Commodity Correlation Risk*

Joseph P. Byrne† and Ryuta Sakemoto‡

1st November 2022

Abstract

It is widely observed that primary commodity prices comove. A parallel literature asserts that correlation risk matters for financial returns. Our novel study connects these topics and presents evidence that commodity correlation risk is both non-constant and important for returns. We reconsider therefore the relationship between primary commodities, risk and macro fundamentals, utilising methods that account for parameter uncertainty and stochastic volatility. We show that correlation risk is positively related to commodity returns and the strongest impact of risk upon return is more recent. We also demonstrate that commodity correlation risk is strongly counter-cyclical, correlation risk predicts returns, our risk measure is unrelated to other risk/uncertainty measures, and that correlation risk is linked to commodity financialization.

Keywords: Primary Commodity Returns; Commodity Correlation Risk; Commodity Comovement.

J.E.L. Classification Codes: E3; F3; F4; G1.

*The authors would like to thank Lamia Bazzioui, Arnab Bhattacharjee, Dimitris Korobilis, Stuart McIntyre and Jun Nagayasu for helpful comments and discussions. We would also like to thank Lawrence Christiano for kindly making his commodity financialization datasets publically available.

†Byrne Address: Department of Economics, Strathclyde Business School, University of Strathclyde, Glasgow, Scotland, UK. Email: <joseph.byrne@strath.ac.uk>.

‡Sakemoto Address: Graduate School of Humanities and Social Sciences, Okayama University, Okayama-shi, Okayama-ken, Japan. Keio Economic Observatory, Keio University, Minato-ku, Tokyo, Japan. Email: <ryuta.sakemoto@gmail.com>.
1 Introduction

Primary commodity price movements are important for consumers’ and firms’ economic decisions, policy makers and increasingly for financial markets. With a view to establishing a parsimonious characterisation, academic studies of commodity markets have uncovered key stylised facts. These facts include strong comovement of commodity prices or returns, which can be approximated by measures of central tendency such as principal components or dynamic factors.\footnote{Illustrative examples of commodity common factor and connectedness research are Cuddington and Jerrett (2008), Byrne, Fazio, and Fiess (2013), West and Wong (2014), Le Pen and Sévi (2017), Diebold, Liu, and Yilmaz (2017a), Fernández, González, and Rodríguez (2018), Alquist, Bhattarai, and Coibion (2019) and Ma, Vivian, and Wohar (2020).} This comovement of commodities relates to a range of macro fundamentals and risks. These key determinants include demand, opportunity cost and a variety of risk measures. Popular approaches to modelling relevant risks include realized stock market volatility and the Chicago Board Option Exchange’s volatility index. Considerable progress has been made understanding the empirical behaviour and determinants of commodities, but several important research questions remain outstanding. These questions include, does commodity comovement or cross correlation vary over time? Can commodity correlation be considered to be an important risk factor for commodity returns? Is commodity correlation risk counter-cyclical? That is, do asset return correlations intensify in business cycle downturns? Is correlation risk distinct from other well known risk factor? Finally, is there evidence of the importance of financialization in the commodity market? This work shall provide answers to these research questions.

Our novel study focuses upon the most appropriate measure of risk for primary commodities, by learning lessons from the literature on asset returns and correlation risk.\footnote{See inter alia Driessen, Maenhout, and Vilko (2009), Pollet and Wilson (2010), Christoffersen, Errunza, Jacobs, and Langlois (2012), Buraschi, Troiani, and Vedolin (2014) and Mueller, Stathopoulos, and Vedolin (2017).} Correlation or dispersion risk has been demonstrated to be important for other asset classes but has not, to the best of our knowledge, been considered for primary commodities. In an asset pricing context, Driessen et al. (2009) show correlation risk is important for stock returns and Pollet and Wilson (2010) observe that, while aggregate risk is nebulous, individual stock return correlation risk can predict aggregate stock returns.
Furthermore, in a study on risk and currencies, Mueller et al. (2017) examines whether correlation risk is priced in foreign exchange cross sectional regressions. In this research international FX correlation risk is measured as the dispersion of currency returns, it provides a positive risk premium which is also counter-cyclical.

Following contributions in the financial economic applications of Driessen et al. (2009), Pollet and Wilson (2010) and Mueller et al. (2017), there are several reasons why correlation risk can matter for commodity markets. Firstly, there is increasing financialization of primary commodity markets, as evidenced by greater synchronisation of the behaviour of institutional investors, see Singleton (2014), Sockin and Xiong (2015), Basak and Pavlova (2016), Le Pen and Sévi (2017) and Kang, Rouwenhorst, and Tang (2020). Financialization also implies commodities display similar behaviour to other asset prices, for example returns comove and display fat tails. To the extent that commodities comove they reduce the idiosyncratic commodity return for an investor and the diversification benefits of individual commodity investment, therefore requiring financial compensation. Secondly, implicit in the commodity comovement literature is the notion that commodities are highly correlated. Indeed this comovement goes beyond that reasonably expected by the similar impact of key macro fundamentals, à la Pindyck and Rotemberg (1990). This can be considered as an excess commodity comovement puzzle and indicates that macro fundamentals relevant for commodities are unspanned by correlation risk and/or correlation risk is time varying. That is, commodities comove beyond a common source of macro shocks. This implies that correlation risk can also be a common sources of shocks impacting returns. Thirdly, correlation risk may matter since the shocks that are important for commodities are not fully captured by empirical factors. In other words, there may be commodity comovement explained beyond dynamic factors or principal components. Commodities have heterogeneous components due to, for example, asymmetric resource constraints in production. Return dispersion is an appropriate risk measure when asset price heterogeneity is important, see Levy (1978) and Goyal and Santa-Clara (2003).³

Utilising correlation risk as an indicator of return dispersion, following the strategy of

³Stivers and Sun (2010) and Garcia, Mantilla-Garcia, and Martellini (2014) present evidence that return dispersion is associated with market states and a risk premium.
Mueller et al. (2017), may be one promising method to model risk in the primary commodity market.

This paper makes the following contributions to the expanding literature on primary commodities. Firstly, we test whether Mueller et al.’s (2017) type correlation risk factor matters for primary commodity returns. We do so examining three representations of commodities: a common factor; grouped by commodity class, that is metals and agriculture; and individual commodity returns. Secondly we examine correlation risk, accounting for different economic conditions in the commodity market, using a vector autoregressive model with time variation in both parameter estimates and residual stochastic volatility. Our empirical model is estimated by Bayesian methods with a Gibbs sampler and a Metropolis-Hastings step. This approach allows us to delineate the impact of each shock over time and fully capture potential structural change. This method has been widely used in other contexts: for instance, Primiceri (2005) and Ellis, Mumtaz, and Zabczyk (2014) investigate monetary policy shocks, Galí and Gambetti (2009) explore technology and non-technology shocks, and Baumeister and Peersman (2013) examine oil supply shocks. Time variation in the impact of shocks is important if correlation risk is only a recent phenomenon. Thirdly, we consider several stylised facts associated with commodity risk. Whether correlation risk is pro- or counter-cyclical, hence if and in what way this commodity risk is linked to measures of demand. We also consider whether commodity correlation risk can predict returns and whether correlation risk is independent of other risk measures. Fourthly, we examine whether commodity correlation risk is linked to measures of commodity financialization developed in Chari and Christiano (2019). If financial markets successfully process information, increasing financialization also potentially explains why correlation risk is tied to returns later in our sample. Finally, we examine the extent to which commodities respond heterogeneously to correlation risk using disaggregate models and Panel VARs. It should also be noted that our work differs from Pollet and Wilson (2010) and Mueller et al. (2017) by examining the commodity market, and from Le Pen and Sévi (2017) on excess commodity comovement by considering correlation risk.
This paper is structured as follows. Section 2 provides a review of relevant literature, Section 3 sets out the empirical model and the time series that we use, Section 4 contains our results, discussion and robustness, and the final Section 5 provides concluding remarks.

2 Review of Primary Commodity Literature

An early stimulus to the considerable research interest in the commodity market was provided by the Prebisch (1950)-Singer (1950) debate on long-run trends in commodity prices and their implications for commodity exporters. It has also been observed that commodity price fluctuations impact economic growth and inflation in both commodity importing and exporting countries, see Cody and Mills (1991) and Brunner (2002). More recently, there has been extensive discussion of excess comovement of commodity prices, see Pindyck and Rotemberg (1990) and whether common factors are observed in metal prices in particular, Cuddington and Jerrett (2008).

Research effort has consequently focused upon the measurement, determinants and consequences of comovement in the commodity market. Common factors in a large number of commodities have been widely observed and utilised, see inter alia Byrne et al. (2013), Gospodinov and Ng (2013), Daskalaki, Kostakis, and Skiadopoulos (2014), West and Wong (2014), Delle Chiaie, Ferrara, and Giannone (2017), Fernández et al. (2018), Alquist et al. (2019) and Ma et al. (2020). Byrne et al. (2013) examines whether a common factor can be identified in over 100 years of commodities from the Grilli-Yang index. Daskalaki et al. (2014) considers common factors and the cross section of commodity returns. West and Wong (2014) models the capacity of commodity common factors to improve predictability of individual commodities. Ayres, Hevia, and Nicolini (2020) find that a change in primary commodity price comovement is a key source of real exchange rate volatility in developed countries. In addition to work on the existence of commodity common factors, the recent research also examines more broadly the importance of

---

4The Prebisch-Singer debate examines trends in the relationship between commodity and manufacturing prices. For recent evidence examining the Prebisch-Singer hypothesis see Yamada and Yoon (2014) and Winkelried (2018).
common factors. Gospodinov and Ng (2013) highlights how common components in commodity convenience yields are important for inflation. Economic activity of emerging market exporters is strongly linked to a common factor in commodity prices Fernández et al. (2018). See also Alquist et al. (2019) for a discussion of global demand and commodity prices in general equilibrium.

While our novel contribution focuses upon the empirical importance of commodity correlation risk, we learn the lessons of the existing literature. For example the literature considers a variety of potential fundamentals and heterogeneity in primary commodities. Interest rates are commonly cited as a worthwhile fundamental to consider, see Frankel (2008). This is because commodity returns are influenced by their opportunity cost and primary commodities can be considered as forward looking assets more generally, whose future value can therefore be discounted via interest rates.

Commodity returns are also influenced by real economic activity or demand, see Pindyck and Rotemberg (1990), Byrne et al. (2013) and Ratti and Vespignani (2015). But which measure of global real economic activity should we use to measure demand for commodities? While modelling demand for crude oil Baumeister and Hamilton (2019) provide a new measure of world industrial production. This demand measure is constructed from real economic activity in OECD and larger non-OECD countries, building upon an existing OECD measure. For an application to global capital flows see Miranda-Agrippino and Rey (2020). When measuring demand for commodities we concentrate on the Baumeister and Hamilton (2019) measure. For further detail see the discussion in Baumeister and Hamilton (2019), Hamilton (2020) and Baumeister, Korobilis, and Lee (2022) and discussion in the data section. Ratti and Vespignani (2015) provides a discussion of the impact of liquidity measures of monetary policy on common factors in commodity prices. It is also well known that there is some heterogeneity in commodities that go beyond a common factor for all commodities, see Ma et al. (2020) and Byrne et al. (2020).

But while existing work on commodity comovement has highlighted particular stylised facts about primary commodity price behaviour, and subsequent research has reinforced
the importance of fundamentals for this common component, see Byrne et al. (2013) and Ma et al. (2020), this work circumvents one key issue. Commodities display excess comovement beyond that related to fundamentals, as best exemplified in the influential work by Pindyck and Rotemberg (1990). Part of our empirical strategy is to formally seek to model the importance of commodity correlation for returns themselves using the correlation risk measure introduced by Mueller et al. (2017).

Financialization of commodity markets have increasingly become an important topic of discussion, see Singleton (2014), Cheng and Xiong (2014), Adams and Glück (2015), Bessler and Wolff (2015), Sockin and Xiong (2015), Hamilton and Wu (2015), Basak and Pavlova (2016) and Le Pen and Sévi (2017). In a significant theoretical contribution to commodity market research, predicated on the increased involvement of institutional investors in commodities, Basak and Pavlova (2016) argue that financialization can have a pervasive impact upon a wide range of commodity returns. Singleton (2014) highlights the importance of speculative activity driving oil prices away from fundamentals. According to Singleton (2014), these findings could be explained by the importance of risk for commodities. Using large approximate factor models, Le Pen and Sévi (2017) identified that there is a time varying comovement of commodity returns in excess of fundamentals. This excess commodity comovement can be linked to the financialization of commodities. Informational noise due to speculative trading can lead to a loosening of the link between commodity returns and fundamentals according to Sockin and Xiong (2015). This is very much in keeping with the notion that fundamentals may have a time varying impact upon commodity returns, based upon the extent to which demand signals are more or less clear. Juvenal and Petrella (2015) identified a role for financialization in the oil market, beyond the role played by demand. Our research will therefore contribute to this expanding literature on commodities, financialization and risk.
3 Empirical Methods and Data

3.1 Econometric Model

In this section we set out our main econometric methods. These methods shall be applied in our results section to examine the relationship between commodity returns and correlation risk factors. Our benchmark approach considers a data vector which contains demand \((dy_t)\) a measure of commodity return central tendency \((dc_t)\) and commodity correlation risk \((cor_t)\). Our empirical model in Equation (1) facilitates parameter uncertainty by allowing parameters \((A_{j,t})\) and residual volatility \((\Sigma_t)\) to vary over time \(t\). We therefore estimate the following model using Bayesian methods, see Primiceri (2005) and Dieppe, Legrand, and von Roye (2018),

\[
Z_t = A_{1,t} Z_{t-1} + A_{2,t} Z_{t-2} + \ldots + A_{p,t} Z_{t-p} + e_t; \quad e_t \sim \mathcal{N}(0, \Sigma_t)
\]  

(1)

Equation (1) is a vector autoregressive model, with a \((N \times 1)\) data vector for the core commodity time series \(Z_t = (dy_t, dc_t, cor_t)\), in which \(N = 3\). We focus upon these three variables in our benchmark model since we need to estimate a lot of parameters in Equation (1). See, for instance, Peersman et al. (2021). Given the susceptibility of financial assets to rapidly changing relationships, we deploy a model robust to potential breaks. Therefore time varying parameters are \(\beta_t = \sum_{j=1}^{p} A_{j,t}\). Time varying parameters can be justified for asset prices by the theory of Bacchetta and Wincoop (2004), Veldkamp (2005) and Sockin and Xiong (2015). The relationship between fundamentals and asset prices is time varying because: investors emphasize different fundamentals in each period due to scapegoating, Bacchetta and Wincoop (2004); abundant information in good times relative to bad times, Veldkamp (2005); and/or there exists information limitations more generally Sockin and Xiong (2015).

Key parameters of interest to be estimated within the model are time varying VAR coefficients \(\beta_t\), the time varying residual covariance matrix \(\Sigma_t\) and parameter covariance matrix \(\Omega\). In this model \(\beta_t\) follows an autoregressive formulation, as depicted by Equation (2), while \(\beta_t\) residuals \(u_t\) have random covariance matrix \(\Omega\).

\[
\beta_t = \beta_{t-1} + u_t; \quad u_t \sim \mathcal{N}(0, \Omega)
\]  

(2)
With regard to the stochastic volatility component of the model, Equation (1) residuals $e_t$ are a $N \times 1$ vector and distributed as $\mathcal{N}(0, \Sigma_t)$ with diagonal matrix $\Sigma_t$. Importantly, our general time varying parameter model allows our residuals to capture stochastic volatility (i.e. $\Sigma_t$). Stochastic volatility is important within this context, since commodity returns are similar to many other assets and display periods of elevated volatility, while in other periods this goes into abeyance. We illustrate this in our results.\footnote{More generally, Diebold, Schorfheide, and Shin (2017b) is an example of the usefulness of incorporating stochastic volatility to improve model properties and predictions for asset returns.}

Our time varying parameter VAR with stochastic volatility (TVP-VAR+SV) is estimated using a Gibbs sampler with a Metropolis-Hastings step, using 15,000 repetitions and 10,000 burn-in. We then extract shocks from the model to allow interpretation of the data dynamics based upon impulse responses. The VAR lag length $p$ we use when implementing Equation (1) is equal to two.\footnote{Such a lag length allows us to trade of reasonably rich dynamics with a parsimonious model that reduces the computational burden of time varying parameters and errors.}

Further details of our econometric model are provided in the Model Appendix A.

### 3.2 Average Correlation and Commodity Market Risk

We consider that correlation risk is potentially important in capturing commodity market risk. We apply to the commodity market a correlation risk measure proposed by Pollet and Wilson (2010) for the stock market. The variance of the commodity market $\sigma^2_{mkt,t}$ is denoted as the value weighted of the product of correlation between commodities $i$ and $j$, and individual standard deviation:

$$\sigma^2_{mkt,t} = \sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,t} w_{j,t} \rho_{ij,t} \sigma_{i,t} \sigma_{j,t} \quad (3)$$

where $w_{i,t}$ is the weight of commodity $i$ at time $t$, $\rho_{ij,t}$ is the correlation between commodities $i$ and $j$, and $\sigma_{i,t}$ is the standard deviation of the commodity $i$ at time $t$. The product of the standard deviations $\sigma_{i,t} \sigma_{j,t}$ is decomposed into the value-weighted cross-sectional average variance for $N$ commodities $\bar{\sigma}_{i,t}$ and the pairwise specific deviations from the average.
average variance $\xi_{ij,t}$ and is denoted as:

$$\sigma_{i,t}\sigma_{j,t} = \bar{\sigma}_{i,t}^2 + \xi_{ij,t}$$  \hspace{1cm} (4)

where $\bar{\sigma}_{i,t}^2 = \sum_{i=1}^{N} w_{i,t} \sigma_{i,t}^2$. Substituting Equation (4) into Equation (3) and the variance of the commodity market is rewritten as:

$$\sigma_{mkt,t}^2 = \bar{\sigma}_{i,t}^2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,t} w_{j,t} \rho_{ij,t} + \sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,t} w_{j,t} \rho_{ij,t} \xi_{ij,t}.$$  \hspace{1cm} (5)

Following Pollet and Wilson (2010), we assume the second term is equal to zero, and then the variance of the commodity market depends upon the average variance and the value weighted of the product of correlation as:

$$\sigma_{mkt,t}^2 \approx \bar{\sigma}_{i,t}^2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,t} w_{j,t} \rho_{ij,t}.$$  \hspace{1cm} (6)

Equation (6) indicates that the correlation component plays a key role in the commodity market risk. This is also related to the finding of Mueller et al. (2017) who present evidence that the correlation risk measure for the FX market is linked to expected FX returns. According to theoretical and empirical evidence, we use the correlation measure and estimate commodity market risk.

### 3.3 Data

We now discuss in detail the data utilised in this study. The main focus of this work is to test the connection between commodities and correlation risk. We use 19 primary commodities included in the Grilli and Yang (1988) primary commodity price data set and are widely used for the primary commodity price studies, see for instance, Bleaney and Greenaway (2001), Byrne et al. (2013), and Yamada and Yoon (2014). Following Grilli and Yang (1988), we focus upon primary commodities and not explore energy commodities which have a different impact upon business cycles, see, for instance Baumeister and Peersman (2013). We obtain primary commodity prices from the World Bank commodity price data (The Pink Sheet), which covers a longer time period. We calculate real com-
Note: This figure illustrates three commodity correlation risks (all commodities, metals and agricultural materials) which are calculated as the differences between the 90th percentile and 10th percentile. The vertical lines indicate National Bureau Economic Research recessions.

Commodity prices by subtracting US CPI inflation and extract the first principal component of the returns of commodity prices. Further details of the commodities are provided in the Data Appendix B. Correlation risk $\text{corr}_{ij,t}$ for commodity returns $i$ and $j$ is estimated in Equation (7) by their ratio of realized covariance $\text{rcov}_{ij,t}$ to their realized volatility $\text{rv}_{i,t}$,

$$\text{corr}_{ij,t} = \frac{\text{rcov}_{ij,t}}{(\text{rv}_{i,t}\text{rv}_{j,t})^{1/2}}$$  \hspace{1cm} (7)

It is worthwhile investigating commodity correlation risk partly because this risk has been shown to price other asset classes. Commodity returns could be detrimentally impacted by an increase in correlation risk, since it reduces individual commodity’s diversification benefits. Following Mueller et al. (2017) we consider two measures of correlation risk: the first based upon dispersion of the 75th percentile to the 25th percentile; the second the difference between the 90th percentile and 10th percentile. We can see considerable
Table 1: DESCRIPTIVE STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand ((dy_t))</td>
<td>0.349</td>
<td>1.134</td>
<td>2.866</td>
<td>−6.478</td>
</tr>
<tr>
<td>Commodity Prices ((dc_t))</td>
<td>0.007</td>
<td>1.081</td>
<td>3.352</td>
<td>−4.850</td>
</tr>
<tr>
<td>Correlation Risk ((cor_t))</td>
<td>−0.001</td>
<td>2.077</td>
<td>5.090</td>
<td>−11.388</td>
</tr>
<tr>
<td>Interest Rates ((r_t))</td>
<td>1.402</td>
<td>3.034</td>
<td>13.163</td>
<td>−8.225</td>
</tr>
</tbody>
</table>

Note: Sample period is 1970Q1 to 2019Q4. Commodity returns is the first principle component of 19 commodity returns.

Variability in correlation risk over time, the dispersion of 75-25th and 90-10th is highly correlated although there are some differences between commodities if we group by metal and agriculture.7 Figure 1 illustrates three commodity correlation risks and we note that the commodity correlation risk using all commodities rises during the U.S. recessions.

We also consider standard determinants of commodities, consistent with the consensus in the existing literature. Demand is measured by world industrial production, see Baumeister and Hamilton (2019). Industrial production is for OECD countries and selected non-OECD countries (i.e. Brazil, China, India, Indonesia, Russia and South Africa), extracted from OECD Main Economic Indicators. Real interest rates are calculated by subtracting consumer price inflation from nominal U.S. three month T-Bill rates. The frequency of the data is quarterly and the span is from 1970:Q1 to 2019:Q4. Figure C18 presents the growth rates of core data used in this study. We use \(100 \times \ln(X_t/X_{t-1})\) as the growth rate for demand and the principal component of commodity returns. We employ rolling estimation for correlations and the window size is 12 quarters. Table 1 presents summary statistics of our main variables.8

---

7See Figures C16 and C17 in the online appendix.
8Further details of the source of macro fundamentals used in this study are provided in the Data Appendix B.
4 Results and Discussion

The empirical model and commodity returns data introduced in the previous section are now used to examine key research questions on primary commodities. Do commodity returns exhibit stochastic volatility? Is commodity correlation risk non-constant and important for commodity returns and is this relationship time contingent? Do standard fundamentals, for example demand as proxied by real economic activity, still matter once we account for correlation risk? Is correlation risk spanned by other risk/uncertainty measures? Can we predict returns using correlation risk? And does commodity financialization matter for commodity returns?

4.1 Aggregate Commodity Returns

We first discuss the empirical results from our commodity model using aggregate returns. We begin by demonstrating the appropriateness of the stochastic volatility component of our empirical model. There are periods of pronounced increases in volatility of the diagonal elements of $\Sigma_t$ from Equation (1). This can be seen from the estimated residual stochastic volatility in the three diagonal panels for demand, commodities and correlation, in Figure 2. Volatility clusters are closely aligned to the 1970’s commodity spikes and the 2008 financial crisis.

We now move to our paper’s first set of core results, for example on the relationship between aggregate commodity return and risk. We begin by considering unconditional, or time invariant, responses to three different market shocks. The results give an overall sense of the relationship between commodities and risk on average over time. These impulses responses are given in Figure 3, with each column representing one of the three distinct shocks to demand, commodity returns and commodity correlation risk. Model shocks are identified using a Cholesky decomposition. Within the Bayesian VAR, we order the time series as demand, commodity common factor and correlation risk.\footnote{This is partly based upon the Stock and Watson (2005) distinction between slow moving variables,}
Note: This graph provides posterior estimates of stochastic volatility of the residuals from a TVP-VAR+SV model. Endogenous variables considered by this model are demand ($d_{yt}$), commodity returns ($dc_{t}$) and correlation risk ($cor_{t}$). The graphs are for the variance-covariance matrix $\Sigma_t$ in Equation (1). The graphs clearly illustrate the time variation in the residual variances in the model and therefore the appropriateness of our approach. Shaded area is the 16% and 84% posterior credible intervals.

In broad terms, the unconditional results are reasonably intuitive. We see in Figure 3 that a positive shock to demand (first column) shall increase commodity returns (second row). As the shaded response credible intervals does not contain the zero x-axis in this panel, we can be confident the response of commodities to demand is non-zero. We set the credible interval width between 16% and 84%, which is the standard in the literature (e.g., Primiceri (2005) and Baumeister and Peersman (2013)). Notable also is that a positive shock to the commodity return common factor (second column), shall be associated with an immediate increase in risk (third row). Finally we see from the unconditional responses in Figure 3 that a shock to commodity correlation risk (third column) has a more nuanced and somewhat delayed impact upon commodity returns themselves (second row). The response of commodities to risk shocks at the two quarter horizon is around zero and the zero x-axis is within the credible interval, suggesting it is unlikely that the unconditional response is strong at that horizon. The third quarter commodity response however is such as demand, being ordered before fast moving variables like asset prices. We are especially interested in the relationship between commodity returns and correlation risk. We set out in the online appendix that results are robust to a different VAR ordering and sign restrictions.
Figure 3: UNCONDITIONAL COMMODITY RESPONSES TO THREE SHOCKS

Note: This is a graph of time average or unconditional impulse responses to shocks to demand, returns and risk. Our Bayesian VAR model contains a system with demand ($d_y$), commodity returns ($d_c$) and risk ($cor$). The first column has a shock to demand, the second column has a shock to commodities and the third column has a shock to correlation risk. The rows represent the response of demand, returns and risk respectively. The response horizon is up to 12 quarters. Shaded area is the 16% and 84% posterior credible intervals.

Positive and zero x-axis is marginally below the credible interval, giving some confidence that the response to the risk shock at the third quarter horizon was more likely to be greater than zero. Having set the scene, we now make fuller use of our time varying parameter methods and consider in particular whether these unconditional responses to a correlation shock are supported by the time specific impulse response evidence.

Our core aggregate results indicate that there is also time contingent evidence of the importance of commodity correlation risks. This evidence is from the conditional impulse responses of commodity returns to correlation risk shocks at two and three quarter horizons in Figure 4, consistent with the unconditional results. The shorter two quarter horizon response in the top panel Figure 4 is suggestive of a marginal effect from correlation shocks upon returns for most of the sample period and a negative impact early in the 1980s. In contrast, the longer three quarter response horizon of returns to a risk shock, in the bottom panel of Figure 4, is indicative of a strongly positive effect later in the sample periods, since the shaded response credible intervals do not contain the zero x-axis. For the time varying response at the three quarters horizon there is evidence of a stronger and
Figure 4: CONDITIONAL COMMODITY RETURN RESPONSE TO RISK SHOCKS

Note: This is a graph of impulse responses of primary commodity returns to a correlation risk shock for our TVP-VAR+SV model. The first row has a response at the two quarter horizon (horz=2) and the second row has a response at the third quarter (horz=3). As can be seen from the graph, the responses are different. For instance returns are positively related to risk later in the sample period for the longer response horizon. The shaded area is the 16% and 84% posterior credible intervals.

positive risk effect upon commodities from around 2005. Later in the paper we identify that the stronger response of returns to risk overlaps a period of increased financialization in commodity markets. We also link financialization to returns themselves.

The bottom panel of Figure 4 indicates that the positive impact of commodity correlation risk on returns arose in the 1990s and 2000s, especially after the global financial crisis, and has endured since then. Given that our commodity sample does not include precious metals, such as platinum, gold and silver, our results can not be explained by commodity investment as a store of wealth. Furthermore, greater risk generally can be associated with increases in prices of precious metal commodities if they act as a financial hedged (e.g. Baur and McDermott (2010)). Instead, the recent financialization of primary commodities, excluding precious metals, is associated with increasing institutional investment, Singleton (2014) and Le Pen and Sévi (2017). In such an environment moreover, macro events can provide clearer signals to drive commodity returns due to an easing of informational asymmetries, Bacchetta and Wincoop (2004) and Veldkamp (2005) and Sockin and Xiong (2015).
Figure 5: CONDITIONAL COMMODITY RESPONSE TO THREE SHOCKS

Note: This graph sets out nine impulse responses from a TVP-VAR+SV model, estimated between 1970Q1 and 2019Q4. There are three variables in the system: demand $d_{yt}$, commodity returns $dc_t$ and risk $cor_t$. The first column has a shock to demand, the second column has a shock to commodities themselves and the third column has a shock to correlation risk. The response horizon is two quarters, and is set out for each time period. The shaded response area is the 16% and 84% posterior credible intervals.

It is worthwhile considering the behaviour of commodities further in the light of comparable correlation risk in the stock market. Our risk measure captures both a high correlation and high return dispersion. Both are related to a higher risk price in the stock market. The aggregate correlation is a suitable market risk proxy, as proposed by Pollet and Wilson (2010). The aggregate correlation is also associated with idiosyncratic risk. Levy (1978) proposed a theoretical model, in which investors do not hold a well diversified portfolio and hence idiosyncratic risk is a more appropriate risk measure than market risk. Goyal and Santa-Clara (2003) confirm that idiosyncratic volatility is associated with future return. Moreover, return dispersion is linked to future economic states as shown by Stivers and Sun (2010), since firm characteristics vary with the business cycle, see Gomes, Kogan, and Zhang (2003). This stock market discussion also relates to the commodity market after the recent period of financialization. This is in the sense that there is a positive relationship between elevated correlation risk and higher commodity returns. That is based upon the idea that commodity returns response to a correlation shock is stronger in the later sample period with the longer response horizon in Figure 4.
Figure 5 provides a broader set of results from our main time varying parameter Bayesian estimation results, by setting out the responses of all variables in our system to all three shocks. The nine panels in Figure 5 show the response at the two quarter horizon of demand \((dy_t)\), commodities \((dc_t)\) and risk \((cor_t)\) to shocks to these three variables. Two quarter responses were considered to be representative of the responses of some of the other key relationships. From Figure 5, first column and second row, we see that demand and commodities are positively correlated, and the response credible intervals are outwith the zero axis for practically the entire sample. In addition commodity correlation risk shocks are negatively associated with global demand, in the bottom left panel. This result mirrors the counter-cyclical behaviour of FX correlation risk highlighted by Mueller et al. (2017).

In contrast, correlation risk does not respond to commodity returns in the bottom middle panel of Figure 5. Given we model commodity returns using a principal component of 19 returns, our results may be sensitive to aggregation in this summary measure. In the following sections we address issues related to the heterogeneity of commodities, by making different trade-offs between the usefulness of individual commodity return granularity and the commonalities that may be associated with different groups of commodities.

### 4.2 Agricultural and Metal Commodity Returns

We next examine whether commodities behave as groups in terms of their relationship to fundamentals and correlation risk. In particular groups of agricultural and metal commodity price returns. Reasons \textit{a priori} to believe that commodities exhibit heterogeneity with regards to fundamentals and risk is the extent to which metals are durable and/or storable, and the extent to which they are used in manufacturing or construction sectors.\footnote{Physical inventories are important sources to determine heterogeneous commodity risk premia (Gorton et al. (2013) and Bakshi et al. (2019)).} Prices of industrial metals may be linked to business cycles and are strong predictors for economic growth and equity market returns (see Fama and French (1988), Fernandez-Perez et al. (2017), Jacobsen et al. (2019)). While agricultural commodities are less durable and more sensitive to consumer demand/preferences. We consider whether the behaviour of commodities can be grouped together in Figures 6 and 7. Our evidence suggests that
Figure 6: CONDITIONAL AGRICULTURAL RESPONSES

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand $d_{yt}$, the second column has a shock to agricultural commodities $dca_t$ and the third column has a shock to correlation risk $cora_t$. The response horizon is two quarters. The first row is the response of $d_{yt}$, the second row $dca_t$ and the third row response is $cora_t$. Correlation risk has an impact on agricultural return towards the end of the sample period. Shaded area is the 16% and 84% posterior credible intervals.

Figure 7: CONDITIONAL METAL RESPONSES

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand $d_{yt}$, the second column has a shock to metal commodities $dcm_t$ and the third column has a shock to correlation risk $corm_t$. The response horizon is two quarters. The first row is the response of $d_{yt}$, the second row $dcm_t$ and the third row response is $corm_t$. Correlation risk has no clear impact on metal return. Shaded area is the 16% and 84% posterior credible intervals.
there are some obvious heterogeneity between the impact of shocks to agricultural and metal commodities. If anything agricultural commodities responses are consistent with the earlier results. That is agricultural commodities respond positively to demand shocks and correlation risk is counter-cyclical, see Figure 6. Correlation risk also has a positive impact upon commodity returns towards the end of the sample period, consistent with the notion that this happened during a period of greater commodity financialization. In Figure 7 metal prices are also strongly impacted by demand shocks but metal prices are relatively less impacted by correlation shocks.

4.3 Individual Commodity Returns

We have presented evidence that there may be some heterogeneity in commodity data. Furthermore, Appendix Figures C1 and C2 indicate that individual commodities are not homogeneously associated with fundamentals. In particular, metals behave similarly, for example, they are idiosyncratically correlated with each other and with our measure of demand. Agricultural commodities are somewhat more diffuse in their behaviour. Consequently we consider the impact of general shocks on individual commodities with a Panel VAR model. We estimate the model using Zellner and Hong (1989) random effects Panel VAR, which combines individual commodity and sample average information, with further details of the approach provided in Appendix A.

From the results it would appear that many individual commodities are impacted by both demand and correlation risk shocks, see Figures C13-C15. However, agricultural commodities are relatively more prone to correlation shocks while metal commodities are more susceptible to demand shocks. Overall 13 out of 19 commodities are impacted by demand shocks, in the sense that the zero axis is outwith confidence bands, while the majority of individual commodities are impacted by correlation risk (i.e. 10 out of 19). We can therefore evidence that commodity correlation risk also matters for individual commodities. Hence the result is not merely on average or an artefact of aggregation.
<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
<th>(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH</td>
<td>-0.43</td>
<td></td>
<td></td>
<td></td>
<td>-0.06</td>
<td></td>
<td></td>
<td>-0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.59)</td>
<td></td>
<td></td>
<td></td>
<td>(0.46)</td>
<td></td>
<td></td>
<td>(0.60)</td>
<td></td>
</tr>
<tr>
<td>GARCH-US</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td></td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.01)</td>
<td></td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Policy Uncertainty</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.03)</td>
<td></td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>VIX</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macro Uncertainty</td>
<td></td>
<td>6.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5.41)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(9.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default Spread</td>
<td></td>
<td>0.49</td>
<td></td>
<td></td>
<td>0.55</td>
<td></td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.53)</td>
<td></td>
<td></td>
<td>(0.57)</td>
<td></td>
<td>(1.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TED Spread</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.22)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>Observations</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.1</td>
<td>-0.8</td>
<td>-0.9</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-2.7</td>
</tr>
</tbody>
</table>

Note: This table shows relationships between the commodity correlation risk and the other risk variables. We regress the commodity correlation risk onto the other risk variables. All risk variables show autocorrelations and we take first differences and obtain stationary variables. The commodity correlation risk is calculated by all commodity variables and the difference between the 90th percentile and 10th percentile. We use the following risk variables: GARCH risk using the aggregate commodity index (GARCH); GARCH risk using U.S. stock market (GARCH-US); US Economic Policy Uncertainty Index (Policy Uncertainty, Baker et al. (2016)); VIX; macro uncertainty index (Macro Uncertainty, Jurado et al. (2015)); Default Spread; TED Spread; and Factor indicates the first principal component of these risk variables. The standard errors are reported in parentheses and obtained by the Newey and West (1987) procedure. The sample period is from 1990Q1 to 2019Q4. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. 

Electronic copy available at: https://ssrn.com/abstract=4265924
4.4 Correlation Risk and Other Risk Measures

Having established the importance of our measure of commodity correlation risk for commodity returns, we investigate whether it entails different information from other widely used risk and uncertainty measures. We therefore consider whether there is an empirical link between commodity correlation risk and the following eight risk measures: the commodity market and the U.S. stock market volatility estimated by a GARCH model (e.g., Glosten et al. (1993) and Byrne et al. (2020)); U.S. economic policy uncertainty index (Baker et al. (2016)); VIX (e.g., Ang et al. (2006) and Adrian et al. (2019)), a macroeconomic uncertainty index (Jurado et al. (2015)); the default spread (e.g., Fama and French (1989) and Welch and Goyal (2008)); the TED spread (e.g., Brunnermeier et al. (2009)); and the first principal component of these seven measures. Commodity market volatility is estimated using the non-energy commodity index in the World Bank commodity price data and stock market volatility is estimated by the value weighted index of the Center for Research in Security Prices (CRSP). The default spread is calculated as the Baa corporate bond yield relative to yield on the 10 year T-Bill and the TED spread is calculated as the difference between the three month Eurodollar LIBOR rate and the three month T-Bill rate. We note that our commodity correlation risk is not explained by the other risk measures, as reported by Table 2. In the Appendix C we also present evidence based upon Bayesian estimation of the relationship between correlation risk and the other eight risk measures. This is also generally suggestive that correlation risk is unrelated to other measures of risk, although one exception is a GARCH model of commodities. For further information on the relationship between different risk measures see Figure C19.

4.5 Forecasting Commodity Returns

This section considers forecasts of commodity returns using correlation risk. Pollet and Wilson (2010) and Stivers and Sun (2010) report that stock market correlation risk contains information about future market and factor returns. We investigate whether our commodity market correlation risk carries information about future commodity market returns. We benchmark forecasts of a VAR model containing commodity returns, real
### Table 3: FORECASTING COMMODITY CORRELATION RISK

<table>
<thead>
<tr>
<th>Model Period</th>
<th>( T = 1 )</th>
<th>( T = 4 )</th>
<th>( T = 12 )</th>
<th>( T = 1 )</th>
<th>( T = 4 )</th>
<th>( T = 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.07</td>
<td>0.35</td>
<td>0.70</td>
<td>0.05</td>
<td>0.32</td>
<td>0.68</td>
</tr>
<tr>
<td>MAE</td>
<td>0.07</td>
<td>0.32</td>
<td>0.56</td>
<td>0.05</td>
<td>0.28</td>
<td>0.53</td>
</tr>
<tr>
<td>MAPE</td>
<td>29.35</td>
<td>93.52</td>
<td>103.87</td>
<td>71.775</td>
<td>106.70</td>
<td>106.69</td>
</tr>
<tr>
<td>Theil’s U</td>
<td>0.13</td>
<td>0.71</td>
<td><strong>0.89</strong></td>
<td>0.11</td>
<td>0.74</td>
<td>0.92</td>
</tr>
</tbody>
</table>

| \( h = 4 \)  |             |             |             |             |             |             |
| RMSE         | **0.04**    | 0.34        | **0.69**    | 0.05        | **0.32**    | **0.69**    |
| MAE          | **0.04**    | 0.30        | 0.55        | 0.05        | **0.30**    | **0.54**    |
| MAPE         | **17.94**   | 87.13       | 99.89       | 20.03       | **83.35**   | **95.38**   |
| Theil’s U    | **0.08**    | 0.71        | **0.90**    | 0.11        | 0.75        | 0.93        |

**Note:** This table compares the forecasting performance of three models. The three models are \( Z_3 = (dy_t, dc_t, cor_t) \); \( Z_2 = (dc_t, cor_t) \); \( Z_1 = (dc_t) \). Therefore the first two models include correlation risk, while the third forecast model is a simple autoregressive model. Lower RMSE, MAE, MAPE and Theil’s U have better forecast. Bold if lowest forecast errors among three models. \( h \) is the forecast horizon. Estimation sample period is from 1970Q1 to 2016Q4, with the out of sample period from \( T = 1, 4 \) and \( 12 \) quarters. Models \( Z_2 \) and \( Z_3 \), which incorporate correlation risk, consistently perform better than the benchmark autoregressive forecasting model for commodity returns.

Economic activity and commodity correlation risk (\( Z_3 \)) and compare its performance to an AR model with only commodity returns (\( Z_1 \)). We also consider a model containing only risk and return (\( Z_2 \)). We have several measures of return predictability: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Theil’s U. These are presented for one quarter forecast horizon (\( h = 1 \)) and four quarter forecast horizon (\( h = 4 \)), for up to twelve quarters (\( T = 12 \)). These predictions are presented for one quarter forecast horizon (\( h = 1 \)) and four quarter forecast horizon (\( h = 4 \)), for an up to twelve quarter prediction window (\( T = 12 \)). The results are given in Table 3. We find that models containing commodity correlation risk have incremental predictive performance for future commodity returns. For example at a one step ahead forecasting window, the models incorporating correlation risk (\( Z_2 \) and \( Z_3 \)) perform better than, or at least as good as, a univariate autoregressive model (\( Z_1 \)) in nine out of twelve forecasts in Table 3. For a four step ahead forecast window models with correlation risk perform better or as good as the univariate prediction in ten out of twelve forecasts.
Table 4: COMMODITY RISK AND FINANCIALIZATION

<table>
<thead>
<tr>
<th>Risk Measure</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cor_t</td>
<td>cor_t</td>
<td>cora_t</td>
<td>cora_t</td>
<td>corm_t</td>
<td>corm_t</td>
</tr>
<tr>
<td>Open Interest</td>
<td>-0.42</td>
<td>-0.97**</td>
<td>1.22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.43)</td>
<td>(1.25)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Financial Flows</td>
<td>-0.53</td>
<td>0.14</td>
<td>2.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(1.19)</td>
<td>(2.19)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.12</td>
<td>-0.15</td>
<td>-0.06</td>
<td>-0.14</td>
<td>-0.09</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.20)</td>
<td>(0.24)</td>
<td>(0.26)</td>
<td>(0.56)</td>
<td>(0.54)</td>
</tr>
<tr>
<td>Observations</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>79</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>-0.1</td>
<td>-0.7</td>
<td>1.4</td>
<td>-1.3</td>
<td>-0.6</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

Notes: This table presents the empirical relationship between correlation risk and financialization. Columns (a) and (b) have aggregate correlation risk ($cor_t$) as the left-hand-side variable, agricultural risk ($cora_t$) is in equations (c) and (d), and metal equivalent ($corm_t$) is explained in regressions (e) and (f). Standard errors in parentheses. Financialization measures, from Chari and Christiano (2019), are Open Interest and Net Financial Flows. Both are scaled by commodity production. We also consider nff unscaled, with similar results. The standard errors are reported in parentheses and obtained by the Newey and West (1987) procedure. The sample period is from 1992Q1 to 2011Q4. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

4.6 Commodity Financialization and Commodity Returns

One area of research which has preoccupied researchers is the nature and impact of financialization in the commodity market, see Singleton (2014), Sockin and Xiong (2015) and Chari and Christiano (2019). For example, Chari and Christiano (2019) suggest that increased trading in the commodity futures market impacts outcomes in the spot market.

We consider whether commodity financialisation is related to our commodity market risk measure in Table 4. We relate our measure of correlation risk to Open Interest ($oi_t$) and Net Financial Flows ($nff_t$) from Chari and Christiano (2019), scaled by world production, from the Commodity Futures and Trade Commission (CFTC). These overall activity measures from the financial market are perceived to measure financialization in the commodity market. We use the broad index and link it to our three measures of commodity financialization.
correlation in Table 4. We present evidence that there is a link between risk for agricultural commodities ($cora_t$) and Open Interest in column (c) of Table 4. Links between biofuels and increased speculation in agricultural futures markets, would be consistent with this result. The negative sign is a potential indications that increased dispersion of returns, within our correlation risk measure, is negatively associated with increased financialization. We find less evidence of financialization for metals or Net Financial Flows. While these result as premia facia evidence of the link between financialization, risk and the commodity market more generally, they merit further investigation, see for example Cheng and Xiong (2014) and Chari and Christiano (2019).

4.7 Further Extensions and Robustness

We focus upon three types of robustness for our results: the incorporation of interest rates, following other authors in the literature, examining the impact of changing the measure of demand, changing the nature of shocks and finally examining whether the ordering of common factor and correlation shocks impacts the results. Firstly, it is common to consider that interest rates may have implications for commodity returns Frankel (2008), and this commodity fundamental was considered by Pindyck and Rotemberg (1990) in their classic study of comovement. We present unconditional and conditional evidence by including interest rates in a four variable system with demand, commodity returns and correlation risk. Results are included in an online appendix (i.e. Figures C10 and C11).

Secondly, we examine whether the positive response of correlation risk to commodities is sensitive to the choice of demand measurement. In addition to the use of world industrial production from Baumeister and Hamilton (2019) in core results we also examine the effect on our results when we substitute this for Kilian (2009) global real activity measure and global economic activity from Baumeister et al. (2022). Kilian (2009) is a business cycle indicator based upon shipping demand and Baumeister et al. (2022) which spans multiple dimensions of the global economy (e.g. real activity, financial sector, transportation, uncertainty, weather). Response results at 3 quarter horizons are provided in Figures C3 and C4. Figure C4 provides the greatest corroboration of our existing results that
correlation risk is positively associated with the central tendency of commodities, demand positively impacts commodity return and risk is counter-cyclical, although this as before is somewhat dependent upon time periods. Shipping demand provides some corroboration that correlation risk is positively associated with commodity returns later in the sample in Figure C3, and demand positively drives commodity returns at two quarter horizons, but there is less of a clear signal overall. Given Baumeister et al. (2022) is partly based upon commodities themselves and the criticisms of C3 in Hamilton (2020) largely based upon the method of detrending we would argue that our core measure of demand based upon world industrial production of Baumeister et al. (2022) is to be preferred.

Thirdly, we consider whether our shock identification process is robust. We consider two methods: (i) by utilising Uhlig (2005) sign restrictions; (ii) by changing the ordering of the variables in the VAR. Firstly, the sign restrictions are $dc$ responds positively to $dy$ and $cor$, and $cor$ responds positively to $dc$ and negatively to $dy$. These results are provided in an online appendix and generally confirm the positive response of commodities to a risk shock.\(^{12}\) Secondly, the change in ordering indicates risk is counter-cyclical, commodities respond to demand and risk correlation has an immediate and positive impact upon commodities. The only important difference is that commodities do not have a powerful impact upon correlation risk, since the zero axis is contained with the credible interval.\(^{13}\)

5 Concluding Remarks

Commodity market research has made great strides of late. Prominent results include evidence of commonalities, spillover from commodity indexes to all commodity returns, see Basak and Pavlova (2016) and excess comovement being related to hedging and speculative positions, Le Pen and Sévi (2017). See also Kang et al. (2020) for a recent study of the role of hedgers and speculators in the commodity market. Our work reconsiders commonalities in commodity returns, taking inspiration from the work on stock return correlation risk, see Driessen et al. (2009) and Pollet and Wilson (2010), and the measure-

\(^{12}\)See Online Appendix Figure C5 and C6.
\(^{13}\)See Online Appendix Figure C7 and C8.
ment of the correlation risk factor from Mueller et al. (2017). Correlation risk may matter for commodities since the display time contingent comovement and this comovement shall reduce the diversification benefits of different commodities, especially with increase financialization. We estimate a Bayesian time varying parameter vector autoregression with stochastic volatility. This allows us to account for a time evolving relationship between commodity returns and correlation risk. In addition using stochastic volatility allows us to model the heteroscedasticity implicit in commodity returns. We also account for commodity heterogeneity by grouping returns into metals and agriculture. And we examine the individual commodity responses to demand and correlation risk shocks using Bayesian panel VAR. In terms of our results, we replicate the standard positive response of commodities to demand shocks. We also find evidence that commodities display a positive response to a correlation shock. This would suggest that positive correlations risk shocks are priced by the market. Interestingly commodity return risk is counter-cyclical: that is business cycle downturns are associated with risk spikes. We also present evidence that correlation risk is unspanned by other risk factors, returns can be predicted by correlation risk and correlation risk is linked to commodity financialization.
References


Online Appendices

A Model Appendix

This section provides further detail on the estimation of the Bayesian model. This time varying parameter Bayesian vector autoregression model with stochastic volatility, which allows for time variation in the estimated coefficients and the residual covariance matrix. See Cogley and Sargent (2002) and Primiceri (2005) for further details. We use 15,000 iterations with 10,000 burn-in. We take every one in ten replications. The prior distribution for the residual error term $\Sigma_t$ from Equation (1) in Section 3 is the inverse Wishart distribution. While the prior for the variability of the $\beta_t$ parameters, $\Omega$ from Equation (2), follows an inverse Gamma distribution, I.G.(\(\chi_0\mid \psi_0\)), with shape $\chi_0$ and scaling parameters $\psi_0$ both equal to 0.001.

For the time varying parameter model with the more standard constant variance form, $\Sigma_t = \Sigma$ and core data $Z = (Z_1...Z_T)'$ from Equation (1), the Bayes rule is:

$$\pi(\beta, \Omega, \Sigma|Z) \propto f(Z|\beta, \Sigma)\pi(\beta|\Omega)\pi(\Omega)\pi(\Sigma) \hspace{1cm} (A1)$$

And the likelihood function for $f(Z|\beta, \Sigma)$, with the Equation (1) model in compact form $Z_t = \bar{X}_t \beta + \epsilon_t$:

$$f(Z|\beta, \Sigma) \propto \sum_{t=1}^{T} |\Sigma|^{-1/2} \exp \left( -\frac{1}{2}(Z_t - \bar{X}_t \beta)^\prime \Sigma^{-1}(Z_t - \bar{X}_t \beta) \right) \hspace{1cm} (A2)$$

The joint posterior distribution $\pi(\beta, \Omega, \Sigma|Z)$ in Equation (A1), does not have an analytical solution, hence estimating the model requires the Gibbs sampler with a Metropolis-Hastings step. We consider the more general situation with time specific residual covariance matrix $\Sigma_t$ which is decomposed into as $\Sigma_t = F\Lambda_t F'$. The lower triangular matrix $F$ has ones on its main diagonal and the time specific diagonal matrix $\Lambda_t$ is written with the scaling term $\bar{s}_j$ and the heteroscedasticity term $\lambda_{jt}$ as $\text{diag}(\Lambda_t) = (\bar{s}_1 \exp(\lambda_{1,t}), \bar{s}_2 \exp(\lambda_{2,t}), \cdots, \bar{s}_N \exp(\lambda_{N,t}))$. In Equation (A3) the heteroscedasticity term $\lambda_{jt}$ follows the autoregressive process with the shock term $v_{i,t}$:

$$\lambda_{jt} = \gamma \lambda_{j,t-1} + v_{i,t}; \hspace{0.5cm} v_{i,t} \sim \mathcal{N}(0, \phi_i) \hspace{1cm} (A3)$$
Using $F$, $\lambda_{i,t}$, and $\phi_{i}$, we obtain the Bayes rules as:

$$
\pi(\beta, \Omega, f^{-1}, \lambda, \phi | Z) \propto f(Z | \beta, f^{-1}, \lambda)\pi(\beta | \Omega)\pi(\Omega) \left( \prod_{i=2}^{n} \pi(f_{i}^{-1}) \right) \left( \prod_{i=1}^{n} \pi(\lambda_{i} | \phi_{i}) \right) \left( \prod_{i=1}^{n} \pi(\phi_{i}) \right)
$$

(A4)

where $f$ is the lower elements of the matrix $F$. In Equation (A4), terms $\lambda$ and $\phi$ generate the heteroscedasticity. The VAR has a lag length of 2, based upon a BIC information criteria and computational parsimony. Shocks are identified by Cholesky decomposition, with a VAR ordering of Demand, Commodity Returns and Commodity Risk. A more extensive treatment of Bayesian estimation in this context is set out, in particular, by Primiceri (2005), West and Harrison (2006), Elliott and Timmermann (2016), Dieppe, Legrand, and von Roye (2018) and Chan, Koop, Poirier, and Tobias (2019).

We also consider estimating a Bayesian Panel VAR, following Zellner and Hong (1989). This approach allows for heterogeneity in parameter estimates and impulse responses, associated with individual commodity returns, $dc_{it}$ and aggregate variables demand and correlation risk, where $x_{it} = \begin{bmatrix} dy_{it} & cor_{it} \end{bmatrix}$. The following model is estimated:

$$
dc_{it} = A_{i,1}dc_{it-1} + \ldots + A_{i,p}dc_{it-p} + B_{i}x_{it} + \epsilon_{i,t}; \quad \epsilon_{i,t} \sim N(0, \Sigma_{i})
$$

(A5)

A posterior distribution is obtained by Zellner and Hong (1989) through adopting a Minnesota prior and combining unit specific information and average sample information. Where $dc$ are stacked $dc_{it}$, $\bar{X}$ are stacked right hand side variables in Equation (A5) and $\beta \sim N(\bar{b}, \bar{\Sigma}_{b})$ are stacked coefficients, $dc = \bar{X}\beta + \epsilon$, with the likelihood function is:

$$
f(dc | \beta) \propto \exp \left[ -\frac{1}{2} ((dc - \bar{X}\beta)'\Sigma^{-1}(dc - \bar{X}\beta)) \right]
$$

(A6)

Heterogeneous Panel VAR impulse response functions are provided in Figures C13 and C14. These are consistent with the main results in the paper although they reveal a degree of heterogeneity. In particular, Figures C13 indicates that of those responses outwith confidence bands 8 out of 19 commodities respond positively to a positive correlation shock and two respond negatively. The responses are more definitive for demand shocks, since Figure C14 indicates 11 out of 19 commodities responded positively, which one was negative and one was ambiguous. The latter results are also in line with unconditional correlations in Figure C1, in which rice and tea are not related to demand shocks, while tobacco responds negatively. However, once we account for correlation across shocks with
the PVAR aluminium, lead and zinc are no longer related to demand.

As a final comparison we estimate a pooled Panel VAR results. Here the cross sections are the individual commodities, with the measure of correlation risk and demand. Estimation of the posterior is by a normal-Wishart approach, using the likelihood function. The coefficients are estimated to be the same across cross sections, so the only heterogeneous element is the data. These effectively drop the cross sectional subscripts $i$ from the parameters in Equation (A4). The results are presented in Figure C15 and are comparable with the unconditional VAR in the main text, see Figure 3. The pooled panel VAR estimates indicate that commodities on average respond positively to correlation risk and to demand shocks. For a further discussion of the heterogeneous and pooled panel VAR models see Dieppe et al. (2018) and Canova and Ciccarelli (2013).
B Data Appendix

This section of the appendix provides further details of the data set used in our study. We use 19 commodity return series in the World Bank commodity price data (The Pink Sheet). These are aluminium (alu), copper (cop), lead (lea), tin, zinc (zin), bananas (ban), beef (bee), cocoa (coc), coffee (cof), cotton (cot), maize (mai), palm oil (pal), rice (ric), rubber (rub), sugar (sug), tea, sawnwood (saw), tobacco (tob) and wheat (whe). We divide the primary commodities into agriculture and metals. Agricultural commodities are bananas, beef, cocoa, coffee, cotton, maize, palm oil, rice, rubber, sugar, tea, sawnwood, tobacco and wheat. Metal commodities are aluminium, copper, lead, tin and zinc. We use end of quarter data when available. We calculate real commodity prices by subtracting CPI inflation. We subsequently use this dataset to derive our primary measure of central tendency, i.e. the first principal component of commodity returns. The calculation of correlation risk is based upon Mueller et al. (2017) and we focus on the dispersion between the 75th and 25th percentile. There are similarities between the 75th-25th and 90th-10th percentiles displayed in Figures C16 and C17, although there are differences based upon disaggregate measures metals (MET) and agriculture (AGR). Real interest rates are calculated by subtracting CPI inflation from the 3 month U.S. Treasury Bill rate. Figure C17 also illustrates the low linkage between commodity correlation risk and other standard measures of risk, such as VIX and Jurado et al. (2015). The correlation risk series is somewhat related to commodity GARCH and Economic Policy Uncertainty series.

We examine a variety of different measures of demand. We concentrate on the data from Baumeister and Hamilton (2019), due to the breadth of the measure across countries and over time. For further details see <https://sites.google.com/site/cjsbaumeister/research>. We have also considered the real activity measure as a proxy for demand recommended by Kilian (2009), based upon ocean bulk dry cargo freight rates. <https://sites.google.com/site/lkilian2019/research/data-sets>. Finally we examine whether our results are robust using the Global Economic Conditions (GECON) Indicator Baumeister, Korobilis, and Lee (2022) for a data span commencing in 1973, see Christine Baumeister’s google website. This uses the first principal component extracted from a data set of sixteen variables, see Table 7 Baumeister et al. (2022) for its components.
<table>
<thead>
<tr>
<th>Definition</th>
<th>First Difference</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(dy_t) OECD + Non-OECD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commodity Returns First Principal Component</td>
<td>Yes</td>
<td>Commodity Series/World Bank</td>
</tr>
<tr>
<td>(dc_t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation Risk Eq. (7), Sec. 3</td>
<td>Yes</td>
<td>Authors’ own calculations based on Mueller et al. (2017)</td>
</tr>
<tr>
<td>(cor_t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Interest Rates US 3 month T-Bill Rates Minus CPI Inflation</td>
<td>Yes</td>
<td>FRED</td>
</tr>
<tr>
<td>(r_t)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Sample period is 1970Q1 to 2019Q4. Commodity returns is the first principle component of 19 commodity prices.
C Online Results Appendix: Not for Publication

Figure C1: Individual Commodities and Demand Correlation

Note: This figure illustrates the sorted correlations between 19 individual commodities ($d_{ci}$) and demand ($d_{yt}$). The upper panel is the correlations (rho), while the lower panel are the associated probabilities (pval).

Figure C2: Commodities, Fundamentals and Risk Correlation Matrix

Note: This is a heatmap of correlations between fundamentals demand ($d_{yt}$) and interest rates ($r_t$), the first principal component of commodities ($d_{ct}$), the Mueller et al. (2017) commodity risk measure ($cor_t$), and individual primary commodities ($d_{cti}$). Sample period is 1970Q1 to 2019Q4. Darker colours indicate higher correlations.

Electronic copy available at: https://ssrn.com/abstract=4265924
<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
<th>(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GARCH</strong></td>
<td><strong>-0.46</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>-0.45</strong></td>
</tr>
<tr>
<td></td>
<td>[-.83:.07]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.85:.03]</td>
</tr>
<tr>
<td><strong>GARCH-US</strong></td>
<td>-0.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>[-.48:.34]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.57:.46]</td>
</tr>
<tr>
<td><strong>Pol. Uncert.</strong></td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>[-.01:.01]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.01:.00]</td>
</tr>
<tr>
<td><strong>VIX</strong></td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>[-.01:.08]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.01:.08]</td>
</tr>
<tr>
<td><strong>Mac. Uncert.</strong></td>
<td>6.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>7.99</strong></td>
</tr>
<tr>
<td></td>
<td>[-1.06:13.83]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[6.56:9.41]</td>
</tr>
<tr>
<td><strong>Def. Spread</strong></td>
<td>0.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>[-.15:.15]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.60:.075]</td>
</tr>
<tr>
<td><strong>TED Spread</strong></td>
<td>0.51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>[-.30:.130]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.91:.49]</td>
</tr>
<tr>
<td><strong>Factor</strong></td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>[-.12:.43]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[-.12:.43]</td>
</tr>
</tbody>
</table>

**Note:** This table shows relationships between the commodity correlation risk and the other risk variables. We regress the commodity correlation risk onto the other risk variables. All risk variables show autocorrelations and we take first differences and obtain stationary variables. The commodity correlation risk is calculated by all commodity variables and the difference between the 90th percentile and 10th percentile. We use the following risk variables: GARCH risk using the aggregate commodity index (GARCH); GARCH risk using U.S. stock market (GARCH-US); US Economic Policy Uncertainty Index (Pol. Uncert., Baker et al. (2016)); VIX; Macro Uncertainty Index (Mac. Uncert., Jurado et al. (2015)); Default Spread; TED Spread; and Factor indicates the first principal component of these risk variables. The estimates and 68% credible intervals, in square brackets [], and obtained by the Bayesian Random-walk Metropolis–Hastings sampling estimation with a Zellner g-prior. Bold estimates indicate zero is not contained within critical intervals, hence the explanatory variable is likely to have an association with the dependent variable correlation risk.
Table C2: COMMODITY RISK AND FINANCIALIZATION: BAYESIAN ESTIMATION

<table>
<thead>
<tr>
<th>Risk Measure</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\textit{cor}_t</td>
<td>\textit{cor}_t</td>
<td>\textit{cora}_t</td>
<td>\textit{cora}_t</td>
<td>\textit{corm}_t</td>
<td>\textit{corm}_t</td>
</tr>
<tr>
<td>Open Interest</td>
<td>\textbf{-0.45}</td>
<td>\textbf{-1.00}</td>
<td>1.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-.87:-.02]</td>
<td>[-1.68:-.32]</td>
<td>[-1.19:1.13]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Financial Flows</td>
<td>-0.59</td>
<td>0.07</td>
<td>2.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-1.34:.16]</td>
<td>[-1.13:1.29]</td>
<td>[-.70:4.80]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.11</td>
<td>-0.15</td>
<td>-0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.14</td>
<td>-0.02</td>
<td>-0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log ML</td>
<td>-197.0</td>
<td>-197.2</td>
<td>-230.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-232.0</td>
<td>-303.9</td>
<td>-304.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table presents the empirical relationship between correlation risk and financialization. Regression dependent variables and risk measures are aggregate correlation risk is (\textit{cor}_t), agricultural risk is (\textit{cora}_t) and metal equivalent is (\textit{corm}_t). Financialization measures, from Chari and Christiano (2019), are Open Interest and Net Financial Flows. Both are scaled by commodity production. We also consider nff unscaled, with similar results. The estimates and 68% credible intervals, in square brackets [ ], and obtained by the Bayesian Random-walk Metropolis–Hastings sampling with 10,000 MCMC sample size, and estimation with a Zellner g-prior. Bold estimates indicate zero is not contained within critical intervals, hence the explanatory variable is likely to have an association with the dependent variable correlation risk. The sample period is from 1992Q1 to 2011Q4.
Figure C3: Commodities TVP IRF with Kilian (2009) Demand

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand, the second column has a shock to commodities and the third column has a shock to correlation risk. The horizon is three quarters. Shaded area is the 16% and 84% posterior credible intervals. The measure of demand is from Kilian (2009).

Figure C4: Commodities TVP IRF with Baumeister et al. (2022) Demand

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand, the second column has a shock to commodities and the third column has a shock to correlation risk. The horizon is three quarters. Shaded area is the 16% and 84% posterior credible intervals. The measure of demand is from Baumeister et al. (2022).

Electronic copy available at: https://ssrn.com/abstract=4265924
Figure C5: Commodities Unconditional IRF With Sign Restrictions

Note: This is a graph of unconditional impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand, the second column has a shock to commodities and the third column has a shock to correlation risk. The horizon is three quarters. Shaded area is the 16% and 84% posterior credible intervals. Here we use Uhlig (2005) sign restrictions to identify shocks. These restrictions are $dc_t$ responds positively to $dy_t$ and $cor_t$, and $cor_t$ responds positively to $dc_t$ and negatively to $dy_t$.

Figure C6: Commodities TVP IRF with Sign Restrictions

Note: This is a graph of conditional impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand, the second column has a shock to commodities and the third column has a shock to correlation risk. The horizon is three quarters. Shaded area is the 16% and 84% posterior credible intervals. Here we use Uhlig (2005) sign restrictions to identify shocks. These restrictions are $dc_t$ responds positively to $dy_t$ and $cor_t$, and $cor_t$ responds positively to $dc_t$ and negatively to $dy_t$.
Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, commodity returns and risk. The first column has a shock to demand, the second column has a shock to risk and the third column has a shock to commodities. The horizon is three quarters. Shaded area is the 16% and 84% posterior credible intervals. We assess the robustness of Cholesky identified shocks by changing the ordering of the variables in the VAR.

Figure C8: Commodity Response to Risk Shock; Shock Order Robustness

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with shocks ordered as demand, correlation risk and commodity returns. We therefore assess the robustness of Cholesky identified shocks by changing the ordering of the variables in the conditional Bayesian VAR. The top panel has the response at the one quarter horizon (horz=1) of commodity returns to a risk shock for the full sample. The middle panel has commodity return’s response at two quarters (horz=2). The bottom panel has the response of commodities at three quarters (horz=3). The results indicate that the commodity response is insensitive to the ordering of shocks. Shaded area is the 16% and 84% posterior credible intervals.
Figure C9: **Unconditional VAR with Interest Rates**

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, real interest rates, commodity returns and risk. The first column has a shock to demand, the second column has a shock to interest rate, the third column has a shock to commodities and the fourth to risk. The responses are unconditional. Shaded area is the 16% and 84% posterior credible intervals. The shock is Cholesky identified.

Electronic copy available at: https://ssrn.com/abstract=4265924
Figure C10: Conditional VAR with Interest Rates: Two Quarter Horizon

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, real interest rates, commodity returns and risk. The first column has a shock to demand, the second column has a shock to interest rate, the third column has a shock to commodities and the fourth to risk. The horizon is two quarters. Shaded area is the 16% and 84% posterior credible intervals. The shock is Cholesky identified.

Figure C11: Conditional VAR with Interest Rates: Three Quarter Horizon

Note: This is a graph of impulse responses from a TVP-VAR+SV model, with demand, real interest rates, commodity returns and risk. The first column has a shock to demand, the second column has a shock to interest rate, the third column has a shock to commodities and the fourth to risk. The horizon is three quarters. Shaded area is the 16% and 84% posterior credible intervals. The shock is Cholesky identified.
Figure C12: TVP VAR Model Impulse Responses

Note: This is a graph of impulse responses from a time varying parameter VAR model with stochastic volatility, with demand, commodity returns and risk. The graphs illustrate the responses of the three variables to shocks in the three variables over the entire sample period, for up to 12 quarter response horizons.
Figure C13: PVAR Impulse Responses of Commodities to Risk Shock

Note: This is a graph of impulse responses from a Bayesian Panel VAR model, with demand, commodity returns and risk for each of the individual commodities. The graphs illustrate the responses of the 19 commodities to shocks in correlation risk, for up to 6 quarter response horizons. Shaded area is the 16% and 84% posterior credible intervals.

Figure C14: PVAR Impulse Responses of Commodities to Demand Shock

Note: This is a graph of impulse responses from a Bayesian Panel VAR model, with demand, commodity returns and risk for each of the individual commodities. The graphs illustrate the responses of the 19 commodities to shocks in demand, for up to 6 quarter response horizons. Shaded area is the 16% and 84% posterior credible intervals.
Figure C15: **Pooled Panel VAR Impulse Responses**

*Note:* This is a graph of impulse responses from a pooled Panel VAR model, with demand, commodity returns and risk. The graphs illustrate the responses of the three variables to shocks in the three variables over the entire sample period, for up to 12 quarter response horizons. Shaded area is the 16% and 84% posterior credible intervals.

Electronic copy available at: https://ssrn.com/abstract=4265924
Figure C16: Different Measures of Correlation Risk

Note: This plot provides a graph of Mueller et al. (2017) commodity risk measure ($\text{corr}_\lambda$), based upon various measures. For example, the difference between the 75th percentile and the 25th percentile for all commodities and the difference between the 90th percentile and the 10th percentile (i.e., ALLP75P25 and ALLP90P10, respectively), and the same for metals (METP75P25 and METP90P10), and agriculture (AGRP75P25 and AGRP90P10).
Figure C17: Correlation Risk and Other Risk Measures

Note: This figure illustrates the correlation between Mueller et al. (2017) and other popular measures of risk. (i) All commodity correlation risk, 25%-75% (ALLP7525); (ii) All commodity correlation risk, 10%-90% (ALLP90P10); (iii) Metal commodity correlation risk, 25%-75% (METP75P25); (iv) Metal commodity correlation risk, 10%-90% (METP90P10); (v) Agricultural material commodity correlation risk, 25%-75% (AGRP75P25); (vi) Agricultural material commodity correlation risk, 10%-90% (AGRP90P10); (vii) GARCH risk: Aggregate commodity index (GARCHALL); (viii) GARCH risk: U.S. stock market (GARCHUS); (ix) global economic policy uncertainty index (EPU, Baker et al. (2016)); (x) US economic policy uncertainty index (EPUUS, Baker et al. (2016)); (xi) VIX; (xii) macro uncertainty index (Juarado et al. (2015)). The upper panel is the correlations (rho), while the lower panel are the associated probabilities (pval).
Note: This is a graph of the data used in this study. The data frequency is quarterly for demand, interest rate, the first principal component of primary commodity returns and correlation risk. The sample span is 1970Q1 to 2019Q4.
Figure C19: RISK MEASURES

Note: These figures illustrate the commodity correlation risk and the other risk variables. The commodity correlation risk is calculated by all commodity variables and the difference between the 90th percentile and 10th percentile (ALL). We use the following other risk variables: GARCH risk using the aggregate non-energy commodity index (GARCH); US Economic Policy Uncertainty Index (EPUUS, Baker et al. (2016)); VIX; macro uncertainty index (UNC, Jurado et al. (2015)); Default spread (DEF); and TED spread (TED).