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Warranty service contracts design for deteriorating products with maintenance duration commitments

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ABSTRACT

With the increasing diversification of customers' demand and purchasing behaviors, more and more manufacturers have focused their attention on the warranty service contracts design. The maintenance duration of the sold product, which plays an important role in the normal production and operation process of the user, is frequently taken into consideration in warranty contracts. In this study, we design different warranty contracts with various combinations of maintenance duration and availability requirements. The manufacturer commits to compensate for each overdue repair or failing to satisfy the availability target. The customers' choice behavior is described by the multinomial logit (MNL) model, and customers often form their own minimum acceptable levels (also referred to as reference points) of maintenance duration and availability when making purchasing decisions, which have an impact on the contract choice. The expected warranty servicing profit is maximized to determine the optimal price, maintenance duration and availability. Finally, the proposed warranty contracts are demonstrated by numerical examples. We find that the maintenance duration affects not only the warranty cost but also the customer choice, which further affects the optimal contract pricing and profits.

1. Introduction

1.1. Background and motivation

Nowadays, the technological advances have promoted the emergence of more refined and complex products, such as new-energy vehicles, wind turbines, personal electronics and manufacturing equipment, among others (Darghouth et al., 2017). Many customers of these capital-intensive products have the need to hedge against losses due to failures. However, most manufacturers of large-scale equipment (e.g., medical devices including magnetic resonance imaging systems) do not provide free warranties. The customers need to contract with service providers to reduce operational risk involved breakdown of machinery. The warranty service contract, as a form of insurance, not only satisfies customers' needs for risk aversion but also has become a nonnegligible competitive advantage besides profitability for contract providers. Generally speaking, a warranty service contract starts at the moment of product acquisition and terminates at the end of the warranty expiration. Different warranty contracts have been studied in the literature (Zheng et al., 2020; Huang et al., 2021; Cheong et al., 2021). Among these warranty contracts, the maintenance service strategies sold with warranty have received growing attention. Many studies proposed to perform preventive or corrective maintenance during the warranty period (Wang et al., 2020a; Zheng and Zhou, 2021; Liu et al., 2021), and others focused more on the performance characteristics (e.g. availability, production rate), see, for example, Su and Cheng (2018), Wang et al. (2020b) and Wang et al. (2021b).

For customers, the availability is considered as a critical factor in warranty contracts besides the price. For example, in the wind energy industry, the users of wind turbines usually sign a power purchase agreement (PPA) to specify the minimum delivery limit and penalty for under-delivery, and the availability of a wind turbine depends on the energy output, incorporating the operational availability requirement of wind turbines into the warranty contract is very attractive to wind farm owners (Lei and Sandborn, 2018; Zheng et al., 2020). Similarly, in Pardalos et al. (2013), the wind turbine is required to retain an availability level from 0.95 to 0.97. For some capital-intensive products, their failures or shutdown will cause great production losses to customers, which requires the service providers to offer the warranty contracts including a penalty clause of minimum availability level. In addition to the availability, consumers also require warranty providers to complete each repair within a specified value. To this end, the service

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providers offer each repair duration limit with compensation clauses. For instance, the stipulated repair duration limit of automobile is 20 days or a month, and some cellphones should be repaired within 7 or 15 days (Liu et al., 2021). In our survey on an excavator manufacturer of China, they provide a 72-hour commitment on repair duration in the warranty contract, which means that once the failure is reported, the manufacturer has to complete the repair within 72 h, if not, the service provider will pay an extra amount of money in unit time as a compensation.

From the service providers' perspective, they strive to enhance competitiveness in the marketplace by designing an attractive warranty service contract. To consider the simplified contract settings (e.g. contract price, coverage length) oftentimes fails to satisfy the needs of customers with heterogeneous preferences. As mentioned above, customers' purchase decision is to great extent affected by the support service which includes repair duration, guaranteed level of availability and punitive damages for breach, among others. Therefore, many contractors are increasingly cognizant that they should design flexible contract options to attract diverse customers. In some practical applications, the performance expected by consumers hinges on the service level stipulated by contractors. The customer evaluates the value of different service contract clauses relative to a reference level (Tversky and Kahneman, 1991), here from they put forward the concept of Reference Dependence that the carriers of value are gains (or losses) measured with respect to a reference point. In light of this, we consider the reference effects on the customers' choice in warranty service contracts. Above all, two practical examples of warranty service contracts with maintenance duration commitments are presented as follows.

Service commitments of diesel engine In order to offer better service to the customers, enterprises put forward efficient after-sale service system and affirm their commitments for the products. Yuchai, a global diesel engine manufacturer, has offered a wide service range including service response time and completion time for customers' requirement (Diesel, 2022). More specifically, the pre-specified hours of maximum maintenance duration for diverse failure types are different, the enterprise fulfills the maintenance service overdue commitment and provides customers with RMB1000/day compensation for the delayed work. Among them, the maintenance duration of the agricultural machinery during the busy season is prescribed a shorter period compared with that under normal circumstances. In this case, the content of service commitment clearly specifies maintenance duration and the standard of compensation for delay in completion.

Warranty service contracts A common practice of warranty service contracts with maintenance duration is vehicle maintenance contract. The specific contents including the responsibilities and obligations of service providers and consumers are stipulated in implementation detailed.[1] In this manner, the contract terms prescribe specifications for maintenance duration and commitments for maintenance overdue penalty in the maintenance agreement templates. More often than not, customers with diverse needs will negotiate the concrete maintenance time standards with service provider when signing the contracts. Meanwhile, service providers are willing to offer diversified maintenance time choices to increase the attractiveness to consumers and the profits of the products. Compared with traditional warranty policy, the service providers could adjust the standards of maintenance duration and availability in light of the different customer's needsby offering flexible maintenance commitments.

From above examples, we notice that the maintenance duration commitments are considered in the service contract design, and the commitments can be adapted to different scenarios depending on the customers' requirements. In this case, it is valuable for service contract providers to consider the maintenance duration commitments when design warranty service contracts. To this end, this study proposes a novel warranty service contract design involving the maintenance duration commitments for deteriorating products.

1.2. Literature review

Before delving into more specifies, an overview of the literature pertinent to warranty contracts is given. Afterwards, closely related topics are reviewed in detail.

In the literature, diverse maintenance strategies for products with warranty contract have been widely discussed. A comprehensive review associated with maintenance models in warranty can be found in Shafiee and Chukova (2013). According to the maintenance strategies, warranty contracts can be broadly divided into three categories: warranties with free replacement, minimal repair and imperfect maintenance. Liu et al. (2020) proposed an two-period optimal pricing and production strategies under free replacement warranty for a monopolistic manufacturer. Cheong et al. (2021) studied the dynamic optimization problem of price and two-dimensional warranty policy under the free minimal repair. Zhu et al. (2019) compared different types of warranty policies under the proposed optimization model, which are roughly divided into two categories, i.e., replacement and minimal repair. Hashemi et al. (2022) investigated the maintenance model for a warranted coherent system, they divided the warranty period into two phases: free replacement in the phase I and minimal repair in phase II. In addition, some studies attempt to develop the imperfect maintenance strategies for warranty service. Zhao et al. (2018b) adopted an improvement factor model to describe the random effects of imperfect maintenance. Chien (2019) presented a new way to characterize the effect of maintenance on failure rate. Peng et al. (2021) studied an dynamic maintenance model under two-dimensional warranty contract considering the impact on random and dynamic usage rates. Zheng and Zhou (2021) assumed that the preventive maintenance could only reduce the initial level of the covariate process while cannot affect the product age.

In general, availability, as one of the most concerned factors in the warranty contract design, has been considered in the literature (Liao et al., 2006; Su and Cheng, 2018; Zheng et al., 2020). Qiu et al. (2017) studied the optimal maintenance policy by maximizing the steady-state availability or minimizing long-run cost rate for a repairable system. Shen et al. (2019) also formulated the optimal maintenance policy by minimizing the cost within the constraints of availability and operating time in dynamic environments. Su and Cheng (2018) investigated an availability-based contract with considering the learning effect of maintenance. Jackson and Pascual (2021) studied the optimal pricing problem for an availability-based contract under the effect of different maintenance actions. Besides the availability, the limit of each repair duration is also another critical factor in warranty policy (Wee and Widyadana, 2013; Park et al., 2017; Liu et al., 2021). Park et al. (2013) proposed a renewable minimal repair–replacement warranty policy with the predetermined repair duration limit, and the repair duration is assumed to follow 2-parameter Weibull distribution. Park et al. (2017) considered a periodic preventive maintenance policy that took failure time and repair time of the product into account. Liu et al. (2021) studied the warranty policy with limited maintenance duration and repair numbers from the customer's perspective. However, most existing studies lean on the assumption that the maintenance duration is independent with warranty length.

There is abundant researches on the design of warranty contracts, one of which is the contract choices made by customers with diverse preferences. The multinomial logit (MNL) model, as one of the widely used models in customer choice, has received more attention in the literature. A comprehensive analysis on choice-based revenue management can be found in Strauss et al. (2018). Deprez et al. (2021)

[1] https://amr.hunan.gov.cn/amr/zwx/xxgkmlx/zcfgx/flfggzx/201301/t20130111_10467290.htm (Accessed on 3 December 2022).

designed a pricing scheme for proposed full-service maintenance contract, where the MNL model is used to deliver the probabilities for minor failure types. Wang et al. (2021a) adopted the MNL and nested logit models to study the joint decisions on price, quality and service duration. Wang et al. (2020c) used the MNL model to describe the customer choice behavior on extended warranty menu which offered multiple options with different lengths and prices. Based on this, some researchers have further investigated the effects of reference dependence on customer choice behavior. Jindal (2014) estimated the risk and product preference of purchasing the extended warranty through the survey design, and the effect of reference dependence was also considered in extended warranty choices. Wang (2018) combined the prospect theory (e.g., reference prices) and customer choice models to solve the assortment planning and pricing problems. Wang et al. (2021c) addressed the optimal pricing and inventory policies with considering the reference price effect on the customers' purchase utility.

In the existing studies, to the best of our knowledge, there is not literature associating the warranty contract design with customer choice. In this study, we design various warranty service contract options to the customers with heterogeneous preferences from the viewpoint of service providers.Specifically, the theory contributions of this study are three folds: (1) Firstly, this study designs a new warranty service contract for deteriorating products with different maintenance duration commitments, including repair duration and availability. (2) Secondly, this study considers the warranty service contracts offering to the customers with different maintenance options, and the reference points of repair duration and availability have an effect on the perceived value through warranty purchase. (3) Finally, this study develops an integrated warranty cost model which combines maintenance cost, overdue payment and refund due to the random repair duration. In addition, this study formulates the design and pricing problem by incorporating the reference levels into the customers' purchase utility under a MNL model.

The reminder of this study is organized as follows. Section 2 describes the formulation of the degradation model and repair duration models, along with the related warranty cost functions. In Section 3, we first present the customer choice model for different maintenance service contracts, then the optimization problem is formulated. A numerical example is conducted in Section 4 to illustrate our proposed model and explore insights. Section 5 concludes this paper and suggests future directions. Technical proofs and simulation algorithm are relegated to the Appendix.

2. Model formulation

In this section, we consider the warranty policies for degrading products bundled with maintenance duration. The degradation process and warranty cost structure are described, and the effects of imperfect maintenance and repair duration are also considered.

2.1. Model assumptions

Let T_i and T_{pi} represent the durations of normal operation and repair, respectively. We assume that the manufacturers offer warranty service with duration W. When the degradation level reaches the failure threshold l_d within the warranty period, the customer will return the product for repair. In general, this repair is imperfect, and the product cannot be restored to the as-good-as-new state. Furthermore, the manufacturers commit to ensure that the maintenance duration and availability do not exceed the pre-specified values during the warranty period, otherwise, the manufacturers need to pay the customer a refund as penalty. The illustration of degradation and repair process is shown in Fig. 1. We can observe that the T_i and T_{pi} ($i = 1,\ 2,\ \cdots,\ N$) are random variables. In addition, to further facilitate the model description and analysis, the following assumptions are considered.

1. The product is only subject to one failure mode-natural degradation. The failure occurs when the degradation level exceeds the fixed failure threshold.
2. The failure is self-announcing, and the inspection is perfect with negligible time.
3. Imperfect maintenance will be carried out upon the product. The maintenance action exerts influence on both degradation level and degradation rate. For convenience, we will use "maintenance" and "repair" interchangeably for the rest of this study.
4. The maintenance duration is nonnegligible, we assume that the duration is a random variable and included in the warranty period.

2.2. The degradation model

In this study, we employ the Wiener process to model the inherent degradation of products. The degradation increment in an infinitesimal time interval can be considered as accumulations of a number of external random shocks which follows normal distribution. The degradation level at time epoch t is characterized by an observable random process $X(t)$ with drift parameter μ and diffusion parameter σ, which is given by

$$X(t) = \mu t + \sigma B(t), \tag{1}$$

where $B(t)$ is the standard Brownian motion. For any $0 < s < t$, the increment $X(t)-X(s)$ follows a normal distribution, that is, $X(t)-X(s) \sim N(\mu(t-s),\ \sigma^2(t-s))$. When the product is repaired, the degradation level and degradation rate will change with the repair times (Pei et al., 2018). As an example, welding of metal products can not only reduce the crack length, but also destroy the physical mechanism from the inside, which accelerates the degradation rate of metal product. The engineering practices have shown that the effects of maintenance activities will intensify with the increase of repair times. Suppose that the ith operating duration after the $(i-1)$th maintenance is T_i ($i = 1, 2, 3, \ldots$), the deterioration process between the $(i-1)$th maintenance and the ith maintenance is described by a Wiener process $X_i(t)$. The drift parameter μ is associated with the repair times that is denoted by μ_i, which represents the influence of maintenance action on the degradation rate. The deterioration state after maintenance is expressed as ηl_d. Given the above, the degradation process for the products after the $(i-1)$th maintenance is defined as:

$$X_i(t) = \eta l_d + \mu_i t + \sigma B(t),\ 0 \le t \le T_i, \tag{2}$$

where η is residual coefficient after the $(i-1)$th maintenance that describes the influence on the degradation level. Various maintenance levels can be modeled by changing the value of η. Specifically, when $\eta = 0$, the maintenance is perfect, the degradation level is restored to zero; when $0 < \eta < 1$, the maintenance brings the degradation level to a state below l_d.

The product will be repaired when the degradation level reaches the failure threshold during the warranty period. The elapse T_i between the $(i-1)$th and the ith maintenance epochs follows the inverse Gaussian distribution with mean $(1-\eta)l_d/\mu_i$ and shape $(1-\eta)^2 l_d^2/\sigma^2$, i.e., $T_i \sim IG(((1-\eta)l_d)/\mu_i, ((1-\eta)^2 l_d^2)/\sigma^2)$. The probability density function (PDF) and cumulative distribution function (CDF) are given as follows:

$$f_{(T_i)}(t) = \left(\frac{(1-\eta)^2 l_d^2}{2\pi\sigma^2 t^3}\right)^{1/2} \exp\left[-\frac{(\mu_i t - (1-\eta)l_d)^2}{2\sigma^2 t}\right]. \tag{3}$$

$$F_{T_i}(t) = \Phi\left(\frac{\mu_i t - (1-\eta)l_d}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\mu_i(1-\eta)l_d}{\sigma^2}\right)\Phi\left(-\frac{\mu_i t + (1-\eta)l_d}{\sigma\sqrt{t}}\right). \tag{4}$$

where $\Phi(\cdot)$ is the standard normal distribution function.

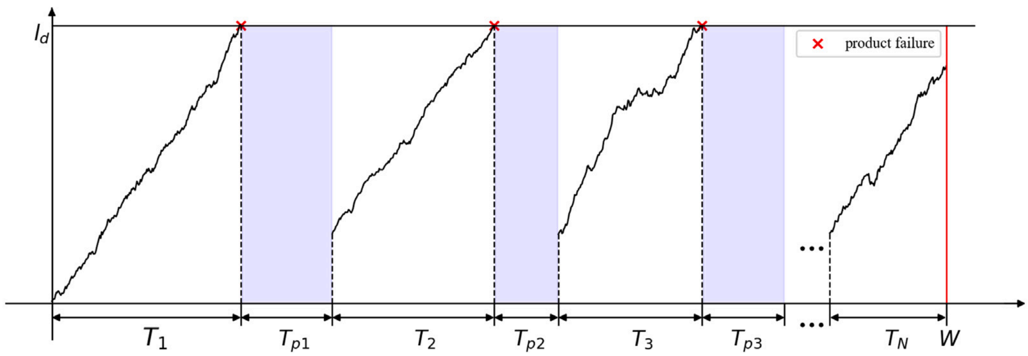

Fig. 1. Illustration of the degradation and maintenance process.

Remark. The random variables T_i, $i = 1, 2, \ldots$ follow the independent IG distributions. For the evaluation of total uptime within a given warranty period, the distribution of $T_k = \sum_{i=1}^{k} T_i$ need to be determined. If the drift parameter μ_i is irrelevant to the repair times, the T_k follows IG distribution, that is, $T_k \sim IG(k((1-\eta)l_d)/\mu, k((1-\eta)^2 l_d^2)/\sigma^2)$. In this study, we assume that the degradation rate is affected by the repair times, the random variable T_k is then modeled by randomly generated value via Monte Carlo (MC) method.

2.3. Warranty cost analysis

The expected warranty cost contains setup cost, repair cost, maintenance overdue penalty and refund for minimal availability level. For a product, a fixed setup cost C_s is incurred once the product is sold. The repair cost C_P is incurred when the maintenance action is implemented, and the cost depends on the maintenance duration rather than being a constant. Overdue penalty C_{P0} is defined as the each compensation paid by manufacturers when the repair duration exceeds the specified value τ_0 after each failure. Refund cost C_τ is the compensation that the availability exceeds pre-specified level A during the warranty period.

We assume that each repair duration is not constant, and the variability is caused by many factors, such as operating environment, maintenance mode, supply of spare parts and logistics. We adopt exponential distribution to characterize the randomness of repair duration T_p (Li and Tomlin, 2022), of which the PDF is $g(t) = \lambda e^{-\lambda t},\ t \geq 0$, where $\lambda > 0$ is the rate parameter. Given the above, we assume that the maintenance cost C_P is a linear function of the repair duration according to Liu et al. (2021), thus the expression of expected maintenance cost is given by

$$EC_P = a\int_0^{\infty} t g(t)dt + C_p, \tag{5}$$

where $a > 0$ represents the cost coefficient associated with the repair duration. C_p is the fixed repair cost. When the repair duration T_{pi} exceeds the specified threshold τ_0, the manufacturers have to pay for the overdue maintenance besides the repair cost. To evaluate the expected penalty cost, similar to Eq. (5), the longer the product is repaired over the warranty period, the more the manufacturer compensates. After all, the overdue maintenance will disarrange the production and living of the product users, the penalty cost increases with the maintenance duration beyond the pre-specified level. Then the expected penalty cost for the ith maintenance is given by

$$EC_{P0} = b\int_{\tau_0}^{\infty} (t-\tau_0)g(t)dt + C_0,\ t > \tau_0, \tag{6}$$

where C_0 is the fixed penalty cost.

Remark. The repair duration T_{pi} has a gamma distribution for $i = 1, 2, \cdots, N$ (i.e., all distributions are independent and have the same scale parameter $1/\lambda$), then the total repair duration follows $\sum_{i=1}^{N} T_{pi}(t) \sim Gamma(N, 1/\lambda)$. The cumulative distribution function is given by $F_{Ga}(t) = \frac{\gamma(N,\ t/\lambda)}{\Gamma(N)}$, where $\gamma(N,\ t/\lambda)$ is the lower incomplete gamma function.

The manufacturers will give a refund C_τ for total overdue maintenance. In other words, the long-term operational availability has to meet the availability requirement for the warranty service contract. Failing to meet the target will incur a refund from the service providers. The long-term operational availability A is defined by the total uptime over the warranty period:

$$A = \sum_{i=1}^{N_1} \frac{T_i}{W} = \sum_{i=1}^{N_1} \frac{W - T_{pi}}{W}. \tag{7}$$

Since the warranty period W is constant, the operational availability is affected by the total maintenance duration $\sum_{i=1}^{N_1} T_{pi}$. Therefore, the predetermined availability target can be translated into the constraint on the total maintenance duration, i.e., $\sum_{i=1}^{N_1} T_{pi} = W(1-A)$. Throughout this study, the threshold of total maintenance duration is assumed to be τ_L, which replaces predetermined availability target for convenience of explanation. When N_1 is the repair times until the end of warranty period. The expression of N_1 is formulated as

$$N_1 = \max\left\{N:\ \sum_{i=1}^{N}(T_i + T_{pi}) \leq W\right\}, \tag{8}$$

where N_1 may be larger or smaller than the repair times when the total maintenance duration exceeds τ_L. When the repair times exceeds the number of failures over the warranty period, the refund cost will not occur. Therefore, the total warranty cost can be divided into two parts: the expected cost under the case that the total repair duration exceeds τ_L and the case otherwise. With the Eqs. (5)–(8), the total expected warranty cost is derived as follows:

$$\begin{aligned} EC_w =& C_s + \sum_{l=1}^{N_1} \Pr(N = l)\left[l(EC_P + EC_{P0}(1 - P(T_{pi} < \tau_0))) + C_\tau\right] \\ &+ \Pr(N > N_1)(EC_P + EC_{P0}(1 - P(T_{pi} < \tau_0)))N_1, \end{aligned} \tag{9}$$

where $P(T_{pi} < \tau_0)$ is the probability that each repair duration does not exceed τ_0. $\Pr(N = l)$ is the probability that total repair duration for the first l times exceeds τ_L, i.e., $\Pr\left(\sum_{i=1}^{N} T_{pi} \geq \tau_L\right) = 1 - F_{Ga}(\tau_L;\ N,\ 1/\lambda)$. The detailed probabilities of $\Pr(N = l)$ and $\Pr(N > N_1)$ are given in Appendix A. The expected warranty cost is derived by Eq. (9), the critical probabilities can be evaluated with different values N_1. In addition, a Monte Carlo (MC) method is adopted to solve this problem and obtain the expected warranty cost. The evaluation process is presented from Fig. 2. After the initialization, the total cost for different warranty contracts is calculated, where the maintenance duration thresholds of different options are pre-specified. The detailed calculation steps are given in Appendix C.

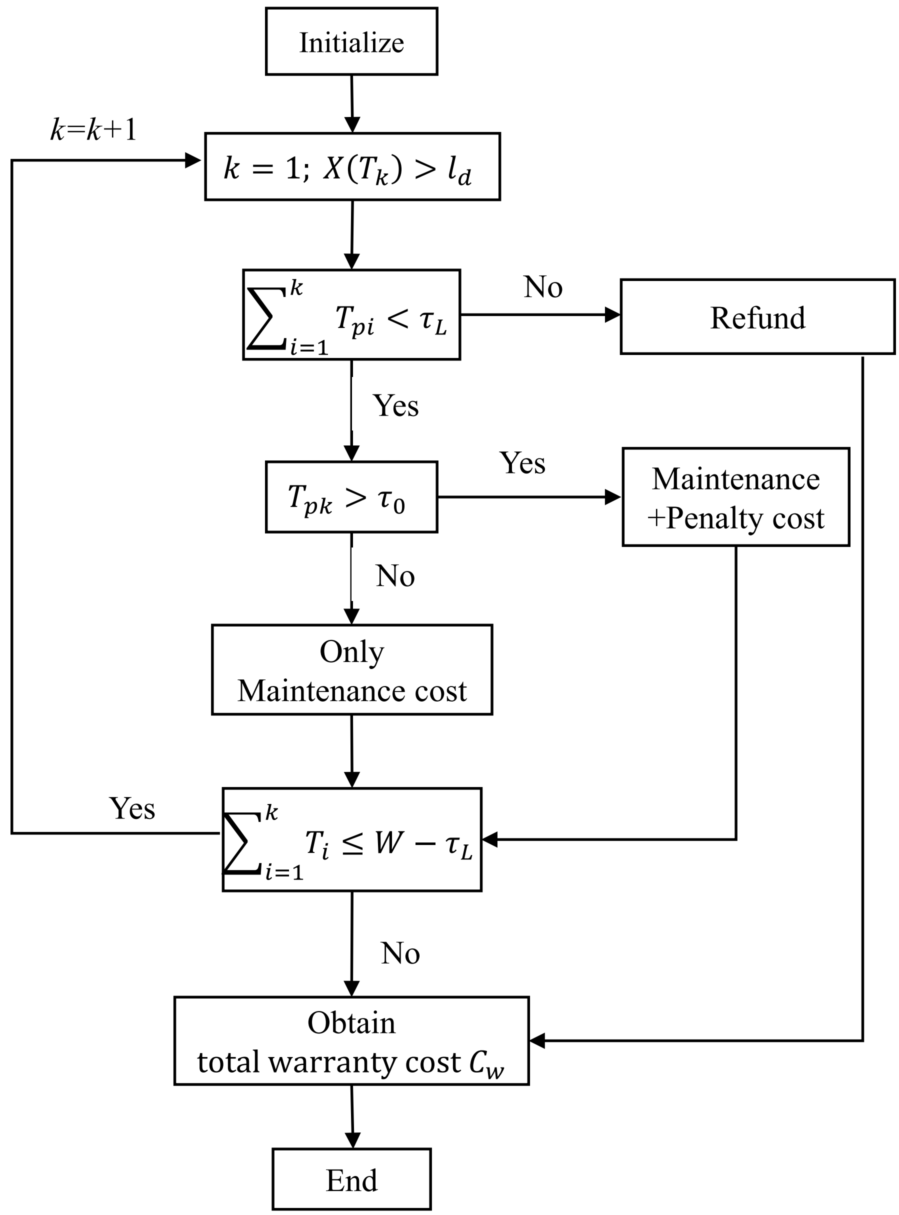

Fig. 2. Procedure of calculating the warranty cost.

3. Service contract optimization

In this section, we firstly combine the customer utility theory with the prospect theory to describe the customer choice behavior. Then an optimization problem is developed to determine the optimal contract choice by maximizing the total expected warranty profit.

3.1. Customer choice model

In reality, a customer who decides to make a purchase is faced with multiple warranty service contracts, and each option j with different price and maintenance duration thresholds, which is represented by $\{\boldsymbol{P},\ \boldsymbol{\tau_0},\ \boldsymbol{\tau_L}\} = \{P_j,\ \tau_{0j},\ \tau_{Lj}\}_{j\in\mathcal{N}}$, where $\mathcal{N} = \{1,\ 2,\ \cdots,\ n\}$. Each and total maintenance duration thresholds are usually pre-specified integers such as (24 h, 20d), (48 h, 20d), and (72 h, 30d). Thus, the warranty service contract is flexible that the customer can make the suitable choice according to their preferences and production level. To model the customer choice behavior, we adopt the widely used MNL model under a random utility maximization (RUM) framework.

Consistent with the prospect theory (Jindal, 2014; Kahneman and Tversky, 1979), we define the value function for different maintenance duration thresholds with two reference levels $r(\boldsymbol{\tau_0})$ and $r(\boldsymbol{\tau_L})$, which is given by

$$v(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L}; \mathcal{N}) = \begin{cases} v_A - \lambda_0(\tau_{0j} - r(\boldsymbol{\tau_0}))^+ - \lambda_\tau(\tau_{Lj} - r(\boldsymbol{\tau_L}))^+, & \tau_{0j} \ge r(\boldsymbol{\tau_0}),\ \tau_{Lj} \ge r(\boldsymbol{\tau_L}), \\ v_A - \lambda_0(\tau_{0j} - r(\boldsymbol{\tau_0}))^+ + \gamma_\tau(r(\boldsymbol{\tau_L}) - \tau_{Lj})^+, & \tau_{0j} \ge r(\boldsymbol{\tau_0}),\ \tau_{Lj} \le r(\boldsymbol{\tau_L}), \\ v_A + \gamma_0(r(\boldsymbol{\tau_0}) - \tau_{0j})^+ - \lambda_\tau(\tau_{Lj} - r(\boldsymbol{\tau_L}))^+, & \tau_{0j} \le r(\boldsymbol{\tau_0}),\ \tau_{Lj} \ge r(\boldsymbol{\tau_L}), \\ v_A + \gamma_0(r(\boldsymbol{\tau_0}) - \tau_{0j})^+ + \gamma_\tau(r(\boldsymbol{\tau_L}) - \tau_{Lj})^+, & \tau_{0j} \le r(\boldsymbol{\tau_0}),\ \tau_{Lj} \le r(\boldsymbol{\tau_L}), \end{cases} \tag{10}$$

where $v(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L};\ \mathcal{N})$ is the value function (Kahneman and Tversky, 1979) that this argument is a gain or loss measured with respect to a reference point. v_A is the deterministic value from consumption of warranty service contracts, which is irrelevant to all alternatives. λ and γ are risk coefficients, which capture the customers' perceived gain or loss due to the effect of reference level. $\gamma < \lambda$ means the customers are more sensitive to loss than gain (i.e., loss-averse customers).

Remark. The utility of different warranty service contracts may depend on the maintenance duration thresholds via two different reference values, and the effect of reference level on value function is represented by the difference between current and reference level. Therefore, it is important to set an appropriate reference level. Similar to the reference price formation, there are multiple ways to formulate a reference level, such as the lowest level, the highest level and weight average level. In practice, the customers choose a warranty contract based on the information about the offer sets (e.g., different maintenance duration levels), they are willing to compare its level with the lowest level, which means that the $r_l(\boldsymbol{\tau_0}) = \min_{j\in\mathcal{N}} \tau_{0j}$ and $r_l(\boldsymbol{\tau_L}) = \min_{j\in\mathcal{N}} \tau_{Lj}$. Thus, the value function used in this study is the first one of Eq. (10).

The customer purchasing utility of warranty service option j is defined as

$$u_j = v(\tau_{0j},\ \tau_{Lj};\ \boldsymbol{r_l}) - P_j + \epsilon_j,\quad j \in \mathcal{N}, \tag{11}$$

where $v(\tau_{0j},\ \tau_{Lj};\ r)$ is the customer-perceived valuation of option j. ϵ_j measures the random part of the utility function, which is unobservable to the manufacturers. We further assume that ϵ_j $(j \in \mathcal{N})$ are independent and identically distributed (i.i.d) Gumbel random variable, as is typically used in the literature (Wang et al., 2021a). The utility of no-purchase (i.e., outside option) is normalized to $u_0 := \epsilon_0$. There is another assumption that the $u_j > 0$, which ensures the customer purchases this warranty service.

Rational consumers always pursue utility maximization, and choose the warranty contract with highest realized utility. Therefore, we adopt MNL model to transform the customers' utility into a choice probability. A customer adopt the option $j \in \mathcal{N}$ if and only if $u_j \geq u_k$ where $\forall k \in \mathcal{N},\ j \neq k$, and the choice probability for option j is given by

$$\begin{aligned} &q_j(\tau_{0j},\ \tau_{Lj};\ \boldsymbol{r_l},\ \mathcal{N}) \\ &= \Pr(u_j \geq u_k, j \neq k,\ \forall k \in \mathcal{N}) = \frac{\exp(u_j)}{1 + \sum_{k\in\mathcal{N}} \exp(u_k)},\ \text{for any } j \in \mathcal{N}, \end{aligned} \tag{12}$$

where the probability for no-purchase option is equal to $q_0 = 1 - \sum_{j\in\mathcal{N}} q_j$. A monotonic property for the choice model with respect to (w.r.t) price and maintenance duration thresholds can also be derived.

Proposition 1. *For the reference levels $(r_l(\boldsymbol{\tau_0}),\ r_l(\boldsymbol{\tau_L}))$, the choice probability for option j is decreasing with P_j given the maintenance duration thresholds $(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L})$. Given the prices and maintenance duration thresholds $\boldsymbol{\tau_L}$, the choice probability q_j is decreasing in τ_{0j} but increasing in τ_{0k} for any other option $k \in \mathcal{N},\ k \neq j$. This situation is similar to τ_L.*

The proof is shown in Appendix B.1. In the MNL model with the lowest reference levels of all options, given the maintenance duration thresholds, the increase of price may result in lower choice probability. Given the availability and price, the probability of choosing contract j decreases if the maintenance threshold τ_{0j} increases, but that for other contracts $k \in \mathcal{N},\ k \neq j$ increases, so that other contracts are more attractive compared with the contract j. Similarly, this is also the case with availability given the prices and maintenance duration threshold. In practice, if the reference levels have an effect on the utility evaluation, the manufacturers will tend to lower the warranty price with the higher reference levels to appeal more customers.

3.2. Warranty profit optimization

As we have addressed before, the optimal contract choice can be determined by maximizing the total warranty profit. We suppose that the market size is normalized to one, then the expected profit under the MNL model with reference levels is expressed as follows:

$$R(\mathcal{N},\ \boldsymbol{P};\ \boldsymbol{r_l}) = \sum_{j\in\mathcal{N}} (P_j - EC_{wj}) q_j(\tau_{0j},\ \tau_{Lj};\ \boldsymbol{r_l},\ \mathcal{N}), \tag{13}$$

where EC_{wj} is the expected warranty cost of option j in Eq. (9), $P_j - EC_{wj}$ is the profit margin of option j.

In what follows, the optimal warranty service contracts (i.e., offer set $\mathcal{S} \subseteq \mathcal{N}$) with offered prices are derived by maximizing the expected profit. This optimization problem is formulated as

$$\begin{aligned} &\max_{\mathcal{S}\subseteq\mathcal{N},\ \boldsymbol{P}} \quad R(\mathcal{S},\ \boldsymbol{P};\ \boldsymbol{r_l}), \\ &\text{s.t.} \quad P_j > 0, \quad 0 < \tau_{0j} < \tau_{Lj} < W,\ j \in \mathcal{N}, \end{aligned} \tag{14}$$

where the expected profit for the offer set $\mathcal{S}$ is defined in Eq. (13). The constraint ensures that the maintenance duration will not exceed the warranty period. Following Wang (2018), this optimization problem is characterized by the following theorems. Therefore, we have a theoretical result on the optimal price with different reference levels in the offer set.

Theorem 1. *The optimal price of option j is $P_j^* = 1 + EC_{wj} + \pi^*$, where π^* is the optimal solution to problem $\pi^* = \sum_{j\in\mathcal{N}} \exp(v_A - \lambda_0(\tau_{0j} - \tau_{0n}) - \lambda_\tau(\tau_{Lj} - \tau_{Ln}) - 1 - EC_{wj} - \pi^*)$.*

The detailed proof is relegated to Appendix B.2. Moreover, the optimal price P_j^* of each option is no higher than that of option n with the lowest reference levels. Besides, for any option j, its profit margin $P_j - EC_{wj}$ is equal to $1 + \pi^*$ ao that it is same for the optimal profit. As shown in Fig. 3, the optimal π^* is increasing with $v(\tau_{0j},\ \tau_{Lj}) - EC_{wj}$, which means that the manufacturers may obtain more profits when the difference between customer perceived value function and cost is larger. Moreover, the optimal offer set to the problem is presented in the following theorem.

Theorem 2. *The optimal offer set is $\mathcal{S}^* = \mathcal{S}_{n-1} = \{1,\ 2,\ \cdots,\ n-1\} \cup \{n\}$, for any $2 \leq n \leq N$.*

The detailed proof of Theorem 2 can be found in Appendix B.3. Note that the optimal offer set includes $n-1$ options with the highest profit margins plus one additional option n which has the lowest reference levels. Since it is optimal to offer all candidate options (that is to say, offer set $\mathcal{S}^* = \mathcal{N}$) corresponding to $n = N$. In reality, the manufacturer may offer limited options ($m \leq n$) of the warranty contract to customers, then the $m-1$ options ($v(\tau_{01},\ \tau_{L1}) - EC_{w1} \geq v(\tau_{02},\ \tau_{L2}) - EC_{w2} \geq \cdots \geq v(\tau_{0(m-1)},\ \tau_{L(m-1)}) - EC_{w(m-1)}$) plus one additional option n will be chosen. If not, the new offer set is more profitable, which further confirms the previous result of Theorem 1.

4. Numerical example

In this section, we consider a numerical example to illustrate the proposed warranty service contract with different combinations of maintenance duration. The sensitivity analysis and managerial insights are also discussed.

4.1. The optimal warranty service contracts

Firstly, we present an example that the warranty service contract of a certain product is designed to provide to the customers. Consider that a firm sells certain type of engine and provides different maintenance service contracts to the customers. The firm intends to provides diversified services contracts and associated warranty prices to maximize the profit. For this purpose, we assume that the performance degradation is characterized by Wiener process, and the maintenance duration follows an exponential distribution. The assumed parameter values are listed in the Table 1 (see Li et al., 2022). Additionally, the degradation rate is associated with repair times i, which is denoted by $\mu(i) = (1+i)\mu$ (see Pei et al., 2018). Other related contract parameters and costs are also presented in the Table 1, where the units for time and money are days and CNY, respectively.

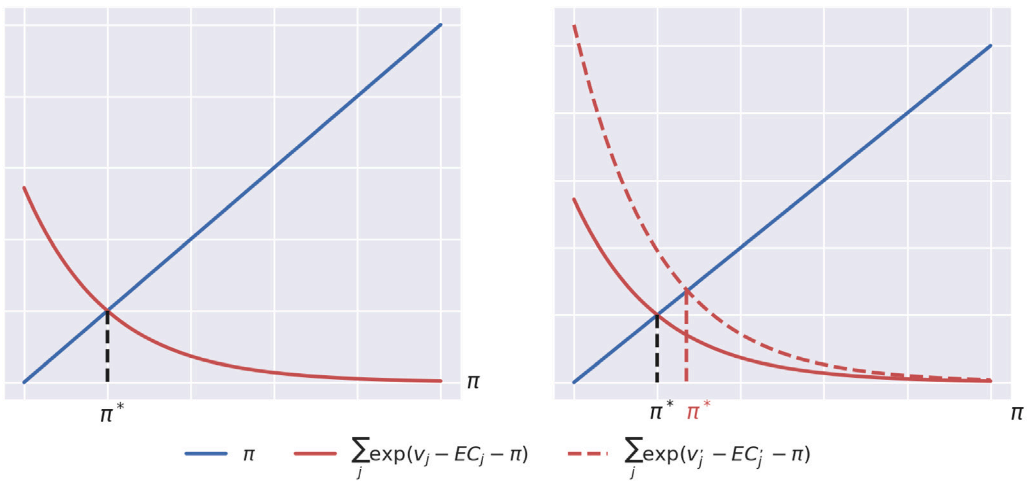

Fig. 3. Illustration of the optimal profit described in Theorem 1.

Table 1
The related parameters of the warranty service contract.

Degradation parameters				Cost parameters (CNY)				Contract parameters	
μ	0.5	σ	1	C_s	200	C_τ	800	v_a	1600
l_d	120	η	0.1	C_p	100	a	30	λ_0	60
λ	0.5	W	360	C_0	50	b	20	λ_τ	5

With the above example, we derive the warranty service contracts by using the proposed method and MC simulation. The corresponding warranty costs are listed in the Table 2, the results of MC Simulation (N=500, 1000, 10,000) and Proposed Method (N=500, 1000, 10,000) are calculated when the number of simulation is equal to 500 and 10,000. From the table, there is no significant difference in computation time between these two methods. Specifically, the computation time of proposed method is relatively higher than that of MC simulation with the same simulations. the MC simulation can derive relatively accurate results when the number of simulations is larger ($N \geq 10{,}000$), meanwhile the standard deviation gradually decreases. Compared with MC simulation, we can conclude that the proposed method can yield more consistent results with a smaller number of simulations. Besides, the proposed method (N=1000) can derive the similar value with less volatility compared with the MC method. In addition, the Proposed Method (N=1000, 10,000) could show a better performance near MC Simulation (N=10,000) when τ_L=20 and τ_L=30 compared with τ_L=15, and the main reason is that the probability that total maintenance duration exceeds τ_L is relatively higher when τ_L=15, hence the final cost would be a little larger than the result obtained by MC Simulation. The optimal warranty contract with different maintenance duration is presented in the Table 3, which ($\boldsymbol{P}$, $\boldsymbol{\tau_0}$, $\boldsymbol{\tau_L}$)={(1664.81, 3, 15), (1492.40, 3, 20), (1431.47, 3, 30), (1627.30, 5, 15), (1449.34, 5, 20), (1391.05, 5, 30), (1619.86, 7, 15), (1445.73, 7, 20), (1386.58, 7, 30)}, the optimal profit is $\pi^* = 287.67$. Furthermore, the expected cost EC_w, price P, perceived valuation $v(\boldsymbol{\tau_0}, \boldsymbol{\tau_L})$ and valuation margin $v(\boldsymbol{\tau_0}, \boldsymbol{\tau_L}) - EC_w$ for various warranty options are demonstrated in Figs. 4 and 5.

From Fig. 4, we observe that the expected cost and price decrease in $\boldsymbol{\tau_0}$ and $\boldsymbol{\tau_L}$. This is because manufacturers need to pay more so as to provide customers with longer maintenance duration and higher availability. The customer perceived valuation is also decreasing in $\boldsymbol{\tau_0}$ and $\boldsymbol{\tau_L}$, which is in accordance with our assumptions. The valuation margin increases firstly, and then decreases in $\boldsymbol{\tau_0}$ and $\boldsymbol{\tau_L}$, as shown in Fig. 5. This indicates that the manufacturers cannot gain more profits from the warranty contract with the lowest $\boldsymbol{\tau_0}$ and $\boldsymbol{\tau_L}$.

4.2. Sensitivity analysis

In this part, we investigate the sensitivity of model parameters on the warranty service contract. Firstly, the effects of different parameter settings on the expected warranty costs are studied, which include the drift parameter of degradation model μ, refund cost C_τ and the key parameter of maintenance duration model λ. In addition, the reference effects on the service contract are also investigated.

The effect of drift parameter μ. The degradation rate is determined by the drift parameter, which describes the inherent properties of the product. Table 4 shows the expected cost variation with different degradation rates. When the degradation rate is small (i.e., $\mu = 0.3$), the expected warranty cost is lowest compared with other cases, while the probability of refund is almost equal to 0. Obviously, the expected cost decreases in $\boldsymbol{\tau_0}$ and $\boldsymbol{\tau_L}$. Likewise, this trend is also applied to the situation with $\mu = 0.5$. When the degradation rate is high (i.e., $\mu = 0.7$), the expected warranty cost increases at first and then decreases with the rising of $\boldsymbol{\tau_L}$, this is because the number of repairs is increasing until a refund C_τ is happened when $\boldsymbol{\tau_L}$ turns from 15 to 20. Therefore, when the product is of good quality (low degradation rate), it can reduce the warranty cost by increasing the pre-specified levels of maintenance duration. Conversely, when the product is of poor quality (high degradation rate), increasing the maintenance duration will result in high warranty cost.

The effect of refund cost C_τ. The refund cost is regarded as a credible signal that the manufacturers convey product quality to customers. Table 4 shows that the expected cost increases in C_τ, while the increase of warranty cost is relatively smaller than the increase of refund. When $\tau_L = 30$, there is little difference in warranty costs with different refund cost C_τ. Therefore, it is a good way for manufacturers to formulate the warranty service contracts with a higher refund amount. On the one hand, it has little influence on the total warranty cost, on the other hand, this signal can convey the information of high quality of products to consumers.

The effect of parameter λ. The λ is a critical parameter in the maintenance duration model, which is determined by the manufacturers. Table 4 illustrates the variation for warranty cost when the parameter λ ranges from 1/3 to 1. In general, the length of repair duration will greatly affect the warranty cost. The maintenance and penalty costs are less likely to occur when each repair duration is shorter, and the probability of a refund is greatly reduced. In contrast, the warranty costs become higher as the repair duration takes longer. For manufacturers, shortening repair duration can greatly reduce warranty costs and increase profits. However, the manufacturers need to put in more efforts to achieve this goal, such as the shorter waiting duration, optimizing spare parts inventory, maintenance management and so on.

Table 2
Simulated warranty cost using different methods.

		N=500		N=1000		N=10,000	
		EC (S.D.)	Time	EC (S.D.)	Time	EC (S.D.)	Time
MC Simulation							
$\tau_0 = 3$	$\tau_L = 15$	1318.02 (359.56)	0.08 s	1155.85 (336.60)	0.12 s	1270.03 (334.95)	0.76 s
	$\tau_L = 20$	1193.97 (262.98)	0.07 s	1389.46 (274.34)	0.12 s	1224.49 (266.27)	0.80 s
	$\tau_L = 30$	1238.07 (248.79)	0.08 s	1230.16 (228.41)	0.12 s	1210.70 (225.33)	0.81 s
$\tau_0 = 5$	$\tau_L = 15$	1189.05 (304.33)	0.07 s	1298.71 (309.55)	0.11 s	1209.11 (313.96)	0.76 s
	$\tau_L = 20$	1273.18 (263.38)	0.08 s	1359.00 (228.91)	0.11 s	1153.99 (240.32)	0.76 s
	$\tau_L = 30$	1246.88 (215.57)	0.07 s	1097.38 (193.98)	0.11 s	1143.20 (193.32)	0.79 s
$\tau_0 = 7$	$\tau_L = 15$	1393.68 (312.43)	0.08 s	1155.82 (311.50)	0.12 s	1191.34 (300.06)	0.71 s
	$\tau_L = 20$	1238.17 (230.82)	0.07 s	1056.50 (205.28)	0.11 s	1131.12 (222.87)	0.77 s
	$\tau_L = 30$	1221.87 (168.71)	0.07 s	1191.54 (167.96)	0.11 s	1115.41 (165.81)	0.75 s
Proposed Method							
$\tau_0 = 3$	$\tau_L = 15$	1378.02 (162.86)	0.12 s	1376.68 (165.26)	0.21 s	1376.54 (167.40)	1.58 s
	$\tau_L = 20$	1209.81 (123.55)	0.12 s	1203.58 (120.11)	0.20 s	1203.52 (118.77)	1.58 s
	$\tau_L = 30$	1138.33 (90.91)	0.12 s	1149.00 (91.49)	0.22 s	1144.66 (92.46)	1.62 s
$\tau_0 = 5$	$\tau_L = 15$	1340.40 (168.97)	0.12 s	1336.10 (162.53)	0.21 s	1335.33 (163.04)	1.67 s
	$\tau_L = 20$	1159.51 (118.43)	0.12 s	1168.06 (115.40)	0.21 s	1163.09 (115.26)	1.59 s
	$\tau_L = 30$	1097.15 (91.96)	0.13 s	1101.83 (87.82)	0.22 s	1102.20 (88.03)	1.59 s
$\tau_0 = 7$	$\tau_L = 15$	1336.76 (159.38)	0.13 s	1335.72 (156.70)	0.23 s	1332.09 (162.82)	1.65 s
	$\tau_L = 20$	1157.33 (107.32)	0.13 s	1155.78 (116.76)	0.21 s	1157.20 (113.00)	1.64 s
	$\tau_L = 30$	1092.03 (94.17)	0.11 s	1096.64 (89.06)	0.21 s	1096.99 (88.93)	1.67 s

Table 3
The optimal warranty service contract with various combinations.

τ_0	3			5			7		
τ_L	15	20	30	15	20	30	15	20	30
P^*	1664.81	1492.40	1431.47	1627.30	1449.34	1391.05	1619.86	1445.73	1386.58
$v(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L})$	1599	1479	1359	1574	1454	1334	1524	1404	1284
$v(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L}) - EC_w$	222.86	275.28	216.20	235.37	293.33	231.62	192.81	246.94	186.09

The reference effects of parameters λ_0 and λ_τ. Despite the significant impact of the above parameters on warranty cost, it is also attracted more attention for manufacturers to investigate the reference effects on the pricing and profits of contract. This part shows that the reference effects have different influences on the contract pricing and profits. If the reference levels of the maintenance duration have a stronger effect on customer evaluation of the service contract, the manufacturers' different contract settings and pricing will greatly affect the customers' choice, which further has an effect on the profit. Table 5 and Fig. 6 show that the influence of reference effects of parameters λ_0 and λ_τ on the optimal price and profit. Although λ_0 and λ_τ both have a significant impact on the price and profit, the degrees of these two effects are slightly different. The price and profit increase in λ_0 and λ_τ, this can be explained by Theorem 2. Therefore, the different reference effects are determined by the values of parameters λ_0 and λ_τ, these specific values are often determined by consumers. In reality, the heterogeneity among consumers often have different effects on the contract choice and further affect the formulation of warranty service contracts. This problem is a key focus that need to be paid attention in our future study.

The purpose of this study is to design a new warranty contract to aid service providers to attract more customers and earn more profits. For this purpose, the study has provided the managerial insights for the service providers in the warranty contracts design.

(1) When the manufacturer has a clear awareness of the production degradation process, the lower the level of maintenance duration is specified, the higher the warranty cost is occurred. Therefore, when the product is high reliability and availability, by increasing the amount of refund, the more customers will be attracted to buy the warranty service contracts. Because there is a low probability that the product failure incurs refund during the warranty period, which has little effect on the total warranty cost. Conversely, the lower reliability of products results in more failures during the warranty period. In this case, it is not necessary to increase refund amount to attract customers. Instead, it is

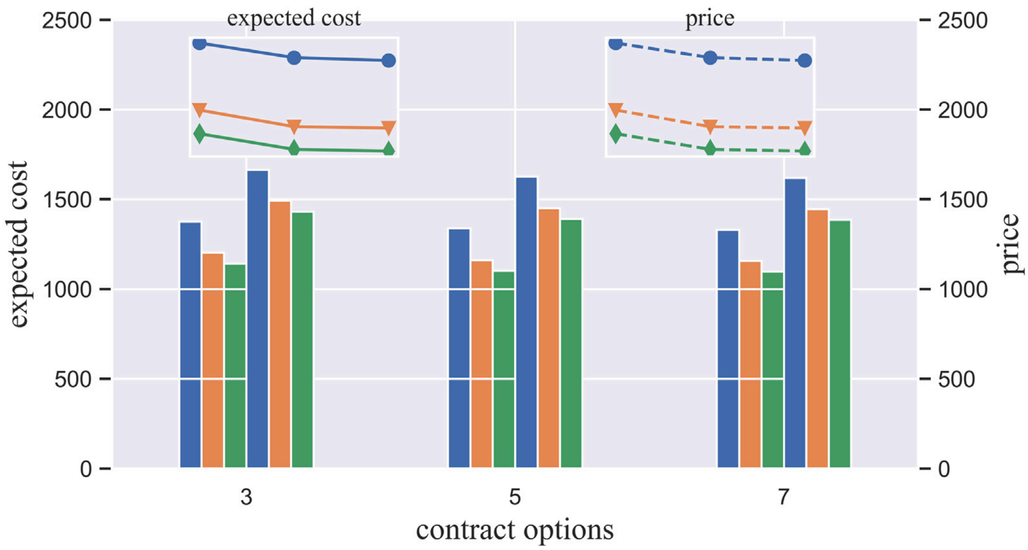

Fig. 4. The expected warranty costs and prices for different contract options.

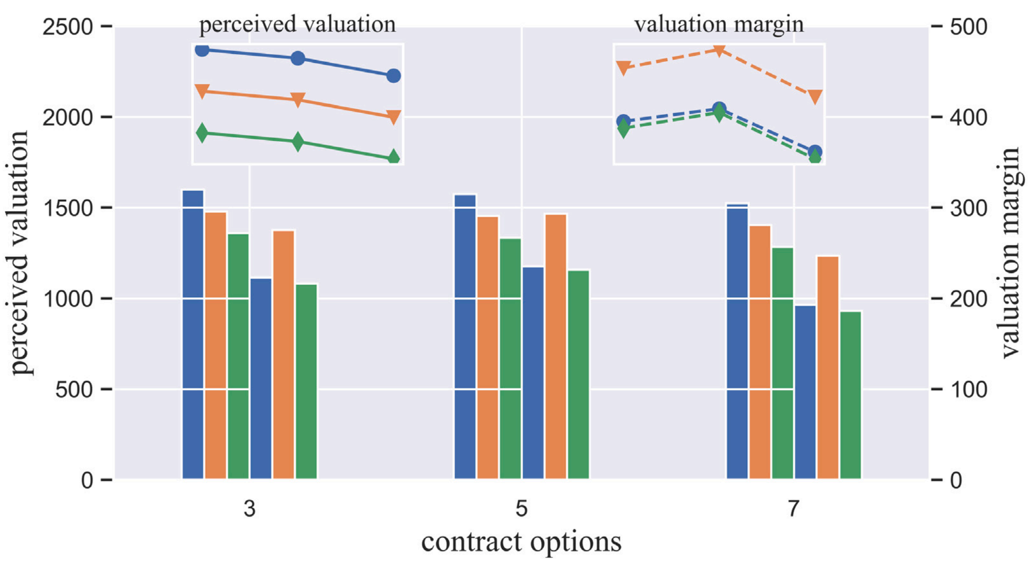

Fig. 5. The perceived valuations and valuation margins for different contract options.

Table 4
The expected cost (and standard deviation) under different parameter settings.

		C_τ		μ		λ		$C_\tau = 800$,
		300	500	0.3	0.7	1/3	1	$\mu = 0.5,\ \lambda = 1/2$
$\tau_0 = 3$	$\tau_L = 15$	1202.86 (105.58)	1272.09 (128.87)	549.07 (55.79)	2536.81 (41.53)	1840.35 (182.91)	968.63 (72.19)	1376.14 (166.38)
	$\tau_L = 20$	1162.36 (98.63)	1176.04 (106.49)	545.26 (53.39)	2735.74 (153.94)	1624.17 (196.01)	964.74 (70.51)	1203.73 (118.61)
	$\tau_L = 30$	1143.60 (91.86)	1142.89 (91.39)	543.43 (52.24)	2317.67 (212.17)	1386.70 (140.56)	964.98 (69.52)	1142.80 (92.18)
$\tau_0 = 5$	$\tau_L = 15$	1163.40 (102.88)	1232.77 (127.00)	533.18 (54.37)	2498.48 (38.79)	1766.03 (181.46)	963.60 (70.77)	1338.63 (162.69)
	$\tau_L = 20$	1116.06 (95.45)	1134.46 (101.52)	528.69 (50.09)	2671.52 (155.33)	1543.16 (191.42)	961.21 (70.59)	1160.67 (115.27)
	$\tau_L = 30$	1099.32 (86.99)	1100.97 (88.27)	528.06 (50.24)	2239.04 (208.44)	1298.28 (130.28)	960.42 (69.84)	1102.38 (88.62)
$\tau_0 = 7$	$\tau_L = 15$	1157.93 (101.89)	1228.05 (127.33)	531.81 (53.47)	2494.66 (38.37)	1743.19 (182.80)	965.21 (71.72)	1331.19 (162.64)
	$\tau_L = 20$	1113.18 (95.58)	1130.04 (102.98)	528.01 (50.24)	2666.96 (153.45)	1516.35 (192.21)	961.48 (71.08)	1157.06 (113.27)
	$\tau_L = 30$	1095.42 (87.35)	1097.38 (87.76)	527.72 (51.00)	2231.96 (207.53)	1270.745 (128.13)	960.63 (70.21)	1097.91 (88.49)

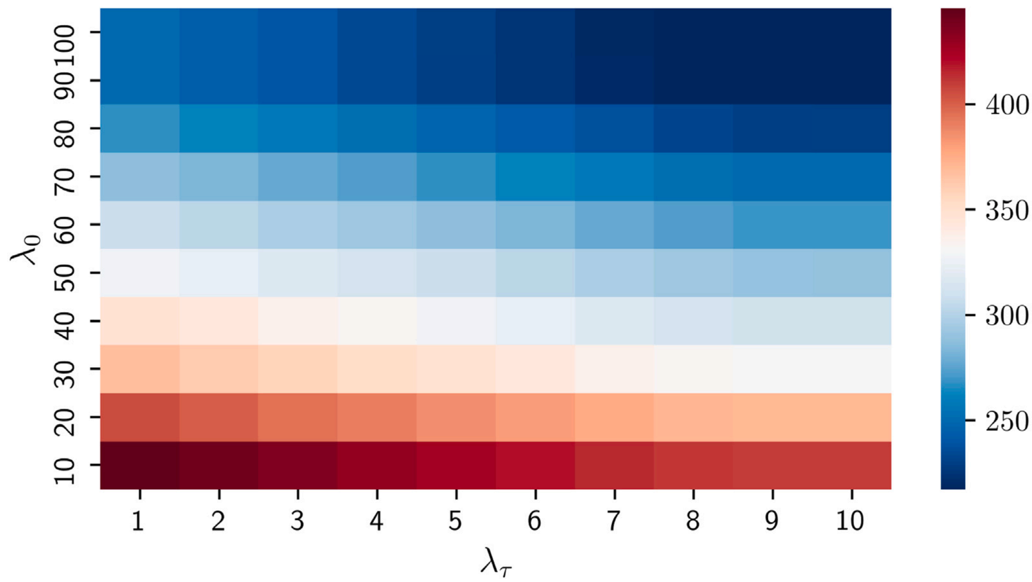

Fig. 6. The profits with different reference effects.

Table 5
The prices with different reference effects.

		$\lambda_0(\lambda_\tau = 5)$		$\lambda_\tau(\lambda_0 = 60)$		$\lambda_0 = 60$,
		20	100	1	10	$\lambda_\tau = 5$
$\tau_0 = 3$	$\tau_L = 15$	1762.80	1607.07	1684.74	1646.82	1664.81
	$\tau_L = 20$	1590.39	1434.66	1512.33	1474.40	1492.40
	$\tau_L = 30$	1529.46	1373.73	1451.40	1413.48	1431.47
$\tau_0 = 5$	$\tau_L = 15$	1725.29	1569.56	1647.24	1609.31	1627.30
	$\tau_L = 20$	1547.33	1391.60	1469.27	1431.35	1449.34
	$\tau_L = 30$	1489.05	1333.31	1410.99	1373.06	1391.05
$\tau_0 = 7$	$\tau_L = 15$	1717.85	1562.12	1639.79	1601.87	1619.86
	$\tau_L = 20$	1543.72	1387.99	1465.67	1427.74	1445.73
	$\tau_L = 30$	1484.57	1328.84	1406.52	1368.59	1386.58

better to increase the pre-specified level of maintenance duration to reduce the cost of overdue penalty, and the manufacturers need to make more efforts to improve the product reliability.

(2) Maintenance duration, as a non-negligible factor in the warranty period, has a crucial impact on the design of the warranty service contracts. Through this study, the service providers should set different combinations of maintenance duration and availability levels when designing the warranty contracts. The concrete options of pre-specified maintenance duration and availability levels are determined by the results of market research and service provider' own maintenance process, inventory management and logistics.

(3) The service providers should take the reference effect of different warranty schemes into account. when faced with multiple warranty options, customers will compare the alternatives and choose the lowest maintenance duration level as a reference. The results of this study indicate that the service providers offer the optimal warranty scheme including options with the highest profit margins plus one additional option with the lowest reference levels. In this way, the service providers could design a more reasonable warranty service contract to maximize profits.

5. Conclusions

In this study, we design the warranty service contract with different combinations of maintenance duration and availability for deteriorating products. The reference levels of maintenance duration and availability affect not only the warranty cost but also the customer choice. The optimization problem of service contract is to determine the price, reference points of each repair duration and availability by maximizing the expected profit, where the reference effect of maintenance duration is incorporated into the MNL choice model. Our numerical example shows that the optimal prices of different warranty service contract options are closely linked to the maintenance duration, while the reference levels are defined by the lowest maintenance duration. The optimal warranty service contract is to offer all candidate options. If the number of options is limited, the optimal offer set includes options with the highest profit margins plus one additional option with the lowest reference levels. Furthermore, the results of sensitivity analysis indicate that the manufacturers should put more attention on the maintenance duration, and the service providers can increase profits by adjusting maintenance duration levels in the warranty contract at the different stages of deteriorating products. The reference effects of maintenance duration and availability also have a significant impact on the warranty cost and contract price.

Several topics that consider more realistic factors deserve further exploration in the future research. In this work, we assume that the refund cost is constant and has no effect on the customer choice, further study could investigate the effect of backup products in the warranty period, the maintenance duration will not only affect the refund cost, but also reduce the utility of customer's purchasing.In addition, the perceived valuation of the warranty service contract is affected by many factors, such as the consumers' heterogeneity and risk attitude. The complex products can be described by different degradation processes with shocks. Meanwhile, the phenomenon of self-healing (or damage annealing) may take place (Liu et al., 2017; Shen et al., 2018; Zhao et al., 2018a; Dong et al., 2021), which is worthy of further research.It is also challenging to study the interactions between degradation process and maintenance duration.

Data availability

Data will be made available on request.

Acknowledgments

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Appendix A. The probability of $\Pr(i = N)$ and $\Pr(N > N_1)$

The concrete probabilities of Eq. (9) are derived as follows:

$$\begin{aligned}
&\Pr(N=1)=\Pr(T_{P1}>\tau_L)=1-F_{Ga}\left(\tau_L;\ 1,\frac{1}{\lambda}\right),\\
&\Pr(N=2)=\Pr\left(\sum_{i=1}^{2}T_{Pi}>\tau_L \text{ and } T_{P1}<\tau_L\right)\\
&=F_{Ga}\left(\tau_L;\ 1,\frac{1}{\lambda}\right)\left(1-F_{Ga}\left(\tau_L;\ 2,\frac{1}{\lambda}\right)\right),\\
&\Pr(N=3)=\Pr\left(\sum_{i=1}^{3}T_{Pi}>\tau_L \text{ and } \sum_{i=1}^{2}T_{Pi}<\tau_L\right)\\
&=F_{Ga}\left(\tau_L;\ 2,\frac{1}{\lambda}\right)\left(1-F_{Ga}\left(\tau_L;\ 3,\frac{1}{\lambda}\right)\right),\\
&\cdots\\
&\Pr(N=N_1)=\Pr\left(\sum_{i=1}^{N_1}T_{Pi}>\tau_L \text{ and } \sum_{i=1}^{N_1-1}T_{Pi}<\tau_L\right)\\
&=F_{Ga}\left(\tau_L;\ N_1-1,\frac{1}{\lambda}\right)\left(1-F_{Ga}\left(\tau_L;\ N_1,\frac{1}{\lambda}\right)\right),\\
&\Pr(N>N_1)=1-\sum_{i=1}^{N_1}\Pr(N=i),
\end{aligned} \tag{A.1}$$

Appendix B. Proofs

B.1. Proof of Proposition 1

First, for given $(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L})$ and reference levels, the choice probability $q_j(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L};\ \boldsymbol{r_l},\ \mathcal{N})$ decreases as P_j increases. Note that $\partial q_j/\partial P_j<0$, this property is proved. Then, for given $\boldsymbol{\tau_L}$ and prices, the monotonicity of the choice probability in τ_{0j} and τ_{0k} is verified with considering which option serves the reference level.

Case 1: Suppose that $\tau_{0j}>\min_{k\in\mathcal{N}}\tau_{0k}=\tau_{0k'}$, which means the option k' serves the reference level of $\boldsymbol{\tau_0}$. Obviously, the choice probability $q_j(\boldsymbol{\tau_0},\ \boldsymbol{\tau_L};\ \boldsymbol{r_l},\ \mathcal{N})$ decreases in τ_{0j} and increases in τ_{0k} for any $k\neq j,\ k'$. As for the $\tau_{0k'}\le\min_{k\in\mathcal{N}\setminus\{k'\}}\tau_{0k}$, the choice probability q_j is equal to

$$\begin{aligned}
q_j&=\frac{\exp(v_A-\lambda_0(\tau_{0j}-\tau_{0k'})-\lambda_\tau(\tau_{Lj}-r(\boldsymbol{\tau_L}))-P_j)}{1+\sum_{k\in\mathcal{N}}\exp(v_A-\lambda_0(\tau_{0k}-\tau_{0k'})-\lambda_\tau(\tau_{Lj}-r(\boldsymbol{\tau_L}))-P_k)}\\
&=\frac{\exp(v_A-\lambda_0\tau_{0j}-\lambda_\tau(\tau_{Lj}-r(\boldsymbol{\tau_L}))-P_j)}{\exp(-\lambda_0\tau_{0k'})+\sum_{k\in\mathcal{N}}\exp(v_A-\lambda_0\tau_{0k}-\lambda_\tau(\tau_{Lk}-r(\boldsymbol{\tau_L}))-P_k)}.
\end{aligned} \tag{B.1}$$

Therefore, the choice probability increases in $\tau_{0k'}$.

Case 2: Suppose that $\tau_{0j}=\min_{k\in\mathcal{N}}\tau_{0k}$. The choice probability q_j is derived as follows:

$$q_j=\frac{\exp(v_A-\lambda_\tau(\tau_{Lj}-r(\boldsymbol{\tau_L}))-P_j)}{1+\sum_{k\in\mathcal{N}}\exp(v_A-\lambda_0(\tau_{0k}-\tau_{0j})-\lambda_\tau(\tau_{Lj}-r(\boldsymbol{\tau_L}))-P_k)}. \tag{B.2}$$

Apparently, the choice probability decreases in τ_{0j} but increases in τ_{0k} for any $j\neq k$.

Similar to the above proof process, the details that choice probability decreases in τ_{Lj} and increases in τ_{Lk} are omitted. Thus, the proposition is verified.

B.2. Proof of Theorem 1

We assume that π^* is the optimal expected profit (i.e., $\max_{\mathcal{N},\ \boldsymbol{P}}R(\mathcal{N},\ \boldsymbol{P};\ \boldsymbol{r_l})=\pi^*$). Then, the equation of π^* can be expressed as follows:

$$\begin{aligned}
\pi^*&=\sum_{j\in\mathcal{N}}(P_j-EC_{wj}-\pi^*)\exp(v_A-\lambda_0(\tau_{0j}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln})-P_j)\\
&=\sum_{j\in\mathcal{N}}\exp(v_A-\lambda_0(\tau_{0j}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln})-1-EC_{wj}-\pi^*).
\end{aligned} \tag{B.3}$$

Note that the option n has the lowest reference levels for all warranty options ($\mathcal{N}$). Then, Eq. (B.3) can be separated for each option, let $\Pi_j(\pi,\ P_j)=\exp(v_A-\lambda_0(\tau_{0j}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln}))-P_j$. Consider that the first-order derivative for $\Pi_j(\pi,\ P_j)$ with respect to (w.r. (t) P_j, we have

$$\frac{\partial\Pi_j(\pi,\ P_j)}{\partial P_j}=(1-P_j+EC_{wj}+\pi^*)\exp(v_A-\lambda_0(\tau_{0j}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln})-P_j) \tag{B.4}$$

It is clear to show that $\Pi_j(\pi,\ P_j)$ is unimodal in P_j, and reaches its maximum at $P_j=1+EC_{wj}+\pi^*$. Therefore, the expected warranty cost EC_{wn} with the lowest reference levels is the highest while $1+\pi^*$ is constant for all options, which further results in the highest price of the option n. Thus the optimal price is $P_j^*\le P_n$. The proof of Theorem 1 is completed.

B.3. Proof of Theorem 2

Suppose that the offer sets $S_n=\{1,\ 2,\ \cdots,\ n\}$ for any $2\le n\le N$, we divide the options into two parts $S_{n-1}=\{1,\ 2,\ \cdots,\ n-1\}$ and the option n serving the lowest reference levels. Let $\max_{S_{n-1}\cup\{n\},\ \boldsymbol{P}}\ R(S_{n-1}\cup\{n\},\ \boldsymbol{P};\ \boldsymbol{r_l})=\pi$, this expression can be rewritten as

$$\begin{aligned}
\pi=&\Pi_n(\pi)=\sum_{j\in S_{n-1}\cup\{n\}}(P_j-EC_{wj})q_j(\tau_{0j},\ \tau_{Lj};\ \boldsymbol{r_l})\\
&=\sum_{j\in S_{n-1}\cup\{n\}}(P_j-EC_{wj})\frac{\exp(v_A-\lambda_0(\tau_{0j}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln}))-P_j}{1+\sum_{k\in S_{n-1}\cup\{n\}}\exp(v_A-\lambda_0(\tau_{0k}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln})-P_k)}\\
&=(P_n-EC_{wn}-\pi)\exp(v_A-P_n)+\\
&\sum_{k\in S_{n-1}}(P_k-EC_{wk}-\pi)\exp(v_A-\lambda_0(\tau_{0k}-\tau_{0n})-\lambda_\tau(\tau_{Lj}-\tau_{Ln})-P_k).
\end{aligned} \tag{B.5}$$

Note that the right-hand side (RHS) $\Pi_n(\pi)$ decreases in π for any given n, while the left-hand side (LHS) is a 45-degree line. Therefore, there exists a unique solution to this optimization problem.

From Eq. (B.5), we observe that the option j is included in the offer set if and only if $P_j-EC_{wj}\ge\pi$. According to Theorem 1, the formulation of $P_j-EC_{wj}=1+\pi$ holds for any $j\in\mathcal{N}$, the optimal solution to above equation is $\{1,\ 2,\ \cdots,\ n-1\}\cup\{n\}$ for $n=2,\ 3,\ \cdots,\ N$. Therefore, we have shown that the optimal offer is $S^*=S_{n-1}\cup\{n\}$.

Appendix C. The details of calculating the expected warranty cost

To derive the total warranty cost of each option, the Monte Carlo (MC) method is performed according to Eq. (9). The details of this simulation are as follows:

(1) Given the values of warranty period W, parameters of degradation model and maintenance duration model ($\mu_i,\ \sigma,\ \eta,\ l_d;\ \lambda$), and cost parameters ($C_p,\ a;\ C_0,\ b;\ C_\tau;\ C_s$), for any k, $0<k<N_1$ (N_1 is given in Eq. (8)), the corresponding warranty cost is computed by following Eq. (C.6), which is given by:

$$\begin{aligned}
C_{wk}=&\{(aT_{pi}+C_p)+[b(T_{pk}-\tau_0)+C_0]I_{\{T_{pk}\ge\tau_0\}}\}(1-I_{\{\sum_{i=1}^{k}T_{pi}\ge\tau_L\}})\\
&+C_\tau I_{\{\sum_{i=1}^{k}T_{pi}\ge\tau_L\}}.
\end{aligned} \tag{C.6}$$

(2) Start with $k=1$ within the range $(0,\ N_1)$, the total warranty cost is calculated according to the following steps:

  (a) For any k, calculate the availability (i.e., total repair duration until k times), if $\sum_{i=1}^{k}T_{pi}>\tau_L$, the warranty cost for the kth maintenance is $C_{wk}=C_\tau$, and the cycle stops, otherwise go to next;
  (b) For every repair duration T_{pk}, if $T_{pk}>\tau_0$, the warranty cost is $C_{wk}=aT_{pk}+C_p+b(T_{pk}-\tau_0)+C_0$, otherwise the warranty cost is $C_{wk}=aT_{pk}+C_p$ and go to the next;
  (c) For $k=k+1$, repeat the steps (a) and (b), the cycle is terminated until the $k\ge N_1$;

(d) Calculate the total warranty cost $C_w = \sum_{k=1}^{N_1} C_{wk} + C_s$ with the warranty period W.

(3) Generate B simulated realizations of the total warranty cost $\hat{C_w}$ that repeat the steps (a)–(d) B times (say $B = 100,000$), then the expected warranty cost is well approximated by average of $\hat{C_w}$ (i.e., $\bar{C_w} = \sum_{i=1}^{B} \hat{C_w^{(i)}}/B$).

Of course, we can obtain the expected warranty cost in Eq. (9) by calculating the probabilities (as shown in Eq. (A.1)). The results of these two methods are listed in Section 4.

References

Cheong, K.F., Zhang, C., Zhang, Y., 2021. Joint dynamic optimization of price and two-dimensional warranty policy. IEEE Trans. Eng. Manage. 68 (2), 528–547.

Chien, Y.-H., 2019. The optimal preventive-maintenance policy for a NHPBP repairable system under free-repair warranty. Reliab. Eng. Syst. Saf. 188, 444–453.

Darghouth, M., Ait-kadi, D., Chelbi, A., 2017. Joint optimization of design, warranty and price for products sold with maintenance service contracts. Reliab. Eng. Syst. Saf. 165, 197–208.

Deprez, L., Antonio, K., Boute, R., 2021. Pricing service maintenance contracts using predictive analytics. European J. Oper. Res. 290 (2), 530–545.

Diesel, Y.C., 2022. Service philosophy and service commitment. https://en.yuchaidiesel.com/service/fu-wu-li-nian.htm. (Accessed 3 December 2022).

Dong, W., Liu, S., Bae, S.J., Cao, Y., 2021. Reliability modelling for multi-component systems subject to stochastic deterioration and generalized cumulative shock damages. Reliab. Eng. Syst. Saf. 205, 107260.

Hashemi, M., Asadi, M., Tavangar, M., 2022. Optimal maintenance strategies for coherent systems: A warranty dependent approach. Reliab. Eng. Syst. Saf. 217, 108027.

Huang, H., Liu, F., Zhang, P., 2021. To outsource or not to outsource? Warranty service provision strategies considering competition, costs and reliability. Int. J. Prod. Econ. 242, 108298.

Jackson, C., Pascual, R., 2021. Joint pricing and maintenance strategies in availability-based product-service systems under different overhaul conditions. Reliab. Eng. Syst. Saf. 216, 107817.

Jindal, P., 2014. Risk Preferences and Demand Drivers of Extended Warranties. Mark. Sci. 34 (1), 39–58.

Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk. Econometrica 47 (2), 263–291.

Lei, X., Sandborn, P.A., 2018. Maintenance scheduling based on remaining useful life predictions for wind farms managed using power purchase agreements. Renew. Energy 116, 188–198, Real-time monitoring, prognosis and resilient control for wind energy systems.

Li, T., He, S., Zhao, X., 2022. Optimal warranty policy design for deteriorating products with random failure threshold. Reliab. Eng. Syst. Saf. 218, 108142.

Li, C., Tomlin, B., 2022. After-sales service contracting: Condition monitoring and data ownership. Manuf. Serv. Oper. Manag. 24 (3), 1261–1885.

Liao, H., Elsayed, E.A., Chan, L.-Y., 2006. Maintenance of continuously monitored degrading systems. European J. Oper. Res. 175 (2), 821–835.

Liu, Z., Diallo, C., Chen, J., Zhang, M., 2020. Optimal pricing and production strategies for new and remanufactured products under a non-renewing free replacement warranty. Int. J. Prod. Econ. 226, 107602.

Liu, P., Wang, G., Su, P., 2021. Optimal maintenance strategies for warranty products with limited repair time and limited repair number. Reliab. Eng. Syst. Saf. 210, 107554.

Liu, H., Yeh, R.-H., Cai, B., 2017. Reliability modeling for dependent competing failure processes of damage self-healing systems. Comput. Ind. Eng. 105, 55–62.

Pardalos, P.M., Rebennack, S., Pereira, M., Iliadis, N.A., Pappu, V., 2013. Handbook of Wind Power Systems. Springer.

Park, M., Jung, K.M., Park, D.H., 2017. Optimal maintenance strategy under renewable warranty with repair time threshold. Appl. Math. Model. 43, 498–508.

Park, M., Mun Jung, K., Park, D.H., 2013. Optimal post-warranty maintenance policy with repair time threshold for minimal repair. Reliab. Eng. Syst. Saf. 111, 147–153.

Pei, H., Si, X., Hu, C., Wang, Z., Du, D., Pang, Z., 2018. A multi-stage Wiener process-based prognostic model for equipment considering the influence of imperfect maintenance activities. J. Intell. Fuzzy Systems 34 (6), 3695–3705.

Peng, S., Jiang, W., Zhao, W., 2021. A preventive maintenance policy with usage-dependent failure rate thresholds under two-dimensional warranties. IISE Trans. 53 (11), 1231–1243.

Qiu, Q., Cui, L., Gao, H., 2017. Availability and maintenance modelling for systems subject to multiple failure modes. Comput. Ind. Eng. 108, 192–198.

Shafiee, M., Chukova, S., 2013. Maintenance models in warranty: A literature review. European J. Oper. Res. 229 (3), 561–572.

Shen, J., Cui, L., Ma, Y., 2019. Availability and optimal maintenance policy for systems degrading in dynamic environments. European J. Oper. Res. 276 (1), 133–143.

Shen, J., Cui, L., Yi, H., 2018. System performance of damage self-healing systems under random shocks by using discrete state method. Comput. Ind. Eng. 125, 124–134.

Strauss, A.K., Klein, R., Steinhardt, C., 2018. A review of choice-based revenue management: Theory and methods. European J. Oper. Res. 271 (2), 375–387.

Su, C., Cheng, L., 2018. An availability-based warranty policy considering preventive maintenance and learning effects. Proc. Inst. Mech. Eng. O J. Risk Reliab. 232 (6), 576–586.

Tversky, A., Kahneman, D., 1991. Loss aversion in riskless choice: A reference-dependent model*. Q. J. Econ. 106 (4), 1039–1061.

Wang, R., 2018. When prospect theory meets consumer choice models: Assortment and pricing management with reference prices. Manuf. Serv. Oper. Manag. 20 (3), 583–600.

Wang, R., Ke, C., Cui, S., 2021a. Product price, quality, and service decisions under consumer choice models. Manuf. Serv. Oper. Manag. msom.2020.0947.

Wang, X., Li, L., Xie, M., 2020a. An unpunctual preventive maintenance policy under two-dimensional warranty. European J. Oper. Res. 282 (1), 304–318.

Wang, Y., Liu, Y., Chen, J., Li, X., 2020b. Reliability and condition-based maintenance modeling for systems operating under performance-based contracting. Comput. Ind. Eng. 142, 106344.

Wang, X., Liu, B., Zhao, X., 2021b. A performance-based warranty for products subject to competing hard and soft failures. Int. J. Prod. Econ. 233, 107974.

Wang, X., Zhao, X., Liu, B., 2020c. Design and pricing of extended warranty menus based on the multinomial logit choice model. European J. Oper. Res. 287 (1), 237–250.

Wang, Q., Zhao, N., Wu, J., Zhu, Q., 2021c. Optimal pricing and inventory policies with reference price effect and loss-Averse customers. Omega 99, 102174.

Wee, H.M., Widyadana, G.A., 2013. A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule. Omega 41 (6), 941–954.

Zhao, X., Guo, X., Wang, X., 2018a. Reliability and maintenance policies for a two-stage shock model with self-healing mechanism. Reliab. Eng. Syst. Saf. 172, 185–194.

Zhao, X., He, S., Xie, M., 2018b. Utilizing experimental degradation data for warranty cost optimization under imperfect repair. Reliab. Eng. Syst. Saf. 177, 108–119.

Zheng, R., Zhang, Y., Gu, L., 2020. Optimal post-warranty maintenance contracts for wind turbines considering availability. Int. J. Green Energy 17 (7), 373–381.

Zheng, R., Zhou, Y., 2021. Comparison of three preventive maintenance warranty policies for products deteriorating with age and a time-varying covariate. Reliab. Eng. Syst. Saf. 213, 107676.

Zhu, X., Jiao, C., Yuan, T., 2019. Optimal decisions on product reliability, sales and promotion under nonrenewable warranties. Reliab. Eng. Syst. Saf. 192, 106268.