Acceptance Reliability Sampling Plan for Items with Two Failure Modes

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Abstract

Existing reliability sampling plans reported in the literature assume that the items have only one failure mode. However, in practice, the items can fail due to two or more failure modes. In this paper, we study acceptance reliability sampling plan for items, which can fail due to the normal ageing and an external fatal shock during field operation. Two-stage reliability sampling plan that takes into account these two factors is developed. The lifetimes of items in a population before and after the acceptance test are stochastically compared.

Keywords: Failure mode; variables reliability sampling plan; life test; shock; stochastic comparison of lifetimes

1. Introduction

The manufactured products are usually grouped into lots by a supplier. A consumer can draw a random sample from a lot and, based on the reliability/lifetime test, the consumer can decide to accept or reject this lot of products. Thus, the information obtained through testing of some items from a lot is the basis for acceptance or rejection of the whole lot.

The acceptance sampling plans have become an effective and convenient tool in statistical quality control providing a method that screens out the products of poor quality. A general introduction to the theory and practice of acceptance sampling plans can be found in Stephens [1] and Montgomery [2]. Specifically, when the lifetime of a product is the main characteristic of its quality, the corresponding sampling plans are called the life test reliability sampling plans.

Various types of acceptance reliability sampling plans have been suggested and studied in the literature. For instance, in life testing, a fixed number of items are often tested for some fixed period of time (Type I censoring) or until some fixed number of items on test fail (Type II censoring) (Cha [3]). Initially, reliability sampling plans were implemented for items with lifetimes described by the exponential distributions. See Epstein [4], Epstein and Sobel [5,6], Blugren and Hewette [7], Fairbanks [8] for different acceptance reliability sampling plans in

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this case. Ferting and Mann [9] and Schneider [10] have used the Weibull distribution, which is popular in practice, as the baseline lifetime distribution for the corresponding acceptance reliability sampling plans. More advanced reliability sampling plans have been developed later. See, for instance, Seidel [11], Edgeman and Salzberg [12], Pérez-González and Fernández [13], Kim and Yum [14], Lam and Choy [15], Tsai and Wu [16], Balakrishnan et al. [17] to name a few. Aslam et al. [18] has developed a time-truncated skip-lot sampling plan (type V) for an accelerated life test.

Cha [3] has studied the reliability improvement of a population of items after the tests as compared with reliability characteristics before the tests. Recently, Cha and Finkelstein [19] have developed a reliability sampling plan for discrete lifetime models.

As listed above, most reliability sampling plans reported in the literature assume that the items can fail due to only one failure mode. However, frequently in practice, failures can occur due to two or more failure modes. In this paper, we study acceptance reliability sampling plan for items which can fail due to the normal ageing and an external fatal shock during field operation. We assume that the items are subject to the external shock process following the nonhomogeneous Poisson process (NHPP). On each shock, if the strength of the item is greater than the magnitude of the shock, then the item survives, or it fails otherwise. Taking into account the two failure modes, a two-stage reliability sampling plan is developed.

Furthermore, the lifetime of the population before the acceptance test and that of the population which have passed the sampling test are stochastically compared and the reliability improvement due to the test is analyzed.

As far as we know, the acceptance reliability sampling plans reported in the literature have been developed *only for the items with one failure mode*. However, items with multiple failure modes can be frequently encountered in practice as well. Therefore, the goal and contribution of this paper is in developing *a new variables acceptance reliability sampling plan for items with two different failure modes*. Moreover, our approach is general in the sense that it can be applied to any parametric distribution (whereas in most publications, the plans are distribution-specific). We believe that this as an important contribution of our study.

This paper is organized as follows. In Section 2, we describe the failure model with two failure modes and the structure of the population of manufactured items. Basic concepts for stochastic orders and related properties to be used in the rest of the paper are also introduced. In Section 3, a two-stage reliability sampling plan is designed and the feasibility of the proposed sampling plan is briefly discussed. Besides, an algorithm for obtaining the relevant parameters of this plan is suggested. In Section 4, we discuss reliability improvement of items in the population after the acceptance test. For this, the lifetime of the population before the acceptance test and that of the population that has passed the sampling test are stochastically compared. A numerical example which supports our theoretical findings is also provided. Finally, in Section 5, the concluding remarks are given.

2. Items with Two Failure Modes and Structure of the Population

In this section, we describe the lifetime of items with two failure modes and the corresponding population structure. For a convenient description of the model, first, we assume that a population of items is a homogeneous one. We assume that the item can fail due to: (i) normal failure mode caused by the internal ageing of items; (ii) failure mode due to external shocks. The failure model due to external shock considered in this paper is similar to the stress-strength model in reliability (see, e.g., Rao et al. [20], Srinivasa et al. [21], Rao et al.

[22]). Denote the lifetime of an item in the absence of the failure mode (ii) by T_N. The corresponding survival function is

$$P(T_N > t) = \exp\{-\int_0^t r(s)ds\},$$

where $r(s)$ is the failure rate function for T_N.

Denote the lifetime of an item in the absence of the failure mode (i) by T_S. We will define now the distribution of T_S. Assume that an item is subject to external shocks and the shock process is the nonhomogeneous Poisson process (NHPP) $\{N(t), t \geq 0\}$ with rate $\nu(t)$ (Cha and Finkelstein [23]). Denote by S_i the magnitude (stress) of the i th external shock. Assume that $S_i, i = 1,2,...$ are i.i.d. random variables with the common cumulative distribution function (Cdf) $M(s) = \Pr(S_i \leq s)$ ($\overline{M}(s) \equiv 1 - M(s)$) and the corresponding probability density function (pdf) $m(s)$. Let R be the *unobserved* random strength of the item with the corresponding Cdf, survival function and the pdf, $G(r)$, $\overline{G}(r)$, $g(r)$, respectively. On the i th shock, the item survives if $S_i \leq R$ and fails if $S_i > R$, '*independently of everything else*', $i = 1,2,...$ When R is deterministic: $R = r$, the survival probability of our system in $[0,t), t > 0$, is given by (see Cha and Finkelstein [24]).

$$P(T_S > t) = \sum_{n=0}^{\infty} (M(r))^n \frac{\left(\int_0^t \nu(x)dx\right)^n}{n!} \exp\{-\int_0^t \nu(x)dx\}$$

$$= \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\},$$

and the corresponding failure rate function is given by

$$\lambda_S(t,r) = \overline{M}(r)\nu(t),$$

respectively (see also Cha and Finkelstein [24]). When R is random,

$$P(T_S > t) = \int_0^{\infty} \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g(r)dr,$$

and the corresponding failure rate is (see also Cha and Finkelstein [24])

$$\lambda_S(t) = -\frac{P'(T_S > t)}{P(T_S > t)}$$

$$= \frac{\int_0^{\infty} \overline{M}(r) \cdot \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g(r)dr}{\int_0^{\infty} \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g(r)dr} \nu(t) = \tilde{p}(t)\nu(t),$$

where

$$\tilde{p}(t) \equiv \frac{\int_0^\infty \overline{M}(r)\cdot \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g(r)dr}{\int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g(r)dr} .$$

Denote by T the lifetime of the item in the presence of the two failure modes (i) and (ii). Then, as the lifetime model in this case is the competing risks model, the survival function of T is given by

$$P(T > t) = P(\min\{T_N, T_S\} > t) = \exp\{-\int_0^t r(s)ds\} \exp\{-\int_0^t \tilde{p}(s)\nu(s)ds\} .$$

In the above, we have described the lifetime of the item with two failure modes belonging to a homogeneous population. However, in practice, a population of manufactured items is often composed of two ordered subpopulations – the subpopulation with relatively high reliability (to be called the main or strong subpopulation), which contains items having longer lifetimes and the subpopulation with relatively poor reliability (to be called the 'freak' or weak subpopulation), which contains items having shorter lifetimes. In practice, items in the weak subpopulation can be produced along with the items of the main subpopulation because of, e.g., faulty resources and components, human errors, unstable production environment, etc. (see Jensen and Petersen [25], Kececioglu and Sun [26]). Thus, in this paper, we assume that the population of manufactured items is composed of two ordered subpopulations, i.e., the strong and the weak ones. Let us call it the *mixed population*. The composition of our mixed population is as follows: the proportion of the strong subpopulation is ϕ, whereas the proportion of the weak subpopulation is $1-\phi$. Denote by T_{N1} and T_{N2} the lifetimes due to the normal failure mode of the item (in the absence of failure mode (ii)) from strong and weak subpopulations, respectively. Also, denote by T_{S1} and T_{S2} the lifetimes due to the shock failure mode of the item (in the absence of failure mode (i)) from strong and weak subpopulations, respectively. Then, the survival functions of T_{N1} and T_{N2} are assumed to be given by

$$P(T_{Ni} > t) = \exp\{-\int_0^t r_i(s)ds\} , \quad i = 1, 2 ,$$

where $r_i(s)$ is the failure rate function of T_{N1}, $i = 1, 2$, respectively. On the other hand, the survival functions of T_{S1} and T_{S2} are assumed to be given by

$$P(T_{Si} > t) = \int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g_i(r)dr = \exp\{-\int_0^t \tilde{p}_i(s)\nu(s)ds\} , \quad i = 1, 2 , \qquad (1)$$

where

$$\tilde{p}_i(t) \equiv \frac{\int_0^\infty \overline{M}(r)\cdot \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g_i(r)dr}{\int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g_i(r)dr}$$

and $g_i(r)$, $i = 1, 2$, are the pdf of random strength R_i of the item belong to the strong and weak subpopulations, respectively. The corresponding Cdf and the survival function are denoted by $G_i(r)$, $\overline{G}_i(r)$, $i = 1, 2$, respectively. Note that, in Eq. (1), the distribution of the magnitude of the external shock $\overline{M}(r)$ and the intensity of the external shock $\nu(t)$ are common for the two subpopulations because items from the two subpopulations are operated under the same environment. Then, the survival functions of the lifetimes T_i, $i=1,2$, of the items from strong and weak subpopulations in the presence of the two failure modes (i) and (ii) are given by

$$P(T_i > t) = \exp\{-\int_0^t r_i(s)ds\}\int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\}g_i(r)dr$$

$$= \exp\{-\int_0^t r_i(s)ds\}\exp\{-\int_0^t \tilde{p}_i(s)\nu(s)ds\},\ \ i=1,2.$$

As mentioned before, items from the strong subpopulation have longer lifetimes, whereas those from the weak subpopulation exhibit shorter lifetimes. To represent this relationship properly, we need some conditions on the functions $r_i(s)$ and $g_i(r)$ in the expression for $P(T_i > t)$, $i=1,2$. For this and for our discussion on stochastic comparison in this paper, we need to introduce some basic concepts of stochastic orders that will be employed in what follows. For more details, the readers could refer to Shaked and Shanthikumar [27].

Definition 1. *Let* Z_1 *and* Z_2 *be two nonnegative continuous or discrete random variables with respective failure rate functions* $\gamma_1(t)$ *and* $\gamma_2(t)$, *such that*

$$\gamma_1(t) \ge \gamma_2(t),\ \textit{for all}\ \ t \ge 0.$$

Then Z_1 *is said to be smaller than* Z_2 *in the sense of failure rate order, which is denoted by* $Z_1 \le_{fr} Z_2$.

Definition 2. *Let* Z_1 *and* Z_2 *be two nonnegative continuous or discrete random variables with respective survival functions* $\overline{H}_1(t)$ *and* $\overline{H}_2(t)$, *such that*

$$\overline{H}_1(t) \le \overline{H}_2(t),\ \textit{for all}\ \ t \ge 0.$$

Then Z_1 *is said to be smaller than* Z_2 *in the sense of usual stochastic order, which is denoted by* $Z_1 \le_{st} Z_2$.

Definition 3. *Let* Z_1 *and* Z_2 *be two nonnegative continuous (discrete) random variables with respective probability density functions (probability mass functions)* $h_1(t)$ *and* $h_2(t)$, *such that*

$$\frac{h_1(t)}{h_2(t)}\ \ \textit{is decreasing for all}\ \ t \ge 0.$$

Then Z_1 *is said to be smaller than* Z_2 *in the sense of likelihood ratio order, which is denoted by* $Z_1 \le_{lr} Z_2$.

Due to Shaked and Shanthikumar [27], it holds that

$$Z_1 \le_{lr} Z_2 \Rightarrow Z_1 \le_{fr} Z_2 \Rightarrow Z_1 \le_{st} Z_2. \tag{2}$$

Furthermore, for stochastic comparison of populations to be discussed in the next section, we need the following lemma. The proof can also be found in Shaked and Shanthikumar [27].

Lemma 1.
(i)Let Z_1 *and* Z_2 *be two nonnegative random variables satisfying* $Z_1 \le_{st} Z_2$. *Then* $E[Z_1] \le E[Z_2]$.

(ii)If $Z_1 \le_{st} Z_2$ *and* $k(\cdot)$ *is any increasing [decreasing] function, then* $k(Z_1) \le_{st} [\ge_{st}] k(Z_2)$.

Throughout this paper, we assume that

(i) $r_1(t) \le r_2(t)$ (failure rate order), for all $t \ge 0$,

and

(ii) $\overline{G}_1(r) \ge \overline{G}_2(r)$ (usual stochastic order), for all $r \ge 0$,

which describe two aspects of stochastic ordering of the two subpopulations, i.e,, the stronger subpopulation has the smaller failure rate with respect to the normal failure mode and the larger threshold with respect to the shock failure mode.

Then,

$$P(T_{N1} > t) = \exp\{-\int_0^t r_1(s)ds\} \ge \exp\{-\int_0^t r_2(s)ds\} = P(T_{N2} > t), \text{ for all } t \ge 0. \quad (3)$$

Furthermore,

$$P(T_{Si} > t) = \int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g_i(r)dr = E[\exp\{-\overline{M}(R_i)\int_0^t \nu(x)dx\}],$$

where $\exp\{-\overline{M}(r)\int_0^t \nu(x)dx\}$ is an increasing function of r. Then, from the assumption $\overline{G}_1(r) \ge \overline{G}_2(r)$, for all $r \ge 0$, and Lemma 1,

$$P(T_{S1} > t) = E[\exp\{-\overline{M}(R_1)\int_0^t \nu(x)dx\}] \ge E[\exp\{-\overline{M}(R_2)\int_0^t \nu(x)dx\}] = P(T_{S2} > t), \quad (4)$$

for all $t \ge 0$. Then, from (3) and (4),

$$S_1(t) \equiv P(T_1 > t) = \exp\{-\int_0^t r_1(s)ds\}\int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx\} g_1(r)dr$$
$$\ge \exp\{-\int_0^t r_2(s)ds\}\int_0^\infty \exp\{-\overline{M}(r)\int_0^t \nu(x)dx g_2(r)dr = P(T_2 > t) \equiv S_2(t),$$

for all $t \ge 0$, which implies that $T_1 \ge_{st} T_2$.

Denote by $f_i(t)$ and $\lambda_i(t)$ the pdf and the failure rate functions which corresponds to $S_i(t)$, $i = 1,2$, respectively, i.e., $f_i(t) = -S'_i(t)$ and $\lambda_i(t) = f_i(t)/S_i(t)$, $i = 1,2$. In our population, the proportion of the strong subpopulation is ϕ, whereas the proportion of the weak subpopulation is $1-\phi$. Then, the mixture (population) survival function is given by

$$S_\phi(t) = \phi S_1(t) + (1-\phi)S_2(t). \quad (5)$$

Denote by T_ϕ the lifetime of the item which corresponds to the mixture distribution in (5). Then the population failure rate that describes T_ϕ is defined as

$$\lambda_\phi(t) = \frac{\phi f_1(t) + (1-\phi) f_2(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)}$$
$$= \frac{\phi S_1(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)} \lambda_1(t) + \frac{(1-\phi)\, S_2(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)} \lambda_2(t).$$

In this paper, the quality of a lot will be defined in terms of the mean time to failure of the item drawn from the lot, which is given by

$$\mu(\phi) \equiv E[T_\phi] = \int_0^\infty S_\phi(t)dt = \phi \int_0^\infty S_1(t)dt + (1-\phi)\int_0^\infty S_2(t)dt.$$

Note that the causes for the heterogeneity in the population (faulty resources and components, human errors, unstable production environment, etc.) are variable and, the proportion of the items from the strong subpopulation ϕ in a lot may also change depending on them. Intuitively, the larger ϕ results in the better quality of the corresponding lot, which can be formally shown by considering the function

$$\mu(\phi) = \phi \left(\int_0^\infty S_1(t)dt - \int_0^\infty S_2(t)dt \right) + \int_0^\infty S_2(t)dt$$

that is increasing in ϕ as $\int_0^\infty S_1(t)dt - \int_0^\infty S_2(t)dt \geq 0$ and thus, a lot having large ϕ is a lot of good quality, whereas a lot having small ϕ is a lot of poor quality.

3. Two-Stage Reliability Sampling Plan

We will now design a sampling plan for items with two failure modes to assure that the mean time to failure of an item is larger than the predetermined level. Denote by μ_1 and μ_2, $\mu_1 > \mu_2$, the mean time to failure of an item in a lot specified as the higher quality level and that specified as the lower one, respectively. The consumer requires that the lot acceptance probability should be smaller than the specified consumer's risk β at the lower quality level μ_2, whereas the producer requires that the lot rejection probability should be smaller than the specified producer's risk α at the higher quality level μ_1.

As $E[T_\phi]$ is monotonically increasing in ϕ, the lower quality level μ_2 and the higher level μ_1 can equivalently be defined in terms of the value of ϕ: the lower quality level is defined by $\phi = \phi_2$ and the higher quality level is defined by $\phi = \phi_1$, where ϕ_1 and ϕ_2 ($\phi_1 > \phi_2$) are two values which satisfy $E[T_{\phi_i}] = \mu_i$, $i = 1,2$, respectively.

It should be emphasized that, in the field operation, items along with a normal failure mode can fail due to shocks of *a random magnitude*. However, the laboratory environment under which the reliability sampling test is usually performed is a controlled environment and such external shocks do not exist. Thus, the reliability sampling plan will be designed in such a way that, at the first stage, a simulated shock test with a fixed magnitude is performed and, at the second stage, a normal truncated life test is carried out. The detailed procedure for this two-stage reliability sampling plan is as follows.

[Two-Stage Reliability Sampling Plan]

(Step 1) The total number of n items are randomly chosen from the lot to be tested.
(Step 2) At the first stage, n items are exposed to a shock with constant magnitude $s_0 > 0$. The failed items are discarded and items that have survived the shock are used for the second stage of the test.
(Step 3) At the second stage, items that have survived the first stage are tested during the time interval $(0, t_0]$, where t_0 is a predetermined testing time.
(Step 4) If the number of items that have failed during the two-stage test exceeds the threshold number c (integer), then the lot is rejected; otherwise, the lot is accepted.

The diagram for the two-stage reliability sampling plan is given in Figure 1.

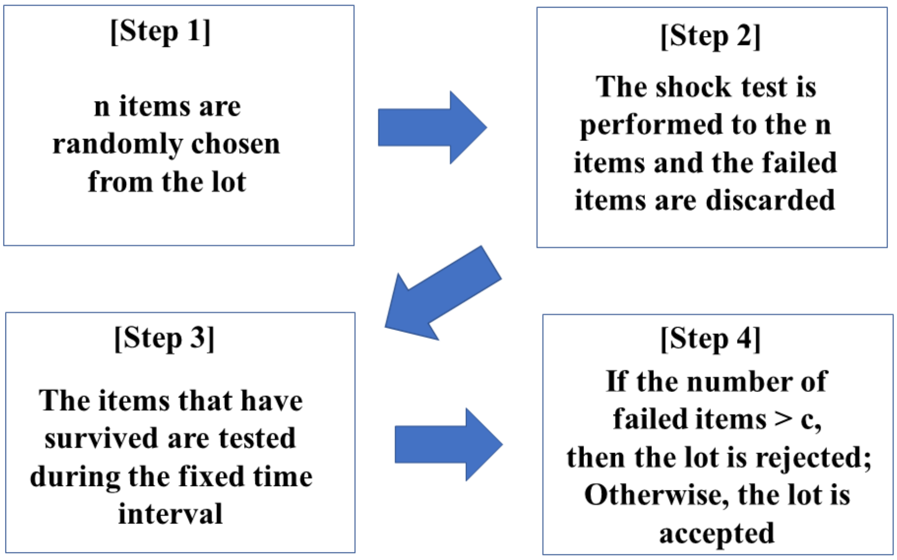

Figure 1. The diagram for the two-stage reliability sampling plan

Note that, the important aim of the sampling test is to distinguish the lots of good quality and those of bad quality. For this, through the sampling test, the items from strong and weak subpopulations should be distinguished efficiently. For example, in the first stage of the reliability sampling plan, if the shock magnitude s_0 is too large, all tested items would fail. If the shock magnitude s_0 is too small, none of the tested items would fail. In these cases, the first stage of the reliability sampling test will not be, obviously, efficient. Therefore, for this stage, we suggest to determine the shock magnitude s_0 in the following way

$$s_0 = \arg\max_{s>0}(\bar{G}_1(s) - \bar{G}_2(s)) = \arg\max_{s>0}(G_2(s) - G_1(s)),$$

that is, we choose s_0 which maximizes the difference of the survival probabilities (failure probabilities) of items from strong and weak subpopulations. By this we ensure that s_0 achieves the maximal 'difference' between items from the two subpopulations that is crucial for the test.

Observe that the probability that an item from the strong subpopulation will survive the two-stage reliability sampling test is

$$\rho_1(s_0,t_0) \equiv \overline{G}_1(s_0)\exp\{-\int_0^{t_0} r_1(s)ds\},$$

whereas the probability that an item from the weak subpopulation will survive the two-stage reliability sampling test is

$$\rho_2(s_0,t_0) \equiv \overline{G}_2(s_0)\exp\{-\int_0^{t_0} r_2(s)ds\}.$$

Assuming that the lot size is large enough, the acceptance probability of the lot as the function of ϕ is given by

$$L(\phi) = \sum_{i=0}^{c}\binom{n}{i}(1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])^i(\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0))^{n-i}.$$

As ϕ is the proportion of the strong subpopulation, for the proposed sampling test to be 'reasonable', $L(\phi)$ should be increasing in ϕ. The following theorem states that the proposed sampling plan satisfies this requirement. Before stating the theorem, we need a simple preliminary lemma. In the following, denote by $Z_{n,p}$ the binomial random variable with the number of trials n and the success probability p.

Lemma 2. *If* $p_1 < p_2$, then $Z_{n,p_1} \le_{lr} Z_{n,p_2}$.

Proof.

The ratio of the probability mass functions of Z_{n,p_1} and Z_{n,p_2} for $p_1 < p_2$ is given by

$$\frac{\frac{n!}{i!(n-i)!}p_1^{\,i}(1-p_1)^{n-i}}{\frac{n!}{i!(n-i)!}p_2^{\,i}(1-p_2)^{n-i}} = \left(\frac{1-p_1}{1-p_2}\right)^n\left(\frac{p_1}{p_2}\frac{1-p_2}{1-p_1}\right)^i,$$

which is decreasing in i. Then, according to Definition 3, $Z_{n,p_1} \le_{lr} Z_{n,p_2}$.

■

Theorem 1. *If* $r_1(t) \le r_2(t)$, *for all* $t \ge 0$, *and* $\overline{G}_1(r) \ge \overline{G}_2(r)$, *for all* $r \ge 0$, $L(\phi)$ *is increasing in* ϕ, *that is,*

$$L(\phi) \le L(\phi'), \textit{ for } \phi < \phi'.$$

Proof.

If $p_1 < p_2$, from Lemma 2 and relation (2), $Z_{n,p_1} \le_{st} Z_{n,p_2}$ and, according to Definition 2, we can order the corresponding Cdfs as

$$\sum_{i=0}^{c}\frac{n!}{i!(n-i)!}p_1^{\,i}(1-p_1)^{n-i} \ge \sum_{i=0}^{c}\frac{n!}{i!(n-i)!}p_2^{\,i}(1-p_2)^{n-i}, \text{ for any } 0 \le c \le n.$$

Under the assumption that $r_1(t) \le r_2(t)$, for all $t \ge 0$, and $\overline{G}_1(r) \ge \overline{G}_2(r)$, for all $r \ge 0$, $\rho_1(s_0,t_0) \ge \rho_2(s_0,t_0)$ and thus, for $\phi < \phi'$,

$$1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)] \ge 1-[\phi'\rho_1(s_0,t_0)+(1-\phi')\rho_2(s_0,t_0)].$$

Thus,

$$L(\phi)=\sum_{i=0}^{c}\binom{n}{i}(1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])^i(\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0))^{n-i}$$

$$\leq\sum_{i=0}^{c}\binom{n}{i}(1-[\phi'\rho_1(s_0,t_0)+(1-\phi')\rho_2(s_0,t_0)])^i(\phi'\rho_1(s_0,t_0)+(1-\phi')\rho_2(s_0,t_0))^{n-i}=L(\phi').$$

■

Note that the proposed sampling plan is characterized by two parameters (n,c). As both n and c are integers, one cannot obtain parameters (n,c) that exactly meet the consumer's and producer's risks. Thus, we will find the integers n and c that will attain the nearest acceptance probabilities:

$$L(\phi_1)\approx 1-\alpha \quad \text{and} \quad L(\phi_2)\approx\beta. \tag{3}$$

The following proposition can help in obtaining parameters (n,c) that agree with (3). The proof is similar to that in Cha [3] and, therefore, is omitted.

Proposition 1. *The acceptance probability*

$$L(\phi)=\sum_{i=0}^{c}\binom{n}{i}(1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])^i(\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0))^{n-i}$$

is decreasing in n *for any fixed* $\phi>0$ *and* c.

Due to Proposition 1, the search of parameters (n,c) could be significantly simplified. For the fixed $n\geq 1$, define now $c(n)$ as a nonnegative integer which satisfies the following equation

$$L(\phi_1)=\sum_{i=0}^{c}\binom{n}{i}(1-[\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0)])^i(\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0))^{n-i}=1-\alpha.$$

As $L(\phi_1)$ decreases in n for any fixed $\phi_1>0$ and c due to Proposition 1, we have: $c(n)\leq c(n+1)$, $n=1,2,\ldots$, which gives a lower bound in the following sequential procedure. To find parameters (n,c) satisfying the two equations in (3), the following sequential procedure can be applied:

(Step 1)

Fix $n=1$ and find the integer $0\leq c(1)\leq 1$ such that

$$\sum_{i=0}^{c(1)}\binom{1}{i}(1-[\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0)])^i(\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0))^{1-i}=1-\alpha.$$

If $\sum_{i=0}^{c(1)}\binom{1}{i}(1-[\phi_2\rho_1(s_0,t_0)+(1-\phi_2)\rho_2(s_0,t_0)])^i(\phi_2\rho_1(s_0,t_0)+(1-\phi_2)\rho_2(s_0,t_0))^{1-i}=\beta$, then choose $(1,c(1))$ as the desired parameters; otherwise go to Step 2.

(Step 2)

Fix $n=2$ and find the integer $c(2)$, where $c(1)\leq c(2)\leq 2$, such that

$$\sum_{i=0}^{c(2)} \binom{2}{i} (1-[\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0)])^i (\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0))^{2-i} = 1-\alpha .$$

If $\sum_{i=0}^{c(2)} \binom{2}{i} (1-[\phi_2\rho_1(s_0,t_0)+(1-\phi_2)\rho_2(s_0,t_0)])^i (\phi_2\rho_1(s_0,t_0)+(1-\phi_2)\rho_2(s_0,t_0))^{2-i} = \beta$, then choose $(2, c(2))$ as the desired parameters; otherwise go to Step 3.

(Step 3)

Fix $n=3$ and find the integer $c(3)$, where $c(2) \le c(3) \le 3$, such that

$$\sum_{i=0}^{c(3)} \binom{3}{i} (1-[\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0)])^i (\phi_1\rho_1(s_0,t_0)+(1-\phi_1)\rho_2(s_0,t_0))^{3-i} = 1-\alpha .$$

If $\sum_{i=0}^{c(3)} \binom{3}{i} (1-[\phi_2\rho_1(s_0,t_0)+(1-\phi_2)\rho_2(s_0,t_0)])^i (\phi_2\rho_1(s_0,t_0)+(1-\phi_2)\rho_2(s_0,t_0))^{3-i} = \beta$, then choose $(3, c(3))$ as the desired parameters; otherwise go to the next Step 4 with $n=4, \ldots,$ and so on.

As an illustrative example, let $r_1(t)=0.1,\ t \ge 0$, $r_2(t)=1.0,\ t \ge 0$, $\overline{M}(r)=\exp\{-r\}, r>0$, $\overline{G}_1(r)=\exp\{-0.1r\}, r>0$, $\overline{G}_2(r)=\exp\{-2r\}, r>0$. Then, the assumptions (i) $r_1(t) \le r_2(t)$, $t \ge 0$, and (ii) $\overline{G}_1(r) \ge \overline{G}_2(r)$, for all $r \ge 0$, are satisfied. In this case, to fine the shock magnitude s_0 satisfying

$$s_0 = \arg\max_{s>0}(\overline{G}_1(s)-\overline{G}_2(s)) = \arg\max_{s>0}(G_2(s)-G_1(s)),$$

the graph of $\overline{G}_1(s)-\overline{G}_2(s)$ is obtained in Figure 2.

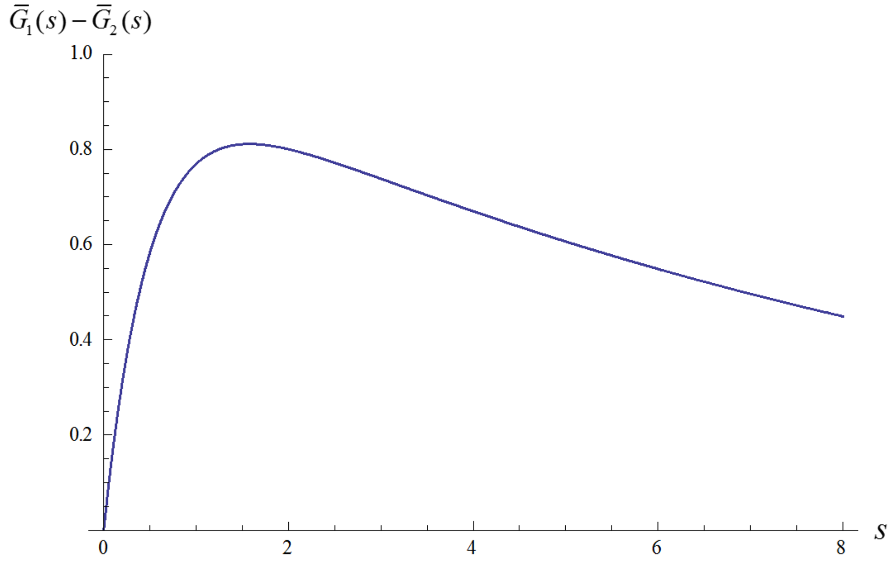

Figure 2. The graph for $\overline{G}_1(s)-\overline{G}_2(s)$

It can be seen that s_0 is given by $s_0 = 1.58$. Let $t_0 = 1.0$, $\alpha = 0.05$, $\beta = 0.10$. Then, following the sequential procedure described above, the parameters (n, c) have been obtained for $\phi_1 = 0.950$ and different values of ϕ_2.

Table 1. The parameters (n,c) for $\alpha = 0.05$, $\beta = 0.10$.

ϕ_1	ϕ_2	(n,c)
	0.700	$(n=56, c=20)$
	0.675	$(n=48, c=18)$
0.950	0.650	$(n=37, c=14)$
	0.625	$(n=29, c=11)$
	0.600	$(n=26, c=10)$

Observe that when the difference between ϕ_1 and ϕ_2 is larger, the smaller number of testing items n are required, and vice versa. This is since it is easier to identify lots of good and bad quality when there is a larger difference in the quality levels.

4. Analysis of the Reliability Improvement after the Acceptance Test

In this section, we discuss reliability improvement in the population after the acceptance test. It should be noted that, in practice, the proportion of the strong subpopulation ϕ and that of the weak subpopulation $1-\phi$ can often change from lot to lot. This can happen, e.g., due to unstable production environment which is varying in time, the proportion of faulty resources and components is subject to change in time, etc. Therefore, it is natural to assume that the proportion of the strong subpopulation is a random variable to be denoted by Φ with the pdf $\pi(\phi)$ and support in [0,1]. The following discussion will address this setting.

First, we will describe the population distribution before the reliability testing procedure. Let T be the lifetime of an item randomly selected from the population before testing. Then the corresponding mixture survival function of T, $S_m(t)$, is

$$S_m(t) = \int_0^1 [\phi S_1(t) + (1-\phi) S_2(t)] \pi(\phi) d\phi, \tag{4}$$

with the pdf $f_m(t)$

$$f_m(t) = \int_0^1 [\phi f_1(t) + (1-\phi) f_2(t)] \pi(\phi) d\phi. \tag{5}$$

and the failure rate $\lambda_m(t)$

$$\lambda_m(t) = \frac{f_m(t)}{S_m(t)} = \frac{\int_0^1 [\phi f_1(t) + (1-\phi) f_2(t)] \pi(\phi) d\phi}{\int_0^1 [\phi S_1(t) + (1-\phi) S_2(t)] \pi(\phi) d\phi}$$

$$= \int_0^1 \left[\frac{\phi S_1(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)} \lambda_1(t) + \frac{(1-\phi)\, S_2(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)} \lambda_2(t) \right] \frac{[\phi\, S_1(t) + (1-\phi) S_2(t)] \pi(\phi)}{\int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi) S_2(t)] \pi(\phi) d\phi} d\phi$$

$$= \int_0^1 \lambda_\phi(t) \pi(\phi \mid t) d\phi, \qquad (6)$$

where

$$\pi(\phi \mid t) = \frac{[\phi\, S_1(t) + (1-\phi) S_2(t)] \pi(\phi)}{\int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi) S_2(t)] \pi(\phi) d\phi},$$

which is the conditional distribution of Φ given $T > t$, $(\Phi \mid T > t)$ and $\lambda_\phi(t)$ is the corresponding conditional failure rate given by the first multiplier in the integrand of the second row of (6). General introductions to mixture distributions can be found in Cha and Finkelstein [28] and Finkelstein and Cha [29].

We will describe now the lifetime distribution of items in the population that is composed of lots that were accepted in the reliability sampling test. Denote by T^* the lifetime of an item randomly selected from this population. The survival function, the probability density function and the failure rate function of the population conditional on the acceptance are denoted by $S_m^{\ *}(t)$, $f_m^{\ *}(t)$ and $\lambda_m^{\ *}(t)$, respectively. Furthermore, we denote by $\pi^*(\phi)$ the conditional pdf of $(\Phi \mid N \le c)$ and by $\Pi^*(\phi)$ the corresponding Cdf, where N is the number of items which have failed during the test. We also denote by $\Psi(k; n, p)$ the Cdf of the binomial random variable with parameters (n, p), that is,

$$\Psi(k; n, p) \equiv \sum_{i=0}^{k} \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} .$$

Theorem 2. *The survival function, the probability density function and the failure rate function for items in the population of lots accepted in the sampling test are given by*

$$S_m^{\ *}(t) = \int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi) S_2(t)] \frac{\Psi(c; n, 1 - [\phi \rho_1(s_0, t_0) + (1-\phi) \rho_2(s_0, t_0)]) \pi(\phi)}{\int_0^1 \Psi(c; n, 1 - [\phi \rho_1(s_0, t_0) + (1-\phi) \rho_2(s_0, t_0)]) \pi(\phi) d\phi} d\phi,$$

$$f_m^{\ *}(t) = \int_0^1 [\phi\, f_1(t) + (1-\phi)\, f_2(t)] \frac{\Psi(c; n, 1 - [\phi \rho_1(s_0, t_0) + (1-\phi) \rho_2(s_0, t_0)]) \pi(\phi)}{\int_0^1 \Psi(c; n, 1 - [\phi \rho_1(s_0, t_0) + (1-\phi) \rho_2(s_0, t_0)]) \pi(\phi) d\phi} d\phi,$$

$$\lambda_m^{\ *}(t) = \int_0^1 \left[\frac{\phi S_1(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)} \lambda_1(t) + \frac{(1-\phi)\, S_2(t)}{\phi\, S_1(t) + (1-\phi) S_2(t)} \lambda_2(t) \right]$$

$$\times \frac{[\phi\, S_1(t) + (1-\phi)S_2(t)]\Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])\pi(\phi)}{\int_0^1 [\phi S_1(t) + (1-\phi)S_2(t)]\Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])\pi(\phi)d\phi} d\phi\,, \qquad (7)$$

respectively.

Proof.
Clearly, for the accepted lots, the distribution of Φ should be changed to the conditional distribution of $(\Phi \mid N \le c)$, as the number of items which have failed during the test is less than or equal to c if a lot is accepted. As the conditional probability of $P(N \le c \mid \Phi = \phi)$ is given by $P(N \le c \mid \Phi = \phi) = \Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])$, the joint distribution of $(N \le c, \Phi = \phi)$ is

$$\Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])\pi(\phi)\,,$$

and, therefore, the conditional pdf of $(\Phi \mid N \le c)$ can be obtained as

$$\pi^*(\phi) \equiv \frac{\Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])\pi(\phi)}{\int_0^1 \Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])\pi(\phi)d\phi}\,.$$

Thus, the role of $\pi(\phi)$ in (4), (5) and (6) in the population before the test is now taken by $\pi^*(\phi)$ in the population which is composed of lots accepted in the reliability sampling test. Then, the desired results is obtained by substitution of $\pi(\phi)$ in (4), (5) and (6) with $\pi^*(\phi)$, respectively.

■

We will now compare stochastically the population before the test and the population which is composed of lots accepted in the reliability sampling test. For this, we compare the proportions of the strong subpopulation in the populations before and after the test and the lifetimes of items randomly selected before and after the test.

Theorem 3. *If* $r_1(t) \le r_2(t)$, *for all* $t \ge 0$, *and* $\overline{G}_1(r) \ge \overline{G}_2(r)$, *for all* $r \ge 0$, *the following results hold:*
(i) $\Phi \le_{lr} (\Phi \mid N \le c)$;
(ii) $T \le_{fr} T^*$, *that is,*

$$\lambda_m(t) \ge \lambda_m^*(t), \text{ for all } t \ge 0.$$

Proof.
In the proof of Theorem 1, it has been shown that $\Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])$ is increasing in ϕ and, thus, the ratio $\pi^*(\phi)/\pi(\phi)$

$$\frac{\pi^*(\phi)}{\pi(\phi)} = \frac{\Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])}{\int_0^1 \Psi(c;n,1-[\phi\rho_1(s_0,t_0)+(1-\phi)\rho_2(s_0,t_0)])\pi(\phi)d\phi}$$

is increasing in ϕ and $\pi(\phi)/\pi^*(\phi)$ is decreasing in ϕ. This implies that $\Phi \le_{lr} (\Phi \mid N \le c)$.

We will now prove that $T \le_{fr} T^*$. Observe that, from (6), $\lambda_m(t)$ can be written as:

$$\lambda_m(t) = E_{(\Phi|T>t)}[\lambda_\Phi(t)],$$

where '$E_{(\Phi|T>t)}[\bullet]$' stands for the conditional expectation with respect to the conditional distribution of Φ given $T>t$, which is given by

$$\pi(\phi \mid t) = \frac{[\phi\ S_1(t) + (1-\phi)S_2(t)]\pi(\phi)}{\int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi)S_2(t)]\pi(\phi)d\phi}.$$

Similarly, $\lambda_m^*(t)$ in (7) can be expressed as:

$$\lambda_m(t) = E_{(\Phi|T^*>t)}[\lambda_\Phi(t)],$$

where '$E_{(\Phi|T^*>t)}[\bullet]$' represents the conditional expectation with respect to the conditional distribution of Φ given $T^*>t$, which is specified as:

$$\pi^*(\phi \mid t) \equiv \frac{[\phi\ S_1(t) + (1-\phi)S_2(t)]\Psi(c;n,1-[\phi\rho_1(s_0,t_0) + (1-\phi)\rho_2(s_0,t_0)])\pi(\phi)}{\int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi)S_2(t)]\Psi(c;n,1-[\phi\rho_1(s_0,t_0) + (1-\phi)\rho_2(s_0,t_0)])\pi(\phi)d\phi}.$$

Observe that

$$\frac{\pi(\phi \mid t)}{\pi^*(\phi \mid t)} = \frac{\int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi)S_2(t)]\Psi(c;n,1-[\phi\rho_1(s_0,t_0) + (1-\phi)\rho_2(s_0,t_0)])\pi(\phi)d\phi}{\int_0^1 [\phi \mathrm{S}_1(t) + (1-\phi)S_2(t)]\pi(\phi)d\phi} \times \frac{1}{\Psi(c;n,1-[\phi\rho_1(s_0,t_0) + (1-\phi)\rho_2(s_0,t_0)])}$$

is decreasing in ϕ, implying that

$$(\Phi \mid T^* > t) \ge_{lr} (\Phi \mid T > t), \text{ for all } t \ge 0.$$

Also, note that

$$\lambda_\phi(t) = \frac{\phi S_1(t)}{\phi\ S_1(t) + (1-\phi)S_2(t)}\lambda_1(t) + \frac{(1-\phi)\ S_2(t)}{\phi\ S_1(t) + (1-\phi)S_2(t)}\lambda_2(t)$$

is decreasing in ϕ as

$$\frac{\phi S_1(t)}{\phi\ S_1(t) + (1-\phi)S_2(t)}$$

is increasing in ϕ and $\lambda_1(t) \le \lambda_2(t)$, for all t. Then, according to Lemma 1,

$$\lambda_m(t) = E_{(\Phi|T>t)}[\lambda_\Phi(t)] \ge E_{(\Phi|T^*>t)}[\lambda_\Phi(t)] = \lambda_m^*(t), \text{ for all } t \ge 0.$$

■

From Theorem 3, we can see that both the proportion of the strong subpopulation in the population and the lifetime of the items have increased after the test.

Remark 1. Intuitively, the reliability characteristics of lots after the test that accepts not more than c failures in the test should improve, which can be loosely written as $\Phi \le (\Phi \mid N \le c)$ and $T \le T^*$. However, the main questions arise: in what stochastic sense these inequalities hold and what are the assumptions? Theorem 3 gives answers to these questions. Thus, similar to Bayesian reasoning, the additional information updates the distribution of Φ in a specified way.

Example 1. As in the example in Section 3, let $r_1(t) = 0.1, t \ge 0$, $r_2(t) = 1.0, t \ge 0$, $\overline{M}(r) = \exp\{-r\}, r > 0$, $\overline{G}_1(r) = \exp\{-0.1r\}, r > 0$, $\overline{G}_2(r) = \exp\{-2r\}, r > 0$, with $t_0 = 1.0$, $\alpha = 0.05$, $\beta = 0.10$ and $v(t) = 1, t \ge 0$. We assume that Φ follows the beta distribution with parameters $(4,1.8)$, i.e.,

$$\pi(\phi) = \frac{\Gamma(5.8)}{\Gamma(4)\Gamma(1.8)} \phi^3 (1-\phi)^{0.8}, \ \ 0 < \phi < 1.$$

The two quality levels are specified as $\phi_1 = 0.950$ and $\phi_2 = 0.650$. Then, from Table 1, the two parameters of the reliability test are given by $(n = 37, c = 14)$. Under the given setting, first we compare the proportions of the strong subpopulation in the populations before and after the test. Define the Cdfs of Φ and $(\Phi \mid N \le c)$ as $\Pi(\phi) \equiv \int_0^\phi \pi(\rho) d\rho$ and $\Pi^*(\phi) \equiv \int_0^\phi \pi^*(\rho) d\rho$, respectively. The Cdf's and the pdf's of Φ and $(\Phi \mid N \le c)$ are plotted in Figures 2 and 3. We can see that the random variable $(\Phi \mid N \le c)$ is stochastically larger than Φ ((i) of Theorem 3). Also note that, the proportion of Φ which is smaller than the lower quality level $\phi_2 = 0.650$ has dramatically decreased after the test.

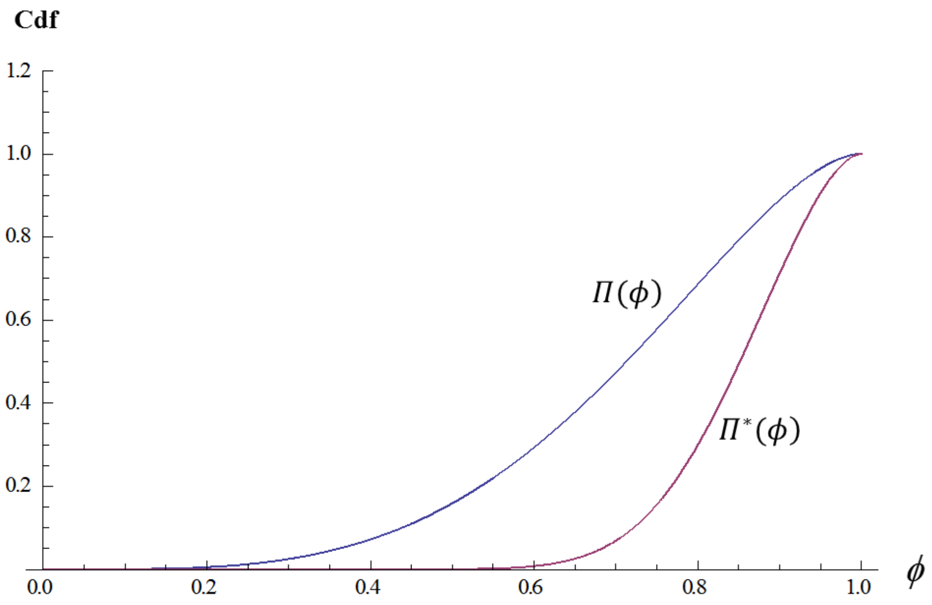

Figure 3. The Cdf's $\Pi(\phi)$ and $\Pi^*(\phi)$.

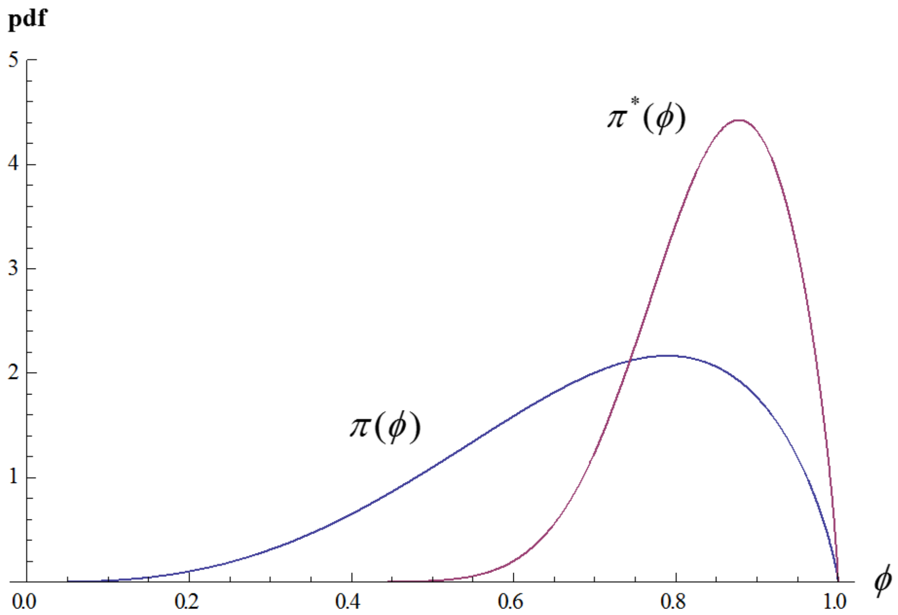

Figure 4. The pdf's $\pi(\phi)$ and $\pi^*(\phi)$.

The failure rate functions $\lambda_m(t)$ and $\lambda_m^*(t)$ are plotted in Figure 5. We can see that $\lambda_m(t) \geq \lambda_m^*(t)$, for all $t \geq 0$, implying that T^* is larger than T in the sense of the failure rate order, which illustrates (ii) of Theorem 3.

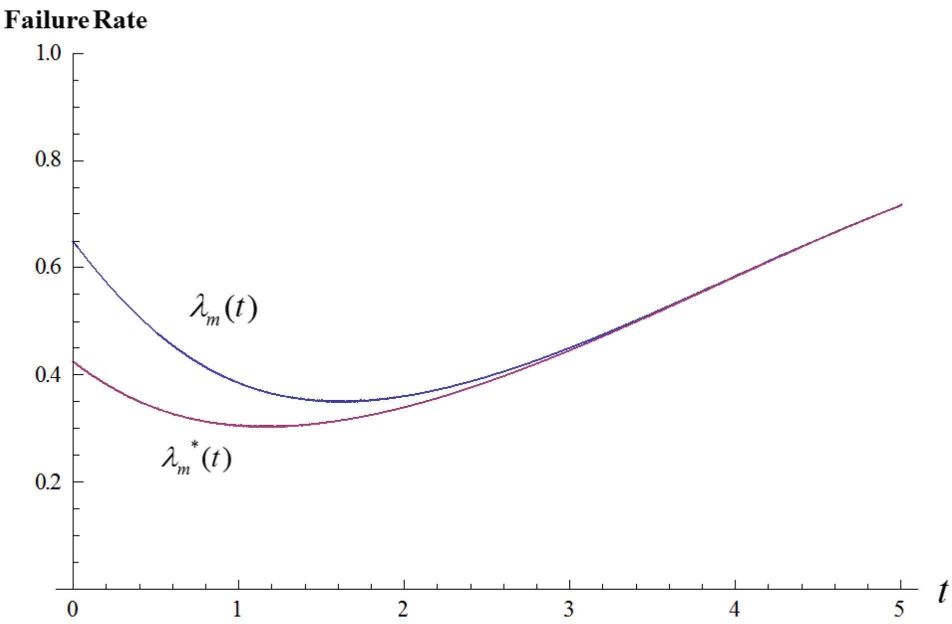

Figure 5. Failure rate functions $\lambda_m(t)$ and $\lambda_m^*(t)$.

5. Concluding Remarks

Existing reliability sampling plans reported in the literature assume that items can fail due to only one failure mode. However, frequently in practice, failures can occur due to two or more failure modes. In this paper, we have proposed the acceptance reliability sampling plan for items which can fail due to the normal (internal) ageing and external fatal shocks during a field operation.

We have discussed the feasibility of the proposed plan and provided an algorithm for obtaining the corresponding parameters. Reliability improvement in the population after the acceptance test has been also discussed by stochastically comparing the lifetimes in the population before the acceptance test and that in the population of items that have passed the sampling test. Specifically, we show that these two population's lifetimes are ordered in the sense of the failure rate ordering. Our findings have been supported by the relevant numerical example.

It is important to note that the exact values for n and c matching the requirement for α (producer risk) and β (consumer risk) cannot be obtained. Therefore, the producer's and the consumer's risks are not exactly achieved. This can be considered as the limitation of our study. ~~however, this is the inevitable problem that cannot be improved in this field.~~

As listed earlier, most reliability sampling plans reported in the literature assume that the items can fail due to only one failure mode. However, frequently in practice, failures can occur due to two or more failure modes. In this case, the conventional reliability acceptance testing plans for items with one failure mode cannot be applied. The proposed study can be applied to such cases. Ji, I think this paragraph should be deleted. Everything is OK without it (see the first paragraph).

To the best of the authors' knowledge, this is the first work to study the reliability sampling plan for items with two different failure modes. Furthermore, in most of the existing studies, acceptance reliability sampling plans have been developed for specific parametric distributions. In this paper, we do not specify the lifetime distribution and thus, the results obtained in this paper can be regarded to be general in the sense that they can be applied to any parametric distributions.

Neutrosophic statistics, introduced by Smarandache [30], is the extension of classical statistics and is applied when the data is coming from a complex process or from an uncertain environment. Recently, neutrosophic statistics theory has been applied to the inspection, inference, and process control (see, e.g., Aslam [31-33], Chen et al. [34]). The current study can be extended using neutrosophic statistics in the future research.

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