ψ, based on,[10–13] is given by

$$\partial_t\psi = i\nabla_\perp^2\psi - i\left(s|F|^2 + \beta_{col}|\psi|^2 - iL_3|\psi|^4\right)\psi. \tag{8}$$

Eq. (8) provides a Gross-Pitaevskii based description of the dynamics of the BEC field ψ, where we have assumed that the light is far-detuned from the atomic transition so that spontaneous emission can be neglected and the

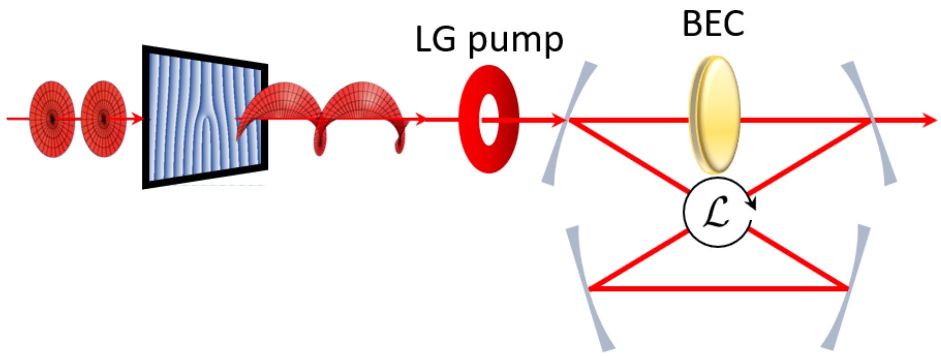

Figure 1. Schematic set-up. A gaussian laser beam is incident on a spatial light modulator (SLM), which adds OAM m, before entering a unidirectional ring cavity, consisting of four highly reflective mirrors and interacting with a pancake-shaped BEC.

excited atomic state adiabatically eliminated.[11] Parameter $s = \pm 1$ represents the sign of the light-atom far-detuning: here we choose $s = -1$ which corresponds to fields that are far-red detuned, so that the BEC atoms are attracted to optical maxima. Parameter β_{col} represents the interatomic scattering of the atoms within the BEC, providing an attractive or repulsive nonlinearity depending on the BEC's scattering length, a_s. We consider a BEC of Caesium atoms as this has a scattering length which may be tuned using magnetic fields around its Feshbach resonance.[14] We include the effect of three-body loss in high-density regimes via L_3,[15,16] but neglect terms corresponding to dipole-dipole forces,[12] and optical saturation.

We consider an optical pump carrying OAM as in Eq. (2) and a pancake-shaped atomic cloud described by a Thomas-Fermi distribution:

$$\psi(r) = 1 - r^2 / \left(2w_\psi^2\right), \tag{9}$$

with w_ψ the waist size of the BEC in terms of the optical beam waist w_0.

For weakly repulsive interactions, we find that the BEC atoms are initially attracted onto the ring of maximum intensity of the optical pump before undergoing a spontaneous pattern formation into bright peaks around it. Both optical and atomic ring lattices rotate azimuthally with angular velocity $\omega = 2m/(R^2)$ as expected.[4] Notably we also see a transfer of phase onto the atoms, as shown in Fig. (2).

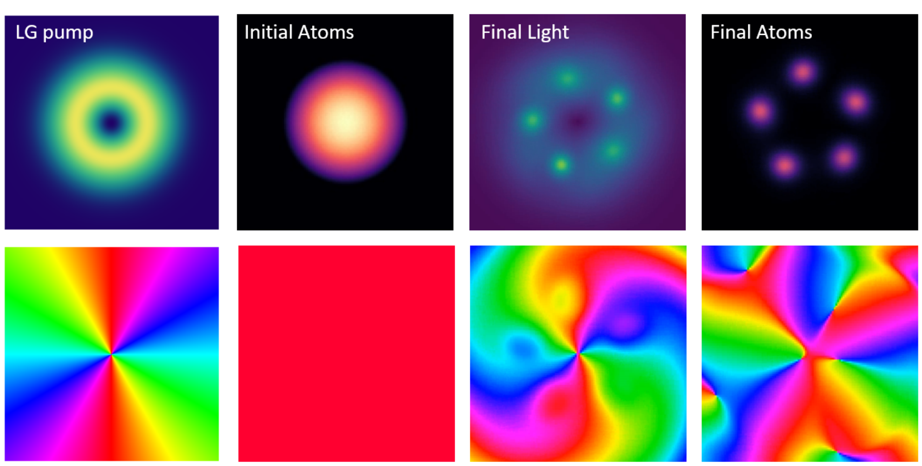

Figure 2. Amplitudes (top) and phases (bottom) of optical pump with OAM, $m = 2$, and initial BEC wavefunction (left); final optical and atomic fields at $t = 10^4$ (right).

The BEC offers more control parameters than the Kerr medium. In particular, the interatomic scattering value (via parameter β_{col} in Eq. (8)) affects the number and size of the atomic peaks, and also the time required for the atomic lattices to form. Moving from attractive to repulsive interactions, we find that the lattices take longer to form and have fewer and broader peaks. For more strongly repulsive regimes we find that there is no longer any lattice formation or phase transfer. We note, however, that the rotation speed of the lattice seems to be largely unaffected by the scattering length.

3.1 Dynamic atomic lattices

One advantage of our proposed set-up is the potential to create BEC rotating atom lattices whose velocity and transverse size can be precisely controlled via the OAM and beam waist of the optical pump.[4,6,17] This is further enhanced by the fact that the optical pump is easily reconfigurable, allowing real-time changes to the atomic lattices with a level of control that would not be possible with conventional static trapping approaches. Complex pump configurations offer the possibility of creating concentric rings of atomic lattices, with independent control of the speed and direction of each ring.

Finally, by changing the cavity detuning θ, we are able to realise atomic lattices of any desired peak number, or uniform rings of (rotating) atoms. These atomic cavity solitons again rotate with angular velocity $\omega = 2m/(R^2)$.

4. CONCLUSIONS

We have used a 1D Lugiato-Lefever model to demonstrate the formation of ring optical lattices in self-focussing nonlinear (Kerr) optical cavities pumped by beams carrying orbital angular momentum, m. We have shown that the lattices rotate around the axis with angular velocity $\omega = 2m/(R^2)$, meaning that the lattice size, speed and rotation direction are controlled by the size and OAM of the optical pump.

Replacing the Kerr medium with a pancake-shaped cloud of ultracold atoms, we couple the Lugiato-Lefever model to a Gross-Pitaevskii equation to model an optical cavity containing a BEC and demonstrate the formation of ring atomic lattices that also have angular velocity $\omega = 2m/(R^2)$. Our results apply over a wide range of interatomic scattering lengths, which affects the size and number of atomic peaks but not their velocity. Our results are dynamically reconfigurable and do not require complex trapping potentials.

ACKNOWLEDGMENTS

We acknowledge support from EPSRC (EP/R513349/1) via a Doctoral Training Partnership and from the European Training Network ColOpt, which is funded by the European Union (EU) Horizon 2020 program under the Marie Skłodowska-Curie Action, Grant Agreement No. 721465. AMY thanks SPIE for support to attend Photonics West 2023.

REFERENCES

[1] Turing, A., "The chemical basis of morphogenesis," *Philosophical Transactions of the Royal Society of London Series B* **237**(641), 37–72 (1952).

[2] Lugiato, L. A. and Lefever, R., "Spatial Dissipative Structures in Passive Optical Systems," *Phys. Rev. Lett.* **58**, 2209–2211 (May 1987).

[3] Allen, L., Beijersbergen, M. W., Spreeuw, R. J. C., and Woerdman, J. P., "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," *Phys. Rev. A* **45**, 8185–8189 (1992).

[4] Yao, A. M., Gibson, C. J., and Oppo, G.-L., "Control of spatially rotating structures in diffractive Kerr cavities," *Opt. Express* **27**, 31273–31289 (Oct 2019).

[5] Barnett, S. M. and Zambrini, R., [*Orbital Angular Momentum of Light*], 277–311, Springer New York, New York, NY (2007).

[6] Baio, G., Robb, G. R. M., Yao, A. M., and Oppo, G.-L., "Optomechanical transport of cold atoms induced by structured light," *Phys. Rev. Res.* **2**, 023126 (May 2020).

[7] Halkyard, P. L., Jones, M. P. A., and Gardiner, S. A., "Rotational response of two-component bose-einstein condensates in ring traps," *Phys. Rev. A* **81**, 061602 (Jun 2010).

[8] Amico, L., Anderson, D., Boshier, M., Brantut, J.-P., Kwek, L.-C., Minguzzi, A., and von Klitzing, W., "Colloquium: Atomtronic circuits: From many-body physics to quantum technologies," *Rev. Mod. Phys.* **94**, 041001 (Nov 2022).

[9] Amico, L., Boshier, M., Birkl, G., et al., "Roadmap on Atomtronics: State of the art and perspective," *AVS Quantum Science* **3**(3), 039201 (2021).

[10] Saffman, M., "Self-induced dipole force and filamentation instability of a matter wave," *Phys. Rev. Lett.* **81**, 65–68 (Jul 1998).

[11] Saffman, M. and Skryabin, D. V., [*Coupled Propagation of Light and Matter Waves: Solitons and Transverse Instabilities*], 433–447, Springer, Berlin, Heidelberg (2001).

[12] Henderson, G. W., Robb, G. R. M., Oppo, G.-L., and Yao, A. M., "Control of Light-Atom Solitons and Atomic Transport by Optical Vortex Beams Propagating through a Bose-Einstein Condensate," *Phys. Rev. Lett.* **129**, 073902 (Aug 2022).

[13] Zhang, W., Walls, D. F., and Sanders, B. C., "Atomic soliton in a traveling wave laser beam," *Phys. Rev. Lett.* **72**, 60–63 (Jan 1994).

[14] Di Carli, A., Henderson, G., Flannigan, S., Colquhoun, C. D., Mitchell, M., Oppo, G.-L., Daley, A. J., Kuhr, S., and Haller, E., "Collisionally Inhomogeneous Bose-Einstein Condensates with a Linear Interaction Gradient," *Phys. Rev. Lett.* **125**, 183602 (Oct 2020).

[15] Altin, P. A., Dennis, G. R., McDonald, G. D., Döring, D., Debs, J. E., Close, J. D., Savage, C. M., and Robins, N. P., "Collapse and three-body loss in a ^{85}Rb Bose-Einstein condensate," *Phys. Rev. A* **84**, 033632 (Sep 2011).

[16] Köberle, P., Zajec, D., Wunner, G., and Malomed, B. A., "Creating two-dimensional bright solitons in dipolar Bose-Einstein condensates," *Phys. Rev. A* **85**, 023630 (Feb 2012).

[17] Baio, G., Ackemann, T., Oppo, G.-L., Robb, G. R. M., and Yao, A. M., "Rotating and spiraling spatial dissipative solitons of light and cold atoms," *Phys. Rev. A* **105**, 023318 (Feb 2022).