Generation of high-winding-number superfluid circulation in Bose-Einstein condensates

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We experimentally and numerically demonstrate a method to generate multiply quantized superfluid circulation about an obstacle in highly oblate Bose-Einstein condensates (BECs). We experimentally achieve pinned superflow with winding numbers as high as 11, which persists for at least 4 s. Our method conceptually involves spiraling a blue-detuned laser beam, around and towards the center of the BEC, and is experimentally implemented by moving the BEC in a spiral trajectory around a stationary laser beam. This optical potential serves first as a repulsive stirrer to initiate superflow, and then as a pinning potential to transport the superfluid circulation within the BEC. The spiral technique can be used either to generate a high-winding-number persistent current, or for controlled placement of a cluster of singly quantized vortices of the same circulation. Thus, the technique may serve as a building block in experimental architectures to create on-demand vortex distributions in BECs.

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I. INTRODUCTION

Highly oblate dilute-gas Bose-Einstein condensates (BECs) [1], in which fluid dynamics occur primarily in two dimensions, have opened up experimental studies of two-dimensional (2D) quantum turbulence [2–5], as well as theoretical and numerical studies of point vortex models and the complex collective behavior of a distribution of many vortices [6–8]. In these scenarios, the initial placement of the vortex cores determines the system's quantum phase profile and hence the subsequent fluid flow and vortex dynamics, and vortex behavior can highlight fundamental differences between superfluids and classical viscous flows [7,9]. Yet interest in vortex dynamics extends well beyond basic aspects of superfluidity. Vortex dynamics also play a role in analog cosmology [10,11], where large-quanta vortices are used to mimic rotating black holes [12] and to study ergoregion instabilities [13]. As experiments progress to include quantum mixtures and binary superfluid dynamics [14–17], vortices once again become highly relevant as probes of the macroscopic quantum state [18]. Of particular recent interest is whether vortices survive in quantum-fluctuation-enhanced regimes such as the Lee-Huang-Yang gas [19,20] or quantum droplets [21–24]. These states are formed in quantum mixtures where the net mean-field interaction is tuned closed to zero such that beyond-mean-field effects like quantum fluctuations play an enhanced role in governing the system's behavior.

Exploring such superfluid physics with BECs requires developing experimental techniques for deterministic vortex generation with control over placement, vorticity, and direction of circulation. This is particularly relevant for studies of quantum turbulence where one might want to reproducibly generate a many-vortex state to look for signatures of quantum turbulence in subsequent vortex dynamics. Progress towards a flexible experimental vortex architecture has been made in highly oblate single-component BECs regarding deterministic placement of individual vortex cores [25], with subsequent refinement made possible by the development of arbitrary configurable optical potentials [26,27]. However, a fully flexible architecture would benefit from additional experimental techniques that allow for controlled placement of a cluster of a fixed number of vortices, all with the same sign of circulation. In this paper, we present a controlled vortex generation method that can generate large net superfluid circulation and multiply charged vortex states with observed winding numbers up to 11. The vortices are pinned to the beam and can be moved to a desired location within the BEC, or can be released from the beam for studies of vortex dynamics.

Various methods have been proposed and used to create quantized vortices in BECs. Early techniques included density and phase engineering in a two-component condensate [28], and rotating the confining potential [29–32]. In the absence of pinning potentials, and in a radially symmetric harmonic trap, such rotation leads to the formation of a vortex lattice of singly quantized vortices all with the same sign of circulation. For sufficiently high rotation speeds in cylindrical traps, the system should undergo a transition to a giant vortex state, with all the vorticity contained in a hole along the rotation axis [33]. While this transition has yet to be observed experimentally, recent experiments have observed the formation of a ring of supersonic rotating superfluid confined in an anharmonic

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trap, an intermediate step towards the giant vortex state [34]. Forced aggregation of vortices into one giant circulation has been achieved by applying a focused laser beam at the center of the rotating BEC [35]. Multiply charged vortex states have also been created through topological phase imprinting methods [36–39], in which winding numbers of 2 and 4 were obtained. In general, unpinned multiply quantized vortices are unstable in single-component BECs, and tend to break apart into singly quantized vortices. However, persistent currents have been demonstrated in toroidal geometries where the presence of a central pinning potential keeps the current from dissociating into individual vortex cores [2,40–43]. Persistent currents with winding numbers of $q = 10$ [41] have been obtained via topological phase imprinting, and winding numbers of $q = 12$ [43] have been obtained using a paddle stirrer to initiate circular superflow around the toroid. Much of this early work focused on introducing vorticity into the BEC, but did not focus on controlled placement of individual vortices or large-net-vorticity clusters. More recently, digital micromirror devices combined with high-numerical-aperture objectives have enabled optical traps and stirring potentials with greater resolution and enhanced dynamic control over vortex generation and placement [4,5,26]. Thus the development of a wide variety of techniques for controlling winding number and vortex cluster placement continues to be highly relevant and desirable.

The paper is organized as follows. In Sec. II, we introduce the conceptual foundation for our technique. In Sec. III we describe the details of our experimental studies and observations. In Sec. IV we present results from corresponding simulations of the 2D Gross-Pitaevskii equation that illuminate the process for creating pinned superfluid circulation, or multiply quantized vortices. In Sec. V we explore the relationship between stirring speed and the number of pinned vortices. In Sec. VI we discuss prospects for extending the technique to multiple stirring beams and hard-wall trapping potentials. Section VII concludes the article.

II. CONCEPT

Conceptually, our method involves spiraling a repulsive optical potential formed by a blue-detuned laser beam around and inwards towards the center of the BEC as depicted in Fig. 1. This optical potential serves first as a repulsive stirrer to initiate superflow, and then as a pinning potential that transports the center of the superfluid circulation to the center of the condensate. In our experimental implementation, the BEC is moved in a spiral trajectory around a stationary laser beam, as will be described in Sec. III.

The spiral sequence begins with the blue-detuned laser beam placed just at the edge of our highly oblate axially symmetric (about the $\hat{z}$ axis) BEC [see density profile (i) in Fig. 1(a)]. This stirring beam acts as a repulsive obstacle that is spiraled inwards in the $\hat{x}$-$\hat{y}$ plane towards center of the BEC at $x = y = 0$. Figure 1(a) shows the simulated 2D density profile of the BEC at relevant time points within the spiral sequence. In this example the beam travels counterclockwise. The radial and angular trajectories of the beam are shown in Figs. 1(b) and 1(c), respectively. As the stirring beam initially spirals inwards, it pushes fluid out of the way and initiates

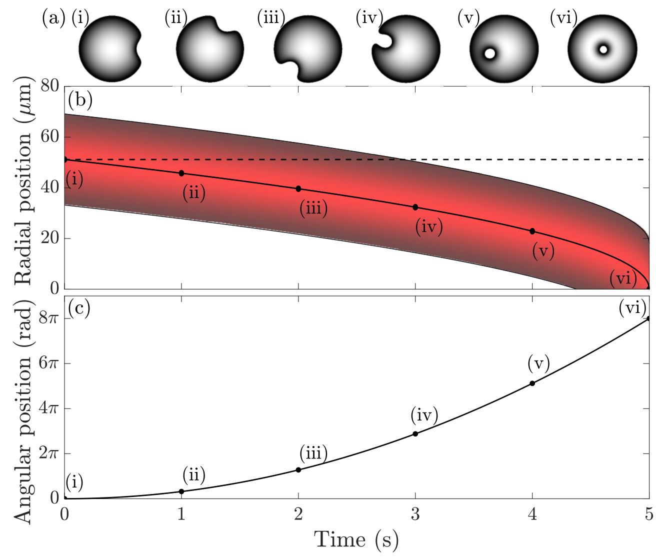

FIG. 1. Spiral beam trajectory $\vec{s}(t) = \{r(t), \theta(t)\}$, with radial position $r(t) = R_0\sqrt{1 - t/\tau_s}$, angular position $\theta(t) = 2\pi N_s(t/\tau_s)^2$, and spiral parameters $N_s = 4$, $\tau_s = 5$ s, and $R_0 = R_r$. See text for definitions of parameters. (a) Simulated 2D density profiles of the BEC in the presence of the beam are shown at 1-s intervals [density profiles (i)–(vi) correspond to the labeled time points in plots (b) and (c)]. (b) Radial beam position $r(t)$ as a function of time during the spiral trajectory. Red shading indicates the radial extent of the Gaussian beam. The horizontal dashed line indicates the radial Thomas-Fermi radius of the circular condensate, R_r. (c) Angular beam position $\theta(t)$ as a function of time during the spiral trajectory.

local flow. Given that the beam initially moves along the edge of the BEC, in a clockwise (CW) or counterclockwise (CCW) direction, the induced superflow direction is fixed by the direction of motion of the stirring beam. Thus, a beam spiraling CCW with respect to the center of the BEC [see Fig. 1(a)] also initiates CCW local superflow around the beam. This is in contrast to the methods in Refs. [25,44,45] that nucleate vortex dipoles, pairs of vortices with equal but opposite-signed circulation. We note that the method proposed in Ref. [45] bears some similarity to our technique in that (after vortex dipole nucleation) a laser beam is used to pin one of the vortices and then guide it to the center of the BEC. However, due to the fundamental difference in the vortex creation process, the method of Ref. [45] does not readily extend to multiply quantized vortices.

Eventually the beam moves sufficiently inwards that on the outside edge of the beam, the fluid behind the beam merges with the fluid in front of the beam, yielding connected circular flow around the beam [see density profile (v) in Fig. 1(a)]. At this point circulation has been brought inside the BEC and pinned to the beam. The winding number associated with the pinned circulation is fixed by the velocity of the now-continuous superfluid flow. Thus the winding number of the multiply quantized circulation that is to be pinned to the beam can be controlled by varying the speed of the stirring beam. For sufficiently high beam powers and low spiraling speeds the circulation remains pinned to the beam as the beam continues its trajectory to the center of the condensate. Once the pinning beam has reached the desired final position, it can be left in place at full power such that the condensate is in a toroidal geometry with a high-winding-number persistent current [40]. Alternatively, the beam's power can be ramped off, allowing placement of a cluster of singly quantized vortices of the same sign of circulation, with control over the placement of the cluster's centroid.

The particular trajectory $\vec{s}(t) = \{r(t), \theta(t)\}$ used in our experiment was chosen such that the speed of the beam would mimic the variation of the speed of sound in the condensate, i.e., low at the edges and highest at the center. Here $r(t) = R_0\sqrt{1 - t/\tau_s}$ plotted in Fig. 1(b) is the radial distance of the beam's center from the center of the BEC, and $\theta(t) = 2\pi N_s(t/\tau_s)^2$ plotted in Fig. 1(c) is its angular displacement with respect to the axis defined by the center of the BEC and the initial beam position. Here R_0 is the initial radial position of the beam, N_s is the number of 360° rotations within the spiral, and τ_s is the total time duration of the spiral trajectory. The speed of the circular superflow initiated by the beam, and the associated winding number, are fixed by the spiral parameters N_s and τ_s. We emphasize that the exact nature of the trajectory and the diameter of the stirring beam are not critical for the success of the method so long as the initial motion of the beam is approximately tangent to the outer edge of the condensate. However, it is important to manage the speed of the beam to (1) allow the fluid ahead and behind the beam to merge in a controlled fashion to avoid generating excitations such as dark solitons [46–48], (2) stay below the critical speed for dipole nucleation [44] once the beam has fully entered the condensate, and (3) allow the multiply quantized circulation to remain pinned to the beam as it moves within the condensate.

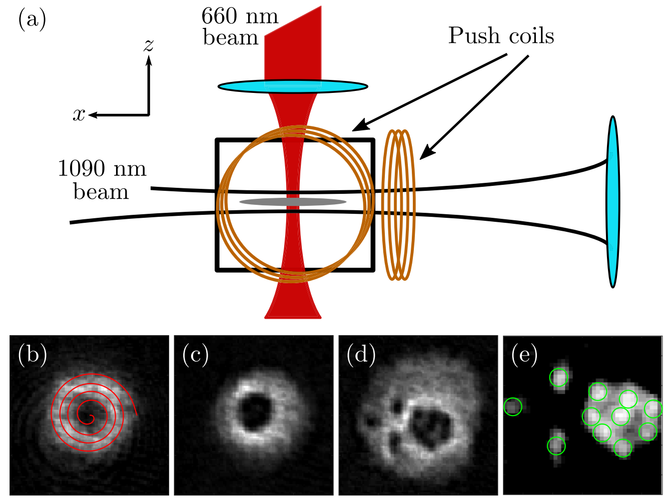

FIG. 2. Experimental setup. (a) Schematic of the experimental setup (not to scale). [(b)–(d)] Axial absorption images showing a 200-μm-square field of view in the horizontal x-y plane. (b) In-trap image of the BEC with the 660-nm blue-detuned beam at its center. Also shown is the effective spiral trajectory (red line) of the 660-nm beam as the condensate is translated by time-varying magnetic fields. (c) BEC after ballistic expansion. The BEC is allowed to expand right after the pinning optical potential is ramped off ($t_d = 0$ ms; see text). The large hole in the center of the condensate is taken as evidence of multiply charged superflow since its size is much larger than a single vortex core. (d) BEC with 11 singly quantized vortices, imaged after ballistic expansion. Here we wait for $t_d = 160$ ms between the 660-nm beam ramp-off and ballistic expansion to allow the superflow to disperse into singly quantized vortices. (e) Residuals from Thomas-Fermi fit to the image shown in (d), zoomed in to the central vortex region. Green circles indicate individual vortex cores.

III. EXPERIMENT

For the spiral technique presented here, we build upon the experimental system described in Refs. [2,44] where time-varying magnetic fields ["push coils" in Fig. 2(a)] are used to move a highly oblate BEC in the $\hat{x}$-$\hat{y}$ plane with respect to a stationary Gaussian obstacle. Briefly, we create ^{87}Rb BECs, confined in a hybrid magnetic-optical harmonic trap with radial (r) and vertical (z) trap frequencies of $(\omega_r, \omega_z) = 2\pi \times (8, 90)$ Hz. The trapped BECs have typical atom numbers of $N_c \sim 2 \times 10^6$, a chemical potential of $\mu_0 \sim 8\hbar\omega_z$, and Thomas-Fermi radii of $(R_r, R_z) \sim (50, 5)\,\mu$m. Thus the BEC is thermodynamically three-dimensional (3D); however, vortex dynamics occur primarily in the $\hat{x}$-$\hat{y}$ plane, and vortex-core excitations such as Kelvin modes are reduced [49]. As in our prior work [2,44], we create a repulsive stirrer with an additional focused blue-detuned laser beam at 660 nm, with a Gaussian $1/e^2$ radius of $w_0 \sim 18\,\mu$m. Figure 2(b) shows a representative *in situ* absorption image of a BEC with the blue-detuned beam centered on the BEC (the final beam position).

In a typical experimental sequence, we form a BEC in the hybrid harmonic trap. We then translate the harmonic trap minimum by $\sim R_r$ over $\sim$1 s, and ramp on the 660-nm beam to a potential height of $U \sim \mu_0$ over $\sim$500 ms, resulting in the initial configuration for the spiral protocol [see density profile (i) in Fig. 1(a)]. We then move the BEC along the spiral trajectory indicated by the red line in Fig. 2(b). As discussed in Sec. II, the position of the focused beam in the rest frame of the BEC is described by $\vec{s}(t) = \{r(t), \theta(t)\}$, with $r(t) = R_0\sqrt{1 - t/\tau_s}$, and $\theta(t) = 2\pi N_s(t/\tau_s)^2$.

After a subsequent hold time $t_h \sim$ 4–7 s, we slowly ramp off the power of the blue-detuned beam ($t_{ramp} \sim$ 0.5–1 s), and then remove all trapping potentials, letting the BEC undergo a period of ballistic expansion prior to imaging. t_{ramp} is determined experimentally with the criteria of being long enough to avoid exciting the BEC, but short enough so that the multiply quantized vortex pinned to the beam does not have time to disperse appreciably into singly quantized vortices. Figure 2(c) shows a representative absorption image of the condensate immediately following the blue-detuned beam ramp-off and subsequent 56 ms of ballistic expansion. The large central region devoid of atoms in the center of the expanded BEC is much larger than what we expect for a singly quantized vortex core. Instead, the giant hole is indicative of a multiply quantized vortex that was created by the spiraling process and then pinned to the blue-detuned beam [42].

To determine the winding number of the pinned superflow, we add an additional short hold time of $t_d \sim$ 150–250 ms after ramping off the blue-detuned beam, and prior to ballistic expansion. As shown in Fig. 2(d), adding the additional dispersal time enables us to resolve the individual vortex cores as the large multiply quantized vortex core dissociates into singly quantized vortices. The dispersal time was chosen empirically to balance the clear observation of vortices that

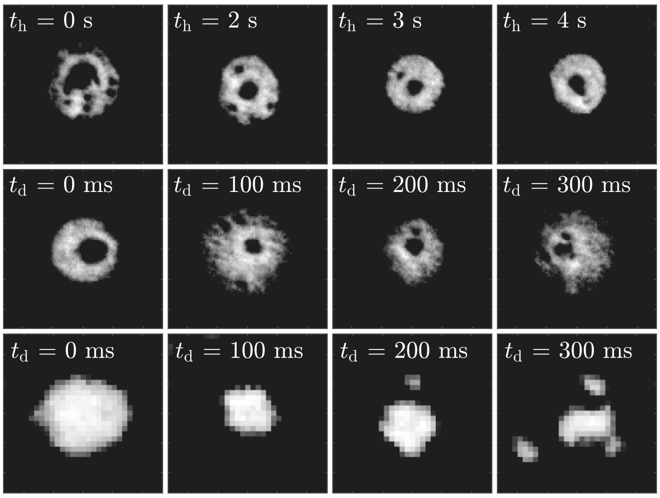

FIG. 3. BEC expansion images for variable hold time t_h and superflow dispersal time t_d. The 200-μm-square axial absorption images (upper and middle row) are taken after a period of ballistic expansion. Top row: Variable hold time, prior to 600-ms beam ramp off. Spiral parameters are $N_s = 4$, $\tau_s = 4.8$ s. Here the BEC is allowed to expand directly after ramping off the beam ($t_d = 0$ ms). Middle row: Variable dispersal time t_d, after the 600-ms beam ramp-off. Spiral parameters are $N_s = 5$, $\tau_s = 3.5$ s. Here we hold the BEC for $t_h = 4$ s, ramp the beam off, and then hold for a variable dispersal time t_d to allow the superflow to dissociate into singly quantized vortices. Bottom row: Residuals after fitting the images in the middle row to a Thomas-Fermi density profile, zoomed in to the central vortex region.

remain closely clustered near the BEC center, with the need to also observe vortices that might leave the pinning potential or central vortex cluster sooner than the others. Figure 2(d) shows a representative high-winding-number superflow after it has dissociated into 11 individual vortex cores. In Fig. 2(e) we show residuals from a Thomas-Fermi fit to the 11-vortex image shown in Fig. 2(d), zoomed in to the central vortex region. Here green circles indicate individual vortex cores.

Figure 3 shows representative images at the end of the spiral trajectory for varying t_h (top row, $t_d = 0$ ms), and varying t_d (middle row, $t_h = 4$ s). The bottom row shows residuals from a Thomas-Fermi fit to the corresponding images shown in the middle row, zoomed in to the central vortex region. We first discuss the role of the hold time prior to the beam ramp-off. For $t_h = 0$ s, we regularly observe both a central current and a number of unpinned vortices. Unpinned cores may be the result of too high of a beam speed during the spiral, resulting in vortices depinning from the beam. Or they may simply be a result of introducing more vorticity than can be stably pinned to the beam [50]. As we increase t_h we find fewer unpinned vortices until finally for hold times $t_h = 3.5$–4.0 s we regularly observe just the central pinned multiply quantized vortex and no unpinned cores. The latter likely leave the condensate due to BEC damping by the thermal atomic background [45,51]. Images of the expanded BEC for short hold times $t_h = 500$ ms show that there are on average six unpinned vortices for $\tau_s = 5$ s ($N_s = 4$) and four unpinned vortices for $\tau_s = 7$ s ($N_s = 4$).

In the absence of a pinning potential (such as our blue-detuned beam), a multiply charged vortex state is unstable [52–55] and will tend to dissociate into individual vortices [55], which then disperse outwards from the center of the condensate [7]. As shown in the middle row of Fig. 3, the vortex cluster appears to begin dissociating *after* we ramp off the pinning beam. Here we hold the BEC for $t_h = 4$ s, ramp the beam off over 600 ms, and then hold for a variable dispersal time t_d to allow the multiply quantized superflow to visibly dissociate into singly quantized vortices. The leftmost image $t_d = 0$ ms shows the multiply quantized vortex prior to dissociation. As we increase t_d, we observe the central superflow break apart into four or five vortices, with individual singly quantized vortices resolvable around $t_d = 200$ ms. To aid in resolving the individual vortex cores at short dispersal times, the bottom row of Fig. 3 shows the corresponding residuals after fitting the images in the middle row to a Thomas-Fermi density profile. The high variance in the experimental data (see Sec. V) makes it difficult to take a consistent time series. However, we consistently observe that the disassociation of the cluster is not instantaneous, consistent with Ref. [7].

Finally, we note that, unlike the spin down of a vortex lattice, the vortex dynamics involved in ramping down a pinning potential with many pinned vortices have not been fully studied. Furthermore, ramping the beam off too quickly can lead to shock waves and complex vortex depinning dynamics. For our proof-of-principle work, we managed the beam ramp-off empirically, through our choice of initial beam height and ramp-down time. A more detailed study of the ramp-down process and subsequent depinning will be the subject of future work.

IV. NUMERICAL SIMULATIONS

To better understand the formation process of the multiply quantized circulation state pinned to the spiraling beam, we performed numerical simulations using split-step Fourier evolution of the 2D Gross-Pitaevskii equation (GPE) [1,25]. The numerical simulations enable visualization of the condensate's phase and speed profiles throughout the spiral trajectory. The 2D GPE captures the largely 2D fluid dynamics of our highly oblate 3D BECs with significantly reduced computing costs.

Following Ref. [25], the condensate dynamics are modeled by the 2D GPE

$$(i-\gamma)\hbar\frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2_{x,y} + V_{\mathrm{ht}} + V_{\mathrm{sb}} + g_{2\mathrm{D}}|\psi|^2\right]\psi,$$

where $V_{\mathrm{ht}} = \frac{1}{2}m\tilde{\omega}_r^2(x^2+y^2)$ is the harmonic trapping potential (in the absence of the stirring beam) for atoms of mass m and $g_{2\mathrm{D}} = \frac{4\pi\hbar^2 a_{\mathrm{sc}}}{m\sqrt{2\pi}\ell_z}$ is the effective 2D nonlinear interaction term, where a_{sc} is the atomic s-wave scattering length. As in Ref. [25], the values for effective length scale $\ell_z \sim 0.51$ μm, effective radial trap frequency $\tilde{\omega}_r = 0.84\omega_r$, and an effective atom number $N_{2\mathrm{D}} = \int|\psi|^2 dxdy = 3.7\times10^5$ were chosen so that observables such as peak density n_0, radial Thomas-Fermi radius R_r, bulk chemical potential μ_0, and the bulk speed of sound $c_0 = \sqrt{\mu_0/m} = 1800$ μm/s are consistent between the effective 2D simulations and the fully 3D experimental observables. This allows us to better compare results from simulations and experiment.

For the GPE simulations we work in the reference frame of the BEC, so that the BEC is stationary and the stirring beam moves along the spiral trajectory. The time-dependent potential due to the blue-detuned stirring beam is

$$V_{\rm sb}(x, y, t) = U \exp\left\{-\frac{2}{w_0^2}[(x - x_{\rm s}(t))^2 + (y - y_{\rm s}(t))^2]\right\},$$

where $x_{\rm s}(t) \equiv r(t)\cos\theta(t)$ and $y_{\rm s}(t) \equiv r(t)\sin\theta(t)$ for $r(t)$ and $\theta(t)$ defined previously are the time-dependent positions of the moving stirring beam. U is the maximum repulsive energy of the Gaussian stirring potential, and w_0 is the $1/e^2$ beam radius. We use imaginary-time propagation of the 2D GPE to generate the initial condition for the BEC in the full potential (harmonic trap + stirring beam). We then model the dynamics using split-step evolution of the 2D GPE with $\gamma = 0.003$ to phenomenologically account for finite-temperature effects such as damping due to the presence of the thermal component. We use a grid spacing of either $\Delta x = 0.23\,\mu$m (Figs. 4 and 5) or $\Delta x = 0.46\,\mu$m (Figs. 7–10). The finer resolution $\Delta x = 0.23\,\mu$m is on the order of the bulk healing length $\xi = \sqrt{\hbar^2/2m\mu_0} = 0.27\,\mu$m and is therefore relevant when considering fluid merging dynamics especially at high merging speeds.

Figure 4 shows representative snapshots of the spiral method for $N_{\rm s} = 4$, $\tau_{\rm s} = 5$ s, $R_0 = R_r$, $w_0 = 18\,\mu$m, and $U = \mu_0$, corresponding to the spiral trajectory plotted in Fig. 1. Columns from left to right show snapshots of 2D density $n(x, y, t) = |\psi(x, y, t)|^2$, condensate phase $\phi(x, y, t)$, the speed profile $v(x, y, t) = \frac{\hbar}{m}|\nabla\phi(x, y, t)|$, and a zoomed-in region of the speed profile at the location of the stirring beam. The speed profiles in the third column from the left are scaled to the effective 3D local speed of sound, $c_{\rm local}(x, y, t) = \sqrt{g\,n_{\rm eff,3D}(x, y, t)/m}$, where $n_{\rm eff,3D}(x, y, t) = n(x, y, t)/\sqrt{\pi}\ell_z$ is the effective 3D atom density. The speed profiles in the fourth column are scaled to the bulk speed of sound, $c_0 = \sqrt{\mu_0/m}$. White arrows indicate the direction of the superflow.

The upper row of Fig. 4 shows the initial condition at $t/\tau_{\rm s} = 0$, with the spiral beam located at the edge of the BEC, $r(t = 0) = R_r$. As the speed profiles show, the inward spiraling beam induces local superflow around the inner edge of the beam; i.e., the beam pushes the superfluid out of the way. Eventually the flow behind the beam merges with that ahead of the beam such that the beam is fully surrounded by a continuous circular superflow. The flow around the beam at this merge time point $t_{\rm m}$ fixes the circulation pinned to the beam. Given the quantized nature of superfluid circulation, this in turn fixes the number of quantized vortices that may become pinned to the beam.

Variations in the spiral parameters such as $N_{\rm s}$ and $\tau_{\rm s}$ change the speed of the stirring potential at the time of merging, $v_{\rm s}(t = t_{\rm m})$, and in turn the speed of the fluid around the potential. As shown in Fig. 5, the flow speed increases with decreasing $\tau_{\rm s}$. Figure 5 zooms in on the region around the spiral beam at the merge time point $t_{\rm m} \sim 0.66\tau_{\rm s}$ for varying $\tau_{\rm s}$ ($N_{\rm s} = 4$). The superflow speed profile becomes less smooth as the speed of the merging fluid increases. In particular, for $\tau_{\rm s} = 3$ s we start to see evidence of a dark soliton excitation forming [46,56], which then evolves into a single unpinned vortex. We note that for $\tau_{\rm s} = 3$ s, vortices depin from the stirring potential towards the end of the spiral trajectory.

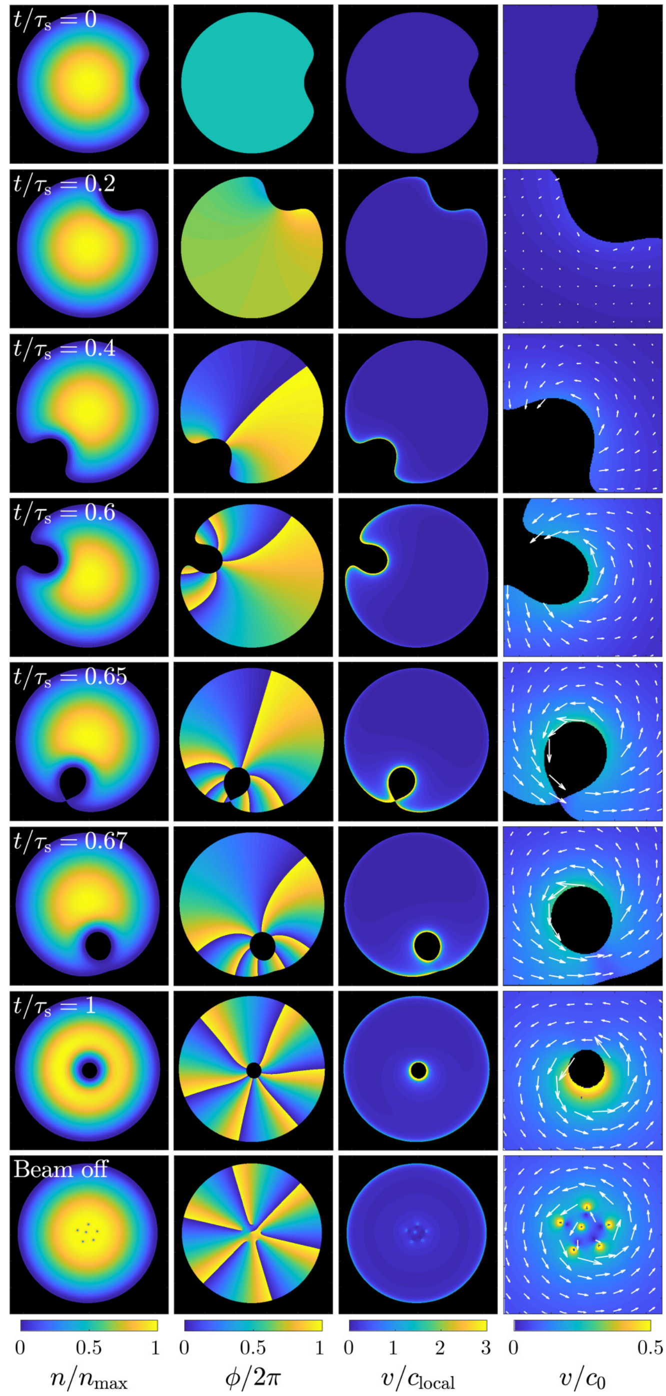

FIG. 4. Snapshots from 2D GPE simulation of the spiral technique at representative times $t/\tau_{\rm s}$. Columns from left to right show snapshots of (i) density, (ii) phase, (iii) the superfluid speed profile scaled to the local speed of sound, and (iv) a zoomed-in region of the speed profile at the location of the stirring beam. The speed profile in column (iv) is scaled to the bulk speed of sound, c_0 (using the peak density n_0). White arrows indicate scaled velocity vectors of the flow. Simulation parameters are $N_{\rm s} = 4$, $\tau_{\rm s} = 5$ s, $R_0 = R_r$, $w_0 = 18\,\mu$m, and $U = \mu_0$.

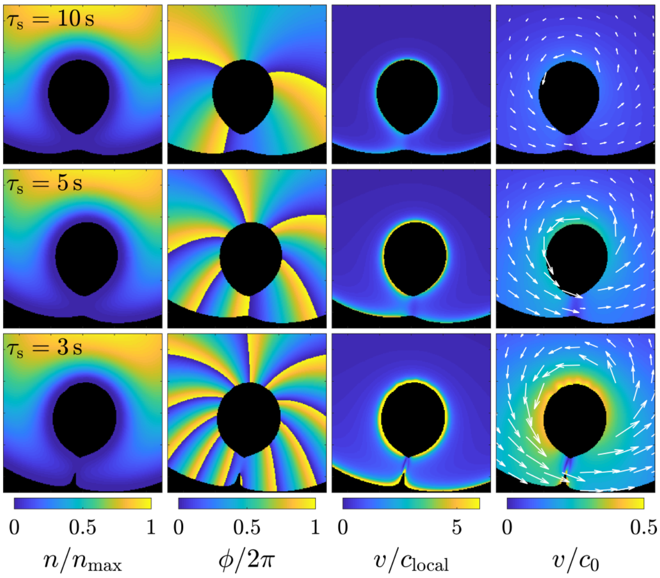

FIG. 5. Snapshots from 2D GPE simulations for varying τ_s (as labeled). Snapshots show the time point $t_m \sim 0.66\tau_s$ where the flow behind the beam merges with that ahead of the beam, resulting in continuous superflow around the beam. Columns from left to right show zoomed-in snapshots of (i) density, (ii) phase, (iii) the speed profile scaled to the local speed of sound, and (iv) the speed profile scaled to the bulk speed of sound, c_0 (using the peak density n_0). White arrows indicate scaled velocity vectors of the flow. Simulation parameters are $N_s = 4$, $R_0 = R_r$, $w_0 = 18\,\mu$m, and $U = \mu_0$. Note the dark soliton appearing in the reconnecting flow for $\tau_s = 3$ s.

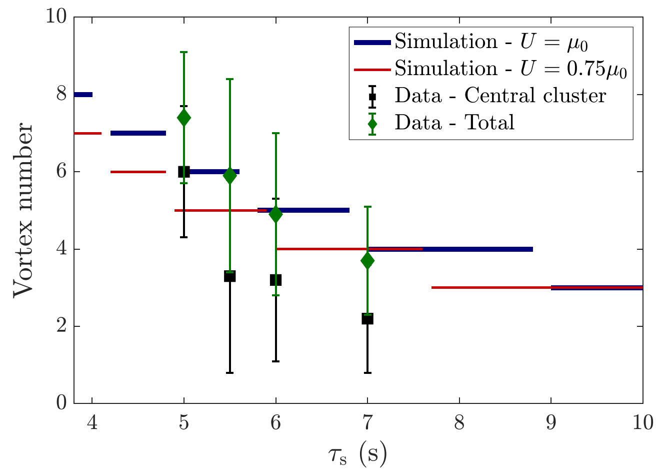

FIG. 7. Average number of vortices observed after ramping off the pinning potential as a function of trajectory duration τ_s, for fixed $N_s = 4$ and beam waist $w_0 = 18\,\mu$m. Black squares and green diamonds correspond to experimental data, while solid blue and red lines are numerical results from the 2D GPE simulations. Black squares show the average number of cores in the central cluster, whereas green diamonds show the average total number of vortex cores. Error bars indicate the standard deviation. Solid blue lines correspond to a spiraling potential $U = \mu_0$, and solid red lines correspond to $U = 0.75\mu_0$. Both experiment and numerics show that the number of vortices generated and pinned using our method decreases as the optical potential moves more slowly through the condensate.

V. CHOOSING VORTEX WINDING NUMBER

The vortex distributions shown in Fig. 6 are representative of the range of multiply quantized vortices created as we vary the spiral parameters N_s and τ_s. Since these individual vortices originate from the large pinned vortex configuration and generally remain clustered, they must have the same sign of circulation, which is determined by the direction of the spiral. We note that regular array configurations of the vortices can sometimes be observed, e.g., for three [Fig. 6(b)], four [Fig. 6(c)], and six windings [Fig. 6(e)], as reported in previous numerical simulations [57,58].

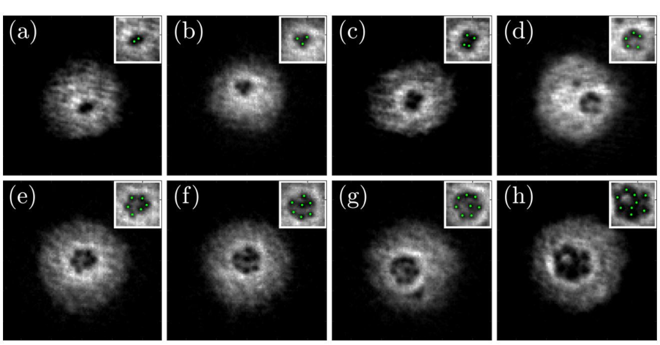

FIG. 6. Representative 200-μm-square axial absorption images of the BEC after ballistic expansion. Expansion of the BEC starts ~160 ms after the pinning potential ramps off. The pinned vortices start to dissociate and individual vortex cores can be resolved. [(a)–(h)] Vortex configurations for two to nine cores, respectively. Vortices are marked by green dots in the inset to guide the eye. Regular array configurations of the vortices can sometimes be observed for, e.g., (b) three, (c) four, and (e) six windings.

Ultimately we want to be able to control the exact winding number and placement of the vortex cluster. To this end, we further explore the relationship between stirring speed and the number of pinned vortices, by varying the time duration of the spiral trajectory. In Fig. 7 we plot the number of vortices observed after ramping off the pinning potential as a function of the spiral trajectory duration τ_s for a fixed $N_s = 4$. Black squares and green diamonds show experimental data; black squares show the average number of cores in the central cluster, whereas green diamonds show the average total number of vortex cores regardless of their position in the condensate. Error bars indicate the standard deviation. The solid blue lines in Fig. 7 correspond to simulations using $U = \mu_0$, and the solid red lines correspond to $U = 0.75\mu_0$.

Results from our 2D GPE simulations are qualitatively consistent with experiment, with longer spiral times resulting in fewer pinned vortices. At the fastest spiral trajectory ($\tau_s =$ 5 s), the mean winding number generated by our method is ~6 to 7.4, while at the slowest ($\tau_s = 7$ s), the mean is ~2.2 to 3.7 vortex cores. The highest winding number that we observed at a single occurrence was 11. The number of vortices that can be stably pinned is limited by the radius of the pinning potential and the angular-rotation frequency of the system [55]. We note that we tend to observe one or two vortices outside of the regular cluster of dissociating vortices even with the use of a hold prior to ramping off the pinning beam [see, e.g., Figs. 2(d) and 6(d)]. These are likely vortices that have left the pinning potential near the start of the beam intensity ramp-down. Refinement of the spiral trajectory and the pinning potential parameters will be the subject of future work.

The GPE simulations indicate that the spiral method should be fairly robust. A $\pm 10\%$ change in atom number N, initial spiral radius R_0, beam waist w_0, height of the blue-detuned potential U, or the aspect ratio of the spiral resulted in variation of the number of pinned cores by ± 1. We found similar variation when shifting the initial beam positions x_0 and y_0 by $\pm 5\ \mu$m out of a Thomas-Fermi radius $R_r \sim 50\ \mu$m. Such simulations of a laser beam spiraling around a stationary BEC can differ from the actual experimental method in that the spiraling BEC experiences an additional inertial force. However, the physical method of vortex generation and pinning depends only on the relative motion of BEC and the stirring beam. To verify this, we performed targeted simulations with a moving BEC and stationary beam, and found no difference in the final number of pinned vortices for trajectories with $\tau_s > 4$ s ($U = \mu_0, N_s = 4$). For faster trajectories, the final number of vortices starts to diverge, but now we are entering the regime where the potential can no longer pin all of the vortex cores regardless of whether the beam or the BEC moves.

In our experiments, the largest impediment to reproducibility was drift in the axial alignment of the blue-detuned potential with respect to the center of the BEC. We experienced both a long-time-scale drift that could be accounted for by periodically centering the beam to the final BEC position, and also shot-to-shot fluctuations due to the BEC receiving an arbitrary kick earlier in the evaporation sequence (likely as a magnetic field turns off). In future implementations, the shot-to-shot fluctuations may be mitigated by ramping on the blue-detuned beam prior to the final stages of evaporation.

VI. EXTENSIONS TO MULTIPLE BEAMS

We use the 2D GPE simulations to explore the potential for extending the spiral technique to multiple beams. For this scenario, the beam position would need to be controlled directly, e.g., with piezocontrolled mirror mounts [25], acousto-optic deflectors [59,60], or digital micromirror devices (DMDs) [26]. Here we have decreased the beam waists to $w_0 = 8\ \mu$m largely to fit more beams within the finite size of the condensate, $R_r \sim 50\ \mu$m. Reducing the beam waist does necessitate a reduction of beam speed at the time when the beam fully enters the condensate, $v_s(t = t_m)$, thus resulting in fewer pinned cores for a given τ_s. However, reducing the beam waist does not fundamentally alter the physical mechanism underlying the spiral technique. For Fig. 8 we simply add a second spiral beam on the opposite side of the condensate. Both beams then follow the same trajectory but on opposite sides of the BEC. Both beams spiral in a CCW direction with the same spiral parameters, and we find each beam generates a doubly quantized pinned circulation with the same sign of circulation.

The unique feature of our spiral technique is that each beam creates and pins vortices all of the same winding number, as opposed to direct nucleation of vortex dipoles [25,44,61]. However, as shown in Fig. 9 we can engineer a scenario where we employ two beams, one spiraling CCW and one spiraling CW. In this scenario, we end up with two regions of opposite-signed multiquanta circulation. We note that this scenario does require modifying the spiral trajectory after the vortices have

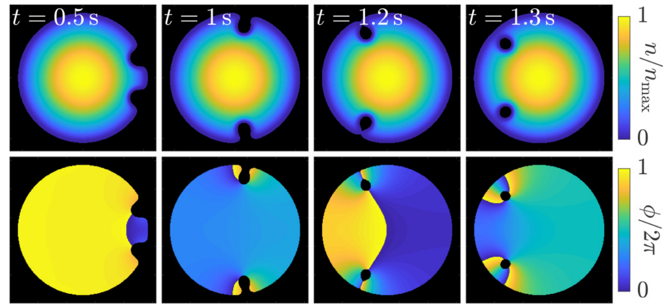

FIG. 9. GPE simulation for two beams, one spiraling clockwise and one spiraling counterclockwise. Simulation parameters are $N_s = 2$, $\tau_s = 3$ s, $w_0 = 8\ \mu$m, and $U = \mu_0$.

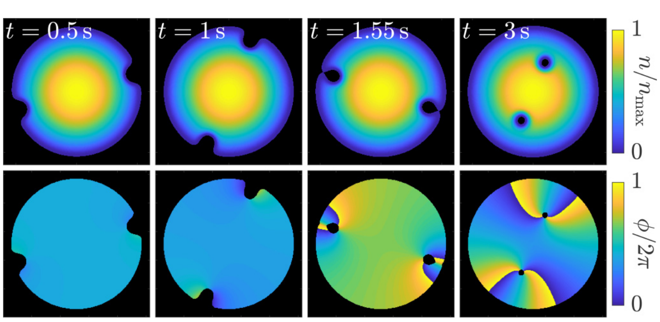

FIG. 8. GPE simulation for two beams both spiraling counterclockwise. Simulation parameters are $N_s = 3$, $\tau_s = 4$ s, $w_0 = 8\ \mu$m, and $U = \mu_0$.

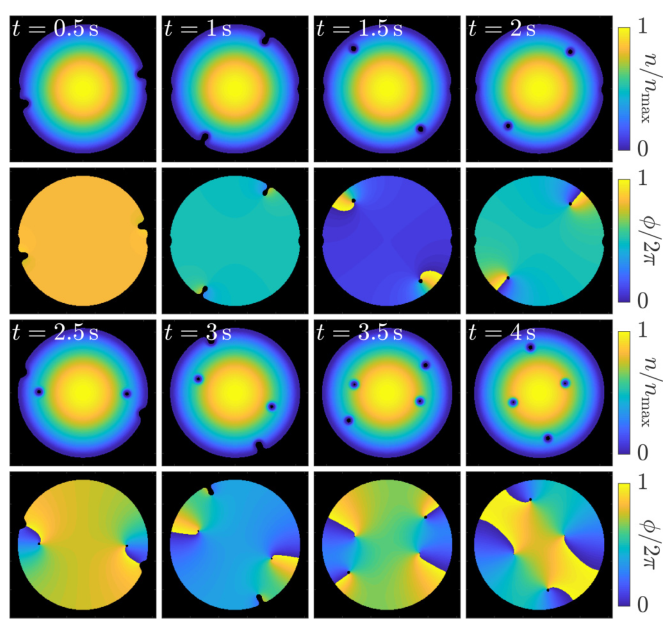

FIG. 10. GPE simulation for four beams. First one pair of beams spirals counterclockwise. After 2 s of evolution, the second pair of beams begins to spiral clockwise. Simulation parameters are $N_s = 4$, $\tau_s = 5$ s, $w_0 = 4\ \mu$m, and $U = \mu_0$.

been generated to avoid a collision of the pinning potentials and subsequent depinning or annihilation of the vortex cores.

In Fig. 10 we extend the technique further to four beams, each with a beam waist of $w_0 = 4\,\mu$m. In this final scenario, first one pair of beams spirals CCW, similar to that shown in Fig. 8. After 2 s of evolution, a second pair of beams begins to spiral CW. The net result is four pinning potentials within the BEC, each guiding a singly quantized vortex. Here the vortices pinned by the second pair of beams have the opposite sign from the vortices pinned by the first pair of beams.

The technique can be readily extended to trapping potentials with hard walls and flat bottoms. However, for smaller beams it is crucial to be able to control the relative position of the trap and the beam. In particular, it is detrimental if the beams dip in and out of the condensate rather than spiraling smoothly inwards. Therefore, extensions of this technique are likely best suited to a DMD-shaped optical potential for the BEC trap as well as the vortex pinning potentials [8,26]. In this scenario both trap and stirring beam are generated from the same laser beam, eliminating unwanted relative motion.

VII. CONCLUSION

We have demonstrated a method for deterministically generating multiply quantized circular superflow in a highly oblate ^{87}Rb Bose-Einstein condensate. By moving a BEC in a spiral trajectory about a stationary optical potential, we experimentally generate multiply quantized circulation with winding numbers as high as 11. By then ramping off the optical pinning potential, a cluster of singly quantized vortices can be released into the condensate. By changing the parameters of the spiral trajectory, we can control the vorticity introduced into the BEC. Our spiral method presents a reliable source of pinned circulation and quantized vortices for superfluid and BEC studies that require superfluid circulation with high winding numbers, and extends readily to multiple stirring beams and hard-wall trapping potentials. This method will be suitable for experimental studies regarding stability and dissociation of multiply charged vortices having higher circulation quanta, as well as experimental studies of 2D turbulence, vortex dynamics, and analog cosmology.

The data presented in this paper are available [62].

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