

Bimodal Vortex Smith-Purcell Radiation

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Abstract—A vortex Smith-Purcell radiation (VSPR) with twin operation points, namely bimodal VSPR, is proposed based on a depth-modulated triple-slots helical grating. Due to the Brillouin-zone folding phenomenon, the dispersion of the helical grating is folded twice in the momentum space. Thus, two spoof surface plasmon (SSP) modes carrying orbital angular momentum can be directly diffracted into the vortex beam. When a ring-shaped e beam flies over the surface of the helical grating, the two SSP modes can be effectively excited owing to the momentum matching. Accordingly, a bimodal vortex Smith-Purcell radiation is achieved when the operating frequencies are in the radiation region, which is validated by the probed intensities and field distributions over the grating. The proposed bimodal vortex Smith-Purcell radiation may facilitate the generation and applications of vortex beams in microwave and terahertz band.

I. INTRODUCTION

Vortex beams are EM waves with helical wavefront and singularity in the phase. Owing to the unique properties of orbital angular momentum carried by them, such as topological protection, mode orthogonality, and chirality, vortex beams have been applied to the fields of communication, optical tweezer, chiral detection, and quantum information encoding[1]. The chirality of the vortex beam can be quantified by a phase index $e^{il\varphi}$, where φ is the azimuthal angle of the cylindrical coordinates and l is the quantum index of orbital angular momentum, i.e., topological charge (TC). In the mid-infrared and millimeter band, the transmission structure, metasurface, and antenna array could be employed to generate the vortex beam. However, on the one hand, current passive devices are constrained by the immature development of terahertz devices and limited conversion efficiency from Gaussian beams. On the other hand, these methods are suffering from the restricted available TC, narrow operation bandwidth, and complicated feed network, which hinders the applications of the vortex beam, especially in the terahertz band [2].

Smith-Purcell radiation is emitted when the e beam flies over the periodic structure, which is a special kind of Cherenkov radiation and is firstly observed by Smith and Purcell[3]. By employing the helical tape, hologram grating, and helical grating, the vortex Smith-Purcell radiation (VSPR) can be readily generated with tunability in operation frequency and TC over broadband, which may remove the barrier of the generation of the vortex beam [4-7]. However, these methods can only generate a vortex beam with a single operation point at a time. For multi-mode applications, the scheme of multi-

operation points is in urgent need of a solution, which might be overcome by the method proposed in this work.

In this paper, a bimodal VSPR, i.e., the VSPR with twin operation points, is proposed for multi-mode application scenarios. The triple-slot helical grating with depth-modulated grooves is employed as the interaction circuit. By exploiting the Brillouin-zone folding phenomenon, twin spoof surface plasmon (SSP) modes carrying orbital angular momentum can be effectively excited by the e beam and directly diffracted into the bimodal VSPR, which is demonstrated by the probed signals and field distributions. Moreover, the tunability of the proposed method is also calculated and discussed. The proposed scheme may facilitate the multi-mode generation and application of vortex beam, which can also be applied to the microwave and terahertz (THz) bands.

II. MODEL AND RESULTS

The schematic of the proposed method is shown in Fig. 1. The triple-slots helical grating is employed as the interaction circuit to support the vortex SSP modes carrying orbital angular momentum. If these vortex SSP modes can be excited by the e beam and be diffracted into the free space, the VSPR with a single operating point can be achieved [5]. To achieve the multi-operation scheme, the Brillouin-zone folding phenomenon is introduced by modulating the depth of the groove. If the depth of one slot is slightly tuned to be smaller than the other two, the dispersion line of the original SSP modes will be folded twice over the momentum space. Thus, the twin vortex SSP modes are formed. When the ring-shaped e beam flies over the helical grating, the twin SSPs mode can be excited when the condition of wavevector matching is satisfied. Furthermore, by tuning the operation voltage of the e beam so that the twin SSP modes are in the Smith-Purcell radiation region, the Bimodal Vortex Smith-Purcell Radiation can be formed accordingly.

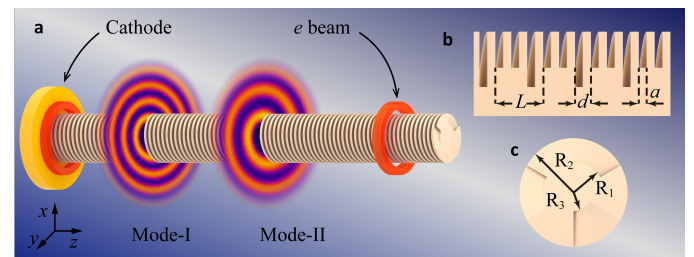


Fig 1. Schematic of triple-slots helical grating. **a** Illustration of the Bimodal Vortex Smith-Purcell Radiation. Twin vortex beams with TC = +1 is emitted when the e beam flies over the helical grating. **b** Cut-view of the helical grating. **c** Structural illustration of three groove.

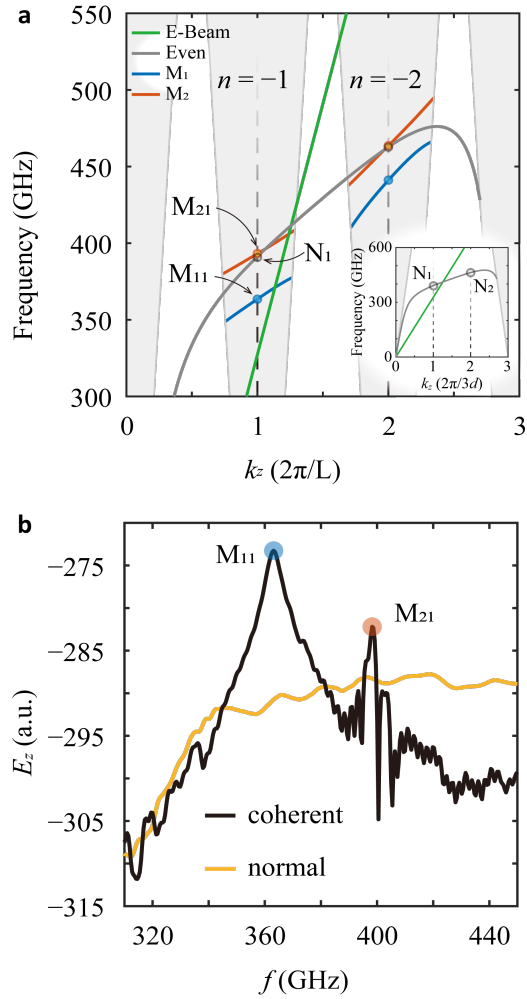


Fig 2. Simulation results of the proposed method. **a** Dispersion relations of the twin vortex SSP modes and e beam. **b** Probed E_z field over the grating.

To further validate the above discussion, the dispersion of the employed helical grating is simulated by CST STUDIO SUITE, and simulation results are shown in Fig. 2a. Owing to the Brillouin-zone folding phenomenon, the twin vortex SSP modes, i.e., M_{11} and M_{21} are supported by the helical grating, which are marked by red and blue solid lines, respectively. The shaded regions represent the light cone and its corresponding folding zone. Since the modes M_{11} and M_{21} are in the fast wave region of -1^{st} space harmonic, they can be readily diffracted into vortex beams and be utilized to generate the bimodal VSPR. When the 20-keV ring-shaped e -beam flies over the grating, the probed signal is illustrated by the black solid line in Fig. 2b. The spectrum result indicates the presence of two radiation peaks at frequencies of 363 GHz and 398 GHz, which are exactly the frequencies of vortex SSP modes M_{11} and M_{21} excited by the e beam. Moreover, compared with the results of the helical grating with uniform slots as illustrated by the yellow solid line, the field intensity is also enhanced due to the effect of field enhancement of the SSP modes.

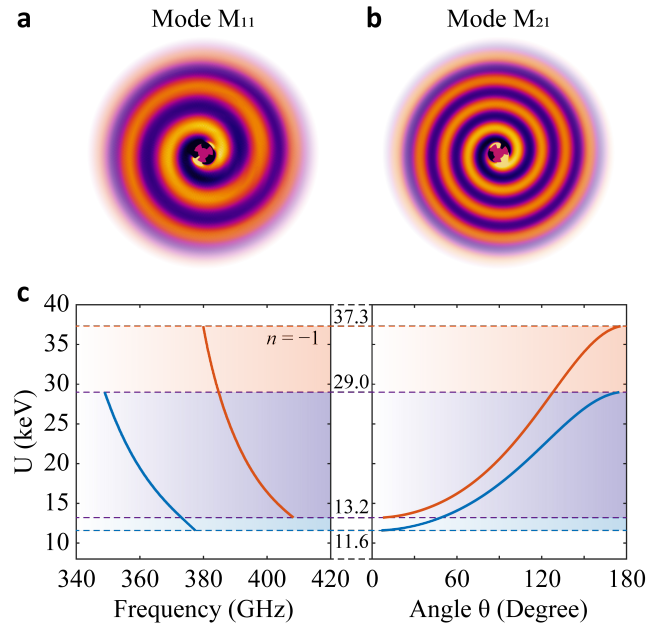


Fig 3. Vorticity and tunability of the bimodal VSPR. **a** and **b** are the E_z distributions of bimodal VSPR. **c** Turnabilities of frequency and radiation angle of bimodal VSPR.

To validate the vorticity of the generate bimodal VSPR, the field distributions of vortex beam converted from the twin SSPs M_{11} and M_{21} are illustrated in Fig. 3a and 3b, respectively. The simulation results indicate that both the TCs of the two vortex beams is -1 , which agrees well with the theory in Ref. [5]. Moreover, the tunability of frequency and radiation direction of the bimodal VSPR is also estimated, as shown in Fig. 3c. When the operating voltage of the e beam simultaneously increases from 13.2 keV to 29 keV, the radiation angle of the two vortex beams can be theoretically tuned from 0 to 180 degrees, thus enabling flexible tunability.

III. CONCLUSION

In this paper, a bimodal vortex Smith-Purcell radiation is proposed based on the twin vortex spoof surface plasmon modes. By exploiting the Brillouin-zone folding phenomenon of the triple-slot helical grating, the twin vortex SSP modes can be achieved and be readily diffracted into the vortex beam, which is validated by the simulation results. Moreover, the tunability of the frequency and radiation angle is also estimated by the theory. The proposed methodology may foster the application of multi-mode VSPR in the THz band.

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REFERENCES

- [1] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," *Physical Review A*, vol. 45, no. 11, pp. 8185–8189, Jun., 1992.
- [2] Y. Shen, X. Wang, Z. Xie, C. Min, X. Fu, Q. Liu, M. Gong, and X. Yuan, "Optical vortices 30 years on: OAM manipulation from topological charge to multiple singularities," *Light: Science & Applications*, vol. 8, no. 1, pp. 90, 2019/10/02, 2019.
- [3] S. J. Smith, and E. M. Purcell, "Visible Light from Localized Surface Charges Moving across a Grating," *Physical Review*, vol. 92, no. 4, pp. 1069–1069, Nov., 1953.
- [4] J.-F. Zhu, C.-H. Du, Z.-W. Zhang, P.-K. Liu, L. Zhang, and A. W. Cross, "Smith–Purcell radiation from helical grating to generate wideband vortex beams," *Optics Letters*, vol. 46, no. 18, pp. 4682–4685, 2021/09/15, 2021.
- [5] Z. W. Zhang, C. H. Du, J. F. Zhu, F. Y. Han, F. H. Li, Z. C. Gao, L. Zhang, A. W. Cross, and P. K. Liu, "A Terahertz Vortex Beam Emitter With Tunable Topological Charge and Harmonic Excitation," *Journal of Lightwave Technology*, vol. 39, no. 19, pp. 6231–6238, 2021.
- [6] M. Wang, F. Liu, Y. Lin, K. Cui, X. Feng, W. Zhang, and Y. Huang, "Vortex Smith-Purcell radiation generation with holographic grating," *Photonics Research*, vol. 8, no. 8, pp. 1309–1315, 2020/08/01, 2020.
- [7] L. Jing, Z. Wang, X. Lin, B. Zheng, S. Xu, L. Shen, Y. Yang, F. Gao, M. Chen, and H. Chen, "Spiral Field Generation in Smith-Purcell Radiation by Helical Metagratings," *Research*, vol. 2019, pp. 3806132, 2019/02/27, 2019.