Considering equity in priority setting using transmission models: Recommendations and data needs


ARTICLE INFO

Keywords:
Transmission modelling
Health economics
Equity
Cost-effectiveness analysis
Priority
Setting

ABSTRACT

Objectives: Disease transmission models are used in impact assessment and economic evaluations of infectious disease prevention and treatment strategies, prominently so in the COVID-19 response. These models rarely consider dimensions of equity relating to the differential health burden between individuals and groups. We describe concepts and approaches which are useful when considering equity in the priority setting process, and outline the technical choices concerning model structure, outputs, and data requirements needed to use transmission models in analyses of health equity.

Methods: We reviewed the literature on equity concepts and approaches to their application in economic evaluation and undertook a technical consultation on how equity can be incorporated in priority setting for infectious disease control. The technical consultation brought together health economists with an interest in equity-informative economic evaluation, ethicists specialising in public health, mathematical modellers from various disease backgrounds, and representatives of global health funding and technical assistance organisations, to formulate key areas of consensus and recommendations.

Results: We provide a series of recommendations for applying the Reference Case for Economic Evaluation in Global Health to infectious disease interventions, comprising guidance on 1) the specification of equity concepts; 2) choice of evaluation framework; 3) model structure; and 4) data needs. We present available conceptual and analytical choices, for example how correlation between different equity- and disease-relevant strata should be...
1. Introduction

Reducing the burden of infectious diseases remains a priority in global health, but public health funding and resources are highly constrained (Chang et al., 2019). Economic evaluations using infectious disease transmission models are increasingly used to prioritise investments in infectious disease control by both national and global funders. Economic evaluations typically compare aggregate costs with aggregate effect and therefore fundamentally assess efficiency, often with no regard for the distribution or who gains and who bears the costs. Whilst we acknowledge that efficiency may have contrasting implications across, for example, economic and epidemiological domains, in this paper we refer to a utilitarian understanding of efficiency seeking to achieve the maximal amount of welfare given available resources or more specifically maximize health given the healthcare budget. In recent years, economists and ethicists have been working together to explore how economic evaluations can incorporate distributional concerns, but as yet these frameworks have not been widely applied to infectious disease interventions (Cookson et al., 2020). Yet equity matters to policy makers and to the public (Herzog et al., 2021; Kupferschmidt, 2020), specifically how benefits and costs of interventions are distributed across population groups within a country, for example by socioeconomic status, or between countries. The COVID-19 pandemic provides a striking example of how the burdens and costs of infectious diseases, and the costs and benefits of interventions addressing transmission, are not only unequally distributed between and within countries, but have specific and broadly predictable impacts, typically on the most disadvantaged populations (Broadbent et al., 2020).

Infectious disease dynamic transmission models, hereon referred to as transmission models, are mathematical representations of infectious disease spread. In these systems, interventions are introduced which change risks and outcomes both for those who receive the intervention and, importantly, those who do not. This second, indirect, effect is an externality, whose value should be considered in economic evaluations. From an economic perspective, the indirect effect also changes the “publicness” of an intervention, since the protective effects of a vaccination programme directed to one population group intrinsically spill over, and therefore are non-exclusive. As different groups mix, the risk of disease in the non-vaccinated group may also fall, ultimately extending the benefits to the whole population through indirect protection and thereby altering both the aggregate outcomes and their distribution. By considering heterogeneity of susceptibility, infectivity, costs, service utilisation, and outcomes across population groups, and simultaneously identifying mixing between population groups, transmission models have the potential to help policymakers understand how interventions are likely to affect the distribution of health and costs. There is, however, little methodological guidance on how to consider equity in transmission models, despite transmission models being increasingly used in priority setting globally.

To date there have been few examples of transmission model-based economic evaluations that explicitly consider equity (Dawkins et al., 2018; Gomes et al., 2019; Verguet et al., 2016; Assebe et al., 2020; Chang et al., 2018; Verguet et al., 2017), despite the growing literature on equity informative economic evaluation (Cookson et al., 2020; Verguet et al., 2016), and a long tradition of employing transmission models to examine changes in the distribution of factors like access to health care, socio-economic conditions, and health outcomes on disease transmission. For example, Munday et al. examined the impact of vaccine coverage across groups depending on the distribution of the risk of infection across those who are vaccinated and those who are not and the level of interaction between the two groups (Munday et al., 2018). Lee et al. considered how influenza vaccines should be distributed across socio-economic groups to prevent the emergence of an epidemic. They showed that selectively targeting poorer communities resulted in greater benefit to wealthier communities than if the wealthier alone were directly targeted (Lee et al., 2011). Johnson et al. considered ethnic differences in both risk of human papillomavirus (HPV) and uptake of vaccination and services such as cervical cancer screening. Their models suggest that vaccination is likely to increase inequalities because vaccine uptake is much higher in the group at lower risk of infection and disease (Johnson et al., 2018). These studies, along with others, demonstrate that modelling the distribution of risk and uptake across population groups is feasible and has the potential to also be applied to assessing the equity of impact and cost-effectiveness of interventions.

The Reference Case for Economic Evaluation in Global Health (Wilkinson et al., 2016) was developed to improve the application, quality, and reporting of economic evaluation in low- and middle-income countries. It is organised by core principles and includes methodological specifications related to these principles. Principle 11 recommends that equity, fairness in the distribution of costs and consequences, should be considered at all stages of an economic evaluation. We aim to provide recommendations for the methodological specifications and reporting standards of Principle 11 to support and enable its application to infectious diseases. We focus on the specific challenges of conducting equity analyses using transmission models and aim to make recommendations for analysts directly applying equity in economic evaluations, alongside identifying important research gaps in methods and data needs.

2. Approach

We conducted a scoping literature review to gain a perspective on current thinking and to create an agenda for a discussion with experts. The scoping review was based on a previously published bibliometric analysis of economic evaluations in global health (Pitt et al., 2016) and a systematic review of economic evaluations using transmission models (Drake et al., 2016) to identify published literature on economic evaluations, focusing on equity considerations when transmission models are used. We reviewed the individual articles in these reviews to assess the state-of-the-art in published models incorporating equity elements. We synthesised the approaches taken into a narrative summary of the theoretical foundations of equity and the different approaches used to date to consider equity in economic evaluation, which was adapted slightly during an expert consultation meeting; the final synthesis is presented in Table 1.

In March 2018, an expert consultation meeting on equity, priority setting, and transmission modelling was held. All attendees of the workshop are listed as co-authors on this manuscript, contributed substantively to it, and critically reviewed the submitted manuscript. Participants included health economists with an interest in equity-informative economic evaluation, ethicists specialising in public health and equity, and mathematical modellers from a variety of disease backgrounds (HIV/AIDS, tuberculosis, neglected tropical diseases, and malaria). Representatives of global health funding organisations (Bill &
3. Results

We summarise the discussion of methodological choices to be specified when including equity in transmission model-based priority setting for infectious diseases. First, we outline different definitions of equity, then we describe how equity has been considered in economic evaluation and transmission modelling to-date, with reference to our literature review (Drake et al., 2019). Given this background, we outline considerations for model structure and data needs for parameterisation.

3.1. Defining equity

The starting point for assessing equity in any priority setting task is to identify which definition of equity is being considered by decision-makers and analysts. While the terms inequity and inequality are sometimes used interchangeably to describe differences in distributions of outcomes, not all inequalities are inequities. Health inequity implies a value judgement about the fairness of systematic differences in population health. As there is no universal consensus as to what can be considered unfair, analysts must clearly identify which form of inequity is being considered, in consultation with those whose decisions they intend to inform. Particular care must be taken when defining equity for where the analysis and funding for any intervention is in part provided externally and foreign funders’ priorities do not necessarily align with those of domestic decision makers. Perspectives on the value judgements embedded in various equity concepts may differ (Revill et al., 2014). Various concepts of what fairness means and frameworks for identifying fair or unfair inequalities have been proposed that can be used as a starting point; for a recent summary of extant approaches taken to
Equity informative economic evaluations typically provide either analyses incorporating values around different distributions of impact and costs, or additional metrics that may be of interest to policy makers, such as poverty cases averted (Verguet et al., 2016). The way policy-makers and analysts define equity should determine the technical decisions in the analysis, such as metrics to be used, approaches for structuring the analysis and the model structure. For example, a transmission model could be employed to understand how to target vaccination by socioeconomic group or geography, with one optimum minimising population-level disease burden whilst another model is constrained to minimise differences in disease burden across strata. Alternatively, one country could seek to minimise the number of cases of an infectious disease, whilst others seek to minimise economic burden or maximise QALYs.

Depending on the type of equity being evaluated, assessing health inequity can include estimating and understanding the distribution of risk factors, health outcomes, costs, and access to care, and understanding the processes influencing choices and decisions to use services. Transmission models of infectious diseases will then have to consider heterogeneity in exposure to pathogens (including variation in transmission rates between groups), susceptibility to infection given exposure, and likelihood of disease progression given infection both before and after an intervention has been implemented.

Different definitions of equity may also require different forms of analysis and model outputs which require additional model development (Cookson et al., 2020). Analysts will have to translate the definition of equity and its ethical foundations into the selection of economic evaluation methods, and where relevant incorporate a social welfare function. In the context of economic evaluation in health care a social welfare function can be loosely defined as a function that allows to compare and rank different distributions of health in a population. This is usually done by aggregating and weighing individual health outcomes in a certain manner. An example of a social welfare function is an approach which values the health of all individuals equally regardless of socioeconomic position. The choice of social welfare function will depend on the target audience and analytic perspective. It is important to consider the perspective of the decision maker (Wilkinson et al., 2016), yet the values of the general population may differ from this and there may be attempts to align the two.

Two economic evaluation approaches that explicitly address equity are Extended Cost-Effectiveness Analysis (ECEA) and Distributional Cost-Effectiveness Analysis (DCEA) (Cookson et al., 2020; Verguet et al., 2016). ECEA disaggregates three groups of outcomes by socioeconomic group: health impacts, household cost impacts, and impact on financial risk protection. Understanding the latter requires defining a ‘catastrophic expenditure’ threshold as a level of patient-incurred costs which severely impact the household financially (Wagstaff, 2008). Such thresholds may be considered important by some policy makers trying to achieve Universal Health Coverage, and more generally may be considered a relevant metric to inform the improvement of outcomes for the least advantaged. DCEA is an approach defined as an umbrella term for “all types of cost-effectiveness analysis that provide information about equity in the distribution of costs and effects, and efficiency in terms of aggregate costs and effects” (Cookson et al., 2020) (p.4). In practice, several DCEAs have been conducted which attempt to show the equity impacts across a range of social welfare functions (Arnold et al., 2020; Aasia et al., 2016a; Love-Koh et al., 2021). Although ECEA may be considered a type of DCEA, DCEAs may differ in the nature or description of outcomes considered.

In summary, the starting point for equity informative evaluation requires defining the underlying concepts of equity that are of interest, and that then drive model design, in terms of structure, data needs and analysis. Given the potential complexity of defining the scope of the analysis, we strongly recommend that modellers, economists, and ethicists work together, collaborating with funders and decision makers, building from the broad evidence base and theoretical literature available (Jansen et al., 2018; Cookson et al., 2017). Further research and application is needed to explore the best processes and approaches for multi-disciplinary collaboration and working with decision makers to elicit definitions of equity, as well as the implications for model development in practice (Morton et al., 2018).

3.2. Approach to balancing equity with efficiency in priority setting

Economic evaluation is commonly implemented within wider priority setting or health technology assessment processes, which may balance efficiency and equity considerations in different ways. A key challenge is how best to weight equity and efficiency as part of these processes, particularly where they conflict with one another. For instance, if there are two weakly connected populations in respect of disease transmission, vaccinating the population with the higher force of infection may be less efficient at low levels of vaccine coverage due to lower herd (indirect) effects. However, this would become more efficient as vaccine coverage increases, because as elimination approaches in both subpopulations, the overall burden averted is higher. In this case an equity goal may clash with an efficiency goal at lower coverage, but not at higher coverage (Klepac et al., 2011).

Depending on the decision process, equity can therefore be characterised either as a constraint on efficiency, or as a parallel objective to be maximised in its own right. Equity may be seen as a one-directional constraint, where policymakers do not necessarily require interventions to reduce inequity but require them so as not to increase inequity. Where the values behind trade-offs are known a priori as part of the priority setting processes, they potentially can be incorporated algorithmically in a modelling exercise, through optimisation; for example, by weighting the benefits accruing among the poorest more than among the richest. When equity is considered to be an objective independent to efficiency, the weighting may also happen ex post to the analysis. In this case, equity outcomes are produced alongside efficiency outcomes and used to inform deliberative processes. Our literature review identified four broad approaches to balancing equity and efficiency in decision processes, noting these may not be mutually exclusive or exhaustive:

- **a) Equity-efficiency trade-off analyses**: Quantify trade-offs in equity against efficiency derived through the same transmission model, using a range of metrics;
- **b) Equity weights and social welfare functions**: Explicitly weigh different distributions of costs and outcomes and incorporate these weights into a single metric;
- **c) Parallel qualitative and/or quantitative criteria**: Allow a quantitative and/or qualitative consideration of equity alongside efficiency objectives;
- **d) Embedded frameworks and descriptive analysis**: Allow the simultaneous qualitative consideration of equity and efficiency objectives, alongside quantitative metrics, in the form of checklists and embedding frameworks or descriptive analyses.

These approaches in turn should define the analytical frame used and employed by the transmission model. Examples of each are shown in Table 1.

3.3. Transmission model structure

Once the definition of equity is defined and the analyst or modeller understands and characterises the decision process as above, the next stage is to design the model. The first step is to determine which population characteristics to represent explicitly in the transmission model structure, and how they interact. In some cases, the population groups...
for which it is important to explore equity implications are the same as those for which it is important to model transmission and health impact; however, this will not generally be the case. In principle, models should aim for the most parsimonious structure given the available data and question being addressed. However, the inclusion of equity in transmission models adds dimensional complexity, which is likely to lead to very large data and computational demands. To capture both equity and transmission across population groups, the model may need to identify differences in populations along three dimensions, as follows:

1) Heterogeneity in risk of infection and/or transmission. These are normally incorporated in models by allowing behaviour or other risk factors to vary between groups to model infection dynamics. This includes access to care or care seeking behaviour for preventative or curative services. Considering these heterogeneities allows for understanding of different dynamics across risk groups, alongside considerations of targeting interventions, for example among high-risk groups.

2) Heterogeneity in risk of disease given infection. Infection and disease are not the same and more vulnerable groups may experience greater health loss for the same degree of infection. Inclusion of heterogeneous risk of disease allows for the consideration of strategies with variation in access to treatment. Additional dimensionality is needed if having the disease increases transmissibility.

3) Heterogeneity in characteristics relevant to equity required to assess the distributional impact of interventions across unequal groups.

Although socioeconomic status, perhaps measured through wealth or income quintiles, is commonly used, other population characteristics may be important to include in transmission models, depending on the pathogen – for example, urban and rural areas or subnational regions, gender, sexual orientation, and ethnicity. The decision about which equity-related characteristics to include should be based on data regarding which important disease, health or social heterogeneities exist by group, or which groups are of specific interest to societies or policymakers.

These dimensions may operate independently, but in some cases risk and equity are correlated and perhaps causally linked. For example, if household income is related to number of children, then income status is related to age, which is related to schooling and infection risk. However, in practice modelling is complicated by endogeneity, where many such relationships are unobserved or inaccurately measured. The structure of the analysis will depend on how fully integrated the equity dimension is within the transmission model and if data allow causal relationships to be measured with confidence. Choices around model structure should be clearly reported and justified. Different structural choices are presented below in Table 2, describing data and model complexity requirements.

A) Cases distributed through equity dimensions post-simulation: Perhaps the simplest approach to include equity dimensions in a model is to produce a post-simulation distribution of cases across equity sub-groups, as in (Verguet et al., 2015, 2013; Rheingans et al., 2012), assuming that the distribution of risk after modelling the impact of an

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intervention would be the same as before, or that the change in risk of relevant outcomes given infection is the same before and after the intervention. The total impact, including impact due to reduced transmission, would be estimated by the transmission model with an equal reduction in infection risk in all sub-groups. This approach is limited, in that transmission dynamics of equity-related sub-groups, for example indirect benefits of reducing transmission in high incidence groups, cannot be incorporated. This approach also requires the assumption that infection risks are not causally related to equity factors.

B) Cases distributed through equity dimensions with parallel unlinked models: A more complex approach is to run parallel, unlinked transmission models for each equity-related sub-group, for example one model per socioeconomic quintile. This approach requires minimal adaptation of the transmission model but allows the incorporation of non-linearities between how equity-related sub-groups may respond to interventions. Each equity-related sub-group is effectively treated as a separate island population, meaning that non-linearities in the relationship between interventions and their effect on transmission would be simulated. Unlike the above example, this means that the post-simulation distribution of disease burden across equity-related sub-groups might differ from the prior situation with respect to the distribution of risk. However, a strong assumption is required around mixing, where between-equity sub-group mixing is assortative (i.e. the rich mix only with the rich). This is a substantial limitation to exploring equity dynamics, as in the presence of between-group mixing the vulnerable group can continually reintroduce infection to the less vulnerable group. In this case, more equitable interventions may benefit the better-off group in the long run.

C) Cases distributed through equity dimensions integrated into model – independent risk and equity heterogeneities: A step further is to integrate equity-related heterogeneity into the transmission model, but not structurally linked with risk-related heterogeneity. For example, a factor which influences susceptibility (e.g. age) is included in the model, but the distribution of this factor is assumed constant across equity-related sub-groups (i.e. the age distribution is the same in all equity sub-groups). Each equity-related sub-group is assigned a transmission sub-model with group-specific prior information on risk. In this case, a contact matrix is defined to specify contact probabilities between different equity-related sub-groups. Structurally, this is more realistic than the previous island populations approach but obtaining data on mixing between-equity-related sub-groups can be challenging. This approach could be important when modelling strategies for targeting the intervention to one or more equity-related sub-groups.

D) Cases distributed through equity dimension(s) integrated into model – correlated risk and equity heterogeneities: Finally, the integration of equity-related heterogeneity into the transmission model completely would allow for equity and risk dimensions to be structurally linked where any number of contact heterogeneities would be fully integrated and quantified. This may be in one dimension, for example allowing pre-existing conditions which influence susceptibility to vary by equity-related sub-groups. Although this model structure is the ideal to explore the link between equity and disease, in practice a) data to infer a causal relationship between factors are rarely available to parameterise all relationships, and b) incorporating increased layers of heterogeneity quickly becomes cumbersome for compartmental models, meaning that individual-based models may be more appropriate, although they have different methodological and practical drawbacks, for example requiring complex calibration procedures. Both approaches would have similarly extensive data requirements.

It is therefore critical to make practical decisions about which heterogeneities matter with regard to equity. For example, the risk of infection may be slightly different between the richest and the poorest in a population, but care-seeking behaviour may differ more significantly, perhaps due to a lack of affordable care or geographical variation in good quality care. It is critical to identify the source of inequities across susceptibility, transmission, care seeking or other factors – modelling full socioeconomic heterogeneity in transmission is more likely to require multidimensional contact matrices and numerous data-free assumptions, compared to disaggregating care-seeking by the same socioeconomic indicators.

3.4. Data requirements

As in any modelling analysis, sufficient quality and quantity of data is paramount. When considering how to integrate equity dimension in models, it is important to note that the data required to support analyses increase with complexity; Appendix 1 provides technical detail on this. Data gaps are of concern in analyses using transmission models, and one first step in the use of modelling is to identify key gaps in data that drive results to policymakers’ queries. If there are substantial data gaps, these should be clearly reported and explored in sensitivity analyses.

On data quality, dimensions of equity can be highly correlated with, or endogenous to, epidemiologically important variations. Almost all data are observational, and it is difficult to isolate how, for example, susceptibility to infection changes in relation to socioeconomic status independent of geography, gender or ethnicity. It is likely that any equity-informative transmission model will make simplifying assumptions using observational data – when model structures are simple, benefits from transparency may outweigh costs from biases due to simplifying assumptions, but this may not be the case with complex model structures.

Although it is more likely that sub-group-level data on disease risk and care seeking are available, more likely to be unknown is the extent to which differential risks of susceptibility, transmissibility, disease progression, and care seeking. However, models can explore combinations of those parameters that are consistent with the observed disease risk. For example, an HPV model showed that the increased risk of HPV infection in smokers compared to non-smokers can be explained by increased sexual activity in smokers and assortative mixing, without needing any biological mechanism between smoking and HPV susceptibility or persistence as was previously thought (Lemieux-Mellouki et al., 2016).

Furthermore, mixing between groups can be altered by the disease and the intervention. Many non-pharmaceutical interventions in the COVID-19 pandemic permitted “essential” workers such as delivery drivers and healthcare providers to work as normal, thus increasing the relative exposure of some professions. Cross-sectional surveys are not sufficient to measure evolving behaviours and responses to interventions and diseases, meaning that data requirements are both longitudinal and detailed. Although the data landscape is changing rapidly, as shown by the OpenSafely collaboration in the UK which has provided unprecedented real-time and granular data during the COVID-19 pandemic (Williamson et al., 2020), such data remain the exception and not the rule, particularly in low- or middle-income settings. On the other hand, at present, there is no clear need for such data, and the development of models and policy that requires these data will drive its collection.

Finally, an additional data requirement to consider the full equity impact of resource allocation decisions is an understanding of how opportunity costs of new interventions, such as disinvestments in current services, are distributed. Although substantial ground has been covered in identifying opportunity costs of health care interventions, much still needs to be researched such as identifying how opportunity costs fall upon particular subgroups.

4. Conclusion

The growing interest in strengthening methods to include equity considerations in priority-setting processes may help guide steps towards achieving the Sustainable Development Goals and help monitor progress towards universal health coverage targets. Infectious disease transmission models are critical to inform priority setting decisions in
many settings; however, to date there has been little inclusion or reporting of equity aspects. Existing methodological frameworks for economic evaluation are compatible with transmission models but require additional care when applied to a disease transmission model.

In this statement, we describe analytical and conceptual choices. We highlight the need for transparency in reporting how and why particular design decisions were taken around analysis structure and encourage analysts to evaluate data requirements comprehensively before undertaking transmission model-based economic evaluations. We propose four steps to guide planning and reporting of such analysis: 1) choice of equity concept(s); 2) choice of approach for assessing equity and efficiency; 3) choice of model structure; and 4) data needs.

Between-country equity considerations fall outside the scope of this article. From a global perspective, it is still unclear how to incorporate equity-informative economic evaluations to be useful in priority setting. This collaboration should be fostered to advance the appropriate application of equity-informative economic evaluation methods suitable for use with dynamic disease models.

Financial disclosures/ funding statements

This work received funding support from Bill & Melinda Gates Foundation, the UK Department for International Development, and the Rockefeller Foundation. All authors have declared that they have no competing interests. GBG is currently employed by Sanofi Pasteur. Sanofi Pasteur did not provide funding for this work and had no role in study design, data collection, data analysis, data interpretation, or writing of the report.

CRediT authorship contribution statement


Declarations of interest

none.

Acknowledgements

The authors are grateful to Richard Cookson, Anthony Culver, Kara Hanson, Francisco Pozo-Martin, and Stephane Verguet, and all participants of the meeting held in London March 2018.

Appendix 1. Data considerations

Contact data requirements

Let $\beta(i,j)$ be the contact rate between groups $i$ and $j$ in the risk dimensions, and $\varphi(a,b)$ be the contact rate between groups $a$ and $b$ in the equity dimensions. Suppose there are $n$ risk groups and $m$ equity groups.

In a first simple approach – where cases are distributed through equity dimensions post-simulation only – $\beta(i,j)$ is defined, which has $n^2$ elements (i.e. parameters).

In the second approach (where cases are distributed through equity dimension(s) with parallel unlinked models), it is possible to define $\beta(i,j)$ for each equity group, $\beta(i,j,a)$ but $\varphi(a,b)$ is undefined. The extended contact matrix has $m^2n^2$ elements.

In the third approach (where cases are distributed through equity dimensions integrated into the model, but risk and equity heterogeneities are assumed to be independent $\beta(i,j,a)$ and $\varphi(a,b)$ are both defined and the risk of infection for an individual in risk dimension i and equity dimension a is given as:

$$k(i,a)\sum_{j} \beta(j,i,a) + \sum_{b} \varphi(a,b)$$

The maximum number of elements for this structure is $m^2n^2 + m^3$.

In the fourth (where cases are distributed through equity dimension(s) integrated into the model and risk and equity heterogeneities are assumed to be correlated) and most complicated approach, the rates of infection between individuals are defined jointly by their risk and equity heterogeneity:

$$k(i,a)\sum_{j} \varphi(j,i,a,b) \text{ where } \varphi(j,i,a,b)$$

is the contact rate between an individual in risk group $i$ and equity group $a$ with an individual in risk group $j$ and equity group $b$. The maximum number of separate parameters is $n^2m^3$.

To give a numerical example, if we had 5 risk groups and 3 equity strata, then the maximum number of parameters for each of the four approaches would be 25, 75, 184 and 225 respectively. One simplification may be to just take a between-subgroup and within-subgroup contact rate since it may be difficult to estimate the full subgroup transmission matrix.