Introduction

Drilling mud plays vital roles in a drilling operation, one of which is the formation of a thin layer of mud cake to prevent the damage caused by mud filtrate (the aquatic phase of drilling mud) penetration through the formation. Hence, understanding the characteristics of mud cake such as thickness, solid content, and filtration rate, can shed light on the theoretical and practical aspects of drilling fluid mechanics. One of the critical parameters in mud loss is the drill string rotation, which imposes shear stress on mud cake and thus reduces the mud cake thickness. This becomes more important when the annular space (e.g. casing drilling operation) is relatively small (Warren, Angman et al. 2000). Several studies have investigated the impact of drill pipe rotation on pressure loss in wellbores containing different types of non-Newtonian drilling fluids such as Yield Power-Law (Ahmed and Miska 2008) or Bingham Plastic (Hemphill, Ravi et al. 2008). There have also been some works on simulation of drilling fluid filtration in wellbore conditions (Vasheghani Farahani, Soleimani et al. 2014, Farahani, Shams et al. 2018, Soleimani, Jahanpeyma et al. 2019). Nevertheless, the detailed information regarding the effect of drill pipe rotation on mud cake and formation damage has not been deeply addressed yet. This study develops a mathematical model to account for the impact of drill pipe rotation on mud cake thickness and filtration radius in an isothermal radial system. We use the Power-Law model to describe non-Newtonian behavior of the drilling fluid, however, the mathematical approach presented here works with any underlying rheological assumption.

Problem Formulation

Figure 1 shows the schematic structure of the annulus containing a centric conduit. The shear stress caused by the pipe/casing rotation can be calculated as follows:

$$\tau = \frac{T}{2\pi h r^2} \tag{1}$$

where T is the torque by the centric conduit, τ is the shear-stress at radius r, and h is the height of the area that is under shear-stress. The rotational speed at radius r can be obtained using $v = r.\omega$, where ω is the angular velocity. Using Eq. (1), the velocity differentiation, therefore, can be written as in Eq. (2).

$$\frac{dv}{dr} = r\frac{d\omega}{dr} + \omega \tag{2}$$

The term $r\frac{d\omega}{dr}$ represents the shear-rate caused by the fluid slippage in annuli. Given that the angular velocity decreases at near-to-wellbore radii, the shear-rate $\dot{\gamma}$ can be defined as follows:

$$\dot{\gamma} = -r\frac{d\omega}{dr} \tag{3}$$

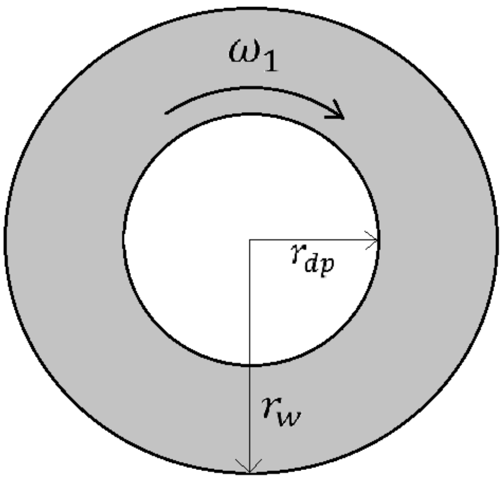

Figure 1 *Schematic representation of centric conduit rotation in annulus.*

The shear-rate at radius r can then be calculated using the shear-stress, which is a function of rheological properties. The rheological behavior of non-Newtonian fluids based on the Power-Law model is described as follows:

$$\dot{\gamma} = (\frac{\tau}{K})^{\frac{1}{n}} \tag{4}$$

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By substituting Eq. (1) and Eq. (3) into the Eq. (4), we have:

$$\frac{T}{2\pi h r^2} = K r^n \left(-\frac{d\omega}{dr}\right)^n \tag{5}$$

where the following boundary conditions at the outer radius of the pipe (r_{dp}) and wellbore radius (r_w) are considered:

$$\omega|_{r=r_{dp}} = \omega_1 \tag{6.a}$$

$$\omega|_{r=r_w} = 0 \tag{6.b}$$

Applying the boundary conditions into the Eq. (5) results in the following equation:

$$\int_{\omega_1}^{0} -d\omega = \left(\frac{T}{2\pi h K}\right)^{1/n} \int_{r_{dp}}^{r_w} \frac{dr}{r^{\frac{2}{n}+1}} \tag{7}$$

Solving the Eq. (7) leads to

$$\frac{T}{2\pi h K} = \frac{2\omega_1}{\left(n\left(\frac{1}{r_{dp}^{\frac{2}{n}}} - \frac{1}{r_w^{\frac{2}{n}}}\right)\right)^n} \tag{8}$$

Dividing both sides of the Eq. (8) by r^2 extracts $\frac{\tau}{K}$, which delivers the shear-rate as a function of r:

$$\dot{\gamma} = \frac{2\omega_1}{\left(\frac{1}{r_{dp}^{\frac{2}{n}}} - \frac{1}{r_w^{\frac{2}{n}}}\right) n r^{2/n}} \tag{9}$$

Substitution of $r_w - T_{mc}$ as the radius (r) gives the shear-rate at wellbore:

$$\dot{\gamma}_{wall} = \frac{2\omega_1}{n(r_w - T_{mc})^{2/n} \left(\frac{1}{r_{dp}^{\frac{2}{n}}} - \frac{1}{r_w^{\frac{2}{n}}}\right)} \tag{10}$$

Numerical Experiments

The equations above are utilized to perform a mathematical analysis of the pipe rotation and discuss its impact on dynamic mud loss, considering the power-law model as the rheological behavior description of the non-Newtonian drilling fluid. The following case studies are considered:

- Case 1: Dynamic mud filtration, pipe rotation effect is neglected.
- Case 2: Dynamic mud filtration, considering pipe rotation.

The dynamic model for drilling fluid filtration presented by Vasheghani Farahani, Soleimani et al. (2014) is used in case 1. Computer code is constructed to calculate the solid deposited volume and mud cake thickness, given the input parameters shown in Table 1. The following correlation has been considered to estimate the mud-loss flow rate as a function of time:

$$q_{loss}(t) = \frac{a}{t^b} \tag{11}$$

where a and b are set to be 10^{-7} and 1.065, respectively. Figure 2 and Figure 3, respectively, represent the solid deposited volume in the wellbore and the mud cake thickness in both cases. In case 1, it can be seen that the required time for mud cake formation is around 3.5 minutes. The solid deposited volume and the mud cake thickness in equilibrium conditions is about $4.5 \times 10^{-3}\ ft^3$ and $6 \times 10^{-3}\ ft$, respectively. In case 2, where the impact of the shear rate on wellbore is considered, mud cake formation stabilizes within about 3 minutes. It can also be seen that the stabilized solid deposited content and the mud cake thickness is about $4.3 \times 10^{-3}\ ft^3$ and $5.6 \times 10^{-3}\ ft$, respectively. Given that the impact of drill pipe/casing rotation and thus the shear rate is considered in case 2, the solid volume content and mud cake thickness are decreased as compared to case 1, as the drilling fluid rotation in annulus reduces solid deposition in the wellbore.

Table 1-Input Variables

Variable	Value
Wellbore radius	0.328 ft
Investigation Interval	1 ft
Formation Porosity	0.2
Filtrate viscosity	1.5 cp
Mud Weight	11.5 ppg
True Vertical Depth (TVD)	5000 ft
Formation Pressure	2800 psi
Formation Permeability	50 md
Reference Permeability of Mud cake	0.00295 md
Compressibility Exponent	0.5
Maximum Particle Radius	800 microns
Solid Content	0.231
Reference Porosity of Mud cake	0.59
δ	0.5
Particle Radius Distribution	$e^{-15R}(1-e^{-5R})$
Shear Rate	20 sec^{-1}
Friction Factor	1.73

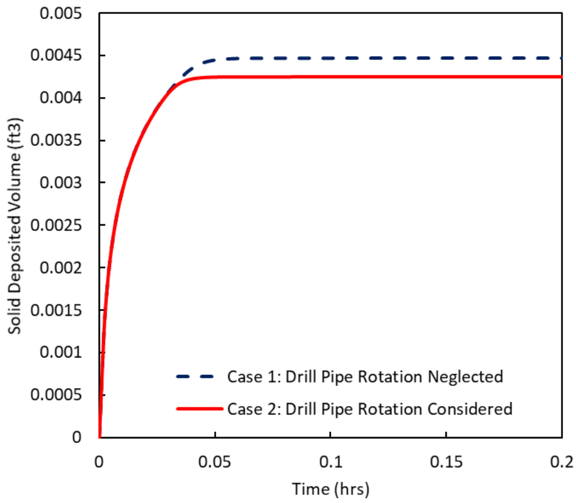

Figure 2 *Solid deposited volume as a function of time.*

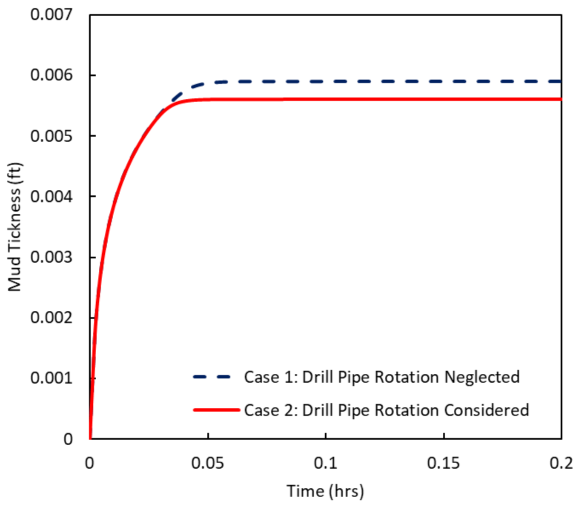

Figure 3 *Mud thickness as a function of time.*

Figure 4 shows the mud cake permeability and porosity changes during the mud cake formation. While both permeability and porosity (mud cake porosity reduction is shown in Figure 4) are decreasing as a function of time, the reduction rate is lower in late times (i.e. when mud cake thickness is stabilized) as compared to earlier times (i.e. mud cake formation period). This is due to the blockage caused by mud cake stabilization, which itself reduces the chance of further changes in permeability and porosity of mud cake.

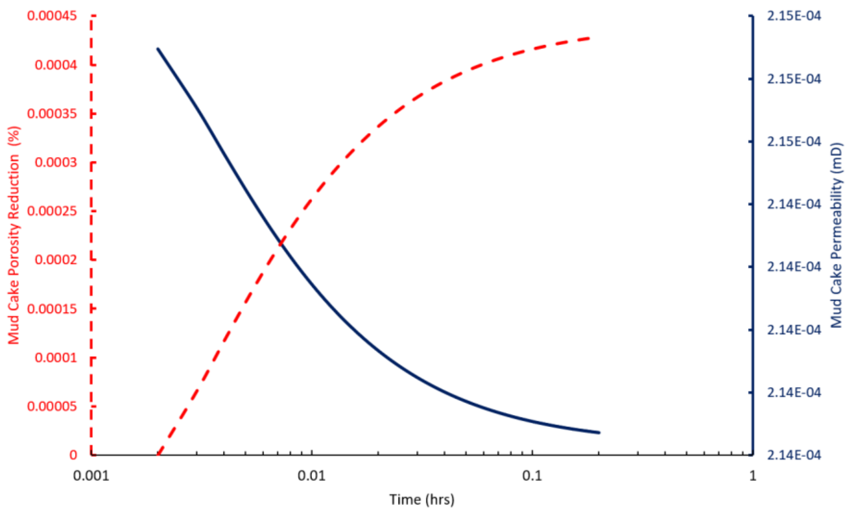

Figure 4 *Mud cake permeability and porosity as a function of time.*

Conclusions

This paper presents a dynamic model to obtain the mud cake thickness and solid deposited volume in a radial system of the annulus and a centric conduit containing non-Newtonian drilling fluid. The here-developed model accounts for the impact of drill pipe/casing rotation and its shear rate on mud cake thickness. Results demonstrate that neglecting the shear rate effect in mud cake formation could lead to erroneous estimation of mud cake thickness, which plays a significant role when the annular space is relatively small.

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