Free-electron-driven Vortex Smith-Purcell Radiation with Higher-Order Topological Charge

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***Abstract*-A free-electron-based vortex Smith-Purcell radiation with the higher-order topological charge is proposed in this paper based on uniform helical grating. Due to the spiral constraint of azimuth, the evanescent field of the electron beam will be diffracted and possess the orbital angular momentum. When the wave vectors of the diffraction wave match those of the free space, the vortex beam will be generated with different topological charges. Moreover, the higher-order topological charge can be affirmatively generated by utilizing the higher spatial harmonics according to Bloch's theorem of the helical periodic system. Moreover, the vortex beam with mixed orbital angular momentum can also be achieved. Furthermore, and the mode purities of these vortex beams are analyzed. The proposed scheme may facilitate the journey of the vortex beam from optical into terahertz band.**

I. Introduction

Terahertz (THz) wave is the electromagnetic (EM) wave whose spectrum ranges from 300 GHz to 3 THz. Due to its properties of strong penetration, non-invasiveness, fingerprint spectrum characteristics, and high-resolution characteristics, THz waves are widely used in the fields of biomedical, communications, astrophysics, and other fields. After more than 20 years of research and development, THz science and technology have also become a growth spurt in imaging, communications, spectroscopy, biomedicine, and other fields [1-3].

Vortex beam is the EM wave with spiral wavefront and orbial angular momentum (OAM). Since the wavefront corresponds to the phase of the wave, there is a phase index $\exp(jl\varphi)$ carried by the vortex beam, where the quantum number l is the value of topological charge (TC). TC can be used to to distinguish different vortex beam patterns. According to the physical characteristics of the OAM, the vortex beam has been widely used in the fields of communications, optical tweezers, quantum information encoding, chiral detection, and so on, since it was first found by Allen [4]. The transmission structure, such as the spiral phase plate and hologram plane, is capable of putting on a spiral phase on the incident Gauss beam and thus forming the vortex beam. While, once the structure is determined, the capability of manipulate the TC is limited. The metasurface, including the transmission structure and reflective one, can also be utilized to generate the vortex beam. However, due to its reliance on the resonance effect, the operation bandwidth of the metasurface-based structure is relatively low. Besides, by arranging the phase of each unit, the vortex beam with a tunable TC can be emitted from the antenna array. Whereas, a complicated feed network is required, which is difficult to be achieved in the THz band. Therefore, generation THz vortex beam with tunable TC is the goal to be realized.

In this paper, by using the helical grating and electron beam, the vortex beam with the higher-order TC is achieved. The dispersion of the helical grating are numerically calculated. When a 6.5-keV electron beam flies over the grating, the radiation regions of higher-order spatial harmonics are also analyzed [5]. According to Bloch's theorem of the spiral system, the vortex Smith-Purcell radiation with higher-order TC will be affirmatively generated. Model and Dispersion

A. Model

The helical grating is utilized in this work as the interaction circuit, as shown in Fig. 1, and the structural parameters are listed in Table 1. The hollow-shaped electron beam is emitted from the left end and flies parallel to the grating in the *z*-direction. Besides, a 0.5-T magnetic field is set along the *z*-direction to overcome the space charge effect.

B. Dispersion

The dispersion relations of the helical grating and a 6.5-keV electron beam are numerically calculated and illustrated in Fig. 2 [6]. The radiation regions of the spatial harmonics from -1^{st} to -7^{th} are marked as red shadow areas separately. The intersection point of the dispersion lines is also marked as point *M1*. Therefore, the bunching frequency of the electron beam is f_{M1} = 0.124 THz. And the wave harmonics are *M2*,

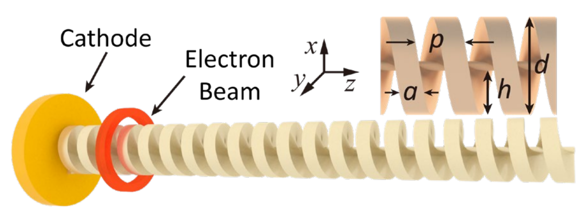

Figure 1. Model of the helical grating. The hollow electron beam, which is marked as a red ring, is emitted from the left end, and travels parallel to the grating in the *z*-direction.

TABLE I
STRUCTURAL PARAMETERS OF THE HELICAL GRATING

Parameter	*p*	*a*	*h*	*d*
Size(mm)	0.30	0.15	0.27	0.60

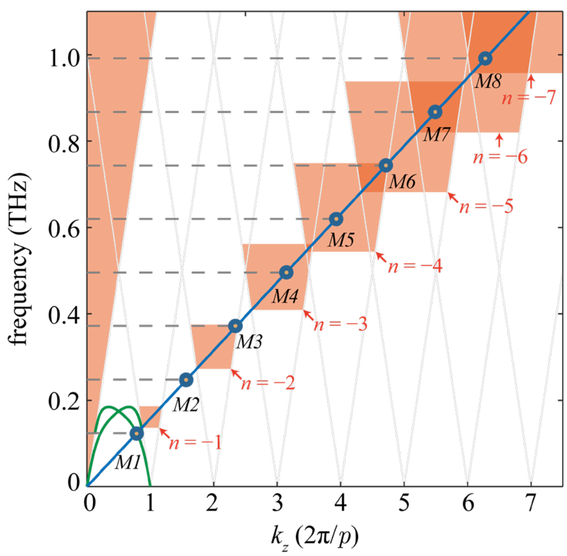

Figure 2. Dispersion characteristic of the helical grating and 6.5-keV electron beam. The red shaded regions are the radiation bands of the spatial harmonics.

M3, *M4*, *M5*, *M6*, and *M7*.

II. Simulation Results and Analysis

According to Bloch's theorem of the spiral system, the *n*-th spatial harmonic of the diffraction wave carries the TC of *n*. Hence in each radiaiton regions of spatial harmonics, the vortex beams with different TCs related to the order of the spatial harmonic will be generated.

The wave harmonics of *M3*, *M4*, *M5,* and *M6* are in the radiation regions of −2nd, −3rd, −4th, and −5th spatial harmonics. Therefore, the vortex beams of TC = −2, −3, −4, and −5 will be generated at f_{M3}, f_{M4}, $f_{M5,}$ and f_{M6}, as shown in Figs. 3(a), 3(b), 3(c), and 3(d), where the field distributions validate the existence of the OAM..

Since f_{M7} is in both the radiation bands of −5th and −6th spatial harmonics, the vortex beam with TC blended by −5 and −6 is formed at f_{M7} as shown in Fig. 3(e). Similarly, since *M8* is located in the radiation bands of −6th and −7th spatial harmonics, the vortex beam with TC blended by −6 and −7 is formed at f_{M8} as shown in Fig. 3(f).

III. Conclusions

In this paper, the vortex Smith-Purcell radiation with higher-order TC is achieved based on the higher-order spatial harmonics. The bunching frequency of the electron beam is set as that of the spoof surface plasmon polaritons of the helical grating. The dispersions of helical grating and 6.5-keV electron beam are numerically calculated. By employing the higher-order spatial harmonics, the vortex beams with higher-order TCs, which are −2, −3, −4, −5, −5&−6, −6&−7, respectively, are achieved in the radiation bands of the higher-order spatial harmonics. The profiles of the E_z component of these vortex beams are also calculated, which validates the existence of the OAM. By tuning the electron energy, the generation and utilization of the higher-order TCs can be further explored. The proposed scheme may facilitate the journey of the vortex beam from the optical into the THz band.

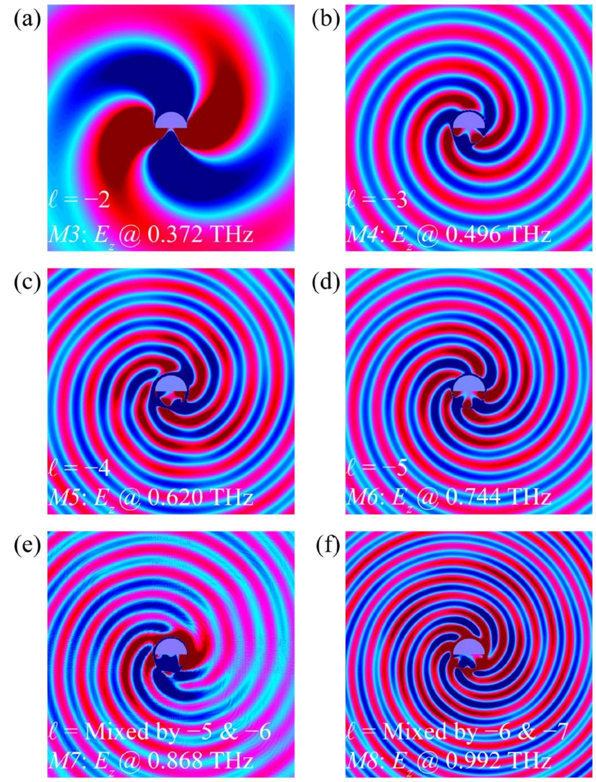

Figure 3. Profiles of the E_z at the frequencies of wave harmonics of *M2*, *M3*, *M4*, *M5*, *M6*, and *M7*.

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