Non-Linear Finite Element Optimization for Inelastic Buckling Modelling of Smooth Rebars

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Abstract: This paper presents an optimization methodology to simulate the monotonic and cyclic response of steel reinforcement smooth bars when subjected to inelastic buckling. A finite element (FE) model of steel rebars, based on non-linear fibre sections and an initial geometrical imperfection, is adopted. The multi-step optimization proposed herein to identify the main parameters of the material constitutive models is based on genetic algorithms (GA) and Bayesian model updating. The methodology consists of comparing available experimental tests from literature with the corresponding numerical results. New empirical relationships and probabilistic distributions of the optimized model parameters, such as post-yielding hardening ratio, isotropic hardening in compression and tension, plus initial curvature, are presented. Finally, utilizing both the GA-based and Bayesian-based calibration, an improvement of an existing analytical model for inelastic buckling of smooth steel rebars is proposed. Such analytical modelling can be efficient and reliable for future building codes and assessment guidelines for existing buildings.

1. INTRODUCTION

Many existing reinforced concrete (RC) buildings and bridges were designed in the 60s and 70s according to obsolete non-seismic codes. Such structures were reinforced with “smooth” rebars (e.g., Arani et al., 2013; Arani et al., 2014; Abbiati et al., 2015; Melo et al., 2015; De Risi et al., 2017) and using poor seismic details (e.g., hook anchoring, short bar lap splice, large stirrups spacing, (e.g., Fabbrocino et al., 2008-Part-I; Fabbrocino et al., 2008-Part II; Verderame et al., 2009-Part I;Verderame et al., 2009-Part II). As a consequence, the global response of RC structures may completely change as RC components could fail under the combined action flexure and shear. Recent earthquakes exemplified the vulnerability of such RC structures, even for low-intensity ground motions. Thus, it is imperative to comprehensively investigate the seismic performance of existing RC structures with smooth rebars. Most of the critical deficiencies of such RC structures could be attributed to the exposure to aggressive environments over their lifetime (e.g., Di Sarno and Pugliese, 2020a-2020b, among others), which have triggered the phenomenon of deterioration (i.e., corrosion). The ageing effects resulted in cracking and spalling of the concrete cover, as well as deterioration of the bond strength between steel reinforcement and concrete (Robushi et al., 2020). Bond deterioration and cover spalling may lead to buckling of longitudinal bars when the structure is subjected to seismic

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loads. Such a phenomenon is often neglected when analysing columns and shear walls, especially in the critical regions at the structural components. As a result, the behaviour of RC elements and, therefore, the global capacity of structures may be overestimated to the actual strength, ductility and, in turn, energy dissipation capacity.
Thus, the choice of an accurate constitutive model for smooth steel reinforcement plays a significant role in investigating the global response of structures, especially in the framework of seismic risk analysis of existing structures and infrastructure. This study presents the optimal parameters characterization for the most-adopted steel constitutive models for smooth rebars using genetic algorithm (GA) and Bayesian updating. A refined finite element (FE) model implemented in the advanced open-source software OpeeSEES (McKenna et al., 2010) is adopted for simulating the actual response of steel reinforcement bars for different bar slenderness, i.e., the L/D ratios. Monotonic and cyclic tests from the literature of steel reinforcement bars are used for the calibration. Firstly, the numerical prediction of the inelastic buckling of ribbed rebars is conducted. The latter allows the validation of the effectiveness and accuracy of the adopted modelling approach by using the original formulation and default parameters of the examined steel constitutive models. Then, the FE model, along with the GA and Bayesian updating, is used to identify the optimal parameters for predicting the cyclic response of smooth steel rebars. Based upon the optimization procedure, regression analyses and probabilistic assessment of the examined parameters are performed. Three available constitutive steel models in OpenSEES (McKenna et al., 2010), namely Steel02 (Menegotto and Pinto, 1973; Filippou et al, 1983), SteelMPF (Menegotto and Pinto, 1973; Filippou et al., 1983; Kolozvari et al., 2015), and ReinfrocingSteel (Kunnath et al., 2009), are investigated herein using the optimization procedure. The mechanical properties and the loading protocol for the smooth rebars are taken from Prota and Cosenza (2009). The key parameters of interest are the hardening ratio (*b*), the initial curvature (R_0) and the isotropic hardening in compression and tension (*a1* and *a3*). The regression analyses and probabilistic assessment of the aforesaid model parameters are used as an effective way to facilitate the implementation of the stress-strain models of smooth rebars in advanced FE software for seismic risk analyses. The optimized variables are then utilized to validate the model further using available experimental monotonic compressive tests. Hence, a comprehensive parametric analysis is performed. The purpose of the parametric study is to develop an improvement of an existing analytical model -- initially developed by Prota and Cosenza (2006) -- for inelastic buckling of smooth rebars, which can be used either for robust seismic analyses or hand calculations for predicting the capacity of RC sections.

2. STATE-OF-ART OF NUMERICAL MODELS AND EXPERIMENTAL TESTS

A comprehensive literature review can be found for the effects of buckling on ribbed rebars, both experimental and numerical (e.g., Dhakal and Maekawa (2002); Bae et al., 2005; Kunnath et al., 2009; Massone and López, 2014; Akkaya et al., 2019, among others). Conversely, limited research has been conducted on the effects of inelastic buckling on smooth rebars (Cosenza and Prota, 2005; Prota et al., 2009). Dhakal and Maekawa (2002) conducted numerical investigations on the post-yielding buckling of reinforcing bars. Based on the numerical simulations, by using different L/D ratios, they provided a stress-strain model to account for the inelastic buckling for reinforcing bars. Then, the proposed method was combined with Menegotto-Pinto (1973) model for steel reinforcement to investigate the reliability of their numerical approach. The results showed an excellent agreement with the experimental test, both under monotonic and cycling loading.

Bae et al. (2005) carried out a large experimental campaign on 162 reinforcing bars tested under monotonic compressive loading. Results showed that L/D ratios greater than 6 demonstrated a negative instability once reached the onset of buckling; the increase in the initial geometrical imperfection resulted in reduced load-carrying capacity; L/D ratios smaller than four did not experience buckling even for large inelastic deformation. Furthermore, they provided a new relationship for the inelastic buckling of reinforcing steel, which was then validated against the experimental results showing a good agreement.

Kashani et al. (2014) carried out an experimental campaign on the effects of corrosion on ribbed rebars, which were then used as a model validation of the proposed buckling approach. A FE model, based on non-linear force-based elements was adopted. Results showed that the model could reliably predict the inelastic buckling response of corroded ribbed rebars under monotonic and cyclic loadings. The cyclic response of the reinforcing steel with different L/D ratios strongly depended upon the strain history, while high values of L/D reduced the onset of the critical load for the inelastic buckling (i.e., L/D = 15 and L/D = 20).

Cosenza and Prota (2005) carried out an experimental campaign on the compressive monotonic behaviour of “smooth rebars” to investigate the buckling effects using different values of the L/D ratio. For low L/D ratios (i.e., L/D = 5), the monotonic behaviour was almost coincident with the tensile tests. From L/D equal to 8, the plateau started decreasing with the increase of the L/D ratio. Finally, L/D greater than 20 showed a critical load close to or smaller than the yielding stress of the smooth reinforcing rebar. A modelling approach for simulating the inelastic buckling of smooth rebars as a function of the L/D ratio was provided. Such an experimental campaign was then extended for the cyclic response of smooth rebars in Prota et al. (2009). They found that for values of L/D ranging between 5 and 7, the monotonic and tensile behaviour was not symmetric due to the progressive effects of buckling. Rebars started buckling earlier with the increase of the L/D ratio and approaching the yielding stress for values greater than 15. Moreover, the progressive effects of inelastic buckling appeared to increase the pinching especially for high values of the strain history. A critical review of the most used constitutive models for predicting the response of ribbed rebars was carried out. The comparison of those constitutive models showed that the non-linear expression (Menegotto-Pinto, 1973; Monti-Nuti, 1982) could sufficiently predict the behaviour of smooth rebars for L/D ratio lesser than 8 when inelastic buckling did not take place. However, for higher values of slenderness (L/D > 8) the formulation could not simulate the actual response of smooth rebars due to the change of curvature for each half cycle.

Although those few studies provided insights on constitutive models for simulating smooth rebars, there is a need to investigate such models by calibrating some of their parameters and using advance non-linear FE approaches. The post-elastic response of reinforcing steel depends on the strain history, so it is path-dependent (Dhakal et al., 2002). However, all previous studies on the cyclic response of smooth rebars referred to the experimental-analytical comparison and, therefore, this paper investigates the numerical-experimental comparison based on the only cyclic/monotonic tests known to the authors.

One of the most relevant methods used for structural optimization and design problems is the so-called Genetic Algorithm (GA). The pioneer of GAs Holland (1975) inspired many researchers to apply GA to many contexts and disciplines as a modern optimization technique. Coello and Christiansen (2000) applied the GA to enhance the design of two typical trusses based on multi-objective optimization. They proposed a new GA-based approach where the populations were generated such that individuals represented only feasible solutions. The technique appeared to be faster and more accurate in optimizing the design of the two trusses. Perry et al. (2006) proposed a new modified GA strategy based on a space-reduction procedure to better identify the parameters of multiple-degree-of-freedom (MDOF) structural systems. They compared this approach to the standard GA, based on numerical simulations of 10-to-20 DOF shear-type structures, by considering structural parameters the mass, the damping and the stiffness. Results showed that the proposed method was able to reduce the average absolute error compared to the classical GA, improving the accuracy for the identification of structural parameters. Numerical modelling and experimental measurement are always affected by uncertainties (Celarek and Dolsek, 2013; Castaldo et al., 2019; Castaldo et al., 2020). As a result, it is paramount to account for modelling uncertainties. One possible solution to characterize these uncertainties is to adopt the Bayesian model updating. Such an approach is often adopted in structural engineering (Gardoni et al., 2002; Jalayer et al., 2010). Jalayer et al. (2010) adopted the Bayesian approach to quantify and update the model uncertainty parameters for the mechanical properties of materials and geometrical properties of construction detailing to assess the structural performance of existing RC structures based on the demand and capacity defined in the modern standard technical codes. A Bayesian framework was used in Gardoni et al. (2002) to construct probabilistic predictive capacity models for RC structural components.

3. Research Significance

One of the most detrimental phenomena for existing substandard RC structures with smooth rebars and subjected to earthquake loadings is the inelastic buckling of longitudinal rebars. Such an inelastic instability may affect the global response of RC structures. As a result, it is essential to approach the inelastic buckling with advanced numerical analyses and machine learning algorithms to produce more reliable model parameters and, in turn, constitutive models to perform accurate seismic analysis and risk assessment. For this purpose, this study presents:

(1) An approach for optimizing the mechanical parameters to model smooth steel rebars under monotonic and cyclic loading. It employs genetic algorithms and Bayesian updating with the integration of a FE model of steel reinforcement. The latter is derived from an advanced open-source program platform for earthquake engineering applications;

(2) Insights on the relationships and probabilistic distributions for the model parameters of the most adopted constitutive models for steel reinforcements as a function of the bar slenderness. Such formulations are beneficial for engineering applications when employed in current stress-strain constitutive models to simulate the inelastic behaviour of smooth rebars;

(3) Improvement of an existing analytical model for inelastic buckling of smooth rebars based on the numerical and experimental results.

4. Optimization Procedure

4.1 The Genetic Algorithm

A GA-based algorithm (Global Optimization Toolbox User's Guide, 2019) is used to calibrate conventional constitutive models for steel reinforcement to describe the inelastic buckling of smooth rebars. To investigate the load-bearing capacity and the seismic vulnerability of RC structures, civil engineers rely on FE approaches and the definition of parameters for existing constitutive models of concrete and steel reinforcement.

The most common approach for such optimization problems relies on the definition of an objective function (e.g., the absolute error of the model and the empirical evidence) that may incorporate some other constrained parameters to obtain a multi-objective optimization. This approach (Figure 1) involves generating initial plausible populations of parameters chosen in pre-defined intervals derived from the literature.

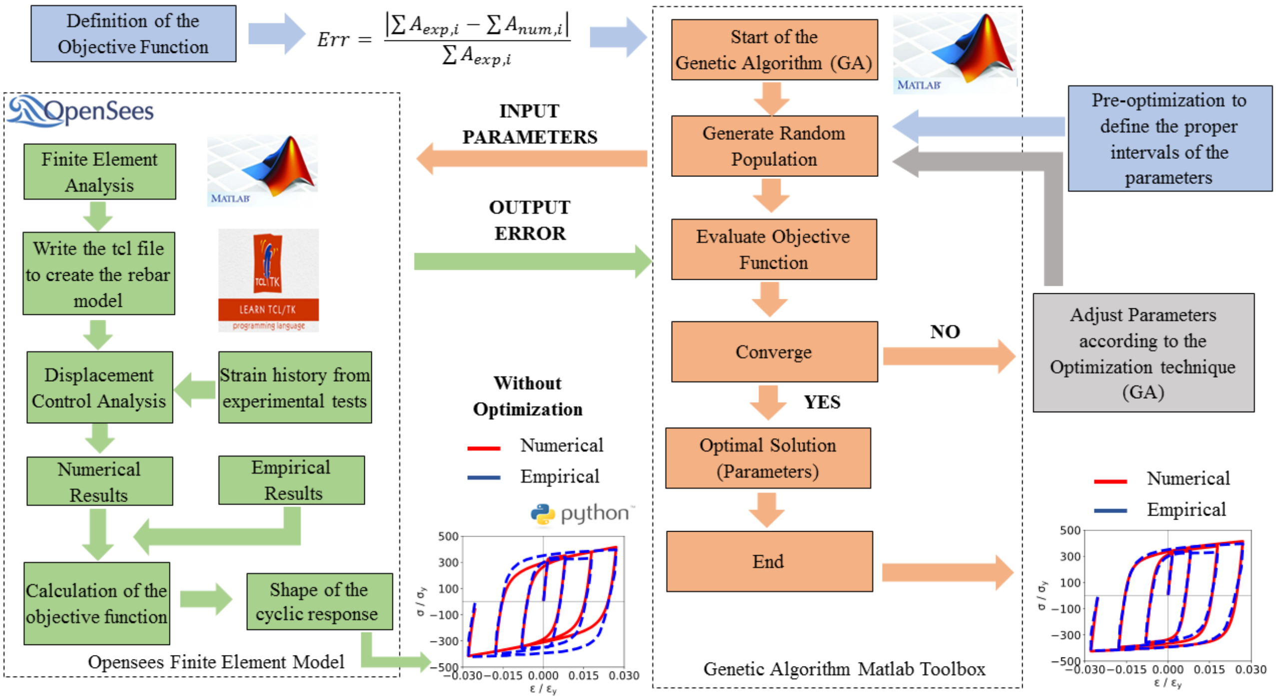

Figure 1. Flow-Chart of the optimization procedure

For instance, the constitutive models named Steel02 and SteelMPF are entirely defined by the following parameters: yielding stress (f_y), hardening ratio (b), elastic modulus (E_s), parameters controlling the transition from the elastic to the plastic branch (R_0, cR_1, cR_2) and the isotropic hardening parameters (a_1, a_2, a_3, a_4). The yielding stress and the elastic modulus are kept constant from the experimental results, as well as cR_1, cR_2 and a_2, a_4 are assumed equal to the default values. Conversely, b, R_0, a_1 and a_3 are used to calibrate the numerical model with the experimental results. Each of this generated population is then employed to construct a numerical (e.g., OpenSEES) model of the smooth rebar under a cycling strain history. Then, the model is subjected to a load that is consistent with the available experimental test in a sort of *virtual* laboratory test. The experimental evidence and the numerical results from the model can then be used to calculate the error function. In this study, the relative error between the hysteretic areas (Equation 1) of the test cycles is used.

$$Err = \frac{|\sum A_{exp,i} - \sum A_{num,i}|}{\sum A_{exp,i}} \tag{1}$$

The relative error defined in (1) targets the energy and it is the objective function to be minimized. Once the analysis is completed and the error computed, the next generation parameters for the next run will be adjusted according to the distance from the optimum. However, one of the main concerns when applying GAs is the inherently random nature of all variables to be optimized. It is generally necessary a pre-optimization process to reduce the variability of such parameters. The pre-optimization is herein used to define proper intervals, depending on the slenderness (L/D) ratio, by running the numerical model with a small number of random populations. The last step allows defining the intervals of the model parameters for the final optimization process with an assumed variation per each variable (Figure 2). Once the variable intervals are defined, the GA performs the analysis until the specified criteria are met.

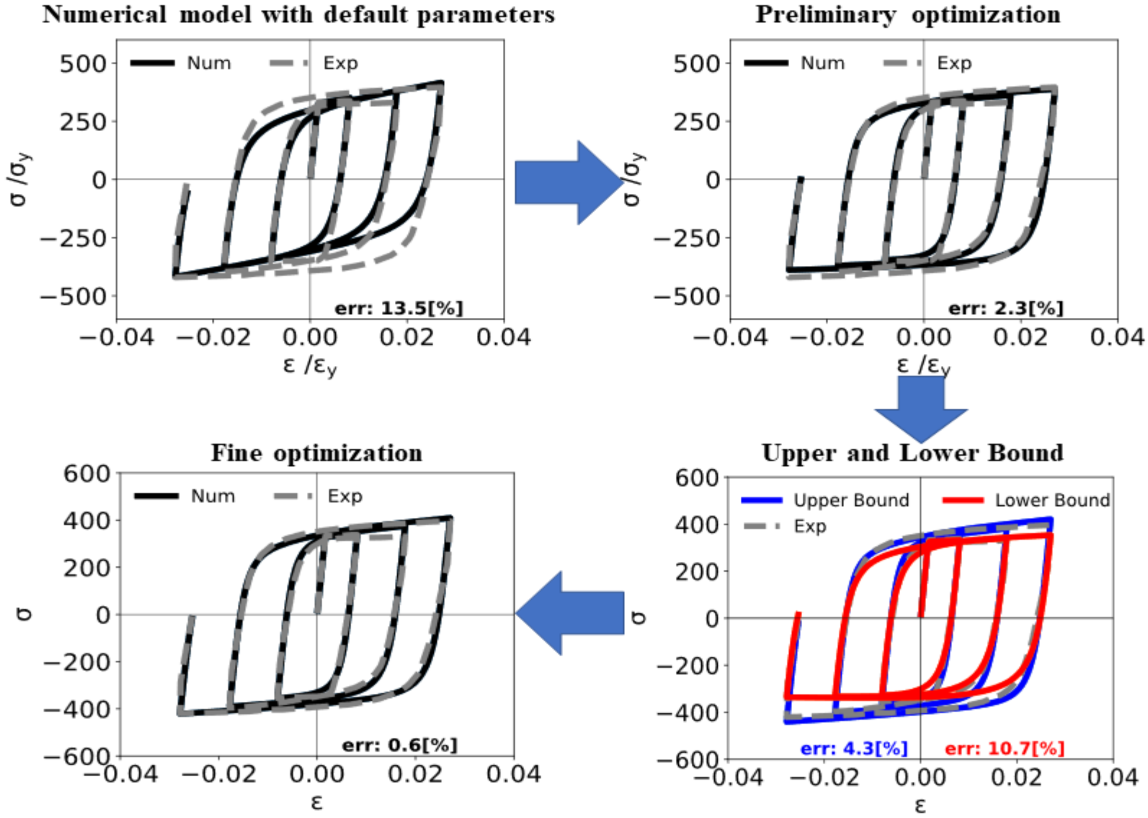

Figure 2. Genetic Algorithm Procedure

4.2 Bayesian Model Updating

Once the model parameters are estimated through the genetic algorithm, it is possible to characterize the numerical modelling and the experimental measurement uncertainties. For this purpose, the Bayesian updating is the recognized tool able to identify the probability for each model parameter $P(\bar{\theta}|D_{exp.})$ (from the numerical model) given the experimental data ($D_{exp.}$). The latter conditional probability can be calculated as follows (e.g., Wasserman, 2013):

$$P(\bar{\theta}|D_{exp.}) = C^{-1}P(D_{exp.}|\bar{\theta})P(\bar{\theta}) \quad (2)$$

$\bar{\theta}$ is the vector containing the model parameters (i.e. *R, a1, a3* for Steel02 and SteelMPF, while *R* for Reinforcing Steel), $P(\bar{\theta})$ is the prior probability, $P(D_{exp.}|\bar{\theta})$ is the likelihood, and C^{-1} is a normalizing factor such that the area under the posterior curve is equal to 1. Based on the linear regression analysis derived from the genetic algorithm for different rebar slenderness (L/D) values, it is possible to derive the prior probability $P(\bar{\theta})$ for each model parameter. The likelihood is a distribution representing the numerical modelling error compared to the experimental counterpart; therefore, a standard normal distribution is adopted, with a mean equal to zero and an unknown standard deviation added to the vector $\bar{\theta}$. The experimental data $D_{exp.}$ can be either a single error term ($\sum A_{exp} - \sum A_{num}$) for each test or a number of error terms for each cycle, assuming that the damage of the rebar at each cycle is a function of only the previous cycle. In this second case, the likelihood is the product of the standard normal pdf with unknown standard deviation calculated for each cyclic error. Equation 2 can be solved numerically. At the end of the analysis, the cumulative density function of each marginal distribution is derived for each model parameter.

5. FINITE ELEMENT MODEL OF THE STEEL BAR

The non-linear behaviour of smooth rebars is modelled through spread plasticity beam-column force-based elements (Spacone et al., 1996a and 1996b). Such an element, available in OpenSEES (McKenna et al., 2010), neglects the effects of shear and bond-slip, but it can simulate the non-linear behaviour of members under coupled bending moments and axial load. It is discretized in different control points known as the integration scheme. Each integration point is characterized by fibre cross-sections that define the non-linear behaviour of the element itself and where the stress-strain relationship of each examined constitutive steel model is employed. The five points Gauss-Lobatto integration scheme is herein used for computing the stress-strain at the section level. Since the aim is to describe the buckling of steel bars, the linear transformation is not adequate to solve the large inelastic displacement-small strain problems. Thus, this study uses the so-called co-rotational formulation (Souza, 2000), which accounts for large displacements while remaining small deformations along with the element.

To capture the inelastic buckling, an initial geometric imperfection is considered in the middle of the element as to force an initial curvature, which turns into a linear transverse deviation of the element to its longitudinal axis (Uriz et al., 2008). This imperfection is taken as L/1000, which is also the maximum value stated by the ASTM A6/A6M (2019). As far as the boundary conditions are concerned, one end is completely fixed (both translations and rotations); the other end is free to move only along the longitudinal axis of the beam. Moreover, a fibre and element discretization schemes were needed to obtain the best trade-off between accuracy and computational demand. From the sensitivity analysis, the maximum number of elements equal to six is deemed acceptable to maintain a realistic aspect ratio for the bar, as well as a circular cross-section with a number of radial fibres equal to 10 and circumferential to 35 guarantees efficiency and small numerical effort (Figure 3).

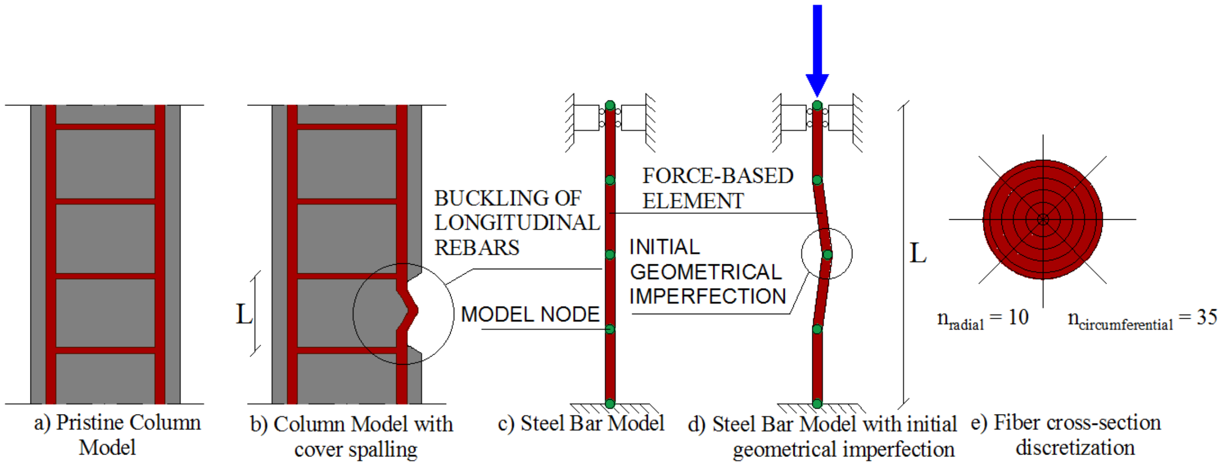

Figure 3. Buckling Model

A larger number of small elements will excessively reduce the bar length and results be no longer realistic. Accurate preliminary tests were carried out and validated against experimental tests from the literature (e.g., Kashani et al. 2014; Imperatore and Rinaldi, 2019). In this study, an adaptive convergence algorithm was needed to perform a non-linear longitudinal displacement control analysis (non-linear static analysis) for all the three examined constitutive steel models by using an initial number of iterations equal to 100. The last observation comes after the convergence issues met for the ReinforcingSteel material available in Opensees. If the convergence is not achieved, the number of iterations is modified to 1000 while several solution algorithms are included to solve the non-linear equations such as Newton, Krylov-Newton (Scott et al., 2010), ModifiedNewton and NewtonLineSearch (Crisfield, 1991).

5.1 Model Validation for ribbed reinforcement bars

Earthquakes typically increase the compressive stresses that RC members are subjected to due to frame effects and the potential vertical components (e.g., Di Sarno et al., 2011). Increasing compressive stresses can induce instability of longitudinal rebars with a consequent significant lateral displacement that may lead to inelastic buckling. Structural models should be able to capture the inelastic buckling to obtain a more reliable and accurate local response of RC components. Typically, buckling affects the post-yielding response of steel rebars which experience relevant softening depending on the L/D slenderness ratio.

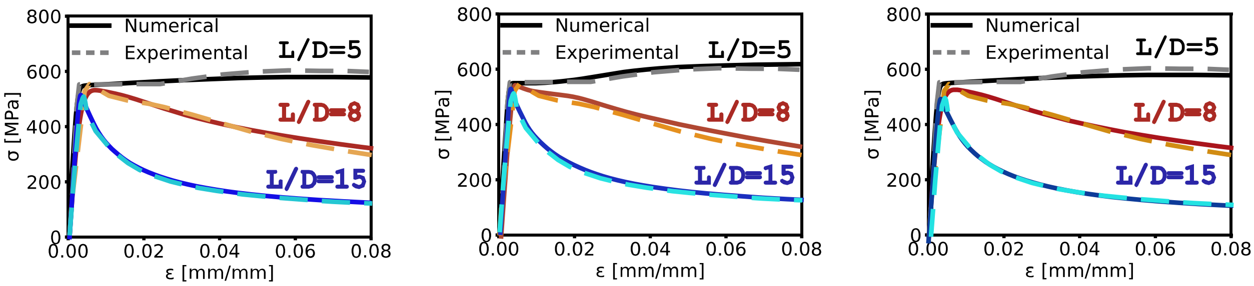

Figure 4. Experimental vs Numerical results. a) Steel02; b) ReinforcingSteel and c) SteelMPF.

Although this study focuses on the effects of the inelastic buckling on smooth rebars, it is essential to validate the accuracy and the consistency of the numerical modelling approach against empirical results of ribbed rebars reported in the literature, since the examined constitutive models were merely calibrated for deformed bars. Despite this, the adopted FE approach can be eventually used for parametric optimization of deformed rebars.

The comparison between the available experimental tests and the numerical approach reveals the ability of the modelling approach to predict the post-yielding softening of steel reinforcement due to buckling, even for small values of L/D, whereas its effects are negligible. Figure 4 show that the proposed finite element approach is able to accurately predict the response of ribbed rebars under the monotonic compressive loading for all constitutive steel models (used in this study) and different values of L/D ratios. The results of the numerical simulations in Figure 4 are calculated using the optimal discretization.

The validation of the current model for ribbed rebars under monotonic compressive loading and affected by the buckling unveils that the modelling approach can be feasible for a parametric optimization to predict the actual response of smooth rebars (Cosenza and Prota, 2009). However, a study conducted by Carreno et al. (2020) shows that some parameters of the Menegotto-Pinto constitutive model should be further investigated and optimized to identify the cyclic response of ribbed rebars due to some discrepancies of the model in OpenSEES (McKenna et al., 2010) to simulate the inelastic behaviour at each half cycle and the strain hardening after the yielding plateau.

6. CYCLIC RESPONSE OF SMOOTH REBARS UNDER INELASTIC BUCKLING

Very few studies have been conducted on the cyclic response of smooth rebars with different values of *L/D* ratios. Cosenza and Prota (2009) highlighted the main differences between the cyclic and monotonic response of smooth and ribbed bars based on an extensive experimental campaign. One of the main aspects referred to the dissipation of the hysteretic energy for both steel reinforcement. Subjected to the same strain history, the cyclic response curve of ribbed rebars was always internal to the previous ones compared to smooth rebars. The last observation is that smooth rebars showed minor stiffness damage (during unloading-loading) than ribbed rebars with a higher curvature for each cyclic response. Lesser curvature produced a much stronger Bauschinger effect for ribbed bars.

6.1 Steel02

Due to the optimal trade-off between simplicity and efficiency of the formulation, one of the most adopted constitutive models for non-linear steel reinforcement response is the model proposed by Menegotto and Pinto (1973), which refers to Steel02 in OpenSEES (McKenna et al., 2010). The stress (σ) and strain (ε) formulation is the following:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}}} \tag{3}$$

Equation (3) represents the curved transition from a straight-line asymptote (with slope equal to the initial stiffness) to another asymptote (with a slope corresponding to the hardening of the steel bar). The ratio between these two asymptotes is the hardening which refers to b in the eq. (3). The parameter R controls the shape of the transition curve (if R = ∞ the curve becomes bi-linear) and allows the representation of the Bauschinger effect for each branch curve. The expression for R depends upon some parameters, such R_0, typically between 10 and 20, cR_1 and cR_2, typically equal to 18.5 and 0.15, respectively, that are experimental parameters (e.g., Menegotto and Pinto, 1973; Filippou et al. 1983). Indeed, Filippou et al. (1983) proposed a refined modification of the original constitutive model for steel reinforcement. Since the isotropic hardening had relevant effects on the bars responsible for the crack closure of RC components under cyclic loading, they proposed a shift of the yielding asymptote position and, then, computing the new intersection point (unloading-loading) through a strain reversal rule. Hence, this shift rule suggested a new formulation with two other parameters a3 and a4, which account for the isotropic hardening in compression and tension (default in Opensees a3 = 0.00 and a4 = 1.00). The last two parameters refer to a1, a2, a3, a4 for Steel02 in Opensees. Further details can be found in Filippou et al. (1983).
Using the FE approach and the default parameters, the comparison between experimental and numerical results (based on the average response of smooth rebars in Prota and Cosenza, (2006)) is carried out for different L/D ratios. Results in Figures 5 show that the use of the default parameters does not sufficiently allow the numerical model to simulate the response of a smooth rebar under cyclic loading, especially for low values of L/D ratio (i.e., L/D = 5,10).
Comparisons between the numerical approach and experimental tests, both in tension and compression, definitely seem to underestimate the energy dissipated for each half cycle. However, the last statement is not directly related to numerical approach, but rather the inability of the constitutive models, based on the provided mechanical properties, to capture the full hysteretic behaviour even when buckling (L/D < 5) does not occur or at least is negligible. On the other hand, for higher values of L/D ratio, where inelastic buckling is most likely to occur, the model prediction overestimates the compressive and the tension behaviour of the steel bar anyhow.

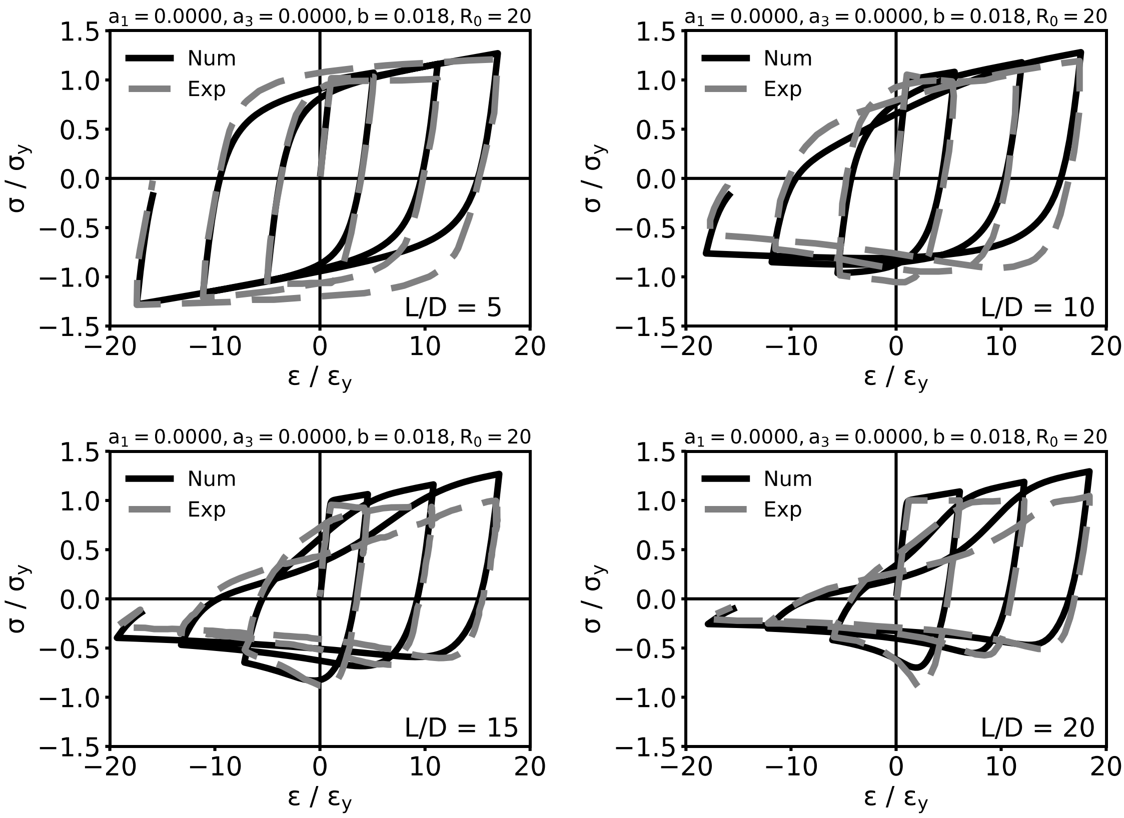

Figure 5. Numerical vs Experimental response of smooth bars with Steel02 in Opensees

Moreover, the cyclic response does not seem conservative enough for the pinching effect, which is relevant for RC components under strong excitations. L/D ratios equal to 10 and 15 still appear to show significant discrepancies with the experimental results, mainly due to the small curvature and not calibrated isotropic parameters assigned to the constitutive models. On the other hand, when the curvature and the isotropic parameters approach the ideal values (i.e., L/D = 20), based on the optimization procedure, the model starts efficiently predicting the cyclic response of smooth rebars.
The above discussion suggests a need for a parametric optimization of the main parameters of such constitutive model, especially for existing RC infrastructures and buildings under strong earthquakes.
The parameters to be optimized are chosen based on the observed experimental data, such as:

- R_0, the curvature is higher for smooth rebars and appears to change for different slenderness ratio;
- b, the hardening ratio; according to the experimental tests, the energy dissipated by smooth bars is certainly higher than ribbed bars; thus, an appropriate estimate of the hardening asymptote is of reasonable importance;
- a_1, a_3, the compressive and tension isotropic parameter; these two parameters allow to capture the higher energy dissipation for each half cycle.

Conversely, some parameters, such as a2, a4 and cR1 and cR2, are kept fixed as in the original formulation, since their change will turn into a complete shift of the curve shape.
The outcomes of the optimization procedure are given in Figure 6.
Results from simulations notably illustrate that the proposed procedure can accurately and adequately predict the response behaviour of smooth rebars under a cyclic strain loading. The hysteretic energy dissipation of each half-cycle, a crucial measure of a component under cyclic loading, is well-predicted, thus, allowing an adequate response if such stress-strain curves are employed in the fibre-

section of RC components. Moreover, the bar-buckling affects the strength capacity and the hysteretic energy dissipation with the increase of the slenderness ratio L/D.

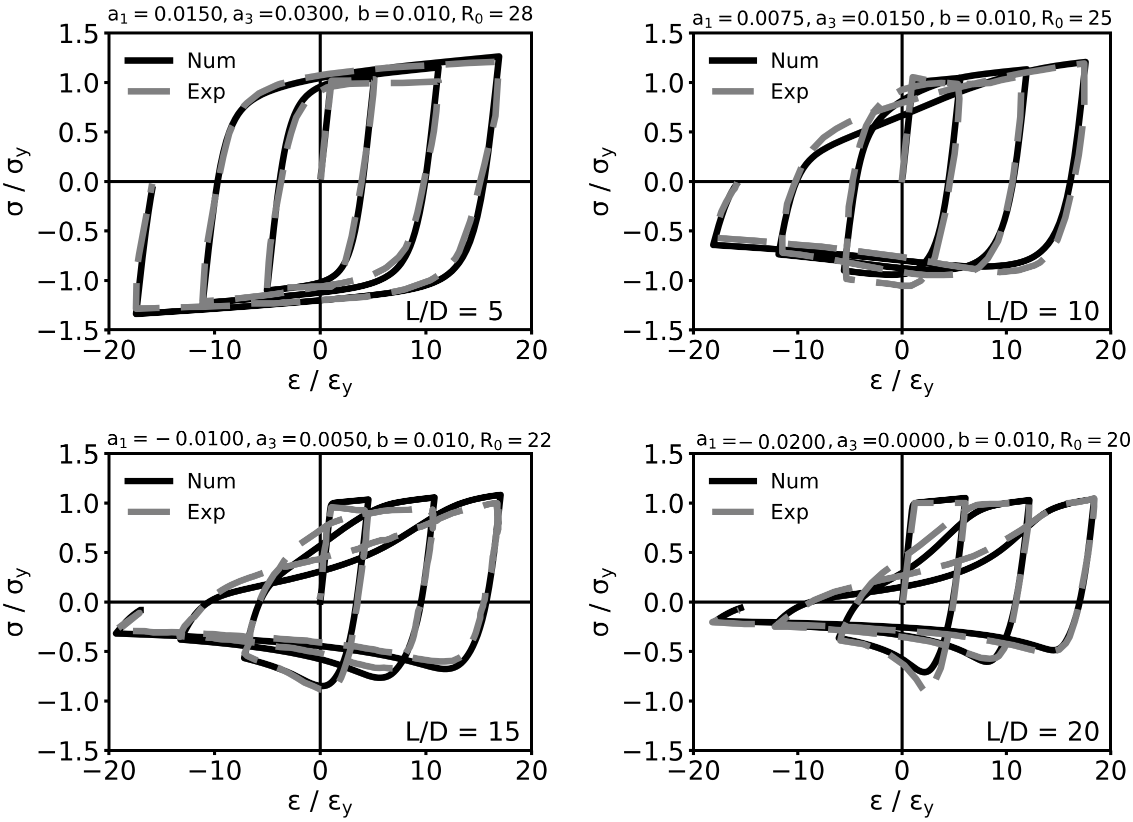

Figure 6. Numerical vs Experimental with optimized parameters

Observations from the optimization procedure show that the value of the hardening ratio appears to be constant regardless of the increase of the slenderness. Particularly, its value is reduced by a factor of 1.8 compared to the initial hardening ratio provided in Cosenza and Prota (2008). R_0, which denotes the value of R during the first loading, ranges between 28 and 20 with the increase of the slenderness, demonstrating almost a linear variation with L/D. The last consideration highlights the increasing effects of the bar-buckling on the initial curvature of the constitutive model. Furthermore, it is worth noting that the increasing value of the slenderness ratio L/D denotes a significant effect on the buckling initiation, which is most likely to occur at lower strain for the same steel bar (but with different L/D). As a result, the post-yielding branch is characterized by an exponential reduction of the strength capacity with a change of the curvature (negative), especially for high values of L/D ratios, as can be seen in Figures 6c and 6d. Regression analysis is then carried out to accurately describe the correlation of the optimized parameters to the slenderness ratio.

Figures 7 show that the linear interpolation seems to be an appropriate estimation of the optimized parameters to define the inelastic cyclic response of smooth rebars. The improved parameters can then be employed to define the stress-strain steel model for the cross-section fibres to obtain a more accurate response of RC components.

Figures 8 shows the numerical simulations from the Bayesian model updating. The marginal distributions of the model parameters (*R_0, a_1, a_3*, in this case) are accurately fitted using normal distributions. However, it should be stressed that the standard deviations are equal to those obtained from the GA application. The last observation comes from the likelihood function, which is defined by only one data (the total error of the full hysteretic curve). Instead, having more data some differences between posterior and prior may arise.

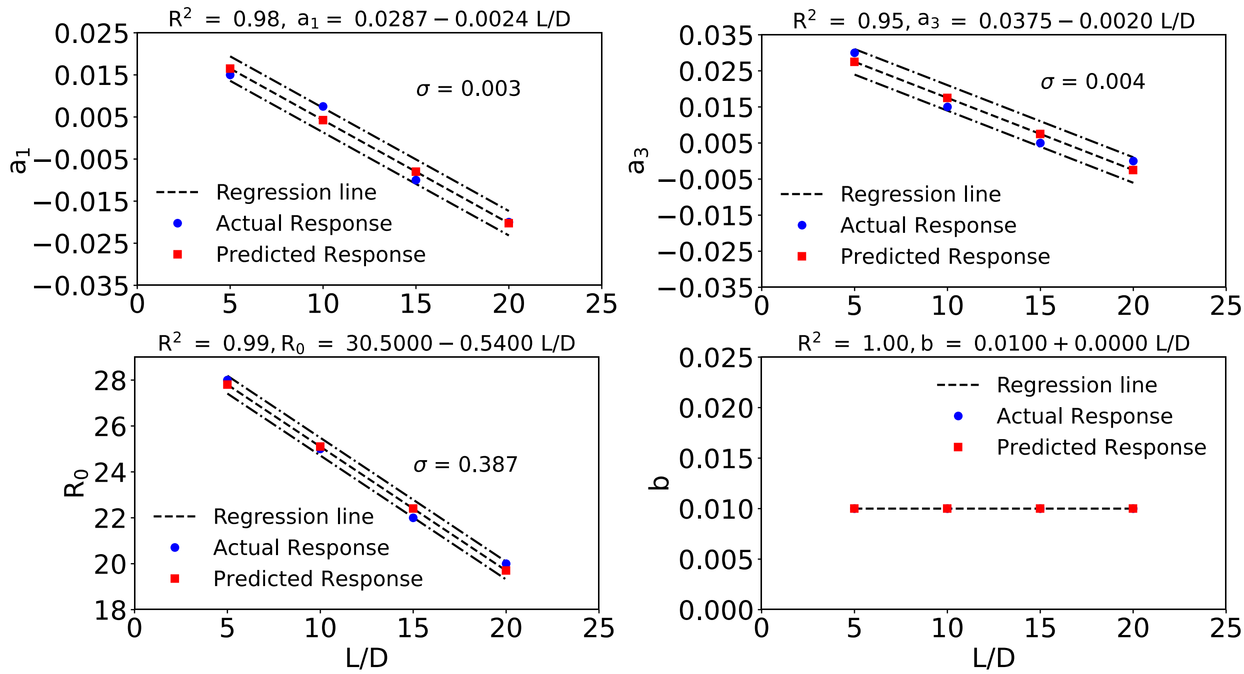

Figure 7.Regression Analysis of the optimized parameters

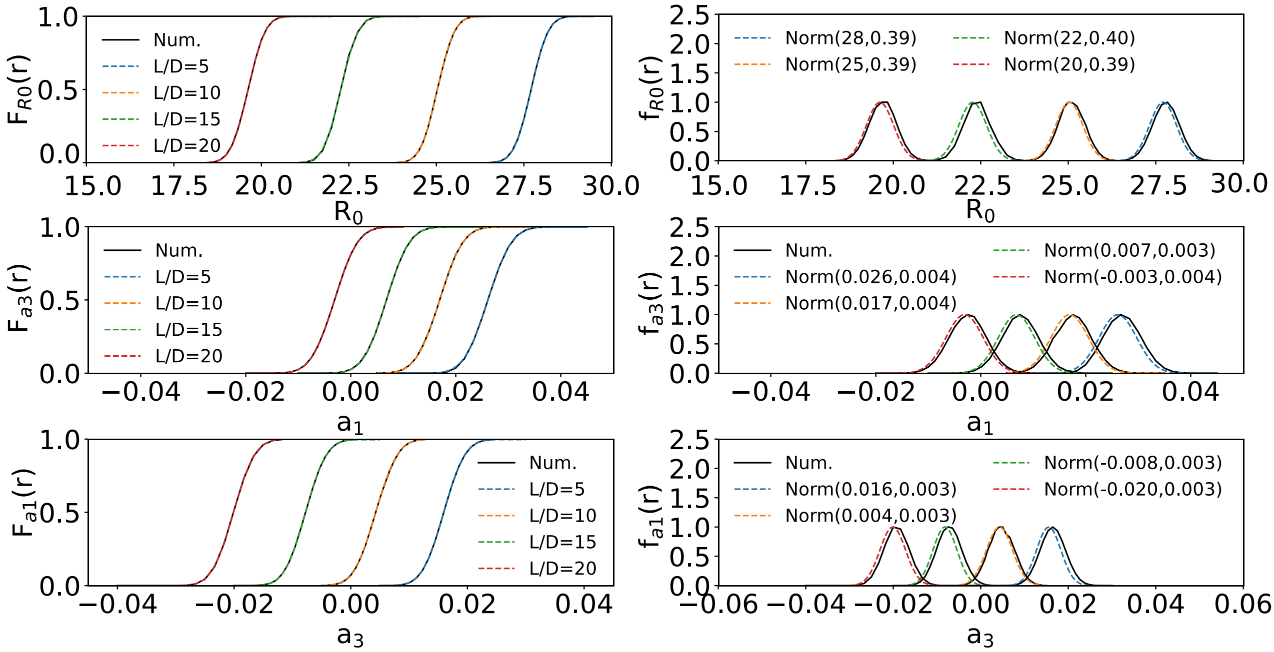

Figure 8. Bayesian model Updating of the model parameters for different L/D ratio

6.2 SteelMPF

Kolozvari et al. (2015) implemented a new material named SteelMPF, which accounts for several new characteristics compared to the existing Steel02. The SteelMPF constitutive model allows defining the yield stress and the hardening ratio in both tension and compression, as well as the pre-yielding cyclic curvature degradation (R) for loading-unloading. Furthermore, they have addressed and solved the overshooting issue of the original formulation in Steel02 when subjected to partial dynamic unloading. The partial unloading developed no longer feasible hardening behaviour of the

model itself and, as a result, it could have caused an inappropriate estimation of the seismic capacity of RC components. The authors remind the original work of Kolozvari et al. (2015) for further details. The current model (e.g., SteelMPF) was still calibrated for ribbed rebars and some default values are available for its use when defining the stress-strain in the fibre-section of RC components. Thus, a comparison between experimental and numerical results via the proposed FE model is needed to investigate if such default parameters are suitable for simulating the cyclic response of smooth rebars. There are two kinds of default parameters, which can be found in Filippou et al. (1983) and Kolozvari et al. (2015), to reproduce the response of ribbed steel reinforcement: a) the first one does not account for the isotropic hardening, both in compression and tension, so the default parameters are a1 = a3 = 0.00 and a2 = a4 = 1.0; b) the second one yields the isotropic hardening, both in tension and in compression, and the default parameters are a1=a3=0.01 and a2=a4=7.0.
To find out if such suggested default values are suitable, both cases, previously mentioned, are herein analyzed and compared to experimental results from the literature.
Figures 9 show that the FE model does not correctly reproduce the inelastic response of smooth rebars when employing default parameters. One of the main reasons for such inconsistency with the experimental outcomes lies in the difference between the constitutive model of the two types of steel reinforcement. For low values of slenderness (e.g. L/D = 5 and 10), the discrepancy with the experimental results derives mostly from the dissipated hysteretic energy, which is underestimated for large values of the strain loading. Conversely, the FE approach seems to overvalue the mean compression stress at large strains. When the slenderness ratio (L/D > 15) approach higher values, whereas the default parameters almost coincide with the optimized ones, the numerical model appears to match the experimental results. However, the mean tension stresses are still higher than the experimental results.

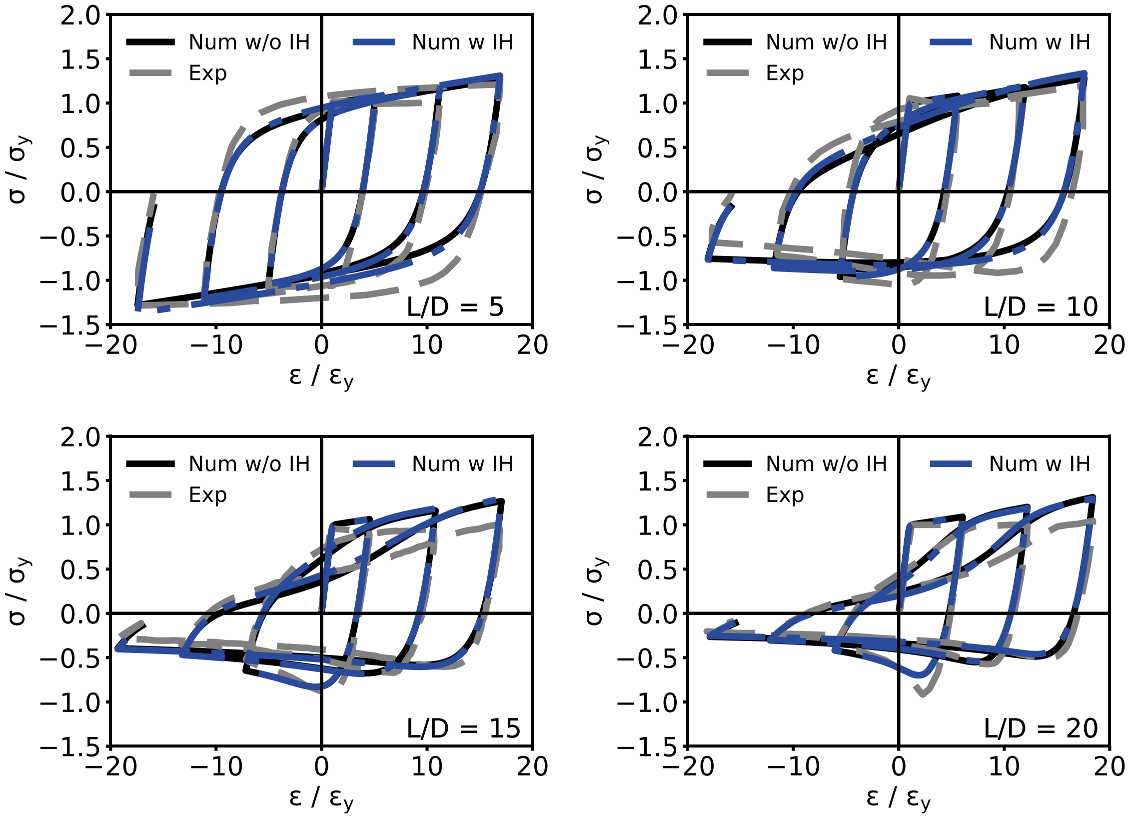

Figure 9. Numerical vs Experimental response of smooth bars with SteelMPF in Opensees. (Keynote: IH – Isotropic Hardening)

Nevertheless, SteelMPF allows the definition of yield stress (f_y) and hardening ratio (b) both for compression and tension, it was chosen to keep both values as one variable to simplify and not overwhelm the optimization procedure. The variables for the optimization process are the following: the isotropic hardening in compression and tension (a_1, a_3), hardening ratio (b) and the initial value of the curved transition (R_0). The results are given in Figure 10.

Figures 10 demonstrate that the joint approach effectively captures the inelastic behaviour of smooth rebars under cyclic loading and different L/D ratios. The compressive and tensile behaviours are accurately reproduced by the FE model of the smooth bar in OpenSEES (McKenna et al., 2010) by optimizing some of the previously mentioned parameters needed to define the constitutive model. The error in the hysteretic energy dissipation is very small for each half cycle, while significant improvement can be seen in the compressive response compared to the existing model Steel02 (Figures 6). It is worth noticing a slight overestimation of the tension stresses for high values of the strain loading. However, this minor variation does not negatively affect the global response of the numerical approach to the experimental results. Indeed, the proposed approach is successfully able to predict the pinching response of the material, which is a relevant aspect for RC components under strong excitations.

A value of slenderness L/D equal to 20 indicates that the numerical approach undervalues the compressive peak stress. However, this is not directly related to the FE model itself but rather to the randomness of the experimental campaign. The experimental compressive stress of L/D = 20 is greater than the corresponding value for L/D = 15. Apart from this observation, the optimization procedure and the proposed FE model can capture the full inelastic response even when longitudinal bars are subjected to strong buckling effects.

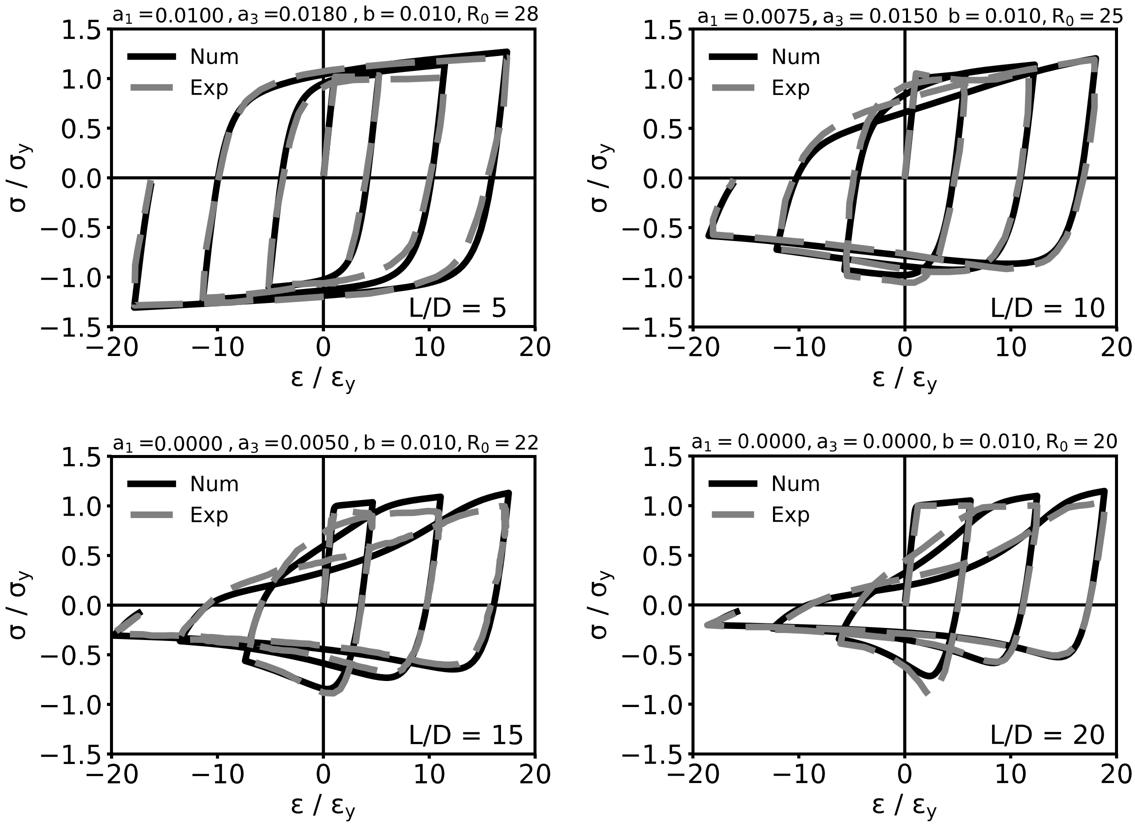

Figure 10. Numerical vs Experimental with optimized parameters

Figures 10 indicate that the increasing value of the slenderness ratio, corresponding more stirrups spacing, significantly affects the dissipation energy of smooth steel reinforcement. The onset of instability reduces the hysteretic energy dissipated at each half cycle, and the strength capacity and stiffness, especially while unloading from the compressive side. The model is also able to reliably predict the change of the curvature with the increase of the L/D ratio as can be seen for L/D =15 and L/D = 20.

A regression analysis (Figures 11) is then carried out to investigate any correlation between the optimized parameters and slenderness ratios. Such correlations aim to facilitate users to simulate the cyclic behaviour of smooth rebars when using the SteelMPF model to assess the inelastic capacity of RC structures.

The linear interpolation in Figures 11 appears to be a proper estimation of the actual response from the numerical simulations. The hardening ratio has not any variation but reduced by a factor of 1.8 compared to the original value as for Steel02. The initial curvature R_0 decreases with the increase of the slenderness ratio, and the values of the isotropic hardening start decreasing with the growing instability of longitudinal rebars. The linear relationships are quite close to the ones provided for Steel02, mainly due to the modification brought to the SteelMPF model and it has produced better results in the inelastic response of smooth bars.

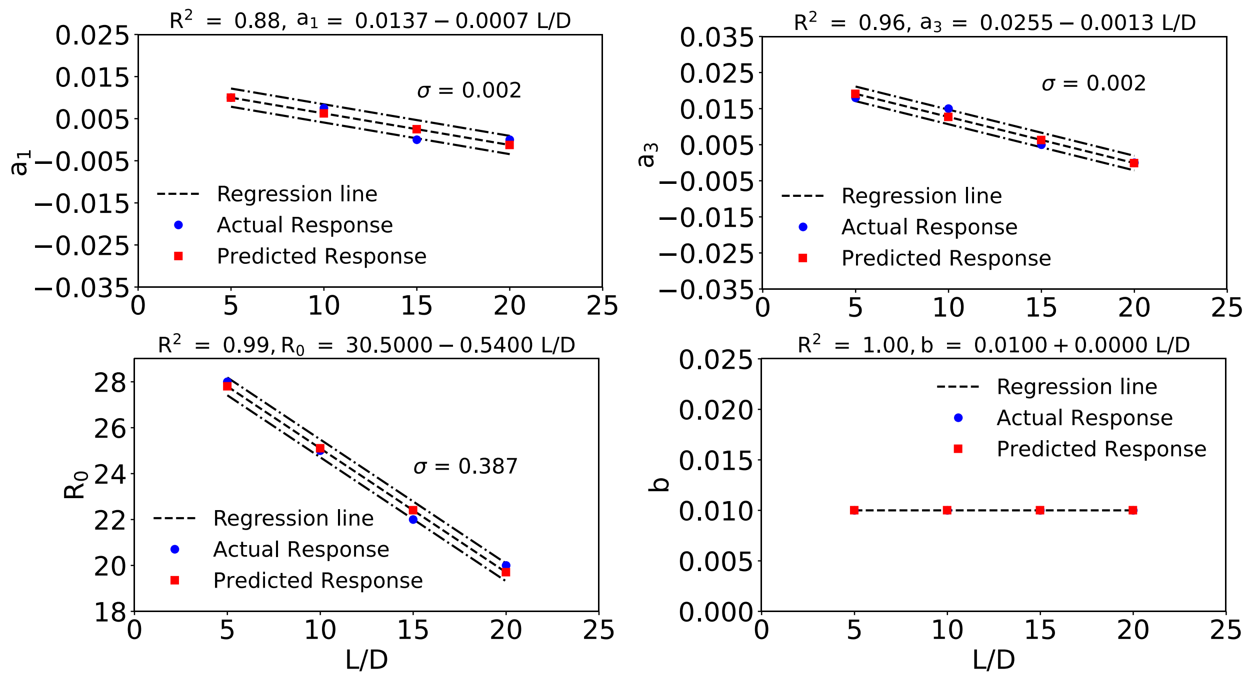

Figure 11. Regression Analysis of the optimized parameters

The numerical simulations from the model updating approach are shown in Figures 12. The marginal probability of each model parameter (R_0, a_1, a_3, in this case) can be adequately fitted through normal distribution with mean and standard deviation almost coincident with the outcomes from the GA.

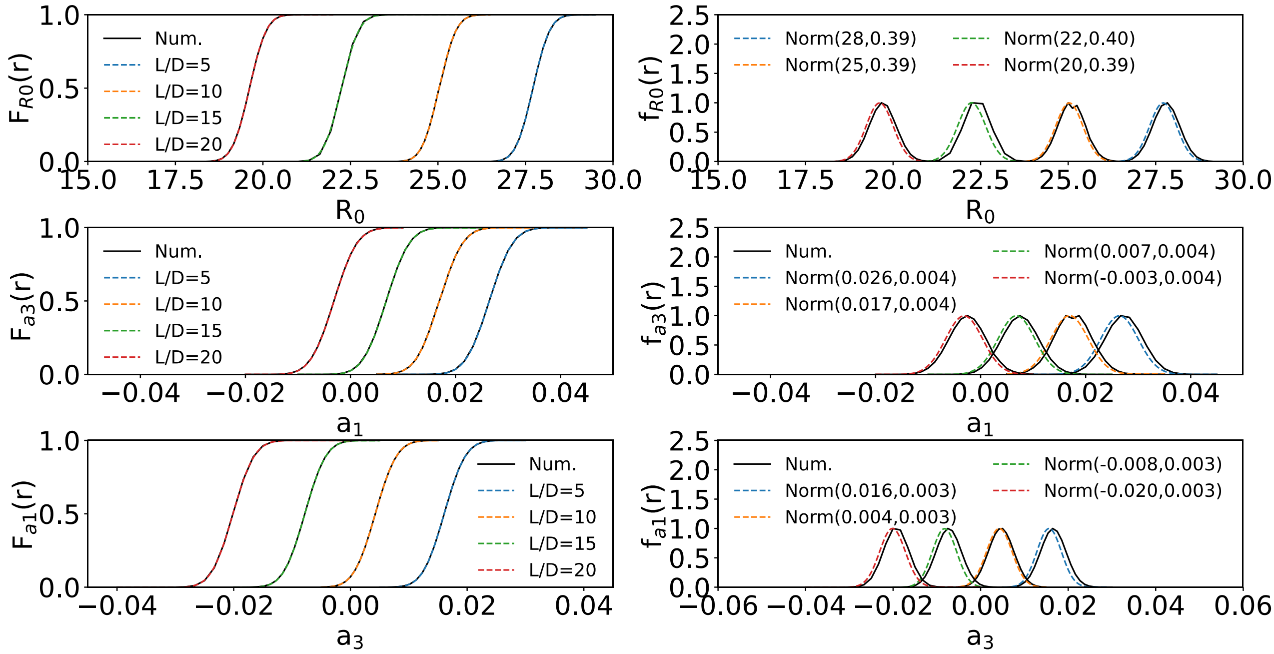

Figure 12. Bayesian Model Updating of the model parameters for different L/D ratio

6.3 Isotropic hardening parameters

Figures 7 and 11 show that the regression analysis for the isotropic parameters (a1,a3), based on the numerical optimization results for STEEL02 and STEELMPF, leads to negative values with the increase of the bar slenderness ratio. It is then recalled that the formulations characterizing the isotropic hardening in tension and compression are as follows:

$$f_{st} = f_y \frac{a_3}{{a_4}^{0.8}} \left[\frac{{\varepsilon_p}^{MAX} - {\varepsilon_p}^{MIN}}{2\varepsilon_y} \right]^{0.8} \tag{4a}$$

$$f_{st} = f_y \frac{a_1}{{a_2}^{0.8}} \left[\frac{{\varepsilon_p}^{MAX} - {\varepsilon_p}^{MIN}}{2\varepsilon_y} \right]^{0.8} \tag{4b}$$

where ${\varepsilon_p}^{MAX}$ and ${\varepsilon_p}^{MIN}$ are the minimum and the maximum strain recorded at the latest strain reversal.

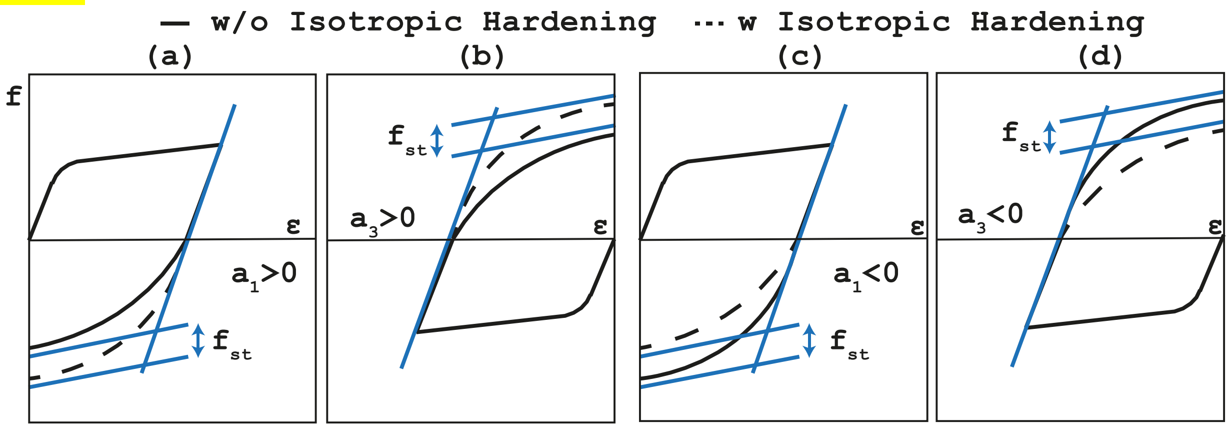

Figure 13. Stress shift of the Hardening Asymptote: (a) and (c) Compression ($a_1>0$ and $a_1<0$); (b) and (d) Tension ($a_3>0$ and $a_3<0$)

Such formulations illustrate that without strong buckling effects (e.g., L/D < 8), the post-yielding stress shift (f_{st}) moves in the positive direction, which implies that each branch curve is external to the previous one reaching out the last reversal strain at higher stresses (positive shift). On the other hand, when the steel rebars are subjected to significant inelastic buckling effects, the post-yielding asymptote tends to be negative and moving downward (Figure 13a and 13b); thus, the following cyclic curve reaches the latest strain point at lower values of the stress (negative shift, Figure 13c and 13d). These observations indicate the physical meaning behind the negative values of the isotropic parameters for high values of the slenderness ratio (L/D > 8).

6.4 Reinforcing Steel

One of the possible constitutive models available in OpenSEES (McKenna et al., 2010) for simulating the response of steel reinforcement in RC cross-sections is the "ReinforcingSteel" material. This model was introduced in Kunnath et al. (2009) and then implemented in the open-source platform Opensees. It includes many features that the previous-examined models (e.g., Steel02 and SteelMPF) do not incorporate, such as buckling and low-cycle fatigue.

The material refers to the original formulation of Chang and Mander (1994) as their monotonic compressive and tensile stress-strain curves are used as boundary limits, while two different formulations are employed to predict the effects of inelastic buckling (Gomes and Appleton, 1997; Dhakal and Maekawa, 2002). The low-cycle fatigue, instead, is considered through the formulation of Coffins-Manson (1971).

The monotonic and cyclic behaviour of the proposed uniaxial material was validated against a set of experimental campaigns reported in the literature. Results from numerical simulation showed that the model was able to capture with good accuracy the response of ribbed rebars.

On the other hand, the constitutive material demonstrated some discrepancies when using the buckling formulation. The results showed that the uniaxial material underpredicted the compressive behaviour of ribbed rebars, mainly due to the independence of the buckling model from the bar diameter, which eventually affects the global compressive response. For many other details, the authors remind the original study of Kunnath et al. (2009).

Since this study investigates the inelastic buckling of smooth rebars, the definition of the ReinforcingSteel material here only refers to the mechanical properties of steel bars without including the buckling and low-cyclic fatigue models.

The uniaxial material in OpenSEES (McKenna et al., 2010) is completely defined by the following mechanical properties: yielding and ultimate stress, strain corresponding to the initial hardening, tangent at the initial hardening (which includes the hardening parameter b), the strain at the peak stress, and Menegotto-Pinto parameters (which refers to R_0).

Figure 14 shows that the FE approach is not sufficiently adequate for predicting the cyclic behaviour of smooth rebars for different slenderness ratio. The numerical simulations demonstrate that the original formulation is not too far beyond the experimental results except for L/D greater than 10 where a slight underestimation for the compressive behaviour can be observed. Hence, based on those comparisons, the optimization parameters are the hardening ratio (b) and the value R_0 of the initial curvature.

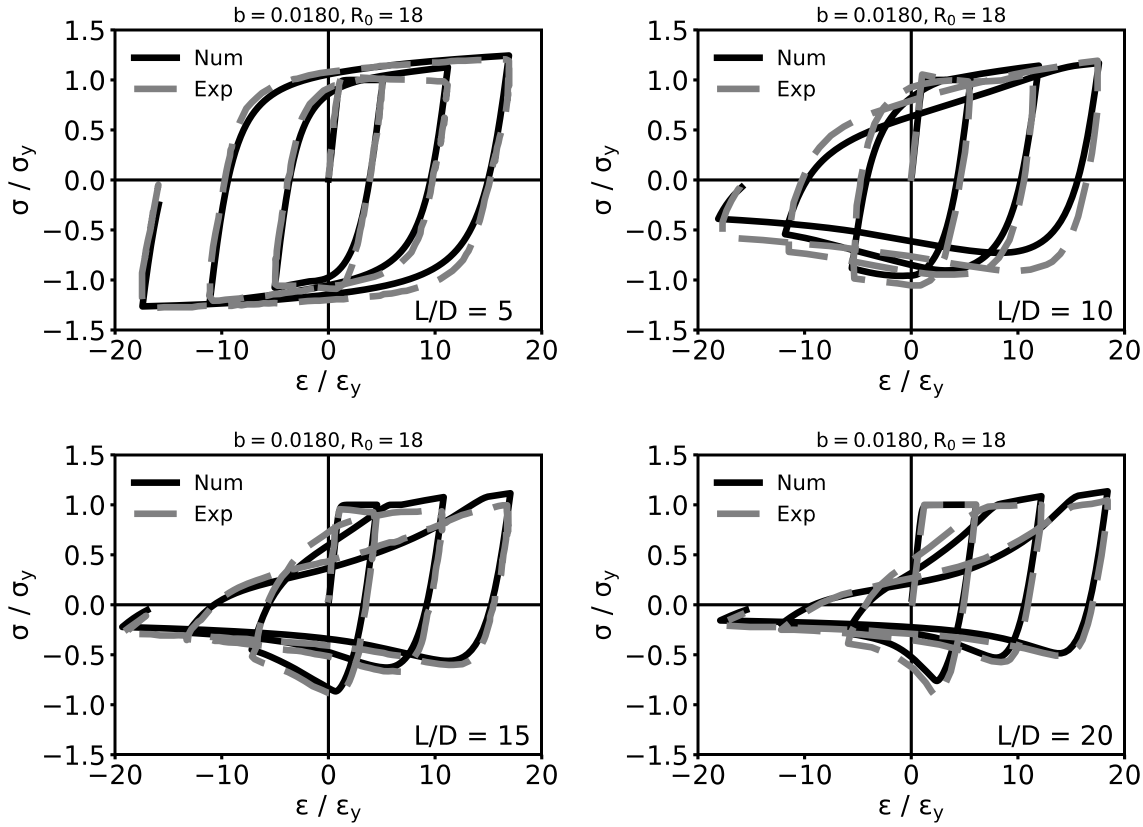

Figure 14. Numerical vs Experimental response of smooth bars with SteelMPF in Opensees

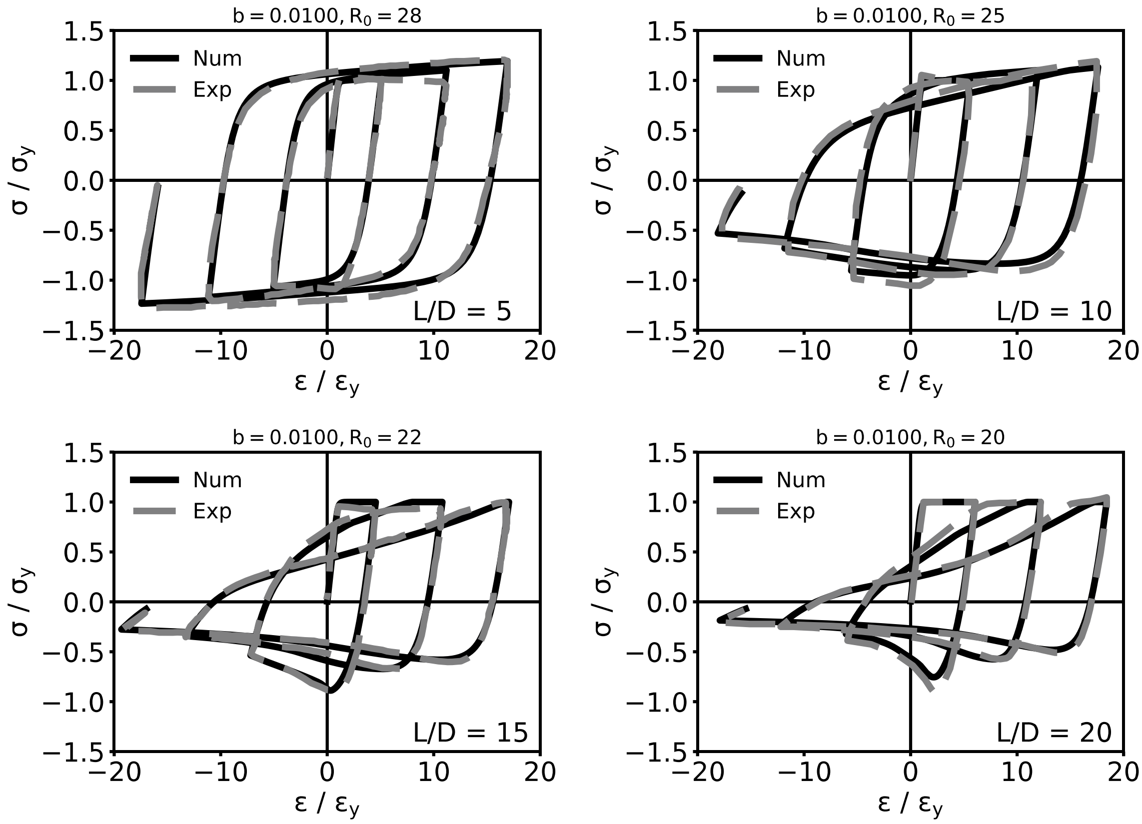

Figure 15. Numerical vs Experimental with optimized parameters

Figures 15 illustrates that the proposed modelling approach can accurately simulate the inelastic response of smooth rebars. Compared to the other two examined uniaxial materials (e.g., Steel02 and SteelMPF), there is an additional improvement at each half cycle of the numerical response. Moreover, the FE model can predict the peak both in tension and compression with excellent accuracy.

The curvature inflection, especially for high values of L/D, and the pinching effects, are effectively reproduced by the numerical simulations. The minor overestimation of the tensile stresses seen for the previously examined uniaxial materials completely disappears as the model can easily follow up the tensile behaviour recorded in the experimental tests. The optimized parameter R_0 decreases with the increase of the slenderness ratio, while the hardening (b) remains constant but reduced by a factor equal to 1.8 compared to the original value.

Finally, there is no need for a regression analysis as the results for the initial curvature and hardening ratio have the same trend seen in Figures 7c-7d and Figures 10c-10d.

Using the linear regression to define the prior as defined in section 4.3, the model updating is applied to account for the uncertainties concerning the numerical modelling and the experimental results. Since the hardening ratio values are constant, the only parameter to be used in the Bayesian approach is the initial curvature *R0*. Figure 16 shows the results of the numerical simulations. The marginal distributions of the model parameter for different values of *L/D* are well fitted by a normal distribution with a mean whose values also verify the accuracy of the genetic algorithm. In this case, as in the likelihood function, only one data is used; the posterior and the prior are identical. Having more data differences may arise. The distributions obtained herein can be used as prior if newly available data will be available.

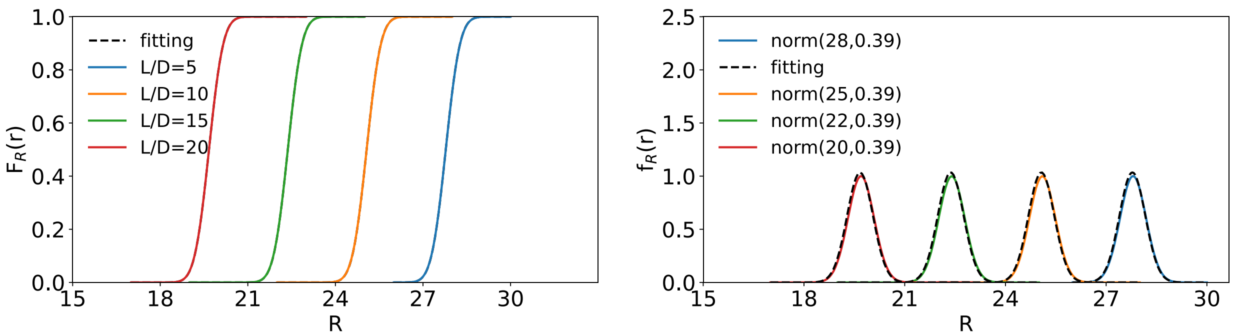

Figure 16. Probabilistic distribution of the model parameters based on the model updating

6.5 Monotonic inelastic Buckling

Using the FE modelling approach of the steel bar, the optimized parameters from the regression analysis and the strain history from experimental results, the monotonic axial compression response of smooth steel reinforcement is here investigated. The non-linear static analysis (in displacement control) is conducted without involving strain reversal rules as suggested in Mau et al. (1989), but rather an increasing compressive loading is applied until failure. The outcomes from the numerical simulation are shown in Figure 17a, 17b and 17c.

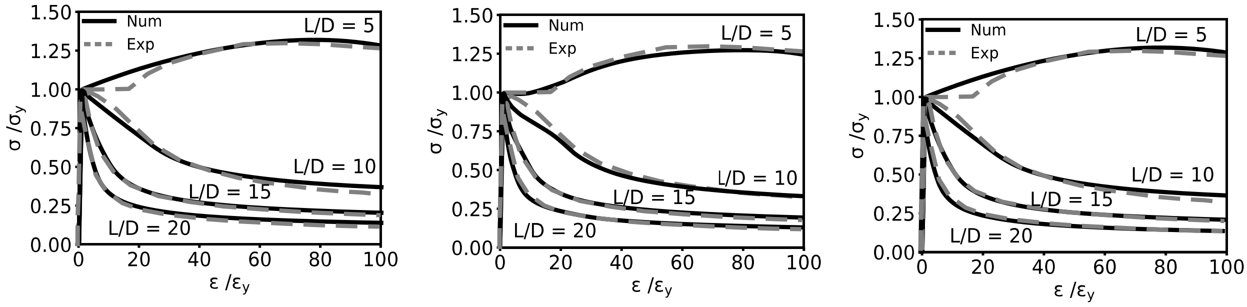

Figure 17. Monotonic numerical vs experimental results. (a) Steel02, (b) ReinforcingSteel and (c) SteelMPF

The latter figure demonstrates that the proposed approach and model parameters can efficiently predict the average compressive stress-strain response of longitudinal smooth bars under severe inelastic instability. Steel02 (Figure 17a) and SteelMPF (Figure 17b) show similar results since they follow the same original formulation (Menegotto-Pinto, 1973 and extended by Filippou et al., 1983). However, the last constitutive models are not able to predict the yielding plateau, but this observation is more related to the concept of material formulation and implementation. Despite this drawback, the model can accurately simulate the overall monotonic compressive behaviour with a slight not conservative response for *L/D* equal to 10. Similarly, the ReinforcingSteel (Figure 17c) material can capture the empirical results and, indeed, adequately simulate the yielding plateau for low slenderness ratio values (e.g., *L/D* = 5). The model seems only to slightly underpredict the monotonic response for an *L/D* ratio equal to 10, but this could lie into the uncertainty of the experimental campaign and observed data.

7. Analytical Model of Inelastic Buckling

Based on an accurate assessment of the stress-strain curves from numerical simulation, all the constitutive models exhibit a compressive behaviour that can be ideally divided into three different stages:

(a) Linear elastic, the stress increases until the yielding of the steel reinforcement without being affected by instability (typically for L/D lower than 20, after that the onset of buckling starts for values lower than the yielding stress);

(b) Stress follows the path of the tensile behaviour of smooth reinforcements until reaching the onset of buckling;

(c) After the onset of buckling, there is an increasing non-linear softening (exponential decrease) that depends on the slenderness ratio L/D.

Numerical methods, based on advanced FE approaches, are not always straightforward to use, so analytical models are simple and effective ways to support civil engineers for performing non-linear fibre-based analyses of RC structures including the effects of inelastic buckling. When the optimization mechanisms are understood and the numerical behaviour aligns with the physical expectations, analytical models can be derived using the numerical tool extensively at a minimal cost. As a result, a numerical parametric study is conducted to investigate the effect of inelastic instability on the yielding plateau and post-buckling softening for different slenderness ratios.

Figure 18 show the results of the numerical FE simulation for low values of slenderness ratio (e.g., L/D = 5, 6 and 7)

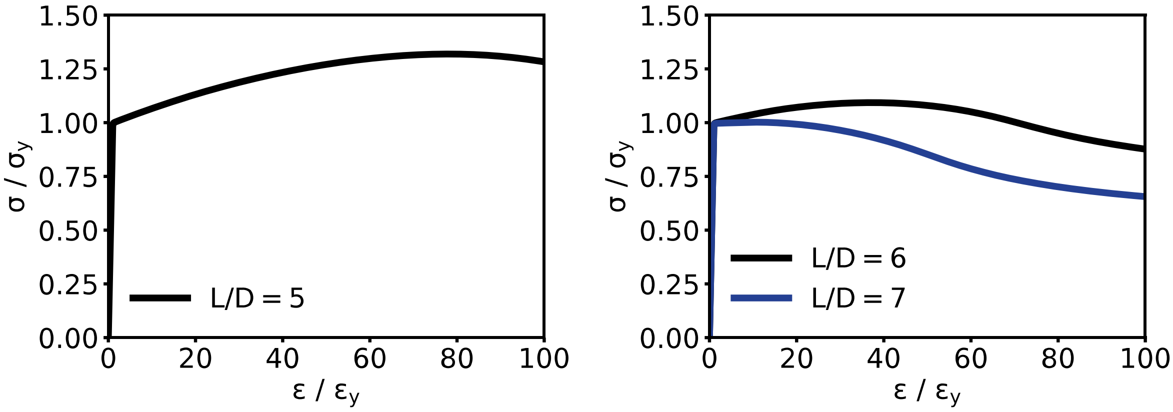

Figure 18. Numerical FE simulation for low values of slenderness ratio (e.g. LD = 5, 6 and 7)

The instability for *L/D* equal to 5 occurs for large values of the strain, whereas the ductility of the longitudinal rebar is very high. Therefore, it can be assumed that the compressive response of the smooth steel bar is coincident and symmetric to the tensile behaviour. Conversely, for slenderness ratios ranging between 6 and 7, the buckling affects the global compressive response; however, the onset of buckling appears at values of $\varepsilon/\varepsilon_y$ equal to 50 and 30, respectively (which corresponds to strains of 4.8% and 8%, respectively). As a result, the steel bar can still exhibit high ductility; thus, it can be safely assumed an elastic-perfect plastic model with a conventional ultimate strain corresponding to the onset of buckling. The softening branch could be neglected for such slenderness ratios as the steel reinforcements in RC columns are not likely to experience such high compressive strains under cyclic loading. The analytical model for the compressive response of the L/D ratios equal to 6 and 7 is given in Figures 19.

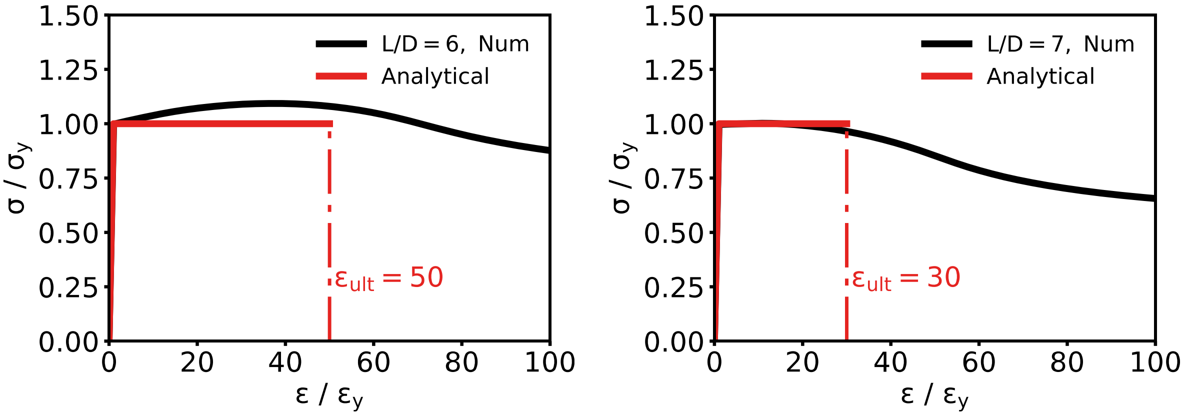

Figure 19. Analytical Model for different L/D (e.g. LD = 6, 7)

Based on the numerical simulation of the FE model with optimized parameters, a parametric study is also conducted to analyse the compressive stress-strain response of the steel reinforcement for slenderness ratios ranging between 8 and 20. The results of the numerical simulation are given in Figures 20.

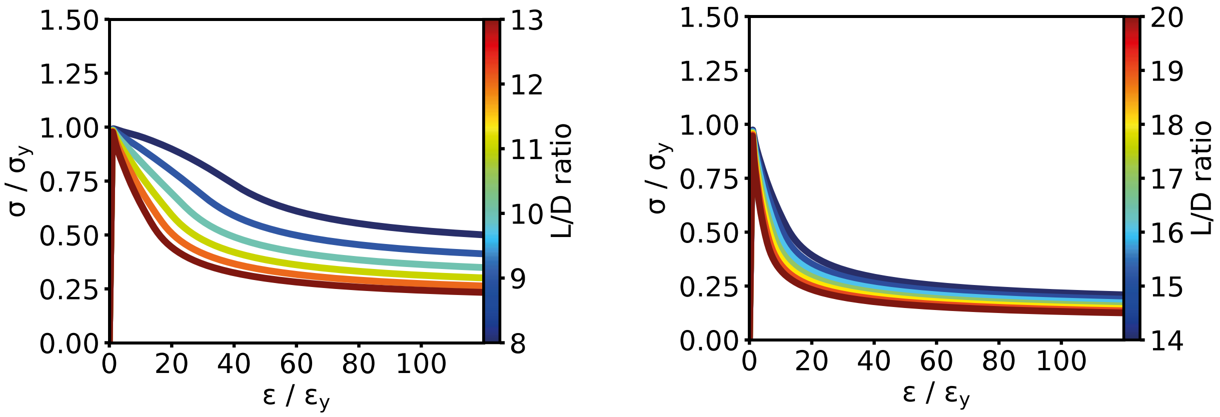

Figure 20. Numerical FE simulation for L/D = [8,20]

The numerical stress-strain responses of smooth rebars unveil that regression analysis is needed to find the best fitting curves for the beginning of buckling and the post-buckling curve.
To generalize the formulation for the best fitting curve, the equation for the onset of buckling should be written as a function of the hardening and the yielding strain of the bar itself. The following formulation (3) can be used:

Hardening strain

$$\frac{\varepsilon_{bh}}{\varepsilon_y} = 1 + \left[\left(\frac{\varepsilon_h}{\varepsilon_y} - 1\right) f\left(\frac{L}{D}\right)\right] \tag{4}$$

where ε_{bh} indicates the onset of buckling, ε_h the hardening strain from the tensile response and ε_y the yielding strain. The above equation (4) has two asymptotes corresponding to 1 and ε_h, for L/D equal to 20 and 8, respectively. Figure 21a depicts the regression analysis for the strain of the buckling commencement. The function of L/D follows a power law with a negative exponent.

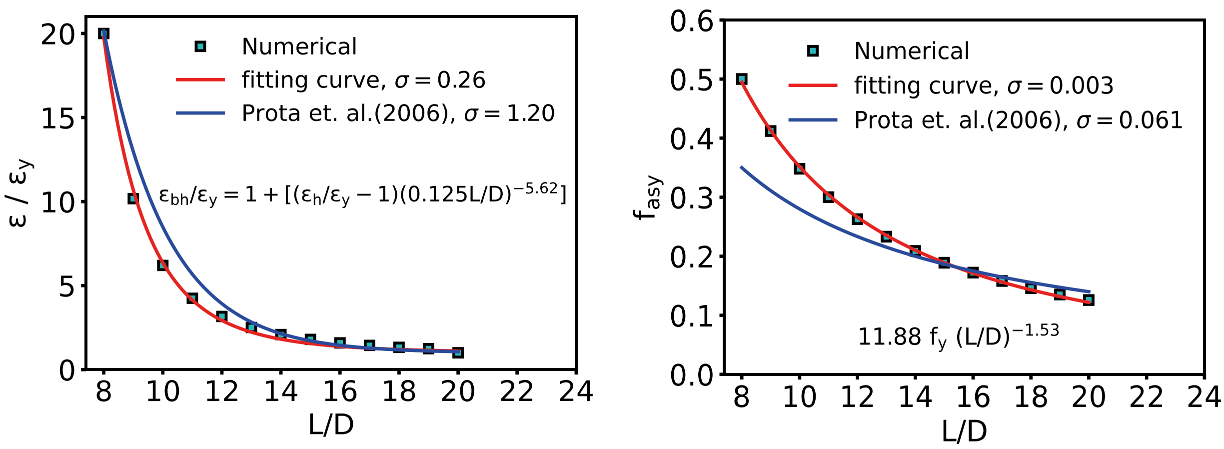

Figure 18. Regression Analysis: (a) the strain at the onset of buckling, (b) horizontal asymptote

Similarly, the investigation of the post-buckling compressive response of smooth bars from the numerical parametric study indicates that all curves tend to a horizontal asymptote for infinite values of strain, while the softening curves follow a decreasing exponential function. Both trends depend on the slenderness ratio. The shape of the horizontal asymptote equation (4) can be assumed as follow:

Asymptote $$f_{asy} = \alpha f_y f\left(\frac{L}{D}\right) \tag{5}$$

Figure 21b shows the result of the regression analysis of the horizontal asymptote for different values of the slenderness ratio (e.g., L/D = [8,20]).

The decreasing exponential function for the post-buckling softening depends on the ratio L/D; therefore, the fitting curves should follow the same exponential formulation, but different values are expected for the regression parameters. As a result of the last observations, it is then straightforward to formulate the post-buckling branch, as follows:

Softening Branch $$f = f_{asy} + \left[\left(f_y - f_{asy}\right) e^{f\left(\frac{L}{D}\right)\left(\frac{\varepsilon}{\varepsilon_{bh}} - 1\right)}\right] \tag{6}$$

Figures 22 show the regression curve for the softening branch. Since the function f(L/D) seems to reach a constant value for large slenderness ratios, the proposed formulation for the post-buckling softening may be used for values of L/D ratio greater than 20.

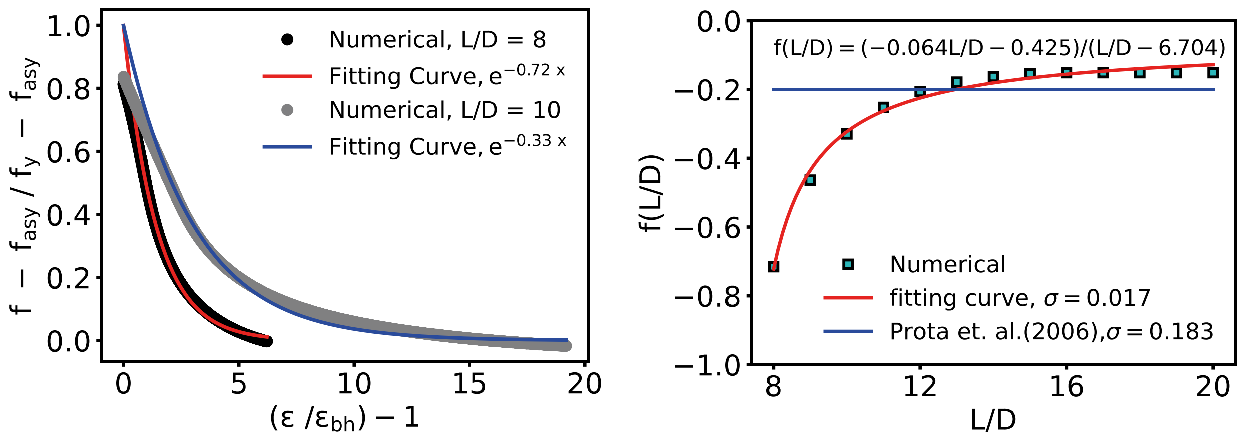

Figure 19. Regression Analysis for the post-buckling softening branch

7.1 Validation of the analytical model

In this section, the analytical model of the inelastic buckling is used to validate its accuracy against the experimental stress-strain response of smooth rebars for different values of L/D ratios (Figures 23).

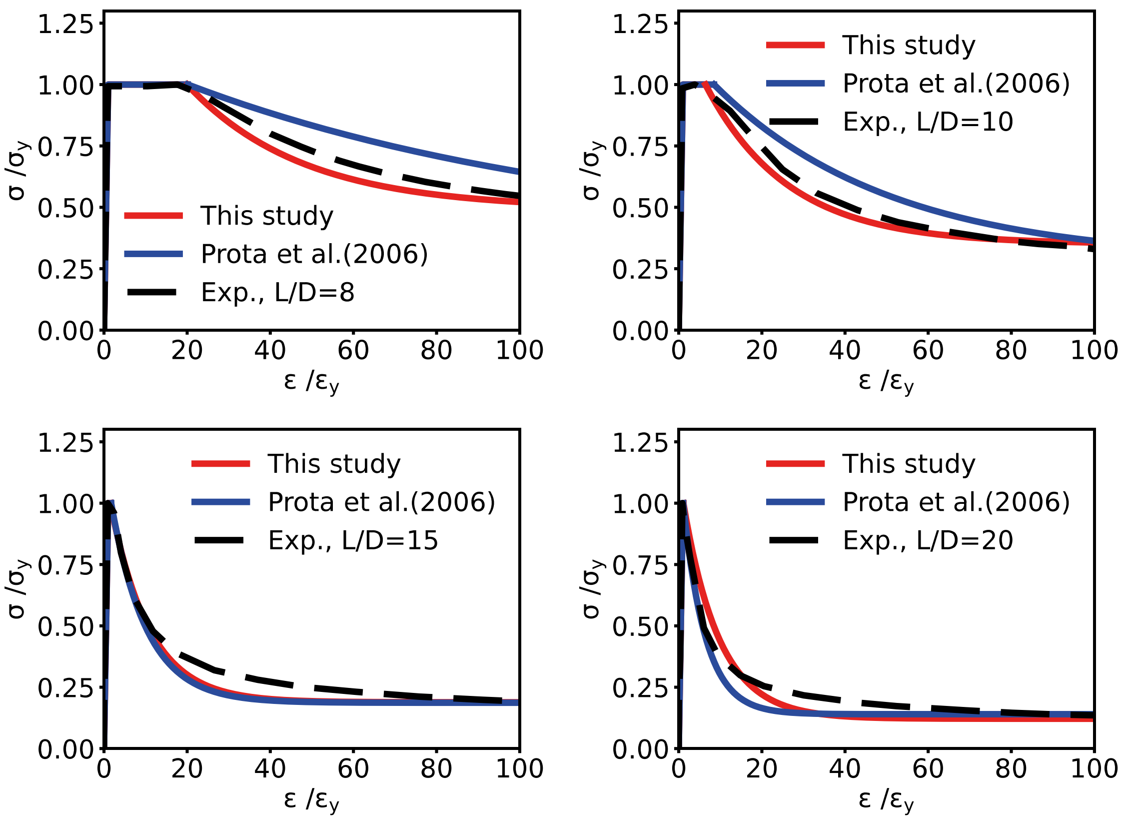

Figure 20. Analytical model vs Experimental results

Comparisons between the numerical simulations and the experimental results (Figures 23) demonstrate that the analytical model can capture the behaviour of smooth steel reinforcement under increasing monotonic compressive loading. The analytical approach seems to slightly underpredict the post-peak softening; this is mainly due to a more conservative approach when fitting the curves for larger bar slenderness values. However, the last observations do not significantly affect the global compressive response of the proposed analytical approach, which can still demonstrate an excellent agreement with the empirical tests.

Finally, the efficient proposed analytical model is shown in Figure 24 to represent all the theoretical monotonic curves with a slenderness ratio between 8 and 20.

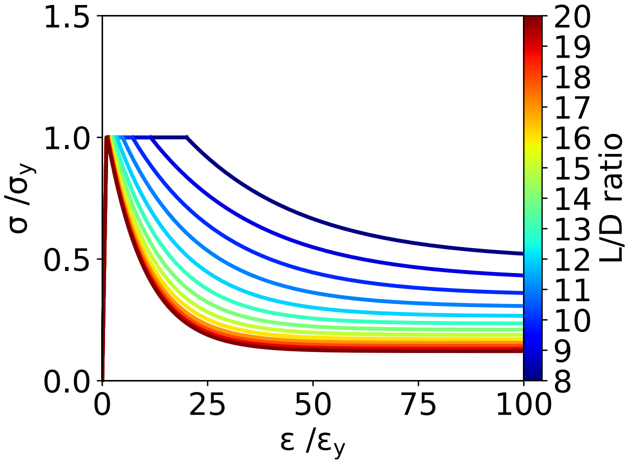

Figure 214. Analytical model for L/D ratio between 8 and 20

Furthermore, numerical and experimental comparisons between the existing analytical model proposed by Cosenza and Prota (2006) and this study are presented. The results demonstrate that for values of the slenderness ratio (*L/D*) greater than 15, typically the case for existing RC buildings where the spacing of the stirrups is very high, both models could predict the experimental tests with slightly conservative solutions. For values of slenderness ratio lesser than 15, that is the typical case of existing RC infrastructures (i.e., existing RC bridge), the actual study showed better prediction accuracy.

The analytical model is given for its implementation in structural engineering software. Nowadays, computer platforms for earthquake engineering applications provide several hysteretic materials (user-defined) for simulating the cyclic behaviour of steel reinforcement bars. Therefore, a backbone envelope curve of the analytical model provided for different slenderness ratio values is beneficial when using such hysteretic models; this allows a better definition of the moment-curvature to evaluate the inelastic behaviour in the critical zone of RC columns with smooth rebars, using either concentrated (plastic hinge) or spread plasticity. The last observation is relevant for the evaluation of the seismic capacity of existing RC structures reinforced with reinforcing plain bars.

It is also worth mentioning that some existing RC columns may exhibit potential rocking behaviour which could govern the deformation capacity, especially in the fixed-end rotation. However, both rocking and inelastic buckling are induced from loss of the bond strength between concrete and steel reinforcement; as a result, they depend upon both the construction details and the mechanisms of transferring the stresses in the RC components. In this case, the proposed analytical model is still beneficial as it could be coupled with approaches that account for the rocking behaviour, as a sort of structural response superposition, to establish which phenomenon is governing the deformation capacity.

8. CONCLUSIONS

In this study, inelastic buckling of smooth longitudinal rebars is addressed using genetic algorithms, Bayesian model updating, and refined finite element (FE) models. The analytic study results contribute to the use of a combination of parametric optimization and FE methods to simulate the monotonic and cyclic behaviour of smooth rebars. First insight on the implementation of three current and most adopted constitutive models of steel reinforcement, available in Opensees (i.e., Steel02, SteelMPF, and ReinforcingSteel), namely calibrated for ribbed rebars, is also presented for smooth rebars. Relationships and probabilistic distributions for the model parameters are provided for each analysed constitutive model to facilitate the stress-strain definition for users. Moreover, an analytical model has also been provided to accurately simulate the monotonic compressive response. The outcomes of this study demonstrate that:

1. The proposed optimization procedure is an efficient and robust technique that accounts for the selected steel constitutive model parameters. It accurately leads to the optimal solution with a reasonable computational cost. Notably, the pre-optimization based on few running of the FE model allows a proper estimation of pre-defined intervals of all parameters to be optimized;
2. The adopted advanced FE model for steel rebars, based on element and cross-section discretization from sensitivity analyses, provided accurate and reliable results, compared to the experimental tests, when simulating the monotonic and cyclic response of ribbed rebar

using default parameters. Numerical simulations for all examined steel constitutive models show that the FE model could adequately predict the onset of buckling and the post-buckling softening of the empirical results. Steel02 and SteelMPF demonstrated a slight underestimation of the hardening for law values of L/D. Conversely, the ReinforcingSteel material could also predict the yield plateau, while a small overprediction in the post-buckling softening branch for L/D equal to 10 was noted. All materials exhibited an excellent match with experimental results for large values of L/D ratio;

3. The joint optimization procedure could produce accurate results as the monotonic and cyclic behaviour are entirely obtained for the full path (compression – tension). The error, referred to the area of each half-cycle, is compatible from an engineering standpoint (or compatible with engineering applications);
4. The hardening ratio seems not to vary for different bar slenderness values but is reduced by a constant factor equal to 1.8, while a linear interpolation was appropriate for the isotropic hardening and the initial curvature of the constitutive models. The homoscedastic statistic model to apply the model updating, based on the regression analyses, further validated the accuracy of the modelling approach accounting even for the uncertainties of the model parameters, numerical modelling, and experimental measurement;
5. The proposed improvement of the analytical model was able to adequately predict the compressive monotonic response of steel smooth rebars when compared to empirical results. A slight conservative solution was shown for the post-buckling softening branch of L/D equal to 15 and 20 without affecting the global compressive response compared to the experimental results. For L/D equal to 8 and 10, the response was in complete agreement with empirical tests. Compared to the existing analytical model, the actual improvement is more accurate for a slenderness ratio lesser than 15, which typically refers to existing RC infrastructure.

Based on the discussion provided above, it is expected that future works should focus on the analytical formulation of the cyclic response of smooth rebars based on different loading strain histories. Thus, it will be possible to account reliably for the path-dependency. If other experimental results are made available, such an approach can be extended to investigate such an issue.

Acknowledgements

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