Gest. Prod., São Carlos, v. 24, n. 4, p. 790-805, 2017
http://dx.doi.org/10.1590/0104-530X1767-16

Optimization approaches to a routing and scheduling problem of oil tankers

Abordagens de otimização para um problema de roteirização e programação de navios petroleiros

Vinícius Picanço Rodrigues[1]
Reinaldo Morabito[1]
Denise Yamashita[1]
Bruno Jensen Virginio da Silva[1]
Paulo Cesar Ribas[2]

Abstract: This study analyzes a routing and scheduling problem of cabotage oil ships motivated by the actual operation of an oil company along the Brazilian coast. Maritime transportation costs from offshore platforms to coastal terminals are an important issue in the search for operational excellence in the oil industry, and the prospects for growth in oil exploration in Brazil have made operations more demanding for agile and effective decision support systems (DSS). This paper presents an optimization approach to deal with this problem consisting of a mixed integer linear (MIP) programming model and an MIP heuristic known as relax and fix. The problem is formulated as a pickup and delivery vessel routing with time windows and heterogeneous fleet which minimizes the costs of fuel consumption of ships and freight contracts. In addition to the usual routing constraints, it also considers specific restrictions of oil maritime transportation problems. Numerical experiments with this approach are presented for a set of real data of the company, confirming that the optimization method is able to find good solutions for moderate-size problem instances.

Keywords: Vehicles routing and scheduling; Pickup and delivery; Maritime transport; Oil industry; Relax-and-fix; MIP heuristics.

Resumo: *Este artigo estuda um problema de roteirização e programação de navios para cabotagem de petróleo, motivado pela operação real de uma empresa no litoral brasileiro. Os custos de transporte desempenham um papel importante na busca pela excelência operacional na indústria de petróleo e as perspectivas de crescimento na exploração de petróleo no Brasil têm tornado as operações mais demandantes de sistemas de apoio a decisões ágeis e eficazes. Neste artigo, apresenta-se uma abordagem de otimização para tratar este problema, composta por um modelo de programação linear inteira mista e uma heurística baseada em programação matemática, conhecida como relax-and-fix. O modelo proposto é inspirado em uma formulação de problemas de coleta e entrega com janelas de tempo e frota heterogênea, que minimiza custos decorrentes do consumo de combustível dos navios e dos contratos de afretamento. Além das restrições usuais de roteirização com coleta e entrega, este artigo considera as restrições específicas deste problema de transporte de petróleo. Experimentos numéricos com esta abordagem são apresentados para um conjunto de dados reais fornecidos pela empresa, os quais comprovam o potencial da abordagem para encontrar boas soluções para instâncias de tamanho moderado.*

Palavras-chave: *Roteirização e programação de veículos; Coleta e entrega; Transporte marítimo; Petróleo; Relax-and-fix; Heurísticas baseadas em programação matemática.*

1 Introduction

Maritime transport moves large volumes of cargo between large distances as, for instance, in certain countries and the import and export operations. In 2012, approximately 8.7 billion tons of goods were transported across the oceans, which means that maritime transport is responsible for about 80% of

[1] Departamento de Engenharia de Produção, Centro de Ciências Exatas e Tecnologia, Universidade Federal de São Carlos – UFSCar, Rodovia Washington Luiz, Km 235, São Carlos, SP, Brazil, e-mail: vinicius@dep.ufscar.br; morabito@ufscar.br; dsyamashita@gmail.com; brunojen@gmail.com

[2] Centro de Pesquisas e Desenvolvimento – CENPES, Petróleo Brasileiro S.A. – PETROBRAS, Av. Horácio de Macedo, 950, Ilha do Fundão, Rio de Janeiro, RJ, Brazil, e-mail: paulo.ribas@petrobras.com.br

Received Nov. 18, 2015 - Accepted Sept. 16, 2016

Financial support: Brazilian National Agency of Petroleum, Natural Gas and Biofuels (ANP), São Paulo Research Foundation (FAPESP) and the Brazilian National Council for Scientific and Technological Development (CNPq).

the volume and 70% of the value of goods traded internationally (UNCTAD, 2012; Diz, 2012). One of the most important economic activities occurring in Brazil's oceans is oil exploration. The country is one of the largest producers of oil, with a daily production of approximately 2.5 million barrels. The estimated reserves exceed the 12 billion barrels and put the country in a privileged position on the global stage. Most of the Brazilian oil is exploited in offshore, which corresponds to about 80% of the total volume operated (ANP, 2012; IEA, 2013).

Another important factor is the exploration of the pre salt layer, which is a pickup of rocks located in very deep waters of great part of the Brazilian coast, which has a great potential for oil exploration. This depth can reach over 7,000 meters - the term "pre" is used because these rocks were deposited before the salt layer of the ocean floor. Petrobras, the largest Brazilian oil company, produces near 470,000 barrels of oil daily from this region and it is estimated that the production will reach 1 million barrels per day in 2017. To stimulate the commercialization of oil products and the discovery of new sources of exploration, the Brazilian oil industry faces a considerable increase in competition and, consequently, the sector's companies have begun to seek improved agility and efficiency in the decision-making process and to develop strategies in order to organize their activities aiming at a better economic outcome, especially related to transport operations of these commodities (Pucu, 2011; PETROBRAS, 2014a, b, d). Thus, the oil sector requires versatile operations from the refineries that are receiving oil to be processed and this, in turn, unfolds in large volume and more rigid deadline requirements for logistics (Christiansen et al., 2007; Hennig et al., 2012).

In this work, we propose and analyze a routing and scheduling problem of oil tankers from offshore platforms to supply terminals in the Brazilian coast. The problem is majorly motivated by operations of an oil company in Brazil. An optimization approach based on mixed integer programming is presented to properly represent the oil pickup at the platforms and delivery at the terminals in the context of this company. A relax-and-fix heuristic is also explored using the GAMS/CPLEX optimization package, with a view to capturing business requirements and specific aspect of the company's operations. Although there are other related works in the literature along this strand of research, authors are not aware of other studies exploring optimization models and solution methods addressing this pickup and delivery problem with all its particular operating features, except for our other work in Rodrigues (2016) that also proposed a time decomposition heuristic procedure for this problem. These studies were conducted in strong collaboration with a Brazilian oil company, so that the problem could be well defined both in the terms of market characteristics and validity of practical operations. The mathematical model captures fundamental characteristics of this real system, therefore making it more difficult to obtain optimal solutions considering the size of the company's real instances, a fact that also motivated the development of MIP-heuristic procedures.

The structure of the paper is established according to the steps taken in our research approach. Initially, Section 2 presents a succinct literature review, while Section 3 describes the characteristics of the oil pickup and delivery problem based on the operations of the case company. Furthermore, Section 4 presents the MIP model that represents the problem, with Section 5 describing the heuristic method based on relax-and-fix we propose in this paper. Section 6 presents and analyzes the results to solve some problem instances based on real data from the company. Some concluding remarks and future prospects are finally presented and discussed in Section 7.

2 Brief review of the literature

Among the reviewed works of the literature related to this study, we highlight Christiansen (1999), which presented a maritime inventory routing problem for picking up and delivering ammonia with time windows, predefined routes, inventory control and minimization of shipping cost, namely Inventory Pickup and Delivery Problem with Time Windows (IPDPTW). Christiansen (1999) and the references therein provide different views on the general modeling of ship routing problems, which served as a basis for this work. Al-Khayyal & Hwang (2007) also studied a maritime inventory-routing problem with heterogeneous fleet and routing cargo, where ships' compartments and the decisions of the quantities to be loaded/unloaded are also considered.

Rocha et al. (2009) dealt with a related allocation problem of oil in the context of a Brazilian oil company, which in addition to the product allocation plans, determines boarding plans for oil tankers from platforms to terminals, in a more aggregate level (tactical), without detailed considerations of the routing and scheduling of ships. Hennig et al. (2012) studied a routing problem in oil maritime transport with load splitting, which employed a specific procedure for pre-generating routes. The work of Christiansen et al. (2013) offered a survey for ship routing and scheduling.

Table 1 provides a summary of the main features of some studies on routing applied to maritime shipping, detailing the main applications, methods and models used. As mentioned, no papers were found in the literature that presented models and solution methods for addressing the present operational problem of ship routing and scheduling applied to oil cabotage with the characteristics of the practical operation here considered, as described in the next section.

Table 1. Some studies on routing applied to maritime shipping.

Author	Application	Model / Method
Dantzig & Fulkerson (1954)	Scheduling of American Navy ships	Linear optimization
Flood (1954)	Transportation of oil by an American military fleet	Linear optimization
Briskin (1966)	Determination of dates and delivery volumes for oil tankers	Scheduling with predefined routes
Appelgren (1969)	Assigning loads to ships with cost minimization	Multi-commodity flow, Dantzig–Wolfe decomposition, and column generation
Appelgren (1971)	Extension of the 1969 article	Method based on cutting plans and branch-and-bound
Bellmore et al. (1971)	Extension of the work of Dantzig and Fulkerson (1954) with heterogeneous fleet	Relaxation of fixed program of visits, decomposition into subgraphs and branch-and-bound
McKay & Hartley (1974)	Oil transportation in military ships around the world	Mixed integer programming and variations with predefined routes
Ronen (1986)	Scheduling for short-term for transport of commodities	Allocation loading algorithms with random bias
Brown et al. (1987)	Oil transportation for export to Europe and the USA from Middle East.	Model based on set partitioning
Perakis & Bremer (1992)	Scheduling of ships in Chevron company	Mathematical formula based on Appelgren (1971) and construction of a "schedule generator" to build feasible programs
Haugen (1996)	Natural gas distribution in Europe	Stochastic dynamic programming applied to programming projects (Stochastic Project Scheduling Problem)
Sherali et al. (1999)	Scheduling oil and natural gas exporters ships in Kuwait	Based on Brown et al. (1987), presents exact and heuristic method for rolling time horizon
Christiansen (1999)	Problem with pickup and delivery of integrated disposal of ammonia in a Norwegian company	Mixed integer model method with column generation
Lasschuit & Thijssen (2004)	Strategic planning of global supply chain of oil and chemical industry	Programming nonlinear mixed integer based on GMOS / NetSim
Persson & Göthe-Lundgren (2005)	Scheduling ships for bitumen runoff	Mixed integer model with valid inequalities and columns generation
Al-Khayyal & Hwang (2007)	Maritime transport of various liquids in bulk	Programming mixed integer linear and non-linear
Gribkovskaia et al. (2007)	Provisioning of oil and gas platforms in the Norwegian Sea	Constructive heuristics and algorithm based on tabu search
Rocha et al. (2009)	Oil allocation to the logistics process of Petrobras	Programming mixed integer and heuristic based on local search
Brønmo et al. (2010)	Scheduling pickups and deliveries of ships with load flexibility	Programming integer with Dantzig-Wolfe decomposition
Kobayashi & Kubo (2010)	Operational scheduling of an oil cabotage fleet in Japan	Programming integer and heuristics developed by the own authors
Andersson et al. (2010)	Bibliographic review of models of inventory routing problem type integrated with inventory management, with emphasis on practical aspects of industrial applications for marine and land modes	
Hoff et al. (2010)	Continuation of the work of Andersson et al. (2010), with a focus on studies that integrate routing problems with fleet composition	
Hennig et al. (2012)	pickup and delivery problem with split loads	Mixed integer programming with pre-generated routes

Table 1. Continued...

Author	Application	Model / Method
Stålhane et al. (2012)	pickup and delivery problem with split loads	Method branch-and-price-and-cut
Song & Furman (2013)	IRP type models	Proposal for a flexible framework to guide the applications of models based on IRP with formulation of multi-commodity flow
Christiansen et al. (2013)	Recent bibliographic survey of routing problems and scheduling of ships and related problems that illustrate the main areas of the area.	
Agra et al. (2012)	Problem of oil distribution of short distance between the islands of the archipelago of Cape Verde	Mixed integer programming model with search strategies to better limits, extension of formulations and valid inequalities
Agra et al. (2013a)	Routing of single product inventory problem with variable rates of consumption and production over the planning horizon	Mixed integer programming model with two discrete formulations and use of relaxations based on sizing to propose valid inequalities
Agra et al. (2013b)	Same problem addressed by Agra et al. (2012)	Formulation based on arc-load flow with use of valid inequalities and hybrid heuristics (rolling horizon, local branching and feasibility pump)
Stålhane et al. (2015)	Pickup and delivery problem with time windows, load combination and synchronized delivery for specialized and unique loads	Method branch-and-price with subproblem based on a shortest path problem with resource restrictions, solved by dynamic programming
Rakke et al. (2014)	Routing problem of inventory for one of the largest global producers of liquefied petroleum gas (LPG)	Formulation based on the decomposition of the problem for patterns of demand with usage of method branch-price-and-cut

3 Problem definition

The problem can be described from its main components: different products, heterogeneous fleet, scattered offshore platforms and terminals along the coast. One of the main challenges faced by the maritime transport operation of offshore oil is the scheduling of ships to meet a set of move requests with origins, destinations and pre-established amount of products, which were defined by the logistics' tactical planning. Pickups at platforms and deliveries at terminals should be performed at predefined time windows over a planning time horizon (e.g., several days to a few weeks). An important aspect of the problem is the consideration of oil inventory on platforms and terminals, an element which is dealt implicitly in this model through the time windows. These time windows are defined in a more aggregated level planning, where the variations and limitations of oil inventory are considered. Platforms hold their own inventory, which may not exceed a specified maximum operating limit (called "*top"* by the operators), since it results in a halt of platform production and generates prohibitive opportunity costs. This situation arises as a major concern for the company's decision-makers. These tops of the platforms are strictly considered when determining the time windows' length, as well as ballast or minimum oil inventory levels of the platforms. The opening of time windows is calculated based on the oil batch (predefined size) to be collected. Once the respective oil amount is available, the windows open. Additionally, the closing of time windows uses the tops of the platforms, which are calculated according to the inventory level, maximum storage capacity and production rate, alongside a safety margin. Similarly, the definition of terminals' time windows takes into account the limitations of inventories and specific demands of terminals.

The problem can be represented as a pickup and delivery vehicle routing with time windows, heterogeneous fleet, multiple deposits (initial locations of each ship), multiple products (each platform produces a different oil), multiple visits to each platform and terminal throughout the planning horizon, with additional restrictions that include specifics of the operation in the case study, as described below. A well-known formulation for the pickup and delivery problem with time windows in the literature (Cordeau et al., 2007) was chosen as a starting point for modeling this problem.

The product consists solely in crude oil, which is divided into over 50 different subgroups. According to

PETROBRAS (2014c), each one of the platforms produces a very specific type of oil, which is different from the others as it is a natural product, whose composition and chemical structure depends heavily on the geographical location of the exploration field. However, it is possible to treat the problem of multiple products as a single-product (single commodity) when using a pickup and delivery formulation, since the origin and destination of each product type are predefined by each pair of origin-destination, with their ordered oil amounts as an input data for the model. It is noteworthy that the maritime transport of oil-derived products rather than crude oil (such as gasoline, diesel fuel, jet fuel etc.) is out of the scope of this paper.

The ships fleet is very heterogeneous, with different operating costs, average speeds and capabilities. Furthermore, the ships feature two other characteristics that define their ability to berth in certain ports, which are the ship draft and its length, or LOA (length overall). The draft is defined as the vertical distance, taken on a transversal plane, between the lower end of the ship and the line determined by the intersection of the surface of the water with the outer surface of the ship's hull. LOA, also known as "length of wheel to wheel", refers to the distance between the salient points of the front and rear of the ship, its total length (SOBENA, 2013). The LOA and the draft of the ship may impose restrictions on their ability to approach a terminal. Figure 1 shows an illustration of these two metrics. Some ships and platforms also present a system called dynamic positioning, or DP, which according to Sørensen (2011) and UFRJ (2013) maintains floating structures in fixed position or pre-defined tracks for marine operation purposes through the use of active thrusters. About 50 different platforms are potentially considered in this paper. There are platforms that allow berthing of conventional ship and platforms that allow only the berthing of ships with DP. To be able to berth on platforms that allow only ships with DP, a maximum capacity on board must be respected in order to reduce the risks of the operation.

There are about 10 company's proper terminals along the Brazilian coast, with around 20 berths in total, distributed heterogeneously by these terminals. The berths are specific locations inside maritime terminals, where ships dock in order to perform the loading and unloading of cargo. Each one of the berths presents physical restrictions for draft and LOA that must be met so that the ships are allowed to berth (each ship occupies a single berth). However, in practice, the draft restriction may be relaxed in some specific cases, and this is done by limiting the load on board to a value lower than the maximum capacity of the ship. Both platforms and terminals are technically called and referred to as "operating sites". The pairs of pickup and delivery are pre-established by tactical planning, but routing and scheduling of ships is configured as a decision to be supported by the model. This modeling approach is called "origin-destination", since each one of the origins (platforms) is pre-matched with its respective destination (terminal). Importantly, this pickup and delivery problem differs from most cases in relation to the maritime transport of oil, which typically involve large distances. In most cases of oil exploration around the world, the transport occurs in several producing companies for several refineries with different rules and responsibility governing the freight. In Brazil, the same company produces, refines and plans the transportation, which considerably increases the possibilities of logistical gains. It is possible to find similar characteristics in the operations performed in the North Sea and the Gulf of Mexico.

4 Problem formulation

The formulation presented here uses a known formulation for the pickup and delivery problem with time windows as presented in Cordeau et al. (2007), complemented with specific aspects directly derived from the company's business rules. As it is common to have multiple origin-destination pairs scheduled involving the same platform or terminal depending on the length of the planning horizon, at each visit the ship collects the ordered amount programmed in

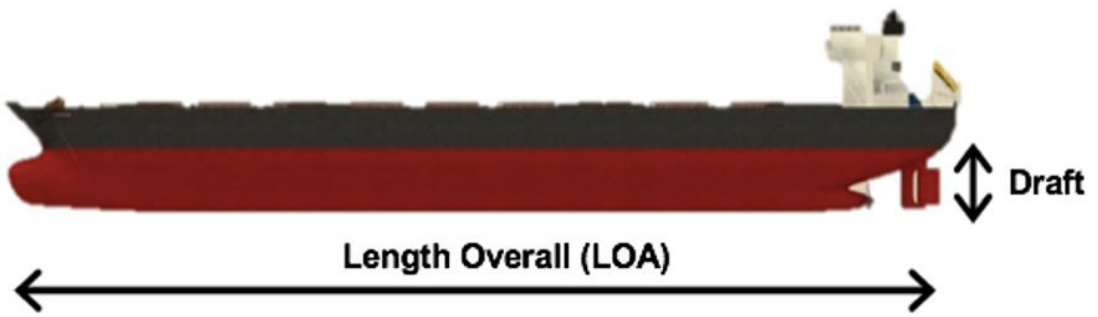

Figure 1. Illustration of an oil tanker with their respective markings of LOA and draft. Source: Adapted from SOBENA (2013).

the batch on the platform and delivers this amount to the corresponding terminal of the origin-destination pair. For modeling convenience, each pickup and each delivery is labeled and represented by a different node, even when the pickups and deliveries are made on the same operating site, forming the graph representing the problem. Therefore, two nodes in the graph may represent the same operating site, whereas they differ with respect to the specific pickup or delivery being performed. Figure 2 shows a schematic problem representation in which the red nodes represent artificial deposits (initial and final locations) of the ship, while the black nodes refer to visits in operating sites.

It can be seen in Figure 2 the strategy of duplication of nodes for scheduled visits. In this case, a ship leaves the origin artificial depot at node 1, performs a pickup on the platform at node 2 and deliver it to the terminal on node 3. Subsequently, the ship visits the same platform for another pickup, now represented by node 4, to deliver the product at the terminal of node 5 and finishes its route in the artificial depot, represented as node 6. Notice that nodes 2 and 4 refer to the same operating site, but designate different pickups within the planning horizon. It is also noteworthy that the initial and final illustrations of fictitious depots exhibit an arbitrary placement only for the purpose of diagrammatic representation. In practice, the ships usually begin and end their work shifts near terminals or platforms, in areas known as "anchoring zones", which host idle ships that are waiting for requests. However, the modeling also considers situations where at the beginning of the planning horizon, some ships may lie outside of the anchoring zones. It should be pointed out that the ships travel by routes over the planning horizon,

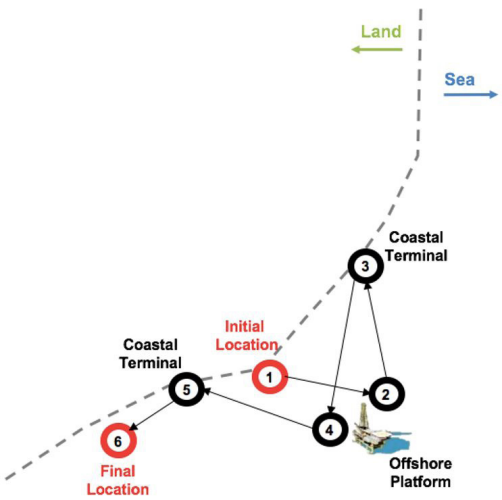

Figure 2. Schematic representation of the replication strategy of nodes and artificial deposits adopted in the problem modeling.

instead of itineraries, since they do not have to start and finish at the same site (depot).

The indices, sets and parameters of the model are:

Indices

- k refers to ships ($k \in K$). The total number of ships is $|K|$;
- i, j refer to operating sites (platforms or terminals) and depots.

Sets

- n is the number of pairs of pickup and delivery (origin-destination);
- $C_1 = \{1,2,...,n\}$ is the set of nodes where pickups are made (origins);
- $C_2 = \{n+1, n+2,..., 2n\}$ is the set of nodes where deliveries are made (destinations); nodes in C_2 are labeling as follows: for each $i \in C_1$ there is a matching node $(i+n)$, wherein node $i \in C_1$ refers to the origin and node $(i+n) \in C_2$ refers to its destination $(i+n)$. Therefore, $n = |C_1| = |C_2|$;
- $C = C_1 \cup C_2$ is the total set of nodes that represent platforms and terminals, $|C| = 2n$;
- $S = \{s_1, s_2,..., s_k\}$ is the artificial set of nodes that indicate the start node S_k of ship k;
- $E = \{e_1, e_2,..., e_k\}$ is the artificial set of nodes that indicate the final node e_k of ship k;
- Set $N = C \cup S \cup E$ represents all nodes, with $|N| = 2n + 2|K|$;
- $A(i,k), \forall i \in C, \forall k \in K$ is a 0-1 matrix that indicates whether the ship k cannot berth at node i (equal to 1), either due to draft, LOA or other reasons, or if it can berth (equal to 0);
- C_{DP} is the set of platforms that feature dynamic positioning;
- K_{DP} is the set of ships k that feature dynamic positioning;
- $CFlex_{ik}$ is the set of pairs (j,k), where that allow draft flexibility for berthing of ship k;

Parameters

- v_k is the average speed, in knots, of ship k;
- $dist_{ij}$ is the distance, in nautical miles, between node $i \in (S \cup C)$ and node $j \in (C \cup E)$. For representing consecutive pickups or deliveries that are performed in the same operating site

(platform or terminal), the corresponding distance is set to zero (i.e. $dist_{ij} = 0$);

- ts_{ik} is the service time, in hours, at node $i \in C$ serviced by ship k;
- $[a_i, b_i]$ is the time window, in hours, on the start of service at node $i \in N$, in which a_i represents its lower bound, and b_i the upper bound;
- d_i is the load amount, in m³, to be picked up or delivered at node $i \in C$, which may have positive or negative values. If this demand is positive, it indicates that in operating site i, d_i units of product must be picked up. If this demand is negative, it indicates that in operating site i, d_i units of product must be delivered. Note that by convention $d_{i+n} = -d_i$;
- Cap_k is the capacity of ship k, in m³;
- Cm_k is the daily fuel consumption of ship k in motion, in monetary units ($), defined in specific freighting contracts by the company;
- Cs_k is the daily fuel consumption of ship k while in standby, in monetary units ($), also defined in specific freighting contracts by the company;
- Ca_j is the fixed cost of berthing in platform j, in monetary units ($). If the ship performs consecutive pickups on the same platform j, a fixed cost is paid only once, when the ship is berthed;
- β is a penalty imposed to consecutive visits of the same ship on different platforms in its itinerary, which is undesirable for the company and should, if possible, be avoided;
- α_1^{jk} is the percentage of the maximum load allowed for ship k to berth at terminal $j \in CFlex_{jk}$;
- α_2^{jk} is the percentage of the maximum load allowed for ship k with DP, $k \in K_{DP}$, to berth at conventional platform $j \notin C_{DP}$; This same percentage is valid for the case where the ship with DP $k \in K_{DP}$ berths on the platform with DP $j \in C_{DP}$;
- α_3^{jk} is the percentage of the maximum load allowed for the conventional ship k, $k \notin K_{DP}$, to berth at platform with DP $j \in C_{DP}$;
- M is a positive sufficiently large number used in the linearization of some restrictions;
- Variables
- x_{ijk} is a binary variable that takes value equal to 1 if ship k runs through arc $(i, j), i \in (S \cup C), j \in (C \cup E)$ and 0, otherwise;
- f_{ik} is a non-negative real variable that indicates the start time of service on node $i \in N$ by ship k;
- y_{ik} is the load amount on ship k at the moment immediately after the scale on node $i \in N$, represented by a non-negative real variable. In this model, for simplicity, it is assumed that the ship begins and ends empty within the planning horizon, but the model can be adapted to consider more general situations;
- Fa_{ijk} is an auxiliary variable model representing the consecutive berthing from platform i to platform j with ship k, with $i \neq j$.

The mathematical formulation is:
Minimize

$$\sum_{i \in N} \sum_{j \in N} \sum_{k \in K} (Cm_k - Cs_k) \frac{dist_{ij}}{v_k} x_{ijk} + \sum_{i \in (S \cup C)} \sum_{\substack{j \in C \\ dist(i,j) > 0}} \sum_{k \in K} Ca_j x_{ijk} + \beta \sum_{i \in C_1} \sum_{j \in C_1} \sum_{k \in K} Fa_{ijk} \quad (1)$$

subject to:

$$\sum_{j \in (C \cup E)} \sum_{k \in K} x_{ijk} = 1 \quad \forall i \in (C \cup S) \quad (2)$$

$$\sum_{i \in (C \cup S)} \sum_{k \in K} x_{ijk} = 1 \quad \forall j \in (C \cup E) \quad (3)$$

$$\sum_{j \in (C_1 \cup e_k)} x_{s_k jk} = 1 \quad \forall k \in K \quad (4)$$

$$\sum_{j\in\left(s_k\cup C_2\right)} x_{ie_k k}=1 \qquad \forall k\in K \tag{5}$$

$$\sum_{i\in N}\sum_{k\in K} x_{ijk}=0 \qquad \forall j\in S \tag{6}$$

$$\sum_{j\in N}\sum_{k\in K} x_{ijk}=0 \qquad \forall i\in E \tag{7}$$

$$\sum_{i\in\left(C\cup S\right)} x_{ihk}-\sum_{j\in\left(C\cup E\right)} x_{hjk}=0 \qquad \forall h\in C;\forall k\in K \tag{8}$$

$$a_i\left(\sum_{j\in N} x_{jik}\right)\le f_{ik}\le b_i\left(\sum_{j\in N} x_{jik}\right) \qquad \forall i\in(C\cup E);\forall k\in K \tag{9}$$

$$a_i\left(\sum_{j\in N} x_{ijk}\right)\le f_{ik}\le b_i\left(\sum_{j\in N} x_{ijk}\right) \qquad \forall i\in S;\forall k\in K \tag{10}$$

$$f_{jk}\ge f_{ik}+ts_{ik}+\frac{dist_{ij}}{v_k}+\left(x_{ijk}-1\right)M \quad \forall i\in\left(S\cup C\right);\ \forall j\in\left(E\cup C\right);\ \forall k\in K \tag{11}$$

$$f_{j+n,k}\ge f_{j,k} \quad \forall j\in C_1;\forall k\in K \tag{12}$$

$$y_{jk}\ge y_{ik}+d_j+\left(x_{ijk}-1\right)M \quad \forall i\in\left(S\cup C\right);\ \forall j\in\left(E\cup C\right);\ \forall k\in K \tag{13}$$

$$y_{jk}\le y_{ik}+d_j+\left(1-x_{ijk}\right)M \quad \forall i\in\left(S\cup C\right);\ \forall j\in\left(E\cup C\right);\ \forall k\in K \tag{14}$$

$$\sum_{i\in(S\cup C)} x_{ihk}=\sum_{j\in C} x_{j,h+n,k} \qquad \forall h\in C_1;\forall k\in K \tag{15}$$

$$y_{jk}\le Cap_k\sum_{i\in(S\cup C)} x_{ijk} \quad \forall k\in K,\forall j\in(C\cup E) \tag{16}$$

$$\sum_{k\in K} y_{s_k k}+\sum_{k\in K} y_{e_k k}=0 \quad \forall k\in K \tag{17}$$

$$x_{ijk}=0,\forall i,k\in A\left(i,k\right) \tag{18}$$

$$y_{jk}\le(\alpha_1^{jk}Cap_k+d_j)+\left(1-\sum_{i\in(C\cup s_k)} x_{ijk}\right)M \quad \begin{matrix}\forall(j,k)\in CFlex_{jk}\\ \forall(j,k)\in A(j,k)\\ \forall k\in K\end{matrix} \tag{19}$$

$$y_{jk}\le(\alpha_2^{jk}Cap_k+d_j)+(1-\alpha_2^{jk})Cap_k\left(1-\sum_{i\in C} x_{ijk}\right) \quad \begin{matrix}\forall j\notin C_{DP}\\ \forall k\in K_{DP}\end{matrix} \tag{20}$$

$$y_{jk}\le(\alpha_3^{jk}Cap_k+d_j)+(1-\alpha_3^{jk})Cap_k\left(1-\sum_{i\in C} x_{ijk}\right) \quad \begin{matrix}\forall j\notin C_{DP}\\ \forall k\notin K_{DP}\end{matrix} \tag{21}$$

$$y_{jk}\le(\alpha_3^{jk}Cap_k+d_j)+(1-\alpha_3^{jk})Cap_k\left(1-\sum_{i\in C} x_{ijk}\right) \quad \begin{matrix}\forall j\in C_{DP}\\ \forall k\in K_{DP}\end{matrix} \tag{22}$$

$$\sum_{i\in C} x_{ijk}=0 \quad \forall j\in C_{DP},\forall k\notin K_{DP} \tag{23}$$

$$x_{ijk} \leq Fa_{ijk} \quad \forall i, j \in C_1, i \neq j, dist_{ij} > 0 \tag{24}$$

$$x_{ijk} \in \{0,1\}, \forall i \in (S \cup C), j \in (C \cup E); \forall k \in K \tag{25}$$

$$f_{ik} \geq 0, \forall i \in N \; ; \; \forall k \in K \tag{26}$$

$$y_{ik} \geq 0, \forall i \in N \; ; \; \forall k \in K \tag{27}$$

In the Objective Function 1, the first part incorporates variable fuel costs incurred during the movement of the ships, the second part represents the fixed costs of platforms berthing and the latest part penalizes consecutive berthings on different platforms due to its high associated cost. Restrictions 2 and 3 ensure that all operating sites are visited exactly once. Restrictions 4 ensure that all ships leave their origin artificial depot, s_k. If a ship does not perform any scale, it leaves the depot and goes directly to its final artificial depot, e_k. Restrictions 5 ensure, in turn, that all ships arrive at their final artificial depot, e_k. Restrictions 6 and 7 ensure that no ship enters the initial artificial depot and no ship goes out of the final artificial depot, respectively. Restrictions 8 ensure that if a ship *k* has reached a node referring to an operating site, this ship has to leave this node - also known as the flow conservation constraint. Restrictions 9 ensure that the upper and lower limits of the time windows of artificial depots and final operating sites are met. Similarly, Restrictions 10 ensure that the upper and lower limits of the time windows of origin artificial depots are met. Restrictions 11 limit the instant when ship *k*, coming from the operating site *i*, starts service on operating site *j*. These restrictions avoid subcycles in the model solution. Restrictions 12, in turn, guarantee that a certain load will have its pickup performed before the respective delivery. Restrictions 13 and 14 refer to the load balance of ship *k*. Note that these restrictions refer to linearization of the equality $x_{ijk}(y_{ik} + d_j - y_{jk}) = 0$, $\forall i \in (S \cup C), \forall j \in (C \cup E), \forall k \in K$. Restrictions 15 ensure that if ship *k* picked up load on node *h* (platform), then it needs to perform a visit on node $h+n$ (terminal). Constraints 16 ensure that the capacity of the ship is respected. The previous observation that the ship begins and ends empty is guaranteed by Restrictions 17. Restrictions 18 prevent ship berth at nodes where there are physical impediments, whether in draft, LOA or other operational rules – these limitations are guaranteed by the definition of matrix $A(i,k)$.

Restrictions 19 limit the load to the case in which the terminal presents draft restrictions for berthing ships. Restrictions 20 are used for the first case of dynamic positioning, on which there is a ship with DP berthing at a conventional or DP platform. Restrictions 21 are applied to the second case, in which a conventional ship berths at a platform with DP. In turn, Restrictions 22 refer to the third case, in which a conventional ship berths in a conventional platform. For the fourth and last case, in which a conventional ship cannot berth in a conventional platform, Restrictions 23 are activated. Restrictions 24 force the value of the auxiliary variable Fa_{ijk} to be 1 if there is an arc connecting two platforms *i* and *j*, $i \neq j$, consecutively for successive pickups, and to be 0, otherwise. This is the term that is being penalized in the objective function. Finally, Restrictions 25 to 27 refer to the domains of the variables of the model.

Model 1-27 properly captures a number of important features of the company's practical problem, namely: the criteria of costs and penalties to be optimized in the routing of ships, considering the fuel consumption of ships in terms of freighting contracts, the limitations of the platforms inventories (tops) through the time windows, the heterogeneity of the ships and their capabilities and limitations, rules of ship berthing on different platforms and terminals considering flexible drafts and dynamic positioning, among others. There are also some simplifying assumptions that were considered in this model. For example, the model does not take into account the compartmentalization of ships to transport different products simultaneously, but considers the ability of the ship in an aggregate form. In some cases, products of different platforms may not be mixed in the same compartment of the ship – some ideas for the extension of the model to incorporate restrictions of incompatible mixture of products transported on a ship were discussed in Rodrigues (2014). The model also admits that the time windows for the platforms and terminals are completely rigid (*hard time windows*) because they are implicitly considering the inventory restrictions of ballast and tops of the platforms and the demands of terminals. Hence, they are no penalties to be applied for any violations in the time windows. As a result of the origin-destination approach, split loads are not allowed - a characteristic that would greatly modify the general description of the problem to a supply-demand approach, where the amount to be picked up or delivered in each visit at operating sites would be modeled as decision variables.

5 Relax-and-fix method

MIP-Heuristics (also known as primal heuristics) are procedures based on mathematical programming that can be implemented directly in the solution methods used by optimization solvers available in commercial software. The main objective is creating alternative paths for finding integer feasible solution in situations when the designed algorithms take too long to find a feasible solution. One of the construction MIP-Heuristics widely known and applied in the lot-sizing and scheduling literature is called relax-and-fix heuristic (Wolsey, 1998).

According to Pochet & Wolsey (2006), in the relax-and-fix heuristic, it is assumed that set of binary variables *y* can be partitioned into *R* disjoint sets of decreasing importance, given by $Q^1,...,Q^R$. From this, the corresponding *R* MIPs can be solved, denoted by MIP^r, with $1 \le r \le R$, to find a heuristic solution to the original MIP. On the first round, MIP^1, an integrality is imposed only to variables in Q^1 and the restrictions of integrality of the remaining variables in *Q* are relaxed. In the following MIP^r, the variables *y* in Q^{r-1}, given by y_j^{r-1}, are fixed at their optimal values obtained in MIP^{r-1} and then the restriction of integrality for variables in Q^r is added. MIP-Heuristics are widely applied in contexts of production planning and scheduling, in problems in which the classic and wide-spread partition of variables is the time index (discretized).

When reviewing the literature on applications of these heuristics in the context of routing and scheduling of vehicles, only one study, made by Uggen et al. (2013), was found. In that paper, the time is discretized to enable the application of the relax-and-fix heuristics similarly to the studies in production planning and scheduling. The authors also stated that they did not found other work in the literature exploring the application of these methods in routing problems, which motivated the present study with the application of relax-and-fix heuristic to the continuous time problem addressed here. The strategy adopted to apply the relax-and-fix heuristic is based on the temporal division of the problem, exploited by traditional implementations of this algorithm to other problems in the literature. As Model (1)-(27) deals with the time in a continuous fashion, its marking can be done through the time windows $[a_i,b_i]$, arranged along the planning horizon and associated with each application to be picked up and delivered (pickup/delivery pairs). Figure 3 shows an illustration of the operation of the relax-and-fix heuristic with time windows partitioning.

Initially, requests are sorted according to the opening of the pickup time window (a_i). Then the *m* first collections are selected, beginning earlier. As these pickups are also associated to their respective deliveries, the algorithm enforces the integrality of the arcs referring to pickups/delivery pairs and relaxes the integrality of the remaining pickup and delivery requests, which have not yet been selected and are in the ordered list. Then, the smaller resulting MIP is solved, and the arcs (with integer value) among previous selected nodes are identified and defined, except those arcs that connect the initial and final depots of ships. If there are no integer arcs, the decomposed problem is infeasible and the algorithm stops. This procedure is repeated until there are no more pickup and delivery requests in the list to be allocated to ships. It is important to highlight that this procedure does not set the variables that connect the initial and final depots of the ship along the algorithm iterations, so that the heuristic is able to insert nodes at any position of the route until the last iteration in which the arcs of the initial and final depots are finally set and defined. A step-by-step description of the algorithm is:

Step 1. Sort pickups in increasing order of a_i ;

Step 2. Select the *m* pickups with the earliest starts, which have not yet been allocated yet;

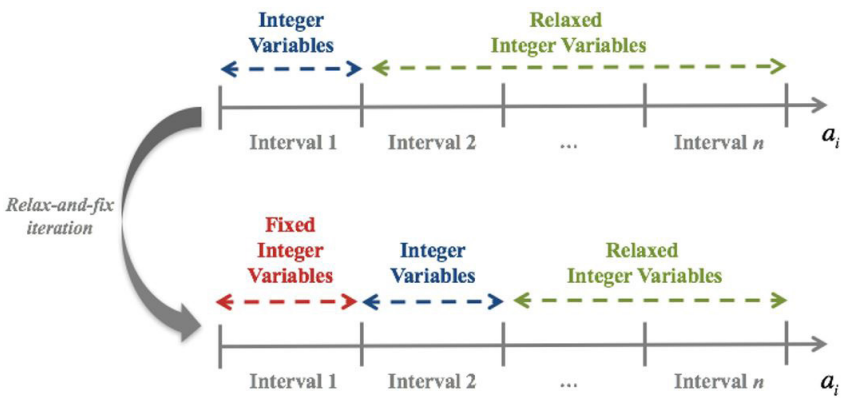

Figure 3. Schematic illustration of the operation of the relax-and-fix heuristic.

Step 3. Identify variables x_{ijk} with $i \in S$ ou $j \in E$;

Step 4. Enforce the integrality of the variables identified in step 3;

Step 5. Enforce the integrality of the pickup-delivery pairs related to step 2;

Step 6. Relax the integrality of the pickup-delivery pairs still not selected in step 2;

Step 7. Solve the resulting MIP;

Step 8. Identify variables x_{ijk} with integer values (direct arcs) that connect the selected pickup-delivery pairs;

Step 9. Fix the integer values of variables x_{ijk} resulting from the MIP solution identified in step 8, except the variables identified in step 3;

Step 10. Fix the arcs of the step 6. If there are no direct arcs, stop;

Step 11. Repeat steps 2-10 until there are no remaining arcs in the ordered list.

Other variations of the relax-and-fix heuristic could also be applied to solve this problem. For example, a variation based on the partition of the groups of ship was also explored in Rodrigues (2014), but the results were not particularly better than the temporal partition based on the algorithm above.

6 Numerical experiments

The Model (1)-(27) of Section 4 and the relax-and-fix heuristic of Section 5 were initially tested by solving various problem instances of small size. The details of these experiments and analyses of these results can be found in Rodrigues (2014). After verifying the consistency and correctness of the model and the heuristic in these small examples, they were tested to solve larger problem instances with data provided by the company, with 16, 22 and 44 pairs of pickup and delivery requests, named, respectively, N16, N22 and N44, all with |K| = 25 ships. These instances correspond to the company's operations in a few days: about three days in N16, seven days in N22 and two weeks in N44 in a certain region, respectively. Table 2 presents the results of the latter experiments with the model. For all tests, a Dell Precision T7600 CPU E5-2680 2.70 GHz (2 cores) and 192GB of RAM workstation with operating system Windows 7 Professional was used. The codes were implemented in GAMS 24.0 with CPLEX 12.5.0 using the default parameters values. The maximum time limit was 18,000 seconds (5 hours), with 32 cores processing option enabled. Furthermore, other experiments were conducted exploring changes CPLEX's parameters beyond the defaults values, such as activation of Local Branching (LB) and Relaxation Induced Neighborhood Search (RINS), change in emphasis from the solver in feasibility and/or optimality and shutdown of preprocessing. However, the results obtained from these experiments did not improve the results of Table 2 significantly.

CPLEX was able to obtain an optimal solution for instance N16, meeting all 16 pickup-delivery requests in less than 1 minute of runtime and using 10 out of the 25 available ships. For instance N22, CPLEX was also able to obtain an optimal solution meeting all 25 pickup-delivery requests, but in a considerably larger time, 5280 seconds (or 1 hour and 28 minutes) and using 13 ships out of the 25. Finally, doubling the number of pickup-delivery pairs to 44, CPLEX was not able to found a feasible solution within the time limit of 5 hours of processing. It is worth to note that these instances are difficult in terms of finding a feasible solution in practice, due to some restrictions involving tight time windows, berthing incompatibilities of several ships at operating sites, among other. The company presents difficulty in finding feasible solutions to these instances in real decision-making situations, without having to partially relax some of the restrictions of the model.

As mentioned, this problem is a particular case of a pickup and delivery routing with time windows and additional dynamic positioning restrictions and limited load for berthing, among others. As the classic pickup and delivery problem is NP-hard in terms of computational complexity theory, the present problem is also difficult to be optimally solved in practice depending on the size of the data set, as well as other pickup and delivery problems with time windows (Savelsbergh & Sol, 1995). CLPEX was still able to optimally solve instances N16 and N22, considered to be of moderate size in the company's planning, with thousands of variables and constraints. Problem instance N44 can be considered as a very large instance, with millions of variables and restrictions. Figure 4 depicts the variation of the model solution values over time, taking N22 as an example. It's noteworthy that the first feasible solution to the model was found within 44 seconds. An optimal solution was obtained only with 2.255 seconds of processing, and it was proven optimal only after 3.025 seconds, totaling the

Table 2. Results of the model using GAMS/CPLEX for problem instances N16, N22 and N44.

Instance	Ships Used	Restrictions	Variables		Gap (%)	Time (s)
			Integers	Continuous		
N16	10	426.480	13.982	345.854	0	57
N22	13	589.362	14.482	446.504	0	5.280
N44	-	1.498.961	116.893	1.003.454	-	-

5.280 seconds of processing in Table 2. This result indicates that CPLEX can find optimal solutions long before it can actually demonstrate that they are proven optimal.

For tests with relax-and-fix heuristics, the same problem instances N16, N22 and N44 were used. For each one of instances, step values (m) were defined as a multiple of the total requests amount of that instance (n), so that each one of the iterations would deal with the same number of requests. The optimality gaps were also calculated based on tests N16 and N22 with the optimal solutions known from Table 2. Notice that this strategy represented by the partition step m refers to the number of requests for pickup and delivery on the instance. Table 3 summarizes the results obtained for these instances with heuristic tests. For the instances N16 and N22, a time limit of 10 minutes (600 seconds) for each heuristic iteration was established. For the instances N16_RF, feasible solutions were found in the last iteration of the algorithm in a short computational time, however above the required by CPLEX to solve the mathematical model for N16, and with optimality gaps larger than 10%. As for instances N22_RF, computational processing times of the heuristic were below the CPLEX to solve the N22 model, but the optimality gaps were also greater than zero. A point to notice is that for each of the iterations performed by the heuristic, the computational implementation of the current heuristic in GAMS spends time to start a preprocessing, accounting for about 2 to 10 seconds, depending on the size of the problem. Obviously, these times are not dedicated to computational processing itself. These times could be reduced by more elaborate computer implementations.

Considering the tests, it is possible to notice that, generally, the larger m, the better the heuristic performance, both in terms of gaps and processing times. This may indicate that it is preferable to use larger steps to minimize the time of reprocessing, as well as prevent the problem to search for local optimal solutions, representing a smaller number of pick-up and delivery requests. In other words, the propensity for infeasibility is higher for instances

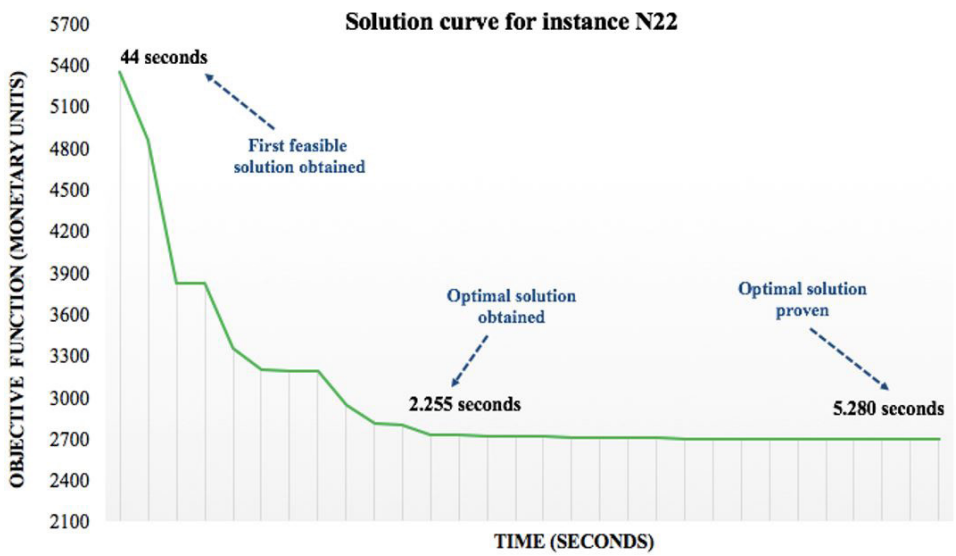

Figure 4. Solution value curve of problem instance N22.

Table 3. Results of the temporal strategy of relax-and-fix heuristic for problem instances N16, N22 and N44.

Instance	Ships Used	Gap (%)	Time (s)
N16_RF_M2	11	16,77	99
N16_RF_M4	11	16,77	56
N16_RF_M8	12	14,36	24
N22_RF_M2	14	11,16	646
N22_RF_M11	13	1,45	200
N44_RF_M2	-	-	-
N44_RF_M4	-	-	-
N44_RF_M11	-	-	-
N44_RF_M22	-	-	-

in which *m* is smaller and the heuristic search solutions with few integer variables. For the largest instance N44, no feasible solutions for any size of the partitioned problem with the temporal strategy were found. Since this is a relatively large and constrained problem from the standpoint of time windows and incompatibilities of berthing of ships, to obtain a feasible solution is very difficult for this instance. Time limits of 10, 30, 60 and 90 minutes for each iteration were also tested, but none of these tests resulted in feasible solutions.

With a view to testing the speed of convergence to the optimal solution of Model (1)-(27) for the solutions obtained by the relax-and-fix heuristic, experiments were set. The solutions with smaller gaps obtained for instances N16 and N22, respectively N16_RF_M8 (14.36% gap) and N22_RF_M11 (1.45% gap), were inserted in the CPLEX solver (CPLEX branch-and-cut method) as initial feasible solutions for instances N16 and N22 and then were ran for optimality. For instance N16, the CPLEX solver with this initial heuristic solution took 70 seconds to find and prove the optimal solution, compared to 57 seconds (Table 2) required by the CPLEX solver without using this initial solution. As for instance N22, CPLEX found the optimal solution and proved its optimality in only 22 seconds, compared to 5,280 seconds (Table 2) required by the CPLEX solver without using initial heuristic solution. These results indicate that a good strategy for solving problems of moderate size would be, initially, run the heuristic and then insert the obtained solution as a starting feasible solution for the model solution via CPLEX.

7 Conclusions and future research

An optimization approach to the routing and scheduling problem of ships for oil cabotage, motivated by the practical problem of an oil company and based on a MIP model and relax-and-fix heuristic, is here presented. This pickup and delivery problem with time windows, heterogeneous fleet, multiple depots (initial and final locations of each ship), multiple products (each platform produces a different oil) and multiple visits on each platform and terminal over the planning horizon. Further restrictions include specifics of the operation of the case under study, as constraints associated with ships with flexible drafts and terminals and devices of dynamic positioning of ships and platforms.

The MIP formulation captures several fundamental features of the problem in the context of oil cabotage. The formulation uses predetermined time windows to relieve the platform's inventory, therefore meeting the oil demands of the terminals. Furthermore, the formulation accounts for the various rules of berthing of ships on different types of platforms and terminals and considerations of fuel consumption of ships. The mathematical formulation achieved optimal solutions to real problems of small to moderate sizes using the GAMS/CPLEX optimization package. This is one of the main results of this paper. However, the model failed to obtain feasible solutions to larger problems. The relax-and-fix heuristic brought an advantage in terms of computational processing to solve the proposed model together with CPLEX, using their solutions as initial solutions in the CPLEX model. This result indicates that this approach has the potential to produce reasonably good solutions for practical problems of moderate size in acceptable computational times. Although it has improved the performance of the model, the heuristic has also failed to find feasible solutions for larger data sets. Generally, the heuristic reached better computational performance in terms of processing time for larger values of the parameter *m*.

Since this is one of the first papers to apply relax-and-fix heuristic to routing problems with continuous time, there are many opportunities for improvement yet to be explored. The implementation of the relax-and-fix temporal heuristic is an improvement for the applications of this heuristic and also provides the basis for future implementations that do not use the widely used time partitioning. Other business rules could be applied, based on these early experiments of implementing outside the "traditional areas" of this heuristic. An important point of discussion is about the specifics of the problem that present infeasibility. In general, this is a way of finding out if the problem is tight in terms of available capacity and resources, thus enabling design strategies with a view to penalizing certain measures, in order to obtain feasible solutions that are adherent to the actual operation, and that can be obtained in short time.

There are several potential future research strands to be derived from this work. They include but are not limited to: (i) improvements in relax-and-fix heuristic, such as implementing backward and overlapping strategies on this temporal partition strategy presented; (ii) implementation of other strategies, such as portioning of ships, based on a set of criteria for the ordering of available ships according to their costs and capacities (some initial results in this research are presented in Rodrigues, 2014); (iii) refinements of the heuristic using new criteria for order selection and partitioning; (iv) combination of the currently proposed heuristics with local search heuristics to improve the feasible solutions, such as fix-and-optimize heuristics. Furthermore, other interesting future research is the application of both relax-and-fix and fix-and-optimize heuristics in alternative formulations of the problem of routing and scheduling of oil tankers exploring discretization of the time, instead of continuous time. Also, the development and applications of other heuristics and metaheuristics to solve larger

problem instances. A motivating line of research would be to modify the current model to include berth limitations in terminals and to explore geographical decomposition of the problem in sub areas to be dealt with as independent, respecting the demand characteristics and planning. Another possible future research is to work on alternative approaches to the problem, for example, by investigating descriptions for the supply-demand type of problem, in which the inventory control across platforms and terminals are considered explicitly in models and amounts to be transported by ships become decision variables, characterizing the problem as a combined problem of routing and inventory (inventory-routing problem; Christiansen, 1999; Al-Khayyal and Hwang, 2007). Furthermore, the integration of routing problem with refineries programming can be explored, providing a more general modelling aspect, in terms of supply chain planning and scheduling, with different levels of decision-making within the company.

Acknowledgements

The authors thank financial the support of the National Petroleum Agency (ANP), FAPESP and CNPq.

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