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Volatility-managed commodity futures portfolios

Abstract

This paper examines whether the volatility management suggested by Moreira and Muir (2017) generates significant benefits both in-sample and out-of-sample in commodity futures markets. The in-sample results show the significant success of volatility management from the 12-month momentum and market portfolio, but the out-of-sample results show that volatility management fails to improve real-time performance, which indicates that in-sample results are not obtainable for real-time investors. To understand the failure of volatility management, we perform the simulation analysis and find that a negative risk-return relation seems to play a pivotal role in addition to strong volatility persistency to make volatility management successful.

JEL classification: G10; G11; G12

Keywords: Commodity futures; Volatility management; Volatility-managed portfolios; Momentum; Portfolio choice

1. Introduction

Moreira and Muir (2017) (hereafter, MM) suggest a simple volatility-managed portfolio can improve profitability in the equity market. The volatility-managed portfolio is a portfolio whose monthly return is scaled by the inverse of its previous month's realized variance. Their motivation of volatility management stems from the empirical fact that variance is highly forecastable at short horizons while returns are not. Specifically, a portfolio with high volatility in the previous month tends to have high volatility in the current month, but its volatility is weakly or not significantly related to its current returns. Consequently, scaling the portfolio with the past variance does not harm future profitability, and more importantly, mean-variance investors could time volatility based on this volatility predictability in a short horizon – taking more risk when volatility is low and less risk when volatility is high. They empirically show that applying this volatility management to various portfolios, such as market, value, and momentum portfolios, increases profitability and expands the investment opportunity from the perspective of a mean-variance investor.

By contrast, Cederburg et al. (2020) (hereafter, COWY) address that MM's findings are in-sample findings that are not obtainable in real-time and examine a set of 103 equity strategies whether they generate significant improvement in profitability by applying the real-time version of volatility management. COWY find that in out of sample, volatility management does not generate significant benefits or improvement except for the equity momentum strategies.

This paper aims to extend the literature on volatility management to commodity futures and answer whether volatility management as suggested by MM in the equity market improves real-time performance in the commodity futures market. The literature has mainly paid attention to the benefits of applying volatility management to equity long-short strategies, such as equity momentum or value strategies (MM;COWY). However, Kang and Kwon (2019) highlight that shorting commodity futures is more implementable with less costs than shorting stocks and also taking leverage is much easier in futures markets than in stock markets, and thus the long-short strategy in futures markets can be of more interests to investors. Furthermore, the literature has consistently shown profitable but unique strategies in commodity futures markets (Kang and Kwon, 2019) and thus we expect that benefits of volatility management in commodity futures markets can be of interest to investors and also still questionable. COWY especially document that MM's findings about the success of volatility management have drawn a considerable attention from investors by referencing the press in The Financial Times on March 9, 2016 (See footnote 2 of COWY). We thus expect that extending the study about effectiveness of volatility management to commodity futures can make a significant contribution to the industry in addition to the literature.

We examine volatility management of the average, basis, momentum, basis-momentum, change in slope, and curvature portfolios suggested by the literature on commodity futures markets. More importantly, we evaluate the benefits of volatility management both in-sample and out-of-sample following COWY. First, from the in-sample comparison between the original and volatility-managed portfolios, our results show that only the 12-month momentum volatility-managed portfolio has a significantly larger Sharpe ratio than its corresponding original portfolio. As the second in-sample analysis, we evaluate whether volatility management significantly extends the mean-variance frontier spanned by commodity factor portfolios and thus generates utility gains for a mean-variance investor. Our empirical findings show that the average, nine-month momentum, and 12-month momentum volatility-managed portfolios generate significant improvement from volatility management.

In both in-sample analyses, our results exhibit the success of volatility management especially in the 12-month momentum portfolio. In the literature on commodity futures, Kang and Kwon (2017) and Kwon et al. (2019) document that the short-term commodity momentum reveals substantial differences with the equity momentum, and the 12-month commodity momentum return has the largest correlation with the equity momentum return. In the equity markets, COWY report that momentum-related strategies exceptionally show significant benefits by applying volatility management in both in-sample and out-of-sample. Collectively, you may wonder whether our success from the 12-month commodity momentum in volatility management can be attributed to the comovement in momentums across diverse assets as Asness et al. (2013) report. To explore the possibility, we test two alternative volatility-managed 12-month momentum portfolios by scaling the 12-month commodity momentum return with the realized variance of the 12-month equity momentum return (SCMOM1) and the first principal component of the realized variances of the 12-month equity and commodity futures momentum returns (SCMOM2), respectively. We find that the latter one (SCMOM2) shows improvement over the original commodity momentum whereas the former one (SCMOM1) does not. Scaling with commodity futures momentum's own volatility generates larger improvement than SCMOM2, but still the difference between the Sharpe ratios of these two volatility-managed portfolios appears to be insignificant. These results suggest a possibility that the in-sample outperformance of the volatility-managed 12-month commodity momentum may be related to the common movement of momentums.

In our out-of-sample analyses, consistent with COWY, we find that volatility management in commodity futures markets does not significantly improve real-time performance. Our results show that the real-time optimal portfolio constructed from the original and volatility-managed portfolios does not show significant improvement over the real-time original portfolio. Comparison with the ex post optimal portfolio also confirms that the ex post results potentially overstate the value of volatility management in practice as COWY noted. The difference between COWY's findings in equity strategies and ours in commodity strategies is the performance of the momentum strategy. In particular, COWY report that momentum-related strategies show significant benefits even in the out-of-sample case by applying volatility management consistent with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). By contrast, in our results, volatility management of commodity futures momentum portfolios does not show any improvement in general. We also consider momentum comovement in the out-of-sample analyses but find that scaling with common volatility does not significantly improve the performance either.

This paper contributes to the literature by bringing up an important question about volatility management. This paper provides empirical evidence that volatility-managed portfolios do not provide a better Sharpe ratio than their corresponding original portfolios, which is counterintuitive considering the short-term persistency of volatility. Volatility management is likely to be successful if volatility is persistent in the short term and the risk-return relation is flat (or at least, non-positive). Previously, for the failure of volatility management, COWY explain that risk-return relations tend to be positive. According to our data, many volatility-managed portfolios fail to significantly improve the Sharpe ratio even though their original portfolios show the volatility persistency and the risk-return relation is even negative, which makes volatility management more favorable. These results suggest that a less impressive success of volatility management in the commodity futures market may be attributed to the weak persistency of volatility. From the simulation analysis, we find that the negative risk-return relation indeed substantially contributes to the outperformance of volatility-managed portfolios, and thus the empirical results that some factors show insignificant improvement even though they have negative risk-return relation may imply that their volatility is not persistent enough. Previously, MM suggest volatility management mainly motivated by the short-term persistency of volatility, but our results address that both having a negative risk-return and high volatility persistency can be important and volatility should be highly predictable to make volatility management successful.

This paper would also contribute to the momentum literature since we find that applying volatility management to the momentum strategy is successful in two different markets – equity and commodity futures markets – from the in-sample analysis and document that this success may be attributed to the common movement of momentums (Asness et al., 2013). However, in out-of-sample, we show that there exists a unique movement of commodity momentum that cannot be captured by the comovement of momentums, and thus using comovement to time volatility significantly worsens the performance of the volatility-managed commodity momentum relative to the one using the commodity volatility as a scaler. These findings can be related to the argument of Kang and Kwon (2017) and Kwon et al. (2019) that commodity momentum is different from equity momentum.

This paper proceeds as follows. Section 2 introduces the data and variables we use in this paper and Section 3 provides direct comparison between the original and volatility-managed portfolios (Section 3.1) and discusses further the empirical findings (Section 3.2). Section 4 studies volatility-managed portfolios from the perspective of a mean-variance investor in-sample (Section 4.1) and out-of-sample (Section 4.2). Section 5 concludes.

2. Data and Variables

We collect the daily settlement prices on 44 US commodity futures contracts from Datastream. In particular, our data include the futures contracts on butter, feeder cattle, live cattle, corn, dry whey, ethanol, lean hogs, cheese, random lengths lumber, milk class III, milk class IV, non-fat dry milk, frozen pork bellies, crude palm oil, oats, rough rice, soybeans, soybean meal, wheat, cocoa, coffee ‘C’, Brazilian coffee, cotton #2, orange juice, sugar #11, coal, Brent crude oil, light crude oil, heating oil, RBOB gasoline, electricity, high grade copper, gold, palladium, platinum, uranium, silver, and natural gas. We also include the futures contracts on aluminum, copper, lead, nickel, and zinc traded in London Metal Exchange as they are traded in the US dollar following the majority of studies (Gorton and Rouwenhorst, 2006; Moskowitz et al., 2012). The sample period is from January 1979 to December 2017.

We compute monthly excess returns on a fully collateralized futures position. In particular, for month t + 1, at the end of month t, we take a position in the nearest futures contract among those whose maturity dates are after the end of month t + 1 and so avoid roll-over during month t + 1.[1] The excess return on commodity futures for month t + 1 is computed by

$$R_{t+1} = \frac{F_{t+1}^{T}}{F_{t}^{T}} - 1$$

where F_t^T indicates the price of the futures at month t whose maturity is month T. Next, we construct various trading strategies based on the literature. Specifically, we first construct long-short portfolios employing various return predictors suggested in the commodity futures literature that are basis, momentum, and basis-momentum.

The basis indicates the slope of the futures term structure which has long been believed as a return predictor based on the theory of storage or the hedging pressure hypothesis. A body of the literature has reported that the portfolio sorted by the basis generates significant returns

[1] Our choice is consistent with that of the majority of commodity studies (Gorton et al., 2012; Hong and Yogo, 2012; Bakshi et al., 2019).

and suggested it as a risk factor in commodity futures markets (Fuertes, et al., 2015; Boons and Prado, 2019; Bakshi et al., 2019). The basis is defined as

$$Basis_t = \frac{F_t^{T_1}}{F_t^{T_2}} - 1,$$

where $F_t^{T_i}$ indicates the *i*th-nearby futures price for month *t*.

Second, the *j*-month momentum – the past *j* month cumulative return – is computed as

$$Momentum_{j,t} = \prod_{s=t-j+1}^{t} \left(1 + R_s^{T_1}\right) - 1,$$

where $R_t^{T_i}$ indicates the *i*th-nearby futures return for month *t*.

The momentum phenomenon is first documented by Jegadeesh and Titman (1993) in the US stock market, but a number of empirical studies have presented the evidence that this phenomenon also exists in commodity futures markets (Erb and Harvey, 2006; Miffre and Rallis, 2007; Kang and Kwon, 2017). The literature on momentum has mainly focused on the 12-month momentum (Asness et al., 2013; Szymanowska et al., 2014; Daniel and Moskowitz, 2016), but commodity futures momentum has revealed substantial differences across choices of ranking periods *j* (Shen et al., 2007; Kang and Kwon, 2017, 2019; Kwon et al., 2019) and thus in this paper we employ various ranking periods in addition to 12 months, which are 1, 3, 6, and 9 months.

Next, we employ the basis-momentum, difference between the past 12-month momentum in a first-nearby and second-nearby futures, suggested by Boons and Prado (2019). Boons and Prado (2019) provide the empirical evidence that the basis-momentum is significantly priced in commodity futures and its pricing effect is robust to and even stronger than that of the basis and the 12-month momentum. Following Boons and Prado (2019), the basis-momentum is defined as

$$BasisMomentum_t = \prod_{s=t-11}^{t} \left(1 + R_s^{T_1}\right) - \prod_{s=t-11}^{t} \left(1 + R_s^{T_2}\right).$$

In addition, following Boons and Prado's (2019) decomposition of the basis-momentum, we further include two components of the basis-momentum that are the change in slope and the average curvature. The change in slope and the average curvature are defined respectively as follows:

$$change\ in\ slope_t = B_{t-12}^{T_2} - B_t^{T_1}$$

$$Average\ curvature_t = \sum_{s=t-11}^{t-1} B_s^{T_2} - \sum_{s=t-11}^{t-1} B_s^{T_1}$$

where B_s^i indicates the slope between the *i*th and *i*+1th-nearby futures prices, $F_t^{T_{i+1}}/F_t^{T_i}$.

Using these return predictors – the basis, momentums, basis-momentum, and two components of the basis-momentum – we form the long-short portfolios. Specifically, in each month, we sort commodity futures by each predictor into quintiles and construct a long-short portfolio buying the top quintile and selling the bottom quintile. For each portfolio, we compute the equal-weighted average of the excess returns. The excess returns on the basis, *j*-month momentum, basis-momentum, change in slope and curvature portfolios are denoted by BS, MOM*j* (for *j* = 1, 3, 6, 9, and 12), BM, SL, and CV, respectively. Lastly, we construct a commodity market portfolio that includes all sample commodity futures following Bakshi et al. (2019). The equal-weighted average of excess returns on this portfolio is named the average factor, AVG.

Based on these factors, we construct the volatility managed version of each factor following MM and COWY. The volatility-managed factor is defined as

$$f_{\sigma,t} = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t, \tag{1}$$

where f_t is the original factor – the excess return on the original portfolio – in month *t* and c^* is a constant variable that makes the full-sample variance of f_t and $f_{\sigma,t}$ the same. $\hat{\sigma}_t^2$ is the realized variance of daily portfolio excess returns in month *t* defined as

$$\hat{\sigma}_t^2 = \sum_{d=1}^{D_t} \left(f_d - \frac{\sum_{d=1}^{D_t} f_d}{D_t} \right)^2 .$$

where D_t is the number of trading days in month *t*. [2]

3. Volatility versus Original factors

3.1. Direct Comparison

To start with, we examine whether the volatility-managed portfolio outperforms its original portfolio. Previously, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) develop the volatility-managed momentum factor and compare its profitability measured by the Sharpe ratio with that of its corresponding original (unscaled) factor. COWY also perform a direct comparison between the comprehensive set of equity factors and their corresponding volatility-managed factors and report that volatility management fails to

[2] In untabulated results, we also employ the realized variance of daily excess returns over the past six months following Barroso and Santa-Clara (2015) and the dynamic strategy suggested by Daniel and Moskowitz (2016). We confirm that the results are qualitatively the same. We appreciate the anonymous referee for the suggestion.

significantly increase the Sharpe ratio in most of the cases. Motivated by these previous studies, we begin with the direct comparison of the original and the corresponding volatility-managed factors in this section.

We examine whether the Sharpe ratio difference between the original and its corresponding volatility-managed factors is statistically significant using the approach developed by Jobson and Korkie (1981). In Table 1, Panels A and B show the mean, standard deviation, and annualized Sharpe ratio of the original and volatility-managed factors, respectively, and Panel C reports the statistical significance of the Sharpe ratio differences with their p-values in brackets.

[Insert Table 1 about here]

Our results in Table 1 reveal that a success of volatility management is very limited in commodity futures factors. Six out of ten test factors show a decrease in the Sharpe ratio by volatility management and one of them (SL) shows that this decrease is even marginally significant (p-value = 0.06). The only success of volatility management can be observed from the 12-month momentum factor (MOM12). The volatility-managed MOM12 shows an impressive improvement in the Sharpe ratio, which is from 0.559 to 0.822.

This rather small success is not surprising in some sense if we recall that among the 103 equity trading strategies 53 strategies have positive differences and only eight of these 53 positive values are statistically significant in the COWY study. They also report that among 50 negative difference cases, four cases have statistically significant values. Thus, one success out of ten test strategies in our results seems comparable to COWY's findings in equity markets.

COWY's results and ours commonly reveal the success of volatility-managed momentum in the equity and commodity futures markets, respectively. COWY report that the majority of the significantly positive Sharpe ratio differences are concentrated in the momentum-related strategies. Specifically, they categorize 103 equity trading strategies into eight groups following Hou et al. (2015) – accruals, intangibles, investment, market, momentum, profitability, trading, and value – and report that all five momentum-related strategies show significant improvement in the Sharpe ratio by applying volatility management. Considering that they find only eight significant increases in the Sharpe ratio, it is impressive that five of these eight successful cases are indeed momentum-related strategies.

Moreover, we find that only the 12-month momentum strategy shows significant improvement in the Sharpe ratio among our five momentum strategies. Our results also show that the Sharpe ratio difference monotonically decreases as the ranking period of momentum decreases. For example, Panel C of Table 1 shows that the Sharpe ratio difference for MOM1 is -0.031 while that for MOM12 is 0.263. Asness et al. (2013) document that there is a comovement in momentum returns across various asset markets including both stock and commodity futures. Measuring the comovement in momentums as the traditional 12-month stock momentum following Asness et al. (2013), Kwon et al. (2019) report that the commodity futures momentum with a shorter ranking period has a weaker relation with the stock momentum. These previous findings suggest that our results may be associated with the comovement in momentums. To check whether the success of the volatility management for MOM12 stems from the comovement with the equity momentum, we construct two alternative volatility-managed MOM12 factors. First, we scale MOM12 with the realized variance of the 12-month equity momentum factor (SCMOM1). Second, we scale MOM12 with the first principal component of the realized variances of the 12-month equity and commodity futures momentum factors (SCMOM2).

[Insert Table 2 about here]

Panel A of Table 2 reports the mean, standard deviation, and annualized Sharpe ratio of SCMOM1 and SCMOM2, respectively, and Panel B of it shows the statistical significance of the Sharpe ratio differences between the original (MOM12) and the volatility-managed (SCMOM1 or SCMOM2) factors with the p-value in brackets. Our results show that replacing the variance of MOM12 with equity momentum variance or the first principal component of the momentum factors as a scaler makes volatility management of the commodity momentum less attractive. If we scale MOM12 with the variance of the equity momentum, then its Sharpe ratio (0.434) is even smaller than that of the original commodity momentum (0.559) though Panel B of Table 2 shows that this difference is not statistically significant. Scaling with the first principal component (SCMOM2) shows better performance than SCMOM1 and also generates a larger Sharpe ratio than the original one, which is 0.752. The difference is significant at the 10% significance level (p-value = 0.09). Comparing to the results in Table 1, SCMOM2 generates slightly lower profitability than the strategy scaling with the commodity momentum's volatility. However, in untabulated results, we find that the difference between the Sharpe ratios of SCMOM2 and the volatility-managed MOM12 is statistically insignificant as the p-value is 0.12. These results suggest that the improvement in the Sharpe ratio for the volatility-managed MOM12 in Table 1 may be associated with the common movement of momentums. Besides, a big difference in the performance of SCMOM1 and SCMOM2 suggests that only the common part in the variances of equity and commodity momentum portfolios matters in volatility management to improve the performance of MOM12 and that the variance of equity momentum itself is not appropriate as a scaler in volatility management for MOM12.

Lastly, Panel D of Table 1 presents the properties of volatility-managed factors. It shows that the original and corresponding volatility-managed factors are highly correlated as in MM and COWY. The correlation coefficients for the original and the volatility-managed factors range from 0.748 to 0.871. The distribution of the volatility-managed portfolio's implied weight on the original portfolio shows some differences from that for the equity portfolios. Our results show that the median positions (c50) tend to be smaller than one, ranging from 0.652 to 0.947, and the 99th percentiles (c99) range from 2.522 to 4.074. On the other hand, for the nine equity factors that MM examine, COWY report that the median positions are around one ranging from 0.81 to 1.11 and the 99th percentiles ranges from 4.45 to 8.64. The difference in the 99th percentiles between equity factors and commodity factors implies that volatility-managed commodity factors take much smaller leverage even in extreme cases. Moreover, the 1st percentiles (c1) also show that the weights for commodity futures factors (0.083 to 0.152) tend to be larger than those for the equity factors (0.03 to 0.06). These overall differences in the distribution of the weights indicate that volatility management for commodity futures factors much less aggressively alters the exposure to the underlying factors over time. This fact is consistent with the finding of Kang and Kwon (2019) that commodity futures momentum strategies are less exposed to the extreme downside risk due to the change in volatility, which may less require volatility management.

3.2. Understanding a small success of volatility management

Volatility management is likely to be successful if volatility is persistent (in the short term) and the risk-return relation is flat as MM's Figure 1 implies.

[Insert Figure 1 about here]

Figure 1 shows that the commodity futures portfolios reveal a feature similar to the one observed in MM. Following MM, we sort months into five states by the previous month's realized volatility and compute the average returns, volatility, and the mean-variance trade-off in the subsequent month. Panels A to C of Figure 1 present the results for the commodity futures market portfolio, and Panels D to F present those for the basis portfolio.

The monotonic patterns in volatility shown in both Panels B and E of Figure 1 indicate the short-term predictability of volatility consistent with MM's findings in stock markets. However, consistent with COWY, in Section 3.1, we find that volatility-managed portfolios do not generally provide significantly higher Sharpe ratios than their corresponding original portfolios. This empirical evidence indeed disproves MM's motivation to manage volatility. For underperformance or no significant improvement of volatility-managed portfolios, COWY examine whether those hypotheses are supported by the data and report that the risk-return relations tend to be positive, which makes volatility management less effective and unattractive.

To better understand our results in Section 3.1, in this section, we first conduct a simple test to see the persistency of volatility and the risk-return relation of our test factors. In particular, we estimate the following time-series regressions[3]:

$$f_{t+1} = a_0 + b_0 \hat{\sigma}_t^2 + e_{0,t+1} \tag{2}$$

$$\hat{\sigma}_{t+1}^2 = a_1 + b_1 \hat{\sigma}_t^2 + e_{1,t+1}. \tag{3}$$

[Insert Table 3 about here]

We estimate equations (2) and (3) for each of our test factors and report the coefficients on the realized volatility in the previous month (b_0 and b_1, respectively) and the adjusted R^2 values in Table 3. For the risk-return relation (equation (2)), our results show that the volatility has quite low predictive power for the subsequent month's return as the R^2 values range from

[3] MM introduces the regression model (2) (equation (14) in MM) a standard risk-return trade-off regression and use it to test the risk-return relation for the equity market portfolio.

0.00% to 5.74% whereas it has much larger predictive power for the subsequent month's volatility. These results are consistent with MM's motivation for volatility management, but we also find some other features contrary to MM's case.

First, in most of the cases, the risk-return relations are statistically significant though the R^2 values are low. More interestingly, except SL and MOM3, the risk-return relations appear to be negative, which makes volatility management more attractive, and six of these cases are statistically significant. In case of SL, in Table 1, it exceptionally shows that the volatility-managed portfolio generates a significantly lower Sharpe ratio than its original portfolio. Our results in Table 3 address that the positive relation between the lagged volatility and the current return might also contribute to it in addition to the weak persistency of volatility. Specifically, the estimated results of equation (3) for SL show that the coefficient on the lagged volatility is substantially small as 0.046 with the R^2 value = 3.13%.

Second and more importantly, however, the results for equation (3) which reveal the extent of the persistency of volatility cast a doubt whether volatility is persistent enough to time volatility. Our results suggest that volatility may not be sufficiently forecastable in many cases as opposed to MM's cases, though the relation between the lagged and the current volatility is much stronger than the relation between the lagged volatility and the current return. In addition to SL, BS and MOM1 also exhibit substantially weak relations between the lagged and current volatility. For example, in case of BS, b_1 appears to be marginally significant as t-statistics = 1.76 and the R^2 value is also extremely low as 0.67%. In case of MOM1, the risk-return relation is even favorable to manage volatility as b_0 is negatively significant, but the weak persistency of volatility might result in an insignificant effect of volatility management.

The risk-return relation of AVG is significantly negative, which makes volatility management even more attractive, and the volatility appears to be the most persistent among our test factors (b_1 = 0.669 and R^2 = 44.75%). In Panel C of Table 1, we find that the Sharpe ratio difference for AVG is positively large as 0.137 but still not statistically significant (p-value = 0.20). In fact, other factors such as BM, CV and MOM9, though having negative b_0 and relatively large b_1, also seem to benefit little from volatility management in Table 1. The only successful case is MOM12, which shows the negative risk-return relation (negative b_0) and the relatively strong persistency of volatility (high b_1 and large R^2).

For comparison, we also estimate equations (2) and (3) for four popular equity portfolios – the market (MKT), size (SMB), value (HML), and momentum (SMOM) portfolios.[4] The results are reported in Table A1 of Appendix. Consistent with COWY, Panel C of Table A1 reveals that among four equity factors, only the equity momentum portfolio shows a significant increase in the Sharpe ratio by managing volatility. Panel D of the table additionally suggests that this success can be attributed to the negative risk-return relation and the volatility persistency as the results from commodity portfolios also suggest. The equity momentum (SMOM) shows a notably large R^2 value even in equation (2), which is 6.46% while others range from 0.01% to 1.60%, and the risk-return relation (b_0) is negative and highly significant. The results for equation (3) also show that the next month's volatility is highly predictable as b_1 = 0.673 and R^2 = 45.28%. HML reveals even stronger volatility persistency which is b_1 = 0.729 and R^2 = 53.18%, but Panel C of Table A1 shows that its volatility managed version does not significantly outperform the original one. The size factor (SMB), which is also documented as an exceptional failure by MM, shows a notably weak relation between the lagged and current volatility (b_1 = 0.262 and R^2 = 6.88%), which imply that the failure of volatility-managed SMB can be due to the weak persistency of volatility.

[4] For comparison, we match the sample period of the equity portfolios with that of the commodity futures portfolios, which is from January 1979 to December 2017. We use the daily and monthly factor return data provided by Kenneth French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

In Figure 1, we confirmed that our test portfolios commonly show a monotonic pattern in subsequent month's volatility as in MM for their equity portfolios, and the positive and significant relation between the lagged and current volatility reported in Table 3 again confirms it. On the other hand, we find in this section that (1) the risk-return relation is statistically significant in most of the cases, and (2) volatility management does not show a significant increase in the Sharpe ratio (Table 1) even though volatility seems relatively persistent and the risk-return relation is even favorable to time volatility. These results raise important and fundamental questions on the presumptions for volatility management – whether volatility is persistent or predictable enough (even in the short term) to time volatility and whether risk-return relation is flat or nonpositive. Since there is no standard or threshold on the volatility persistency that guarantees the success of volatility management, it is hard to answer the question of whether the volatility of those test factors is persistent and predictable enough. In addition, since both equity and commodity futures momentums show relatively strong negative relations between the lagged volatility and the current return compared to other factors, it is also questionable how much this negative relation, which is favorable to volatility management, facilitates the success of volatility management.

To shed a light on these questions, we develop a simulation analysis based on equations (2) and (3). First, we estimate equations (2) and (3) using data, and randomly draw residuals (with replacement) of equations (2) and (3) for T months, which is the number of time-series observation of a test factor. Using the series of randomly drawn residuals, we regenerate the time-series of return and volatility using equations (2) and (3) but with various choices of b_0 and b_1. Next, we construct the time-series of returns on the volatility-managed portfolio and conduct Jobson and Korkie's (1981) test to see whether the volatility-managed factor generates a significantly larger Sharpe ratio than its original factor (and also vice versa). Specifically, we investigate the proportion (%) of the trials that reject the null hypothesis $H_0\colon SR(f_\sigma) \leq SR(f)$

$(H_0: SR(f_\sigma) \geq SR(f))$ against $H_a: SR(f_\sigma) > SR(f)$ $(H_a: SR(f_\sigma) < SR(f))$. The results are reported in Table A2 in Appendix.

To provide explanations for a small success of volatility management in equity markets, COWY develop a bootstrapping analysis, and they address that under the null hypothesis of persistent volatility but no risk-return relation volatility management is attractive as it generates larger Sharpe ratio. COWY conclude that the risk-return relations tend to be positive and thus a small success of volatility management is mainly attributed to this positive risk-return relation. Our simulation analysis is different from that of COWY since we aim to test whether the difference in Sharpe ratio is significant or not in each trial while COWY focus on whether it is positive or not. However, our results provide a consistent implication that if there exists a positive risk-return relation, then a volatility-managed portfolio hardly significantly outperforms its original portfolio. Our results also show that it is even more likely that the original portfolio significantly outperforms the volatility-managed portfolio under the positive risk-return relation.

We also investigate the contribution of the short-term volatility consistency and the risk-return relation using the decomposition suggested by COWY. Specifically, COWY decompose the difference of the average returns on the volatility-managed and original factors as follows (Equation IA1 of COWY):

$$\bar{f}_{\sigma,t} - \bar{f} = cov(\omega_t, f_t) + \bar{f}_{\mathrm{t}}(\bar{\omega}_t - 1).$$

COWY document that if the first component $(cov(\omega_t, f_t))$ is negative then it indicates that the lagged volatility is a positive predictor of the strategy return. Thus, no risk-return relation expects this component to be zero. The second component $(\bar{f}_{\mathrm{t}}(\bar{\omega}_t - 1))$, by contrast, is associated with the predictability of lagged volatility to the current volatility. In untabulated results, we find that the first component of MOM12 is notably large compared to all other test factors. The second component shows a monotonically increasing pattern across the ranking period of momentums, which means that the short-term persistency of volatility also contributes to the success of volatility management of MOM12. However, the first component takes account for 83% of the return difference between the original and volatility-managed MOM12. This indicates that the negative correlation between the lagged volatility and the current return substantially contributes to the success of volatility management of MOM12. These findings highlight the importance of the negative correlation between the volatility and the return consistent with our findings from the simulation analysis.

More interestingly, our results highlight a critical role of the negative risk-return relation. In Table 3, we first find that the risk-return relations tend to be negative and thus it is puzzling that volatility management fails to generate significant improvement in many cases even though the risk-return relation is favorable. Our simulation results show that the probability to generate a significantly larger Sharpe ratio dramatically increases as volatility becomes more persistent only if the risk-return relation is negative. These results imply that some factors show insignificant improvement from volatility management even though they have a negative risk-return relation, such as BM or CV because their volatility is not persistent enough. Previously, MM suggest volatility management is mainly motivated by the short-term persistency of volatility, but our results address that both having a negative risk-return and high volatility persistency can be essential to make it successful (to generate significantly higher Sharpe ratio).

4. Optimal portfolio

Although there are several empirical studies that suggest volatility-managed versions of a popular trading strategy, such as momentum (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016), COWY highlight that MM's finding is especially impressive because it is the first to report that the success of volatility management is not restricted to a particular strategy. MM address that it is rather a pervasive phenomenon by empirically showing that volatility management increases the Sharpe ratios of eight popular trading strategies. The methodology that MM mainly rely on to evaluate the performance of volatility-managed factors is the time-series spanning regression as follows:

$$f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t. \tag{4}$$

MM find that the volatility-managed factors have positively significant alphas in this spanning regression, and interpret these findings as evidence of outperformance of volatility-managed factors over their respective original factors. More precisely, as COWY note, a positive alpha from the spanning regression indicates the expansion of the mean-variance frontier by the combination of the original and volatility-managed portfolios, not by the volatility-managed portfolio itself. In other words, the positive alpha indicates that the optimal ex post combination of the original and volatility-managed portfolios (with positive weights on the volatility-managed portfolios) expands the mean-variance frontier relative to the original portfolios (e.g., Gibbons et al., 1989), and thus the increased Sharpe ratios and utility gains can be obtained from a combination of the original and volatility-managed portfolios rather than from the volatility-managed portfolios alone. Thus, COWY stress that the benefits of volatility management should be evaluated in the context of the mean-variance frontier by comparing the optimal portfolios with and without the volatility-managed portfolio.

In this section, we investigate whether a volatility-managed portfolio expands the mean-variance frontier. More importantly, as COWY criticize MM's approach in the point that the spanning regression using the whole sample period is the in-sample evidence that is not obtainable from the perspective of a real investor, we examine both the in-sample optimal portfolio (Section 4.1) and the out-of-sample optimal portfolio (Section 4.2) following COWY's approach.

4.1. In-sample tests

From the perspective of a mean-variance investor, the vector of a mean-variance investor's ex post optimal weights on the volatility-managed portfolio (x_σ^*) and the original portfolio (x^*) can be given by

$$a = \begin{bmatrix} x_\sigma^* \\ x^* \end{bmatrix} = \frac{1}{\gamma}\hat{\Sigma}^{-1}\hat{\mu}, \tag{5}$$

where γ is the investor's risk aversion parameter[5], $\hat{\Sigma}$ is the variance-covariance matrix, and $\hat{\mu}$ is the vector of mean excess returns. Then, using the full-sample mean and variance, the ex post optimal allocation to the volatility-managed portfolio can be derived as[6]

$$x_\sigma^* = \frac{\hat{\alpha}}{\gamma\hat{\sigma}_f^2(1-\hat{\rho}^2)}, \tag{6}$$

where $\hat{\alpha}$ is the intercept from the spanning regression, $\hat{\sigma}_f^2$ is the unconditional (full-sample) variance of the original portfolio returns, and $\hat{\rho}$ is the correlation coefficient of the original and volatility-managed factors.

[5] Following COWY, we set $\gamma = 5$.

[6] See Section 4.1. of COWY for further details.

Then, the dynamic investment rule can be derived by combining equations (1) and (5) as follows:

$$y_t^* = x_\sigma^* \left(\frac{c^*}{\hat{\sigma}_{t-1}^2}\right) + x^*, \tag{7}$$

where y_t^* indicates the total position on the original factor at month t. Thus, the excess return on the ex post optimal portfolio (the ex post optimal factor) can be computed as $y_t^* f_t$.

[Insert Table 4 about here]

Panel A of Table 4 shows the estimated results of the spanning regression (equation (4)). We also report the appraisal ratio (AR) for each volatility-managed factor following MM and COWY[7] which can measure the extent to which volatility management can be used to increase the slope of the mean-variance frontier. The volatility-managed alpha and AR are annualized.

In Table 1, we find that the volatility-managed MOM12 itself generates a significantly larger Sharpe ratio than the original MOM12, and so it is indeed not surprising that the volatility-managed MOM12 can expand the mean-variance frontier. Moreover, the alpha and AR for MOM12 are notably large compared to other test factors. For example, the alpha is 9.077% with t-statistics = 3.71. We find that the volatility-managed AVG and MOM9 also have positive alphas that are significant at the 10% significance level, but their values are much smaller than that of MOM12 as 1.974% and 3.575%, respectively.

Interestingly, in the equity market, MM also report that the equity momentum factor has a notably large alpha compared to other factors (see Table 1 of MM). Specifically, they show

[7] The appraisal ratio is computed by $AR = \hat{\alpha}/\hat{\sigma}_\varepsilon$ where $\hat{\alpha}$ is the alpha (intercept) from the spanning regression and $\hat{\sigma}_\varepsilon$ is the standard error of it.

that the alpha for the volatility-managed equity momentum is 12.51% while the alphas for other factors range from -0.58% to 5.67%. As we discussed in Table 1, the common success of volatility-managed momentum in equity and commodity futures markets seems interesting, and thus we also conduct the spanning regression test with two alternative volatility-managed MOM12 factors, SCMOM1 and SCMOM2. The results are reported in Panel C of Table 2. The results are similar to our findings from the direct comparison (Panels A and B of Table 2), indicating that SCMOM2 shows improvement while SCMOM1 does not. The alpha and AR for SCMOM1 are substantially small and the alpha is even insignificant (alpha = 1.596 and t-statistics = 0.55). By contrast, SCMOM2 shows a large alpha (7.422), but consistent with our findings in Section 3.1, the results suggest that scaling with commodity futures momentum's volatility generates larger improvement than scaling with the common movement of momentums which is measured by the first principal component of two momentums. However, the alpha for SCMOM12 is still impressive as it is larger than other volatility-managed factors in Table 2 except the volatility-managed MOM12, and so again confirms that the in-sample success of the volatility-managed commodity futures momentum can be attributed to the comovement among momentums.

Panel B of Table 4 presents the estimated ex post optimization parameters for each of the commodity futures factors computed by equations (5) and (6). The normalized relative factor weights w_σ^* and w^* for the volatility-managed and original factors, respectively, are also reported. Not surprisingly, the sign of the (relative) weight on a volatility-managed factor is consistent with that of the alpha in the spanning regression as equation (6) implies. Our results show substantial differences in the optimal weight on the volatility-managed factor across the commodity futures factors, indicating that there is no fixed trading rule across factors as COWY also noted for equity factors.

Panel C of Table 4 reports the Sharpe ratio and the certainty equivalent return (CER) for a mean-variance investor computed following COWY.[8] Though the differences of the Sharpe ratios and CERs differ across the factors, our results show that differences are all positive, indicating that volatility management expands the mean-variance investment frontier. Moreover, again, the volatility-managed MOM12 shows an exceptionally large increase in the Sharpe ratio and CER that are 0.268 and 3.724%, respectively.

In sum, from the in-sample tests, we find that three factors – AVG, MOM9, and MOM12 – earn benefits from applying volatility management. However, these improvements are the ex post results, which potentially overstate the value of volatility management in practice as COWY point out. An investor could only achieve the utility gains by combining the original and volatility-managed versions of a particular factor using weights that are unknown prior to observing the full sample of factor returns. We thus perform the out-of-sample tests considering this real-time issue in the subsequent section.

4.2. Out-of-sample tests

The optimal weights of the original and volatility-managed portfolios in Section 4.1 are determined by the in-sample return moments, and thus it is not available for real-time investors.

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[8] Specifically, the in-sample CER difference can be computed analytically as

$$\Delta CER = \frac{SR(y_t^*)^2 - SR(z^*)^2}{2\gamma},$$

where z^* indicates the optimal portfolio for the investor who does not have access to the volatility-managed portfolio, which is computed by

$$z^* = \frac{1}{\gamma}\frac{\overline{f_t}}{\hat{\sigma}_f^2}.$$

MM's findings provide an important investment implication, but COWY comment that the reasonable out-of-sample versions should be considered for a real-time investor. Thus, in this section, we investigate whether volatility management can benefit even real-time investors following COWY's out-of-sample approach.

A real-time investor would estimate the scaling parameter and the optimal weights on the original and volatility-managed portfolios in each month. In each month *t*, we compute the real-time scaling parameter, c_t, as the constant that makes the original and volatility-managed factors have the same variance over the period preceding month *t*.[9] Next, we estimate the vector of mean excess returns ($\hat{\mu}_t$) and the covariance matric ($\hat{\Sigma}_t$) from the period preceding month *t*. The optimal portfolio weights can be computed as

$$\begin{bmatrix} x_{\sigma,t} \\ x_t \end{bmatrix} = \frac{1}{\gamma} \hat{\Sigma}_t^{-1} \hat{\mu}_t. \tag{8}$$

Then, the investment position in the original factor can be computed as

$$y_t = x_{\sigma,t} \left(\frac{c_t}{\hat{\sigma}_{t-1}^2} \right) + x_t, \tag{9}$$

and so the excess return on this position (real-time combination factor) is $y_t f_t$.

We compare the performance of the real-time combination factor $y_t f_t$ with two benchmarks. The first benchmark is the real-time original portfolio which is optimal for an investor who does not have access to the volatility-managed portfolio. In each month *t*, the allocation on the original portfolio can be computed by

[9] We adopt the COWY's choice for the initial training sample of *K*=120 months (and thus the subsequent out-of-sample of *T-K* months among total *T*-month sample period) and an expanding-window approach.

$$z_t^* = \frac{1}{\gamma}\frac{\overline{f_t}}{\hat{\sigma}_{f,t}^2} \tag{10}$$

where $\overline{f_t}$ and $\hat{\sigma}_{f,t}^2$ are the average and variance of the original factor over the period preceding month *t*. By comparing the performance of the real-time combination and original factor, we can evaluate the benefits of volatility management in real-time.

The second benchmark is the ex post combination factor. After constructing the time-series of a real-time volatility managed factor, we derive the optimal allocation on risky assets using the entire sample period following equations (6) and (7). As COWY document, the ex post results may overstate the value of volatility management in practice, and thus comparing the performance of the real-time and ex post combination factors will be informative to see how the out-of-sample optimization can differ from the in-sample optimization.

The performance of the real-time combination portfolio is evaluated with two metrics. First, the Sharpe ratio difference between portfolios *i* and *j* computed as

$$\Delta SR = \frac{\hat{\mu}_i}{\hat{\sigma}_i} - \frac{\hat{\mu}_j}{\hat{\sigma}_j} \tag{11}$$

is tested by Jobson and Korkie's (1981) approach. Second, the out-of-sample CER gains computed as

$$\Delta CER = \left(\hat{\mu}_i - \frac{\gamma}{2}\hat{\sigma}_i^2\right) - \left(\hat{\mu}_j - \frac{\gamma}{2}\hat{\sigma}_j^2\right) \tag{12}$$

is tested by DeMiguel et al.'s (2009) approach.[10] In Table 5, we report the Sharpe ratio and the CER of the real combination, the original, and the ex post factors and the differences among them in Panels A and B, respectively.

[Insert Table 5 about here]

Consistent with COWY, our out-of-sample results in Table 5 are in stark contrast to the in-sample results in Table 4. In both terms of the Sharpe ratio and the CER, Table 5 shows the poor out-of-sample performance of volatility management. In particular, Panel A presents that compared to both the original and the ex post combination factors, the real-time combination factors fail to improve the Sharpe ratio. In case of SL and MOM6, the results show that the real-time combination factor has even a significantly lower Sharpe ratio than the ex-post combination factor. The real-time combination of MOM12 shows an increase in the Sharpe ratio compared to its corresponding original factor (0.142), but the difference is still statistically insignificant.

In Panel B of Table 5, compared to the original factor, the real-time combination of MOM9 and MOM12 shows an increase in the CER and in case of MOM12, the increase is significant at the 10% significance level. In particular, the CER gain for the real-time combination of MOM12 is 3.1%, which is substantially large compared to other strategies. However, compared to the ex post combination factor, it still shows underperformance (decrease in the CER by 0.86%), indicating that the in-sample results are overstated as COWY criticized. Moreover, considering the insignificance of the increase in the Sharpe ratio for MOM12 in Panel A of Table 5, we may conclude that volatility management fails to improve the real-time performance even for MOM12. Except for these two positive cases, overall results

[10] See further details in Section 4.3.1. of COWY.

in Panel B of Table 5 suggest that the real-time combination factors underperform their corresponding original and the ex post factors in all cases.

In untabulated results, we also examine whether the structural breaks harm the out-of-sample benefits of volatility management. For example, from the in-sample spanning regression tests, we find that AVG, MOM9, and MOM12 generate significant alphas but they show insignificant or much weaker benefits from the out-of-sample analyses. To investigate this, we first conduct the structural break analysis suggested by Bai and Perron (2003) based on the spanning regression model. However, we find that MOM9 and MOM12 both have a relatively small number of breaks – three according to the Bayesian Information Criterion (BIC) or one according to a modified Schwarz criterion (LWZ). AVG has a rather larger number of breaks (five according to the BIC and three according to the LWZ), but considering that our sample period spans 39 years, it still seems to be quite small. Furthermore, to alleviate the possible structural break problem, we construct the real-time volatility-managed factors using a fixed rolling window (120 months) instead of using an expanding window and investigate the improvement from applying volatility management in real time. However, we find that the results are qualitatively the same.

[Insert Table 6 about here]

Lastly, we construct the out-of-sample version of SCMOM2, the alternative volatility-managed MOM12 which shows significant improvement from the in-sample analysis in Section 3, and test its performance in out-of-sample. In Table 6, the results show that the real-time combination of SCMOM2 does not exhibit any significant improvement from the original factor in both terms of the Sharpe ratio and the CER. Compared to the ex post combination of SCMOM2, it also shows a smaller Sharpe ratio (0.754) and the negative CER gain (-1.485%).

The most impressive part in Table 6 is the comparison between the performance of SCMOM2 and that of the volatility-managed MOM12. Our results show that the Sharpe ratio of the real-time combination of MOM12 (0.859) is larger than that of the real-time (0.754) and even the ex post combination of SCMOM2 (0.823). From the in-sample analyses in Section 3, we find that the volatility-managed MOM12 generates the larger Sharpe ratio than SCMOM2, but the difference in Sharpe ratio is not statistically significant. In real-time, however, our results show that the volatility-managed MOM12 has a significantly larger Sharpe ratio and the CER with p-values = 0.09 and 0.05 for the difference, respectively. It also shows no significant difference from the ex post combination of SCMOM2. These results imply that in out-of-sample, timing volatility with the comovement of momentums significantly lowers the performance, and thus highlight the importance of the unique movement of commodity futures momentum rather than the comovement of momentums across assets.

To sum up, our out-of-sample results reveal that volatility management in commodity futures markets does not significantly improve real-time performance. We find that the real-time combination factor does not show significant improvement compared to the real-time original factor. COWY show that in contrast to the impressive in-sample results for the nine equity factors studied by MM, volatility management often harms the real-time performance. They report some positive out-of-sample results from three of nine test factors, but they also document that there is little statistical or economic evidence for the remaining six factors that incorporating volatility management improves real-time portfolio outcomes. Moreover, using the extended sample of 103 trading strategies, COWY additionally provide comprehensive and convincing evidence on the poor out-of-sample performance of combination strategies in the equity market. They find that only eight strategies yield statistically significant Sharpe ratio differences in favor of volatility management. COWY also report that these positive cases are concentrated among momentum-related strategies, in accord with the conclusions from

Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). By contrast, in our result, the real-time combination of commodity futures momentum factors does not show any improvement compared to their corresponding original factors as well as the ex post factors in general. The only exception is the real-time combination of MOM12, which shows the significant CER gains relative to its original factor, but in terms of the Sharpe ratio, we find no evidence of improvement.

5. Conclusion

This paper investigates the benefits of volatility management in various popular trading strategies in commodity futures markets. We test the average, basis, momentum, basis-momentum, change in slope, and curvature portfolios in commodity futures markets both in-sample and out-of-sample. We find a few success from the in-sample analyses, but from the out-of-sample analyses, we find that volatility management fails to improve real-time performance, which indicates that in-sample results can be overstated and also are not obtainable for real-time investors. Our results are consistent with COWY's findings in equity markets, raising a question about the performance of volatility management especially in real-time.

Appendix

A. Volatility-managed equity portfolios

[Insert Table A1 about here]

B. Simulation analysis

To better understand a small success of volatility management, we develop a simulation analysis based on equations (2) and (3). First, we estimate equations (2) and (3) using data, and randomly draw residuals (with replacement) of equations (2) and (3) for T months,[11] which is the number of time-series observation of a test factor.[12] Using the series of randomly drawn residuals, we regenerate the time-series of return and volatility using equations (2) and (3) but with various choices of b_0 and b_1.[13] In particular, we test for b_0 = {$\hat{b}_0$, -0.03, -0.02, -0.01, 0, 0.01, 0.02, 0.03} and b_1 = {$\hat{b}_1$, 0.5, 0.6, 0.7, 0.8, 0.9} where $\hat{b}_0$ and $\hat{b}_1$ are the estimates from the regressions. Next, we construct the time-series of returns on the volatility-managed portfolio and conduct Jobson and Korkie's (1981) test to see whether the volatility-managed factor generates a significantly larger Sharpe ratio than its original factor (and also vice versa). We repeat this for N = 100,000 times for each factor, and thus get how frequently the volatility-managed factor generates a significantly larger (smaller) Sharpe ratio.

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[11] Depending on the length of historical data necessary to compute a variable (for example, MOM12 requires past 12-month data while AVG requires no historical data), T varies from 455 to 467 months.

[12] We independently draw residuals from equations (2) and (3), respectively, but in untabulated results, we also test with drawing residuals in pair. The results appear to be qualitatively similar.

[13] To avoid generating a negative realized variance, we restrict the minimum variance to the minimum historical value of the factor's realized variance.

Under 1% significance level, Panel A (Panel B) of Table A2 shows the proportion (%) of the trials that reject the null hypothesis $H_0\colon SR(f_\sigma) \leq SR(f)$ ($H_0\colon SR(f_\sigma) \geq SR(f)$) against $H_a\colon SR(f_\sigma) > SR(f)$ ($H_a\colon SR(f_\sigma) < SR(f)$). The results show similar patterns across test factors, and thus to save space, we report results only for AVG, BM, CV, and MOM12 in Table A2. In particular, we mainly focus on factors that show relatively high volatility persistency (b_1) and predictability (R^2) from equation (3) in Table 3 but having an insignificant (significant) improvement in Sharpe ratio that are BM and CV (AVG and MOM12).

[Insert Table A2 about here]

First, in case of $b_0 = 0$ indicating no risk-return relation, Panel A of Table A2 shows that the contribution of high volatility persistency on the success of volatility management is substantially small. For example, in case of MOM12 under the assumption $b_0 = 0$, only 16 out of 100,000 simulation trials (0.02%) show that the volatility-managed MOM12 significantly outperforms the original MOM12 even though the volatility is highly persistent as $b_1 = 0.9$. On the other hand, Panel B of Table A24 shows that it is more likely that the original portfolio significantly outperforms its corresponding volatility-managed portfolio under $b_0 = 0$. It also shows small differences across the levels of the volatility persistency (b_1), but compared to the results in Panel A of Table A2, it is clear that the second null hypothesis ($H_0\colon SR(f_\sigma) \geq SR(f)$) is more likely to be rejected, indicating an underperformance of a volatility-managed portfolio is more frequently observed. For example, under $b_0 = 0$ and $b_1 = 0.9$, the results for MOM12 present that about 10% of simulation trials result in rejecting the null.

Table A2 exhibits that the proportion of outperformance of volatility-managed portfolios tends to increase as the risk-return relation becomes more negative in general. Under the positive risk-return relation (positive b_0), our results show 0% outperformance of volatility-managed portfolios in most of the cases and almost no difference across b_1. The most impressive feature is that if the risk-return relation is negative, then the proportion of successful volatility management cases dramatically increases as b_1 increases. For example, as opposed to almost no difference across b_1 under $b_0 = 0$, CV shows that the proportion increases from 2.95% to 33.63% as b_1 increases from 0.226 ($\hat{b}_1$) to 0.900 under $b_0 = -0.010$. Moreover, the proportions of underperformance of volatility-managed portfolios (Panel B of Table A2) are quite small close to zero in all cases under the negative risk-return relation.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Data Citation

[Fama-French Factors] Fama, E.F. and French, K.R.; 1993; http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

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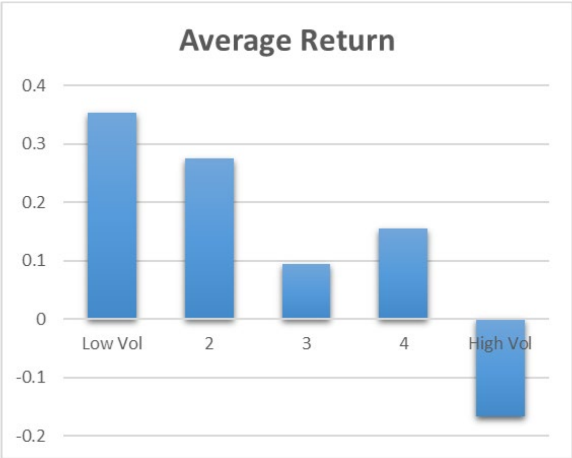
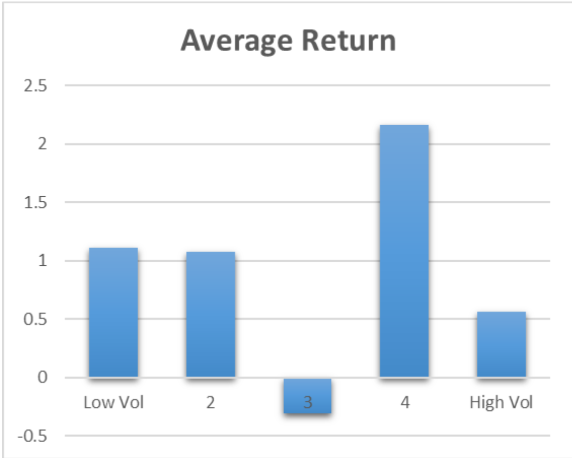
Figure 1. Sorts on the previous month's volatility

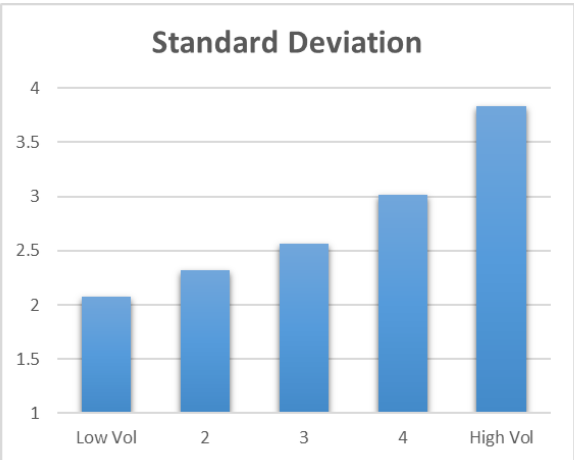
We sort months into five states by the previous month's realized volatility and compute the average returns, volatility, and the mean-variance trade-off in the subsequent month. Panels A to C of Figure 1 present the results for the commodity futures market portfolio and Panels D to F present these for the basis portfolio.



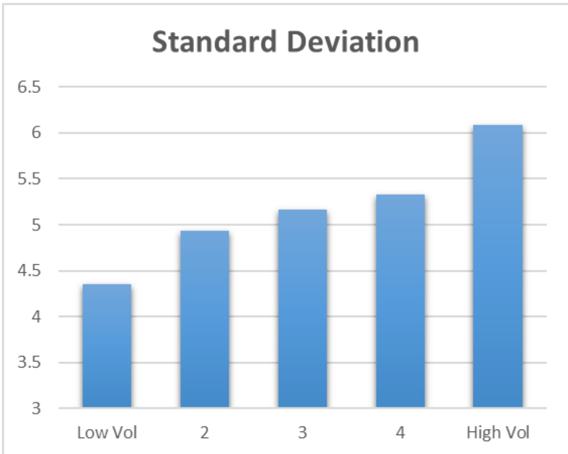
Panel A



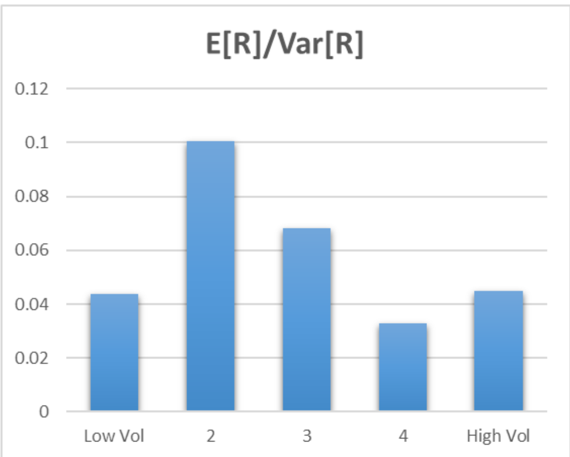
Panel D



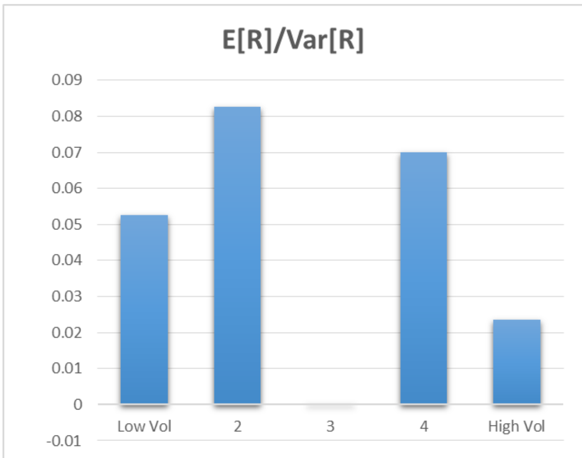
Panel B



Panel E



Panel C



Panel F

Table 1. Volatility-managed and original factors

This table provides a direct comparison of the original and volatility-managed portfolios. Panels A and B show the mean, standard deviation, and annualized Sharpe ratio of the original and volatility-managed factors, respectively. Panel C reports the statistical significance of the Sharpe ratio differences using the Jobson and Korkie (1981) approach, and the numbers in brackets show the p-value. Panel D reports the correlation between each original and corresponding volatility-managed portfolios and the 1st, 50th, and 99th percentiles (c1, c50, and c99, respectively) of the time-series distribution of the volatility-managed factor's implied weight in the original factor ($c^{*}/\hat{\sigma}_{t-1}^{2}$).

	AVG	BS	BM	SL	CV	MOM1	MOM3	MOM6	MOM9	MOM12
Panel A: Original factor										
MEAN	0.143	0.918	0.479	1.499	0.379	1.270	1.407	1.108	0.950	1.061
STD	3.347	6.350	6.326	5.872	6.124	9.171	6.872	6.580	6.553	6.568
SR	0.148	0.501	0.262	0.884	0.215	0.480	0.709	0.583	0.502	0.559
Panel B: Volatility-managed factor										
MEAN	0.276	0.930	0.437	1.192	0.145	1.188	1.389	1.097	1.088	1.559
STD	3.347	6.350	6.326	5.872	6.124	9.171	6.872	6.580	6.553	6.568
SR	0.285	0.507	0.240	0.704	0.082	0.449	0.700	0.578	0.575	0.822
Panel C: Sharpe ratio difference										
diff	0.137	0.006	-0.023	-0.181	-0.132	-0.031	-0.009	-0.006	0.073	0.263
	[0.20]	[0.94]	[0.84]	[0.06]	[0.15]	[0.79]	[0.93]	[0.96]	[0.44]	[0.02]
Panel D: Properties of volatility-managed factors										
Corr	0.775	0.871	0.748	0.837	0.841	0.752	0.773	0.774	0.836	0.756
c1	0.126	0.126	0.083	0.118	0.101	0.091	0.152	0.096	0.125	0.130
c50	0.947	0.861	0.823	0.767	0.670	0.652	0.834	0.814	0.867	0.861
c99	3.506	2.954	3.646	3.033	2.522	4.074	3.155	3.505	3.203	3.839

Table 2. Alternative volatility-managed commodity futures momentum factors

Panel A shows the mean, standard deviation, and annualized Sharpe ratio of two alternative volatility-managed commodity futures momentum factors. Specifically, we construct two alternative volatility-managed MOM12 factors. First, as Asness et al. (2013) suggest, we measure the comovement in momentums with the 12-month equity momentum, we scale MOM12 with the realized variance of the 12-month equity momentum factor (SCMOM1). Second, we scale MOM12 with the first principal component of the realized variance of the 12-month equity and commodity futures momentum factors (SCMOM2). Panel C reports the statistical significance of the Sharpe ratio differences between SCMOM1 (SCMOM2) and the original MOM12 using the Jobson and Korkie (1981) approach, and the numbers in brackets show the p-value. Panel C presents results from spanning regressions of SCMOM1 (SCMOM2) on MOM12 (equation (2)). The estimates of α (Alpha) are reported in percentage per year, and the numbers in parentheses are t-statistics. For each regression, R^2 is the adjusted R^2 value, and the appraisal ratio is computed as the ratio of alpha to root mean square error.

	SCMOM1	SCMOM2
Panel A. Alternative volatility-managed factor		
MEAN	0.801	1.425
STD	6.568	6.568
SR	0.423	0.752
Panel B. Sharpe ratio difference		
diff	-0.136	0.193
	[0.31]	[0.09]
Panel C: Spanning regression		
Alpha	1.596	7.422
	(0.55)	(3.05)
Beta	0.630	0.761
	(17.26)	(24.93)
R^2	0.440	0.579
AR	0.312	1.739

Table 3. Risk-return relations

This table presents the estimated results of equations (2) and (3). The results for the return (variance) as a dependent variable shows the results for equation (2) ((3)). For each model, we report the estimated coefficient on the realized volatility in the previous month (b_0 and b_1) and the adjusted R^2 value (R^2). The numbers in parentheses are t-statistics.

Dependent variable	Estimates	AVG	BS	BM	SL	CV	MOM1	MOM3	MOM6	MOM9	MOM12
Return	b_0	-0.060	0.002	-0.020	0.005	-0.013	-0.006	0.000	-0.008	-0.020	-0.026
		(-3.50)	(0.29)	(-4.68)	(1.68)	(-3.09)	(-3.38)	(0.06)	(-1.09)	(-2.72)	(-5.25)
	R^2	2.57%	0.02%	4.61%	0.62%	2.07%	2.52%	0.00%	0.26%	1.60%	5.74%
Variance	b_1	0.669	0.082	0.481	0.046	0.226	0.124	0.336	0.366	0.429	0.525
		(19.39)	(1.76)	(11.75)	(3.82)	(15.22)	(2.61)	(7.67)	(8.40)	(10.17)	(13.19)
	R^2	44.75%	0.67%	23.39%	3.13%	33.94%	1.52%	11.33%	13.36%	18.51%	27.80%

Table 4. Spanning regressions and in-sample optimization

Panel A presents results from spanning regressions of a volatility-managed factor on the corresponding original factor (equation (2)). The estimates of α (Alpha) are reported in percentage per year, and the numbers in parentheses are t-statistics. For each regression, R^2 is the adjusted R^2 value, and the appraisal ratio is computed as the ratio of alpha to root mean square error. Panel B presents the ex post optimization results that are the scaling parameter ($c*$) for the volatility-managed factor, the ex post optimal total weight in risky assets ($x_\sigma^* + x^*$), and the ex post optimal relative weights in the volatility-managed (w_σ^*) and original factors (w^*). Panel C shows annualized Sharpe ratios and certainty equivalent returns (CERs). The "original" strategy indicates the results for the ex post optimal combination of original factor and risk-free asset, and the "combination" strategy indicates those for the ex post optimal combination of original factor, volatility-managed factor, and risk-free asset. The results in Panels B and C are for $\gamma = 5$ following COWY.

	AVG	BS	BM	SL	CV	MOM1	MOM3	MOM6	MOM9	MOM12
					Panel A: Spanning regression					
Alpha	1.974	1.550	0.944	-0.692	-2.085	2.470	3.540	2.896	3.575	9.077
	(1.69)	(0.89)	(0.40)	-(0.37)	-(1.11)	(0.72)	(1.44)	(1.22)	(1.74)	(3.71)
Beta	0.778	0.872	0.750	0.834	0.841	0.760	0.778	0.773	0.832	0.757
	(26.67)	(38.41)	(24.10)	(32.14)	(33.02)	(24.51)	(26.59)	(26.06)	(32.06)	(24.63)
R^2	0.605	0.761	0.562	0.696	0.707	0.575	0.605	0.597	0.693	0.573
AR	0.937	0.498	0.225	-0.213	-0.628	0.412	0.818	0.692	0.983	2.112
					Panel B: Ex post optimization parameters					
$c*$	5.999	20.453	17.873	18.987	15.349	27.579	22.271	22.533	23.012	24.198
$x_\sigma^* + x^*$	0.420	0.489	0.262	0.851	0.152	0.332	0.666	0.575	0.518	0.691
					Relative weights					
w_σ^*	1.768	0.547	0.343	-0.129	-2.080	0.379	0.475	0.481	0.872	1.187
w^*	-0.768	0.453	0.657	1.129	3.080	0.621	0.525	0.519	0.128	-0.187
					Panel C: Portfolio performance measures					
					Sharpe ratio					
Combination	0.308	0.521	0.270	0.886	0.281	0.497	0.748	0.617	0.577	0.828
Original	0.148	0.501	0.262	0.884	0.215	0.480	0.709	0.583	0.502	0.559
Difference	0.161	0.020	0.008	0.002	0.066	0.017	0.038	0.033	0.075	0.268

					CER (%)					
Combination	0.952	2.714	0.729	7.857	0.790	2.471	5.588	3.802	3.328	6.853
Original	0.218	2.506	0.687	7.819	0.460	2.301	5.028	3.402	2.521	3.130
Difference	0.733	0.207	0.042	0.038	0.330	0.170	0.559	0.400	0.807	3.724

Table 5. Original, real-time and ex post combination strategies

This table presents the annualized Sharpe ratio (Panel A) and CER (Panel B) of the real-time, original, and ex post combination strategies. The "real" strategy indicates the results for the real-time combination of original factor, volatility-managed factor, and risk-free asset, and the "original" strategy indicates those for the real-time combination of original factor and risk-free asset. The "ex post" strategy indicates the results for the ex post combination of original factor, volatility-managed factor, and risk-free asset. The numbers in brackets are p-values for the Sharpe ratio and CER differences. The p-values are computed following the approaches in Jobson and Korkie (1981) and DeMiguel, Garlappi, and Uppal (2009), respectively. The results are for $\gamma = 5$ following COWY.

	AVG	BS	BM	SL	CV	MOM1	MOM3	MOM6	MOM9	MOM12
Panel A. Sharpe ratio										
Real	0.122	0.449	0.120	0.812	0.231	0.478	0.672	0.503	0.534	0.859
Original	0.056	0.657	0.166	0.920	0.153	0.584	0.789	0.592	0.531	0.717
Ex post	0.277	0.667	0.231	0.971	0.358	0.620	0.812	0.626	0.609	0.891
Real – Original	0.066	-0.208	-0.047	-0.108	0.078	-0.106	-0.117	-0.089	0.003	0.142
	[0.77]	[0.11]	[0.56]	[0.21]	[0.39]	[0.33]	[0.25]	[0.24]	[0.98]	[0.26]
Real – Ex post	-0.155	-0.218	-0.111	-0.160	-0.127	-0.142	-0.141	-0.122	-0.075	-0.032
	[0.16]	[0.20]	[0.32]	[0.09]	[0.21]	[0.27]	[0.20]	[0.09]	[0.15]	[0.44]
Panel B. Certainty equivalent return										
Real	-0.085	1.996	-0.155	6.512	0.489	2.163	4.184	2.497	2.855	7.076
Original	-0.065	2.961	0.238	7.153	0.077	3.021	5.442	3.287	2.606	3.983
Ex post	0.768	4.455	0.533	9.435	1.284	3.837	6.600	3.913	3.710	7.932
Real – Original	-0.019	-0.965	-0.393	-0.641	0.411	-0.858	-1.258	-0.790	0.249	3.093
	[0.98]	[0.33]	[0.41]	[0.61]	[0.44]	[0.51]	[0.51]	[0.37]	[0.82]	[0.08]
Real – Ex post	-0.853	-2.460	-0.688	-2.924	-0.795	-1.674	-2.417	-1.416	-0.855	-0.855
	[0.16]	[0.22]	[0.27]	[0.12]	[0.26]	[0.30]	[0.20]	[0.11]	[0.19]	[0.41]

Table 6. Original, real-time and ex post combination strategies of alternative volatility-managed commodity momentum

This table presents the annualized Sharpe ratio (Panel A) and CER (Panel B) of the real-time, original, and ex post combination strategies for SCMOM2. The "real" strategy indicates the results for the real-time combination of original factor, volatility-managed factor, and risk-free asset, and the "original" strategy indicates those for the real-time combination of original factor and risk-free asset. The "ex post" strategy indicates the results for the ex post combination of original factor, volatility-managed factor, and risk-free asset. The numbers in brackets are p-values for the Sharpe ratio and CER differences. The p-values are computed following the approaches in Jobson and Korkie (1981) and DeMiguel, Garlappi, and Uppal (2009), respectively. The results are for $\gamma = 5$ following COWY.

Panel A. Sharpe ratio	
SCMOM2	
Real	0.754
Original	0.717
Ex post	0.823
Real – Original	0.036
	[0.77]
Real – Ex post	-0.069
	[0.24]
Comparison with volatility-managed MOM12	
Real MOM12 – Real SCMOM2	0.106
	[0.09]
Real MOM12 – Ex post SCMOM2	0.036
	[0.67]
Panel B. Certainty equivalent return	
SCMOM2	
Real	5.288
Original	3.983
Ex post	6.773
Real – Original	1.304
	[0.33]
Real – Ex post	-1.485
	[0.26]
Comparison with volatility-managed MOM12	
Real MOM12 – Real SCMOM2	1.789
	[0.05]
Real MOM12 – Ex post SCMOM2	0.304
	[0.83]

Table A1. Volatility-managed equity portfolios

Panels A to C in this table provides a direct comparison of the original and volatility-managed equity portfolios – the market (MKT), size (SMB), value (HML), and momentum (SMOM) portfolios. Panels A and B show the mean, standard deviation, and annualized Sharpe ratio of the original and volatility-managed factors, respectively. Panel C reports the statistical significance of the Sharpe ratio differences using the Jobson and Korkie (1981) approach, and the numbers in brackets show the p-value. Panel D presents the estimated results of equations (2) and (3). The results for the return (variance) as a dependent variable shows the results for equation (2) ((3)). For each model, we report the estimated coefficient on the realized volatility in the previous month (b_0 and b_1) and the adjusted R^2 value (R^2). The numbers in parentheses are t-statistics. The sample period is from January 1979 to December 2017.

	MKT	SMB	HML	SMOM
Panel A: Original factor				
MEAN	0.673	0.130	0.273	0.609
STD	4.396	2.988	2.923	4.471
SR	0.530	0.151	0.324	0.472
Panel B: Volatility-managed factor				
MEAN	0.882	0.054	0.186	1.381
STD	4.396	2.988	2.923	4.471
SR	0.695	0.063	0.220	1.070
Panel C: Sharpe ratio difference				
diff	0.165	-0.088	-0.104	0.598
	[0.17]	[0.42]	[0.44]	[0.00]
Panel D: Risk-return relations				
Return				
b_0	-0.012	0.004	0.002	-0.042
	(-2.75)	(0.32)	(0.19)	(-5.67)
R^2	1.60%	0.02%	0.01%	6.46%
Variance				
b_1	0.556	0.262	0.729	0.673
	(14.44)	(5.86)	(22.98)	(19.62)
R^2	30.95%	6.88%	53.18%	45.28%

Table A2. Simulation analysis

This table summarizes the results from the simulation analysis. First, we estimate equations (2) and (3) using data, and randomly draw residuals (with replacement) of equations (2) and (3) for T months, which is the number of time-series observation of a test factor. Using the series of randomly drawn residuals, we regenerate the time-series of return and volatility using equations (2) and (3) but with various choices of b_0 and b_1. In particular, we test for $b_0 = \{\hat{b}_0$, -0.03, -0.02, -0.01, 0, 0.01, 0.02, 0.03$\}$ and $b_1 = \{\hat{b}_1$, 0.5, 0.6, 0.7, 0.8, 0.9$\}$ where $\hat{b}_0$ and $\hat{b}_1$ are the estimates from the regressions. Next, we construct the time-series of returns on the volatility-managed portfolio and conduct Jobson and Korkie's (1981) test to see whether the volatility-managed factor generates a significantly larger Sharpe ratio than its original factor (and also vice versa). We repeat this for N = 100,000 times for each factor, and thus get how frequently the volatility-managed factor generates a significantly larger (smaller) Sharpe ratio. Under 1% significance level, Panel A (Panel B) shows the proportion (%) of the trials that reject the null hypothesis $H_0: SR(f_\sigma) \leq SR(f)$ ($H_0: SR(f_\sigma) \geq SR(f)$) against $H_a: SR(f_\sigma) > SR(f)$ ($H_a: SR(f_\sigma) < SR(f)$).

	Panel A. $H_a: SR(f_\sigma) > SR(f)$						Panel B. $H_a: SR(f_\sigma) < SR(f)$					
b_0	b_1						b_1					
AVG												
	0.500	0.600	0.669 ($\hat{b}_1$	0.700	0.800	0.900	0.500	0.600	0.669 ($\hat{b}_1$)	0.700	0.800	0.900
-0.060 ($\hat{b}_0$	23.91%	32.63%	42.83%	49.01%	77.68%	98.92%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
-0.030	3.06%	3.78%	4.98%	5.80%	12.83%	51.82%	0.59%	0.40%	0.27%	0.22%	0.05%	0.00%
-0.020	1.18%	1.41%	1.62%	1.81%	3.27%	16.93%	1.58%	1.37%	1.16%	1.00%	0.48%	0.03%
-0.010	0.44%	0.42%	0.42%	0.43%	0.59%	2.02%	3.82%	3.86%	3.64%	3.49%	2.66%	0.80%
0	0.14%	0.12%	0.11%	0.10%	0.08%	0.12%	8.19%	8.81%	9.22%	9.42%	9.80%	8.32%
0.010	0.04%	0.03%	0.02%	0.02%	0.01%	0.00%	15.26%	17.31%	19.27%	20.32%	24.41%	30.08%
0.020	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	24.92%	29.15%	33.10%	35.25%	44.37%	58.59%
0.030	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	36.62%	42.95%	48.70%	51.83%	64.40%	78.97%
BM												
	0.481 ($\hat{b}_1$	0.500	0.600	0.700	0.800	0.900	0.481 ($\hat{b}_1$	0.500	0.600	0.700	0.800	0.900
-0.030	51.05%	53.06%	65.57%	79.51%	91.29%	71.70%	0.00%	0.00%	0.00%	0.00%	0.00%	0.14%
-0.020	22.80%	24.25%	34.64%	50.27%	71.55%	92.03%	0.04%	0.03%	0.01%	0.00%	0.00%	0.00%
-0.020 ($\hat{b}_0$	22.69%	24.14%	34.50%	50.11%	71.44%	92.03%	0.04%	0.03%	0.01%	0.00%	0.00%	0.00%

-0.010	3.69%	3.90%	6.08%	11.10%	23.46%	59.73%	0.55%	0.51%	0.34%	0.18%	0.06%	0.00%
0	0.14%	0.14%	0.14%	0.13%	0.15%	0.18%	6.95%	6.91%	6.87%	6.73%	6.76%	6.35%
0.010	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	28.59%	29.23%	33.54%	40.00%	51.13%	69.26%
0.020	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	50.97%	51.95%	57.84%	64.88%	72.09%	63.28%
0.030	0.00%	0.00%	0.00%	0.00%	0.00%	7.16%	61.63%	62.46%	66.36%	68.06%	63.42%	39.61%
CV												
	0.226 ($\hat{b}_1$	0.500	0.600	0.700	0.800	0.900	0.226 ($\hat{b}_1$	0.500	0.600	0.700	0.800	0.900
-0.030	26.90%	44.47%	56.11%	68.99%	73.14%	48.19%	0.00%	0.00%	0.00%	0.00%	0.00%	0.63%
-0.020	10.62%	18.29%	25.14%	36.58%	53.14%	63.73%	0.07%	0.01%	0.00%	0.00%	0.00%	0.00%
-0.013 ($\hat{b}_0$	4.20%	6.78%	9.10%	13.47%	22.32%	48.08%	0.45%	0.12%	0.07%	0.02%	0.00%	0.00%
-0.010	2.95%	4.47%	5.89%	8.42%	13.48%	33.63%	0.80%	0.31%	0.18%	0.06%	0.01%	0.00%
0	0.67%	0.78%	0.87%	1.03%	1.08%	0.81%	4.38%	3.78%	3.47%	3.18%	2.95%	2.67%
0.010	0.15%	0.16%	0.17%	0.21%	0.25%	0.27%	13.83%	15.50%	17.24%	20.00%	24.81%	35.21%
0.020	0.04%	0.05%	0.05%	0.08%	0.17%	0.48%	28.75%	33.98%	37.64%	41.92%	45.13%	42.63%
0.030	0.01%	0.02%	0.03%	0.06%	0.20%	2.32%	45.12%	51.20%	53.86%	54.40%	47.57%	26.28%
MOM12												
	0.500	0.525 ($\hat{b}_1$	0.600	0.700	0.800	0.900	0.500	0.525 ($\hat{b}_1$	0.600	0.700	0.800	0.900
-0.030	32.04%	35.12%	46.85%	68.73%	91.10%	87.83%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%
-0.026 ($\hat{b}_0$)	19.48%	21.80%	31.33%	52.51%	81.71%	94.84%	0.02%	0.02%	0.01%	0.00%	0.00%	0.00%
-0.020	6.11%	7.10%	11.69%	25.30%	56.24%	93.62%	0.12%	0.10%	0.06%	0.02%	0.00%	0.00%
-0.010	0.17%	0.20%	0.34%	1.15%	6.47%	46.98%	2.38%	2.26%	1.72%	0.97%	0.28%	0.01%
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	19.77%	19.84%	19.97%	18.90%	16.59%	10.40%
0.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	54.31%	55.27%	58.64%	63.41%	68.41%	68.26%
0.020	0.00%	0.00%	0.00%	0.00%	0.00%	0.09%	77.65%	78.83%	82.02%	85.02%	84.46%	56.00%
0.030	0.00%	0.00%	0.00%	0.00%	0.00%	8.98%	86.58%	87.29%	88.57%	87.79%	77.26%	33.08%