Long-range interactions in a quantum gas mediated by diffracted light

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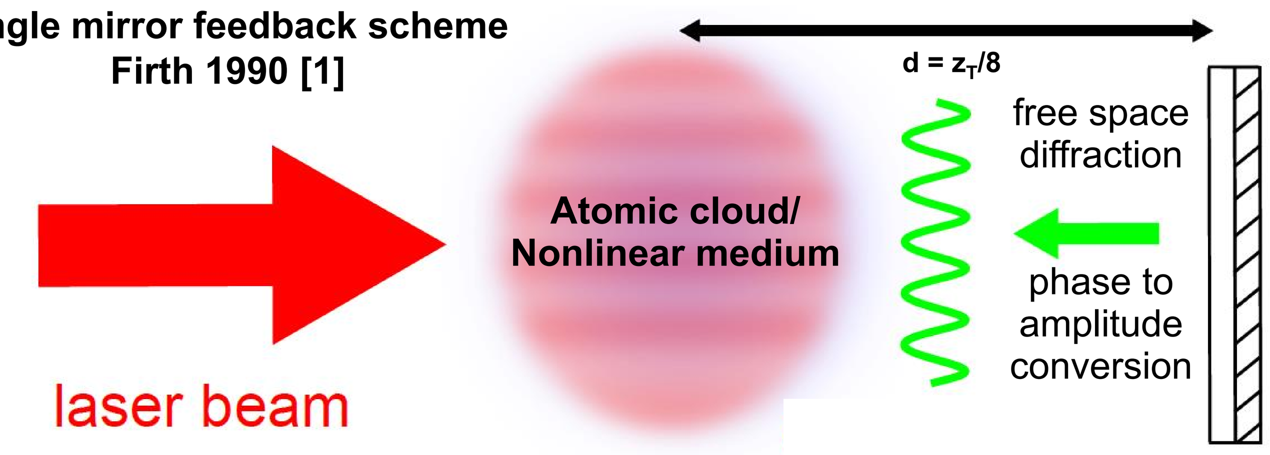
Light-mediated interactions via diffractive coupling:

Single mirror feedback scheme
Firth 1990 [1]



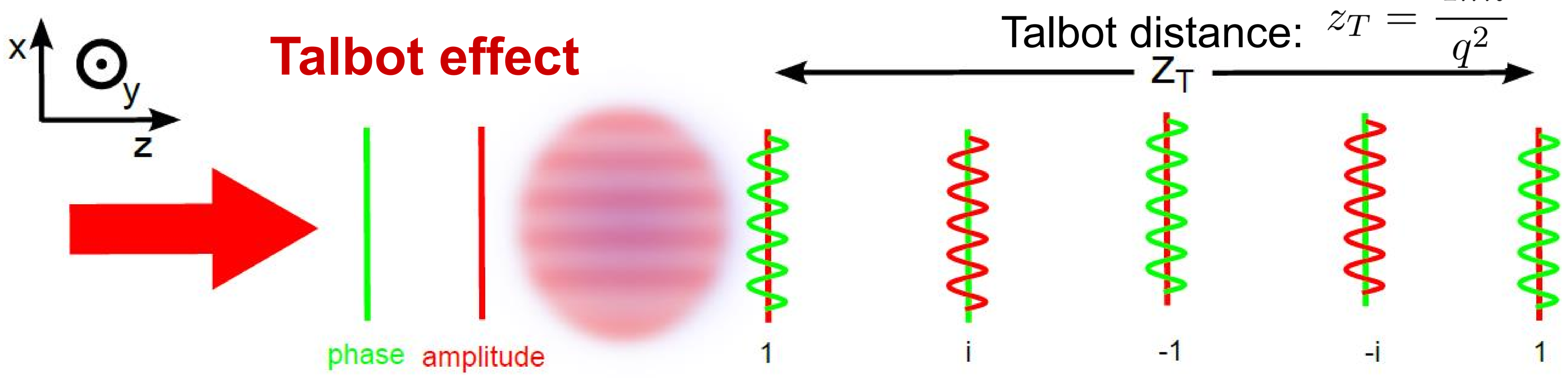
- Single distinguished axis
- No boundary conditions in transverse plane
- Spontaneous breaking of **two continuous symmetries**: translational, rotational

$$\mathbf{E}(\mathbf{r}_\perp, z+l) = \mathbf{E}(\mathbf{r}_\perp, z)e^{ikn_q l},\ n_q = n_0 + \Delta n\cos(\mathbf{q}\cdot\mathbf{r}_\perp) \rightarrow \mathbf{E}(\mathbf{r}) \approx \mathbf{E}_0(1 + i\epsilon\cos(\mathbf{q}\cdot\mathbf{r}_\perp))$$

$$\frac{\partial \mathbf{E}(\mathbf{r})}{\partial z} = -\frac{i}{2k}\Delta_\perp \mathbf{E}(\mathbf{r}) \ \rightarrow\ \mathbf{E}(\mathbf{r}) = \mathbf{E}_0(1 + i\epsilon e^{i\frac{q^2}{2k}z}\cos(\mathbf{q}\cdot\mathbf{r}_\perp))$$

Talbot effect

Talbot distance: $z_T = \frac{4\pi k}{q^2}$



Maximum amplitude modulation: $z = (n + 1/4)z_T,\ n = ..., -2, -1, 0, 1, 2, ...$

Nonlinear light-matter coupling: dipole force → spontaneous crystallization
Cold, thermal atoms [2]

Nonlinear medium: quantum degenerate gas / BEC → Supersolids [3]

Dilute BEC @ T=0
Interatomic scattering a=0

BEC dynamics
$$i\frac{\partial\psi(x,t)}{\partial t} = \left[-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} + \frac{\delta}{2}s(x,t)\right]\psi(x,t)$$
Optical dipole potential

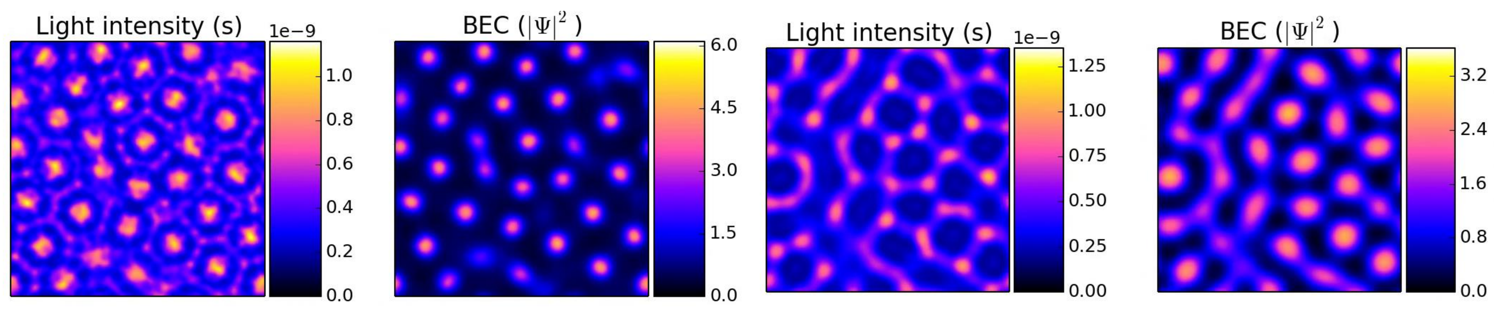
Optical field evolution in BEC (phase encoding)
$$\frac{\partial F(x,z,t)}{\partial z} = -i\frac{b_0}{2\Delta L_z}|\psi(x,t)|^2 F(x,z,t)$$
Atomic density

Optical field evolution in free space (diffractive dephasing)
$$B(q) = \sqrt{R}\,F_{tr}(q)\exp\left(i\frac{q^2 d}{k_0}\right)$$

Interacting intensity
$s(x) = |F|^2 + |B(x)|^2$
sum of forward and backward intensities

Red detuning | Blue detuning

Just after Saturation of instability: Quite regular



Hamiltonian Mean Field (HMF) Model

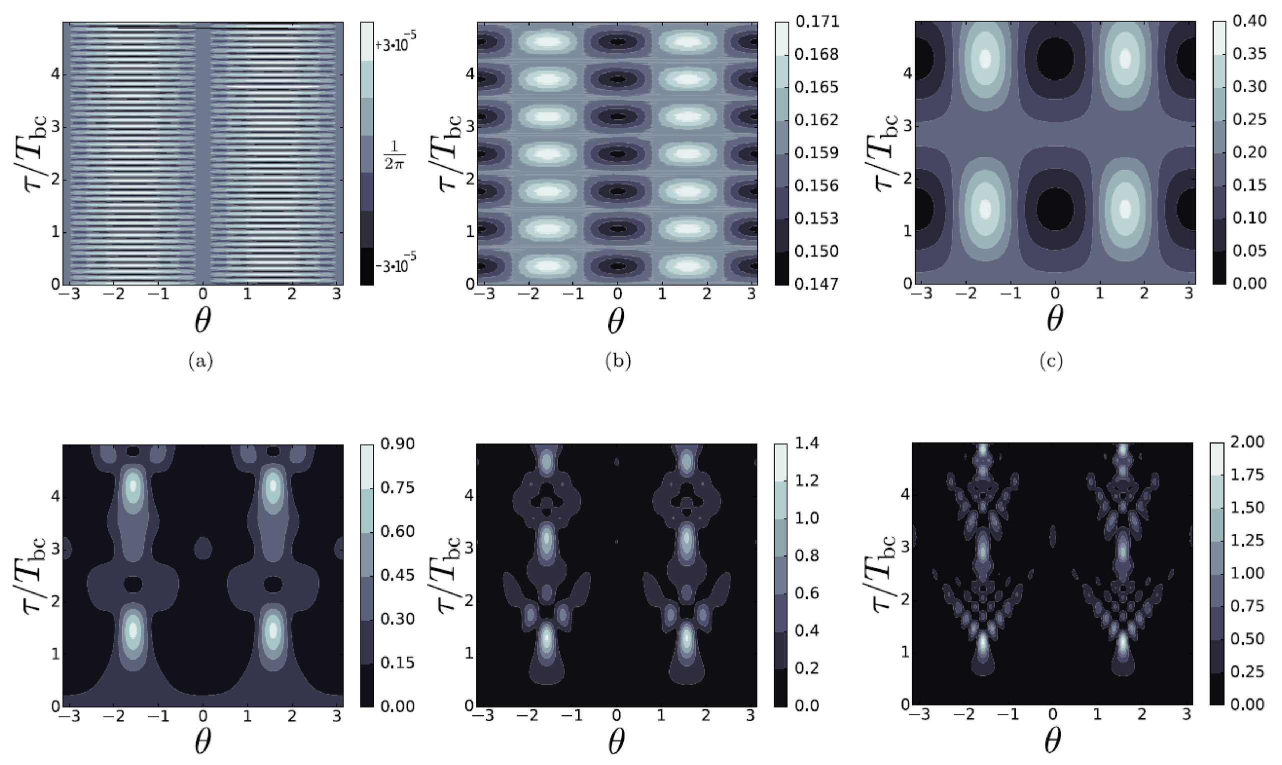
Archetypical model for long-range coupled systems

- N particles on a ring experiencing pairwise cosine interaction
- Self-gravitating systems, X-Y rotors, e.g. [4]

$$H_N = \sum_{i=1}^{N}\frac{p_i^2}{2} + \frac{J}{2N}\sum_{i,j}[1 - \cos(\theta_i - \theta_j)] \qquad H_N = \sum_{i=1}^{N}\frac{p_i^2}{2} + \frac{NJ}{2}(1 - m^2)$$

$$m_x = \frac{1}{N}\sum_{i=1}^{N}\cos\theta_i \qquad m_y = \frac{1}{N}\sum_{i=1}^{N}\sin\theta_i$$

- Extension to quantum HMF: Chavanis [5], Plestid & O'Dell [6]
- Space-time oscillations, chevrons, solitons [6]



Derivation of HMF

Large detuning → small susceptibility

$$F_{tr} = \sqrt{p_0}\exp(-i\chi_0 n(x,t)) \approx \sqrt{p_0}(1 + i\chi_0 n(x,t)); \qquad n(x,t) = |\Psi(x,t)|^2$$

Critical Fourier component
$$n^{(q_c)} = \frac{1}{L_\perp}\int_0^{L_\perp}|\Psi(x,t)|^2 e^{-iq_c x}\,dx$$

Define
$$\theta = q_c x$$

$$n^{(q_c)}|\cos(q_c x - \phi) = \frac{1}{2\pi}\int_0^{2\pi}|\Psi(\theta',t)|^2\cos(\theta - \theta')\,d\theta'$$

Motivates definition of nonlocal potential
$$\Phi(\theta,t) = \frac{1}{2\pi}\int_0^{2\pi}|\Psi(\theta',t)|^2\cos(\theta - \theta')\,d\theta'$$

Effective Gross-Pitaevskii equation
$$i\frac{\partial\Psi'}{\partial t} = -\omega_r\frac{\partial^2\Psi'}{\partial\theta^2} - \epsilon\Phi(\theta,t)\Psi'$$

Ferromagnetic: ε > 0
$$\epsilon = 2\delta R p_0 \chi_0 = \frac{R p_0 b_0 \Gamma}{2}$$

Threshold: ε = 1

Order parameter: Magnetization
$$M = \left|\frac{1}{2\pi}\int_0^{2\pi}|\Psi|^2 e^{i\theta}\,d\theta\right|$$

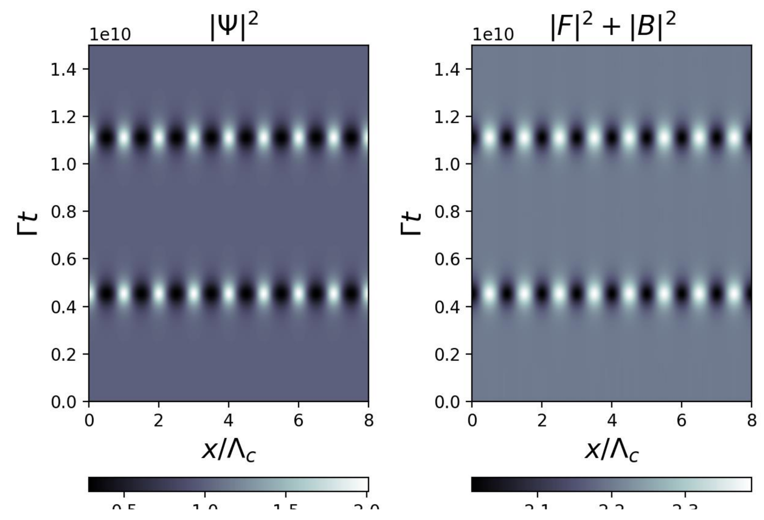
Numerical results: optical model

Weak driving

$b_0 = 20,\ \Delta = 100$

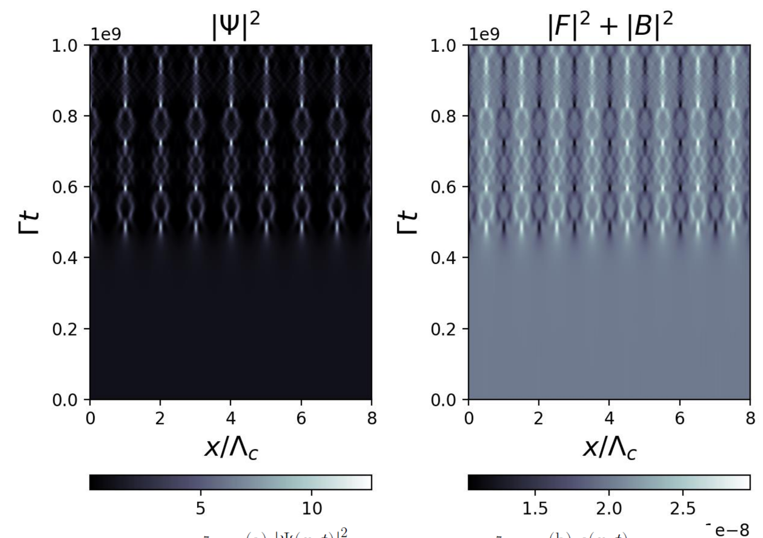
$R = 1,\ \frac{\omega_r}{\Gamma} = 10^{-8}$

$p_0 = 1.1\times10^{-9}$



Strong driving

$p_0 = 1\times10^{-8}$

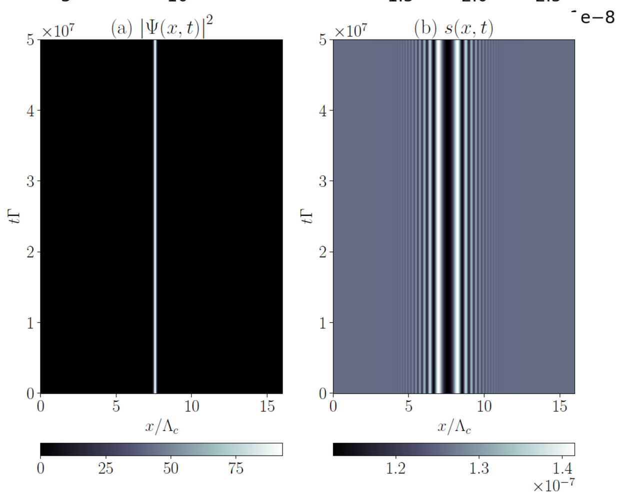


Droplet

$b_0 = 20,\ \Delta = 1600$

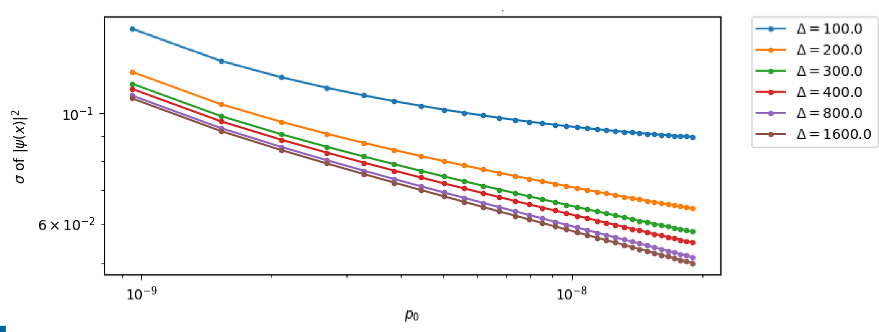
$\omega_r/\Gamma = 1.00\times10^{-7},\ R = 0.99$

$p_0 = 6.3\times10^{-8}$



Scaling of droplet size

$\sigma_x \propto p_0^{-1/4}$



Summary and outlook

- Good agreement between SMF and HMF simulations
- Prospects for experimental realization of archetypical model for long-range coupling
- Thermal atoms plus molasses: Classical HMF
  - Kuramoto model with damping and inertia
  - Brownian Mean Field phase transition
- Connection to
  - Supersolids
  - Time crystals

References and acknowledgements

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