

Ultra-high amplitude isolated attosecond pulses generated in transmission from ultrathin foil regime

Ruyu Pang,¹ Yunliang Wang,^{1,*} Xueqing Yan,^{2,3,4,†} and Bengt Eliasson^{5,‡}

¹*Department of Physics, School of Mathematics and Physics,
University of Science and Technology Beijing, Beijing 100083, China*

²*State Key Laboratory of Nuclear Physics and Technology,
and Key Laboratory of HEDP of the Ministry of Education,
CAPT, Peking University, Beijing 100871, China*

³*Beijing Laser Acceleration Innovation Center, Beijing 100871, China*

⁴*Collaborative Innovation Center of Extreme Optics, Shanxi University, Shanxi 030006, China*

⁵*SUPA, Physics Department, John Anderson Building,
University of Strathclyde, Glasgow G4 0NG, Scotland, UK*

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A coherent synchrotron emission regime of forward high order harmonics generation (HHG) is proposed for the emission of an isolated unipolar half-cycle attosecond pulse by a three-color laser pulse impinging on ultra-thin foil. A theoretical model is proposed for the electron nanobunching mechanism and the forward radiation, which is consistent with the numerical simulation results. As the forward HHG does not need to penetrate from the front to the rear side of the target, the spectrum of the forward HHG has no low-frequency cutoff. The robustness of this regime is verified with different laser and foil plasma conditions as well as the two-dimensional effects. The robustness is also checked with similarity theory, which confirms that isolated attosecond pulses can be efficiently generated when the plasma density and the laser amplitude change simultaneously such that their ratio remains unchanged.

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I. INTRODUCTION

The interaction of high-intensity laser pulses with very thin foil targets has attracted much attention for ion acceleration [1], high order harmonics generation (HHG) [2], and attosecond pulse generation [3]. The latter can provide unprecedented temporal and spatial resolution for the detection of ultrafast atomic and electronic phenomena [4, 5]. Intense attosecond pulses can be generated by the locked frequency HHG from laser interaction with solid targets [6]. The thin foil model has been well developed for HHG from laser-driven thin foils [7, 8], which is a solvable problem in the electrodynamics of HHG from laser-driven thin foil in the relativistic oscillating mirror (ROM) regime [9, 10] and relativistic flying mirror (RFM) regime [11].

The coupling mechanisms in the interaction of intense laser pulses with solid targets is related to the plasma interface steepness and transit from the Brunel mechanism to chaotic dynamic [12]. Compared with laser pulse driven thick targets, the laser pulse impinging thin foil produces forward HHG emission that can be observed at the rear side of the thin foil. Forward HHG generated at the front side of the foil has a low-frequency cut-off in the spectrum because only harmonics above the max-

imum plasma frequency can propagate through the thin foil, which can be used to diagnose the generating plasma [13–16]. The forward harmonics can also be generated at the rear side by the coherent wake emission (CWE) [17, 18].

Forward attosecond pulses are also generated by the coherent synchrotron emission (CSE) [19]. When the laser pulses interact with the thin foil, extremely dense electron nanobunches with a delta-like peak density distribution can be periodically formed at the front surface of the target and accelerated into the transmitted direction by the ponderomotive force of the laser [20], where the spectrum has a low-frequency cutoff as the generated harmonics is filtered by the high-density plasma foil and an attosecond pulse train is emitted in the rear side of the foil. Generally the forward attosecond pulse has the structure of two distinct subpulses, which are emitted by both the primary electron sheet and the secondary electron sheet [3]. However, an isolated attosecond pulse is preferred for many applications of pump-probe techniques. More recently, the direct generation of isolated attosecond pulses in the transmission direction have been suggested, by using a few-cycle laser pulse interacting with a thin foil target, where a portion of backward electron sheet return back to the transmission to form the secondary electron sheet propagating in the forward direction [21, 22]. However, the intensity of the forward attosecond pulse is usually much smaller than that of the incident pulse [3, 21, 22].

An advantage of the CSE mechanism is that the CSE spectrum typically has a slower decay scaling $I(\omega) \propto$

*Electronic address: ylwang@ustb.edu.cn

†Electronic address: x.yan@pku.edu.cn

‡Electronic address: bengt.eliasson@strath.ac.uk

$\omega^{-4/3}$ or $I(\omega) \propto \omega^{-6/5}$ [23] supporting narrower pulses in time compared to the ROM and CWE regimes. Moreover, there is no limitation on the amplitude of the incident pulse in CSE regime, which can withstand arbitrarily high fields. For the HHG in the relativistic regime, the higher laser intensities induce stronger nonlinearities giving rise to higher harmonic efficiency. In the CSE regime, the formation and dynamics of an extremely dense electron sheet is crucial for the emission of attosecond pulses [19].

The aim of this paper is to propose a unique CSE regime in the transmission direction, where the intensity of the generated attosecond pulse can be comparable to that of the incident pulse. The whole thin foil target contributes to the formation of electron nanobunch, of which the extremely dense electron sheet mainly come from the rear surface of the foil. The spectrum of the generated forward harmonic has no low-frequency cutoff as the generated forward harmonic does not need to penetrate from the front to the back of the target. The generated isolated attosecond pulse is a half-cycle unipolar pulse, which may be useful for pump-probe of electron dynamics in solids and atoms by asymmetrical manipulation [21, 22, 24–26].

II. EMISSION MECHANISM OF THE FORWARD ATTOSECOND PULSE

A. Attosecond pulse generation from electron sheet

We consider a new electron nanobunching regime, in which the thin foil target is compressed by a three-color laser pulse to form an electron nanobunch. The formation and dynamics of the electron sheet is controlled by adjusting the relative phases between the three-color laser pulses. This nanobunching regime ensures that only one electron sheet contributes to the transmitted radiation so that an ultra-intense isolated attosecond pulse is generated in the transmission direction without the need for extra filters and gating techniques. As seen in Fig. 1(c), an ultra-high amplitude isolated attosecond pulse is emitted and propagates in the transmission direction, with a squared amplitude of $a_z^2 \approx 24000$ and a corresponding intensity of $4.8 \times 10^{22} \text{ W/cm}^2$. The attosecond pulse is a half-cycle pulse and has a full width at half maximum (FWHM) in time of about 7 attoseconds as shown by the close-up of the inset in Fig. 1(c).

The fundamental mechanism for the isolated attosecond pulse generation is studied using the particle-in-cell (PIC) code EPOCH [27]. A linearly polarized (along z) three-color laser pulse is launched in the x direction, normally incident on the ultrathin foil. The wavelength of the fundamental frequency laser pulse is $\lambda_L = 800 \text{ nm}$ with the angular frequency being $\omega_1 = \omega_L = 2\pi c/\lambda_L$ and the period being $T_L = \lambda_L/c = 2.67 \text{ femtoseconds}$. The second and the third harmonic components have

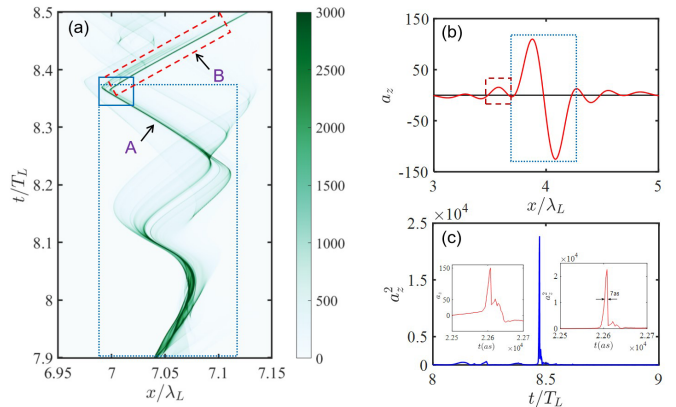


FIG. 1: (a) The spatiotemporal evolution of the normalized electron number density n_e/n_c from time $t = 7.9 T_L$ to $t = 8.5 T_L$. Extremely dense electron sheets are produced in the laser-plasma interaction, which result in forward attosecond pulses generation. (b) The waveform of the normalized electric field $a_z = eE_z/(\omega_L m_e c)$ from the three-color laser pulses before it interacts with the foil. (c) The generated attosecond pulse in the transmission direction. The left inset shows a unipolar profile and the right inset shows a close-up of the attosecond pulse having a FWHM of about 7 attoseconds.

angular frequencies $\omega_2 = 2\omega_L$ and $\omega_3 = 3\omega_L$, respectively. The combined laser pulse has a normalized electric field $a_z = eE_L/(\omega_L m_e c)$ of the form $a_z = a_1 + a_2 + a_3$, where E_L is the electric field amplitude of the laser pulse, m_e is the electron rest mass, e is the unit charge, and c is the speed of light in vacuum. The three laser pulses have a Gaussian temporal envelope given by $a_{1,2,3} = a_{01,02,03} e^{-(t-T_L)^2/\tau^2} \sin[\omega_{1,2,3}(t-T_L) + \phi_{1,2,3}]$, where $\phi_1 = 6.055 \text{ rad}$, $\phi_2 = 5.960 \text{ rad}$ and $\phi_3 = 5.965 \text{ rad}$ are the phases of the fundamental, second, and third harmonic frequency laser pulse, respectively. Here, $\tau = 0.5 T_L$ and the duration $\sqrt{2 \ln(2)}\tau = 0.59 T_L$ is the FWHM in time of the three-color laser pulse. The normalized amplitudes of the fundamental, second, and third harmonic pulses are $a_{01} = 50$, $a_{02} = 50$, and $a_{03} = 48$, respectively. The fundamental and second harmonic frequency laser pulses have the same intensity $I_1 = I_2 = 5.35 \times 10^{21} \text{ W/cm}^2$. The intensity of the third harmonic laser pulse is $I_3 = 4.9 \times 10^{21} \text{ W/cm}^2$. The length of the one-dimensional simulation box is $8 \lambda_L$, which is resolved by 20 000 cells per wavelength and 100 quasiparticles per cell. The ultra-thin plasma foil is located in the region $7 \lambda_L < x < 7.075 \lambda_L$ with a high number density $n_e = 444 n_c$, where the critical number density is $n_c = \omega_L^2 \epsilon_0 m_e / e^2$ with ϵ_0 being the electric permittivity in vacuum. The electrons and ions are uniformly distributed at first.

The unique waveform of the three-color laser pulse is the main factor in the formation and dynamics of an extremely dense electron sheet that is crucial for the emission of the isolated attosecond pulse. For the transmitted radiation, it is necessary to use at least a one-cycle driv-

ing laser pulse to ensure twice-per-cycle oscillation of the electron sheet. Only in this way can the electron sheet move in the direction of transmission. The waveform of the three-color laser pulses with the present carrier envelope phase (CEP) has only one large amplitude cycle that rises and falls faster than for the fundamental frequency laser pulse alone, which is similar to the case of two-color laser pulses[28]. It is the strongest cycle that primarily contributes to the formation of electron sheet moving in transmitted direction, The weak cycles in the rising side of the laser pulse only produce slight perturbations on the plasma front surface.

When the strongest cycle of the three color laser pulses (as marked by a blue dotted rectangle in Fig. 1(b)) interacts with the foil plasma, the majority of the electrons from the thin foil are pushed from their equilibrium position towards the target by the Lorentz force. The Coulomb restoring force due to the displacement of the electrons increases and pulls them back when Lorentz force decreases, which results in the relativistic electrons oscillating twice around the ion background as marked by the blue dotted rectangle in Fig. 1(a). After the relativistic oscillating twice, these electrons pile up and form the self-organized dense electron nanobunch *A* shown in Fig. 1(a). When electron nanobunch *A* return back to bulk plasma, those relatively dispersed electrons on the right side of *A* converge into electron sheet *B* under the action of Lorentz force from the driven laser pulses as marked by the red dashed rectangle box in Fig. 1(b).

The emission process of the attosecond pulse is analyzed by the phase space evolution in Fig. 2, which shows that the formation of the ultra-dense electron sheet *B* occurs at $t = 8.368 T_L$ as the electron nanobunch returns back to target plasma. The position of the electron sheet *B* is illustrated by a green dotted line. After that, the electron sheet *B* is accelerated to ultra-relativistic velocity by the electrostatic force and ponderomotive force, where the strong charge separation field a_x and the laser pulse field a_z are shown in Figs. 2(g), 2(h), and 2(i) during this period. During the acceleration of the electron sheet *B*, the longitudinal momentum of the sheet *B* grows constantly with time, during which the distribution takes the shape of a whip from $t = 8.368 T_L$ to $t = 8.446 T_L$ as shown in Figs. 2(a), 2(b), and 2(c). The transverse momentum decreases with time as shown in Figs. 2(e) and 2(f) at $t = 8.407 T_L$ and $t = 8.446 T_L$, respectively. The lateral acceleration in the z direction reaches a maximum value as the longitudinal velocity reaches a velocity close to c at $t = 8.446 T_L$. The transverse acceleration is due to the weak half-cycle of the laser pulses marked by the red dashed box in Fig. 1(a), which is also confirmed in Fig. 2(i), where the strong transverse field is about $a_z \sim 75$ at time $t = 8.446 T_L$.

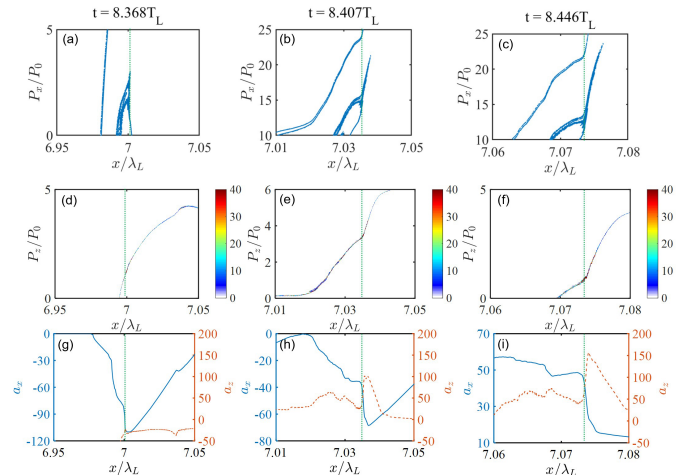


FIG. 2: The first row shows electron longitudinal momentum distribution in phase space in $x - p_x$ plane at different times $t = 8.368 T_L$ (a), $t = 8.407 T_L$ (b), and $t = 8.446 T_L$ (c), where the color bar stands for the number of macro particles. The nanobunching starts at $t = 8.368 T_L$ and the longitudinal velocity of electron sheet *B* reaches its maximum at $t = 8.446 T_L$. The second row (d)-(f) shows electron transverse momentum distribution in phase space in $x - p_z$ plane at the different times. The third row (g)-(i) shows the charge separation field a_x (blue line) and the laser pulse field a_z (red dashed line) acting on electron sheet *B* at the different times. The green dotted line stands for the positions of electron sheet *B* at different time.

B. Theoretical analysis

The high frequency spectrum of the forward pulse is determined by the transverse nonlinear electric current of the ultrathin foil as [7]

$$J_z(x, t) = -2\sigma_0\delta(x - x_0(t))v_z(t). \quad (1)$$

Here we consider a delta-like peak electric current density distribution and $x_0(t)$ is the position of the electron sheet *B* at time t . The transverse velocity is $v_z(t) = A_z(t)/\gamma$ with the relativistic factor being $\gamma = \sqrt{1 + A_z^2}$, where A_z is vector potential of the driven laser pulse and is normalized by $m_e c^2/e$. The dimensionless areal charge density is $\sigma_0 = \pi n_e d_0/\lambda$ with d_0 being the thickness of the foil and λ being the wavelength of the driven laser pulse [7]. The relationship between the dimensionless electric field a_z and dimensionless vector potential A_z is $a_z = -dA_z/dt$. For convenience, we use the dimensionless vector potential A_z to investigate the high frequency spectrum of the forward pulse. The dimensionless vector potential of the three-color laser pulse $A_z = A_1 + A_2 + A_3$ is given as

$$\begin{aligned} A_z(t) = & A_{01}e^{-\frac{t^2}{\tau^2}} \cos(2\pi t + \phi_1) \\ & + A_{02}e^{-\frac{t^2}{\tau^2}} \cos(4\pi t + \phi_2) \\ & + A_{03}e^{-\frac{t^2}{\tau^2}} \cos(6\pi t + \phi_3), \end{aligned} \quad (2)$$

where τ is related to the FWHM $\sqrt{2\ln(2)}\tau$ in time of the three-color laser pulses. t and τ are all normalized by λ/c . The space argument x is normalized by λ . A_{01} , A_{02} , and A_{03} are the normalized vector potential amplitudes of the fundamental, second, and third harmonic pulses, respectively. The first order approximation $\gamma \approx \gamma_0$ gives the transverse velocity as

$$v_z = v_{z0} - \alpha t^2, \quad (3)$$

where the coefficients $v_{z0} = \gamma_0^{-1}[A_{01}(1 - \phi_1^2/2) + A_{02}(1 - \phi_2^2/2) + A_{03}(1 - \phi_3^2/2)]$ and $\alpha = \gamma_0^{-1}[A_{01}(4\pi^2\tau^2 + 2 - \phi_1^2)/2\tau^2 + A_{02}(16\pi^2\tau^2 + 2 - \phi_2^2)/2\tau^2 + A_{03}(36\pi^2\tau^2 + 2 - \phi_3^2)/2\tau^2]$. Here the first power term of time t is omitted as we focus on the high frequency spectrum of the forward pulse. As the electron sheet B has ultrarelativistic velocity, the absolute velocity normalized by c is close to 1. Then by using the relationship $v_x^2 + v_z^2 = v^2$ with v_x being the longitudinal velocity of the electron sheet B , we can obtain the position $x_0(t) = \int v_x dt$ of the electron sheet B as

$$x_0(t) = \sqrt{v^2 - v_{z0}^2}t + \frac{\alpha v_{z0}}{\sqrt{v^2 - v_{z0}^2}} \frac{t^3}{3} - \frac{\alpha^2}{2\sqrt{v^2 - v_{z0}^2}} \frac{t^5}{5}, \quad (4)$$

Here the normalized velocity is close to 1 ($v \sim 1$). The transmitted radiation can be determined by the transverse current distribution $J_z(x, t)$ that is normalized by $en_c c$, where the transverse current density distribution is assumed unchanged. The transmitted field is given as

$$\begin{aligned} E(t) &= -\pi \int J_z(x, t+x) dx \\ &= 2\sigma_0 \pi \int v_z(t+x) \delta(x - x_0(t+x)) dx. \end{aligned} \quad (5)$$

The Fourier transform of the field $E(t)$ gives the spectrum of the transmitted radiation as [29]

$$I(\omega) = 8\sigma_0 \alpha^2 \pi^4 (\beta\omega)^{-\frac{6}{5}} \left| \text{Ai}_2''(\xi) \right|^2, \quad (6)$$

where $\xi = \omega^{\frac{4}{5}}(1 - \sqrt{v^2 - v_{z0}^2})(2\sqrt{v^2 - v_{z0}^2}/\alpha^2)^{\frac{1}{5}}$, $\beta = \alpha^2/2\sqrt{v^2 - v_{z0}^2}$, and $\text{Ai}_2''(\xi) = (1/2\pi) \int \tau^2 e^{i\xi\tau} e^{i\tau^5/5} d\tau$ is the second derivative of the generalized Airy function [29] of the second kind. Here we used the fact that $|\tilde{\delta}(\omega)|^2 \sim 1$ with $\tilde{\delta}(\omega)$ being the Fourier transform of the δ function. **The spectrum of HHG in the transmission direction in simulation shows that attosecond pulse has up to 10 000 order higher harmonic components as shown in Fig. 3(a), which is consistent with the above theoretical prediction confirms the $I(\omega) \propto \omega^{-6/5}$ scaling law for high frequencies in spectrum.** Generally, the spectrum of HHG of the reflected radiation has been shown to have a $I(\omega) \propto \omega^{-6/5}$ scaling law[29]. While, the spectrum of HHG of the transmitted radiation from a thin foil has a $I(\omega) \propto \omega^{-4/3}$ scaling law[20]. For our attosecond pulse from the ultra-thin target, the dense electron sheet B comes mainly from the rear surface of the thin foil as

shown in Fig. 3(b). Accordingly the forward HHG does not need to penetrate from the front to the rear side of the target. In our geometry the high density plasma is no longer a high-pass frequency filter. Then the spectrum of the forward HHG does not have a low-frequency cutoff as seen in Fig. 3(a).

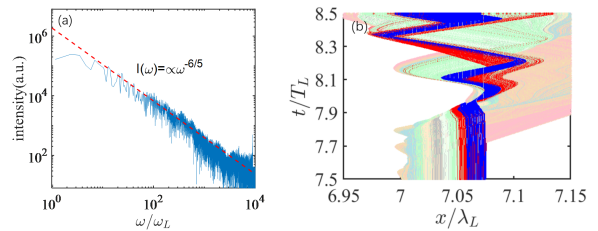


FIG. 3: (a) The harmonic spectra of the transmitted isolated attosecond pulse radiation behind the target. The attenuation of the spectrum follows $I(\omega) \propto \omega^{-6/5}$. (b) Electron trajectory in the $x-t$ plane.

III. THE MECHANISM OF THE ELECTRON NANOBUNCHING

A. Numerical analysis of nanobunching

From the insight of the structure of the dense electron nanobunch A as illustrated in Fig. 4(a), the relatively dispersed electrons on the right side converge into dense electron sheet B just as the bunch reverses its direction towards the target. In the CSE regime, the formation and dynamics of the very dense electron sheet B is crucial for the emission of a forward attosecond pulse.

The physical mechanisms of the electron sheets A and B are illustrated in Figs. 4(b) and 4(c). When the electron nanobunch A is moving in the reflection direction with a negative longitudinal momentum, the resultant Coulomb and Lorentz forces $f_E + f_B$ (represented by red dots in Fig. 4(b)) acting on those electrons on the right side of electron nanobunch A decreases gradually from left to right, which can be seen from three representative space positions marked as a , b , and c in Fig. 4(b), of which the electron longitudinal momentum have little difference as shown in Fig. 4(c). As the Coulomb force f_E (represented by green dots in Fig. 4(b)) acting on those electrons on the right side of electron nanobunch A increases gradually from left to right, the Lorentz force f_B (represented by blue dots in Fig. 4(b)) from laser pulse, which decreases gradually from left to right as the incident laser field is shielded by the dense nanobunches gradually from left to right, plays a leading role in formation of electron sheet B . Since the resultant force $f_B + f_E$ on the electrons is in the right-hand direction contrary to the direction of motion, the relatively dispersed electrons that are on the right of electron sheet A will converge into electron sheet B .

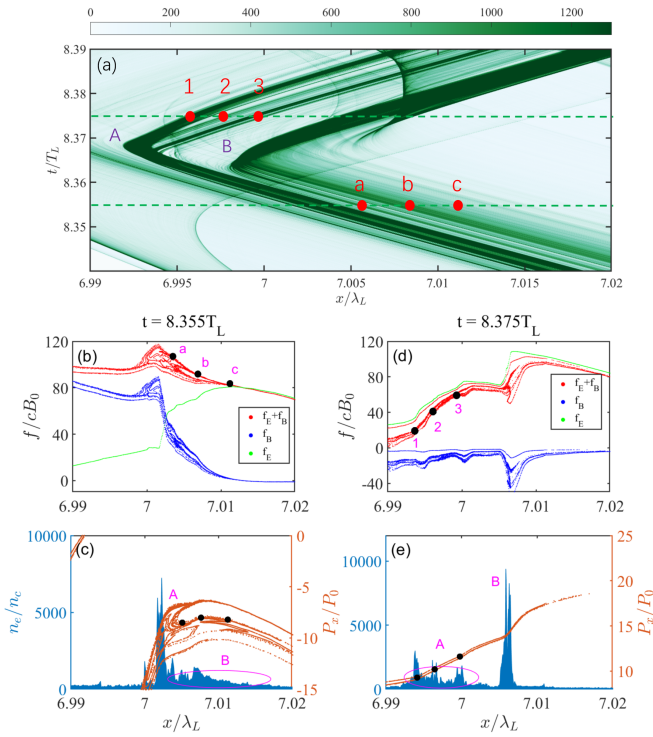


FIG. 4: (a) The close-up of the spatiotemporal evolution of the normalized electron number density n_e/n_c of the blue box in the Fig. 1(a), where we marked six representative space positions $x = 6.995 \lambda_L$, $x = 6.997 \lambda_L$, and $x = 6.999 \lambda_L$ marked as 1, 2, 3 at time $t = 8.375 T_L$ and $x = 7.005 \lambda_L$, $x = 7.008 \lambda_L$, $x = 7.011 \lambda_L$ marked as a, b, c at time $t = 8.355 T_L$ with solid red dots. (b) and (d) show the Coulomb force f_E (green dotted line), the Lorentz force f_B (blue dotted line), and the resultant force $f_E + f_B$ (red dotted line) acting on electron nanobunch B at time $t = 8.355 T_L$ and electron nanobunch A at time $t = 8.375 T_L$, respectively. (c) and (e) show the electron number density and the longitudinal momentum distribution of the nanobunches at time $t = 8.355 T_L$ and $t = 8.375 T_L$, respectively. The force is normalized by cB_0 with $B_0 = m_e \omega / c$. The momentum $P_x = \gamma m_e v_x$ is normalized by $P_0 = m_e c$ with v_x being the longitudinal velocity of electrons.

By contrast, at the time $t = 8.375 T_L$ the resultant force $f_E + f_B$ acting on electrons of nanobunch A increases gradually from left to right, while the magnitude of negative longitudinal momentum increases gradually from left to right. Accordingly, the electron on the right deflects much earlier than the electron on the left of A. At the end of the strong relativistic oscillation twice, the electron nanobunch A is dispersed during acceleration by the electrostatic restoring force after it returns back to target, which can be clearly shown in Figs. 4(d) and 4(e). We select the moment $t = 8.375 T_L$ to show the process of spreading of A, where it can be seen that the positive longitudinal momentum of electrons at the three representative spatial positions (marked as 1, 2, and 3) gradually increase from left to right as shown in Fig. 4(d).

Moreover, the electrostatic field force f_E (represented by green dots in Fig. 4(d)) acting on the electron also increases from left to right. There is little difference in the magnitude of Lorentz force f_B acting on electrons at different positions. Therefore, it can be concluded that electrons on the right will run faster than those on the left, which result in the spreading of A during acceleration. The electron density to the left of electron sheet B is greatly reduced as shown in Fig. 4(e), which reduces the shielding effect on the laser field. As a result, the weak half-cycle of the laser pulse marked by the red dashed box in Fig. 1(a) can easily penetrate nanobunch A and provide a large transverse perturbation on electron sheet B.

B. Theoretical model of nanobunching

We here analytically investigate the formation of the electron sheet B in the laser-plasma interactions in the ultrarelativistic limit. From the numerical simulation results, one can see that the relativistic oscillation is crucial for the formation of electron sheet B. The dynamics of the electron sheet B is governed by the momentum equation

$$\frac{d(\gamma v_x)}{dt} = -2\pi (a_x - v_z B_y), \quad (7)$$

where the electric field a_x , magnetic field B_y , and velocity (v_x, v_z) are normalized by $m_e \omega_L c / e$, $m_e \omega_L / e$, and c , respectively. The time argument t is normalized by λ / c . We assume that the transverse laser field and the transverse momentum of electron remains basically unchanged in the process of electron convergence. Then Eq. (7) can approximately stand for the formation process of electronic sheet. The electrostatic electric field can be written as

$$a_x(x(t), t) = 2\pi \int_{-\infty}^{x(t)} (Z_i n_{i0} - n_e(x', t)) dx', \quad (8)$$

where Z_i is the charge number of ion. The unperturbed ion number density n_{i0} and electron number density n_e are normalized by n_c . The space argument x and x' are normalized by λ . Here we assume that the ions remain stationary under the action of the driven laser pulse. The total derivative of the electrostatic electric field with respect to time is

$$\frac{da_x(x(t), t)}{dt} = 2\pi \left[\frac{dx}{dt} (Z_i n_{i0} - n_e(x, t)) + \int_{-\infty}^{x(t)} \frac{\partial (Z_i n_{i0} - n_e(x', t))}{\partial t} dx' \right], \quad (9)$$

Since the ion number density is constant, Eq.(9) simplifies as

$$\frac{da_x(x(t), t)}{dt} = 2\pi \left[v_x (Z_i n_{i0} - n_e(x, t)) - \int_{-\infty}^{x(t)} \frac{\partial n_e(x', t)}{\partial t} dx' \right], \quad (10)$$

In a fluid picture, the electrons satisfy the continuity equation of the fluid $v_x n_e(x, t) + \int_{-\infty}^x [\partial n_e(x', t)/\partial t] dx' = 0$. Then Eq. (10) can be simplified as $da_x/dt = 2\pi v_x Z_i n_{i0} = 2\pi v_x n_{e0}$ with n_{e0} being the unperturbed equilibrium electron number density satisfying the charge neutrality $n_{e0} = Z_i n_{i0}$. Then from Eq. (7) and the above results, one can obtain the equation governing longitudinal dynamics as

$$\frac{d^2 p_x}{dt^2} = \frac{-4\pi^2 n_{e0} p_x}{\sqrt{1 + p_x^2 + p_z^2}}, \quad (11)$$

where we assumed that the transverse laser magnetic field and the transverse momentum of electrons remain unchanged during the nanobunching when the bunch reverses its direction toward the target. If the longitudinal momentum of the electron sheet B is obtained from Eq.(11), the spatial coordinates of the electron sheet B at any time can be obtained as

$$x(t) = \int_0^t \frac{p_x(t')}{\sqrt{1 + p_x(t')^2 + p_z^2}} dt'. \quad (12)$$

We take time derivative of Eq. (12) and use Eq. (11) in Eq. (12) as

$$\frac{dx(t)}{dt} = \frac{p_x(t)}{\sqrt{1 + p_x(t)^2 + p_z^2}} = -\frac{1}{4\pi^2 n_{e0}} \frac{d^2 p_x}{dt^2}. \quad (13)$$

Integrating Eq. (13) once, one obtain that $x(t) = C_1 - (1/4\pi^2 n_{e0}) dp_x(t)/dt$ with C_1 being the integration constant. Assuming a coordinate system such that $dp_x/dt = 0$ at $t = 0$, we have $C_1 = 0$ and

$$\frac{dp_x(t)}{dt} = -4\pi n_{e0} x(t). \quad (14)$$

We multiply Eq.(11) by dp_x/dt as

$$\frac{1}{2} \frac{d}{dt} \left(\frac{dp_x(t)}{dt} \right)^2 = -4\pi n_{e0} \frac{d}{dt} \sqrt{1 + p_x(t)^2 + p_z^2}. \quad (15)$$

Integrating Eq. (15) once, one obtain that $\frac{1}{2} \left(\frac{dp_x(t)}{dt} \right)^2 = C_2 - 4\pi n_{e0} \sqrt{1 + p_x(t)^2 + p_z^2}$ with C_2 being the integration constant. We assume $p_x(t) = p_{x0}$ at $t = 0$, where $dp_x/dt = 0$. Then the constant is $C_2 = 4\pi n_{e0} \sqrt{1 + p_{x0}^2 + p_z^2}$. We have

$$\begin{aligned} \frac{1}{2} \left(\frac{dp_x(t)}{dt} \right)^2 \\ = 4\pi n_{e0} \left(\sqrt{1 + p_{x0}^2 + p_z^2} - \sqrt{1 + p_x(t)^2 + p_z^2} \right). \end{aligned} \quad (16)$$

Using Eq. (14) in Eq. (16), after simplification one obtains the phase space relation as

$$2\pi n_{e0} x^2(t) + \sqrt{1 + p_x(t)^2 + p_z^2} = \sqrt{1 + p_{x0}^2 + p_z^2}, \quad (17)$$

which gives $x(t)$ as a function of $p_x(t)$ for different values p_{x0} and p_z . Both the phase space and time-space evolution of the electron trajectory are dependent on the initial conditions, which can be obtained from the simulation results shown in Figs. 4(b) and 4(c). The solution of the second order differential Eq. (11) gives the nanobunching process in Fig. 5(a), where we have used the initial condition as $p_z = -2.5$, $p_x = -7.5, -7.6, -7.7, -7.8$, $dp_x/dt = 110, 109, 108, 107$. Fig. 5(b) is used to explain the divergence mechanism of electron nanobunch A as shown in Fig. 4(a), where the initial conditions are taken as $p_z = -4$, $p_x = -10, -9.5, -9, -8.5$, $dp_x/dt = 100, 102, 104, 106$, which can be obtained from Figs. 4(d) and 4(e). The main reason for the divergence of the electron sheet is that the Lorentz forces of the laser field on the electrons at different spatial positions and the longitudinal velocity of those electrons in A are quite different. One can see that the theoretical model agrees well with the simulation results about the nanobunching process of B and the divergence mechanism of A .

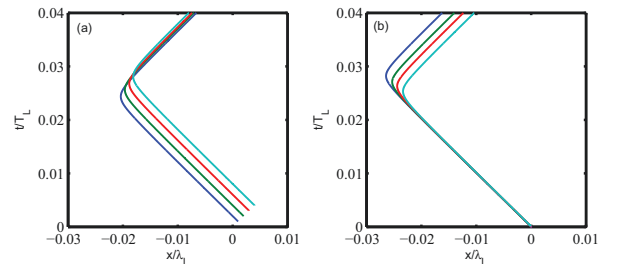


FIG. 5: Electron trajectory in the $x - t$ plane from the analytical model. (a) The nanobunching model for explaining the convergence mechanism of electrons. (b) The model for explaining the divergence mechanism of electron nanobunch.

IV. SIMILARITY LAW OF FORWARD ATTOSECOND PULSE

We verify the robustness of our unique CSE regime of the forward attosecond pulse with the help of similarity parameters $S = n_e/a_m$ with n_e being the normalized electron density by critical density n_c and a_m being the normalized maximum amplitude of strongest cycle of the three-color laser pulses. Similarity theories are useful to describe electron dynamics in laser plasma interaction processes [30], and have been used to investigate the HHG of the reflected radiation in ROM regime [31, 32]. We here show that the forward HHG by the laser driven ultra thin target also has a similarity law in CSE regime.

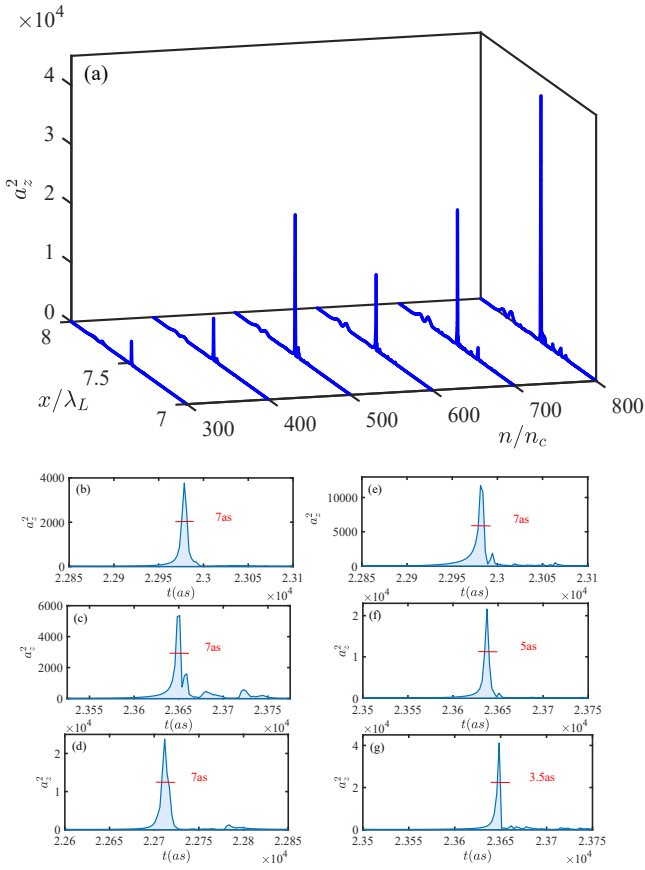


FIG. 6: (a) The similarity law of the forward attosecond pulse generation in CSE regime demonstrated numerically by six cases. The close-up of the attosecond pulse shows a FWHM of (b) 7 as for $n_e = 300n_c$, (c) 7 as for $n_e = 400n_c$, (d) 7 as for $n_e = 500n_c$, (e) 7 as for $n_e = 600n_c$, (f) 5 as for $n_e = 700n_c$, and (g) 3.5 as for $n_e = 800n_c$.

We numerically verify the similarity theory about the forward attosecond pulse emission in the CSE regime. The three-color laser amplitude and plasma electron density parameters are shown in Table I. From Fig. 6(a), one can see the isolated unipolar attosecond pulses emission has similarity states and results with approximately equal similarity parameters $S \sim 3.51$, although a_m and n_e are different in these six cases. When the plasma density and the laser amplitude change simultaneously to ensure similarity parameter is constant, the electron nanobunching dynamics and the forward attosecond pulse emission process remain similar. The three color laser amplitude and the electron density are given in the Table 1, where one can see the value of similar parameters is about $S \sim 3.51$. The phases of the three-color laser pulses are taken as $\phi_1 = 6.055$ rad, $\phi_2 = 5.960$ rad and $\phi_3 = 5.965$ rad. The thickness of the target is 60 nm for all cases. For all six cases, the generated forward attosecond pulses are all isolated and unipolar pulse. From the overall trend of Fig. 6(a), the greater the amplitude of the driving laser pulse leads to the greater the amplitude of the generated

attosecond pulse, of which the duration at FWHM are all sub-10 attosecond as shown by the close-up of the attosecond pulse about the six cases in Figs. 6(b)-6(g). For the case of $n_e = 800n_c$, the attosecond pulse has an intensity of up to $I \sim 8.2 \times 10^{22}$ W/cm² and a FWHM of about 3.5 as.

TABLE I: The similarity parameters $S = n_e/a_m$ of forward attosecond pulse generation in CSE regime. a_m is the maximum amplitude of the strongest cycle of the three color laser pulses.

n_e	a_{01}	a_{02}	a_{03}	a_m	S
300	35	32	33	84.37	3.56
400	44.5	45	43.5	112.8	3.55
500	56.77	56.78	54.51	142.4	3.51
600	68.1	68.6	65.55	171.5	3.50
700	80.1	80	78	201.8	3.47
800	91	92	87	228.9	3.49

The longitudinal dynamics of the ultrathin foil is determined by the combined effects of laser ponderomotive force and electrostatic force. For a given foil thickness d_0 , the value of similar parameters can be estimated by the balance between the laser ponderomotive force and the electrostatic force [7, 32]

$$(1 + R)a_m = 2\pi \frac{d_0}{\lambda} n_e, \quad (18)$$

where R is the laser reflectivity[7] and d_0 is the thickness of the foil target. Here we assume that the whole target is compressed to a so thin thickness that the number density of electrons can be regarded as a delta-like peak density distribution. Accordingly the maximum depletion length is approximately equal to foil thickness d_0 . Then one can get the approximate value range of the similarity parameter S as

$$\frac{\lambda}{2\pi d_0} < S < \frac{\lambda}{\pi d_0}. \quad (19)$$

From Eq.(14), we can obtain the similarity parameter $2.1 < S < 4.2$ with the wavelength $\lambda = 800$ nm and the foil thickness $d_0 = 60$ nm.

In order to further verify the similarity law of forward attosecond pulse generation scheme at lower laser intensity, we consider the three-color laser pulses interaction with near critical density foil plasma. From Table I one see that the similarity parameters S slightly increases with the decrease of the laser pulse intensity, which is due to the fact that the relativistic effects on the plasma density also decreases as the intensity decreases. The self-induced relativistic transparency effect is reduced. As expected, the simulation at lower laser intensity shows that the isolated attosecond pulses can also be emitted

with larger similarity parameters S as shown in Fig. 7. In this CSE regime, the HHG is coherently superimposed to produce high quality isolated attosecond pulses. As the stronger nonlinearities give rise to the higher harmonic efficiency in deed, the higher harmonic efficiency will decrease with the decrease of laser intensity, which will cause the pulse duration to become larger as shown in Fig. 7.

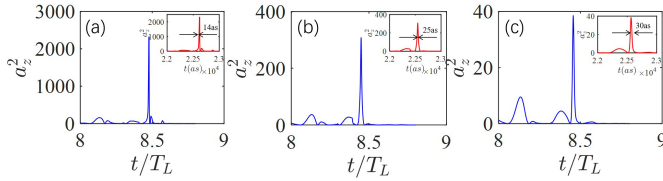


FIG. 7: The forward attosecond pulse generation from the interaction of the three-color laser pulses with foils having different plasma densities. (a) The similarity parameter is $S = 3.63$ and the duration is 14 as for $n_e = 200n_c$, $a_1 = 22$, $a_2 = 22$, and $a_3 = 21$; (b) The similarity parameter is $S = 4.08$ and the duration is 25 as for $n_e = 100n_c$, $a_1 = 10$, $a_2 = 10$, and $a_3 = 9$; (c) The similarity parameter is $S = 4.78$ and the duration is 30 as for $n_e = 50n_c$, $a_1 = 4.4$, $a_2 = 4.4$, and $a_3 = 3.6$.

V. ROBUSTNESS OF THE FORWARD ATTOSECOND PULSE GENERATION SCHEME

We have discussed the robustness of the forward attosecond pulse generation scheme with different laser and foil plasma conditions as well as including multi-dimensional effects. Firstly, we have checked the robustness by varying the density, thickness, and the length of linear density ramp at the front of the target. We keep the other parameters the same as in Fig. 1. One see in Fig. 8(a) that the durations are all below 16 as and the intensity $a_z^2 > 4058$ for plasma density of $440n_c < n_{e1} < 456n_c$. The appropriate foil thickness region can also be determined by the balance between the laser ponderomotive force and electrostatic force. The durations are all below 29 as and the intensity $a_z^2 > 2898$ for the thickness of target in the range $60\text{ nm} < d < 64\text{ nm}$ as shown in Fig. 8(b). When the thickness of the target increases, the duration at FWHM of attosecond pulse will increase from about 10 as to 30 as. When the thickness of the target is too thin, attosecond pulses are no longer generated, which is due to that the driven laser pulse can penetrate the target and the electron nanobunch is no longer formed. The additional preplasma at the front of the foil is also considered by varying the length of linear density ramp as shown in Fig. 8(c), which shows that the durations are all below 35 as and the intensity $a_z^2 > 3329$ for the length L of linear density ramp in the range $30\text{ nm} < L < 100\text{ nm}$.

We also performed a series of simulations to study the effects of different laser parameters on our scheme. In

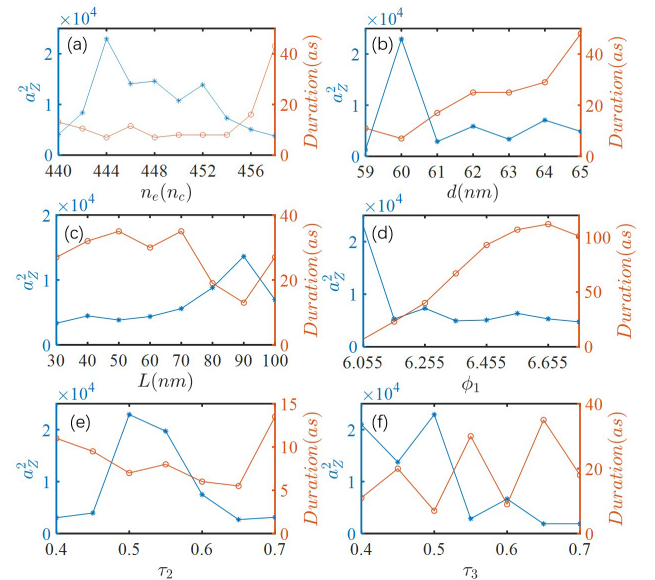


FIG. 8: Durations (as) at FWHM and normalized intensities a_z^2 of the attosecond pulses as functions of (a) the plasma densities n_e , (b) the thickness d of the foil, (c) the length of the linear density ramp of the preplasma, (d) the phase of fundamental frequency laser pulse, (e) the duration $\sqrt{2\ln(2)}\tau_2$ of the second harmonic laser pulse, and (f) the duration $\sqrt{2\ln(2)}\tau_3$ of the third harmonic laser pulse. Here for simplicity we take τ_2 or τ_3 as horizontal ordinate.

Fig. 8(d), the phase ϕ_1 of the fundamental frequency laser pulse varies from $\phi_1 = 6.055$ rad to $\phi_1 = 6.655$ rad, while keeping the other parameters the same as in Fig. 1. With the change of phase ϕ_1 , the duration of the attosecond pulse increases from below 10 as to 112 as and the intensity $a_z^2 > 4746$. We also consider the effects of duration $\sqrt{2\ln(2)}\tau_2$ of the second harmonic laser pulse and $\sqrt{2\ln(2)}\tau_3$ of third harmonic frequency laser pulse as shown in Figs. 8(e) and 8(f), respectively. The durations of attosecond pulse are all below 13.5 as and the intensity $a_z^2 > 2686$ for τ_2 in the range $0.4T_L < \tau_2 < 0.7T_L$ as shown in Fig. 8(e). When τ_3 of third harmonics varied from $0.4T_L$ to $0.7T_L$ as shown in Fig. 8(f), the durations of attosecond pulse are all below 35 as and the intensity $a_z^2 > 1866$.

To consider multi-dimensional effects, we also have performed a two-dimensional (2D) simulation on the transmitted attosecond pulse generation, where the plasma foil is the same as in Fig. 1, but is located between $x = 7.0\lambda_L$ and $x = 7.075\lambda_L$ and between $y = -18\lambda_L$ and $y = 18\lambda_L$. The simulation box is $8\lambda_L \times 36\lambda_L$ containing 40000×7200 cells. The number of particles per cell is 100 in the foil target plasma. The laser pulses have transverse Gaussian profile e^{-y^2/w_0^2} with $w_0 = 6.25\lambda_L$. The longitudinal profile of the laser pulses are the same as that in Fig. 1. The dynamics of the electron sheet and the emission process of the attosecond pulse is almost identical to one dimensional case, because the interac-

tion is too short to cause any multi-dimensional instabilities. The forward attosecond pulse is generated by the electron sheet that is accelerated in longitudinal direction and perturbed transversely. The amplitude of the forward attosecond pulse reaches the maximum at time $t = 8.446 T_L$ and the duration is about 8 as, which is illustrated by snapshots at time $t = 8.446 T_L$ in Fig. 9. We also show the forward attosecond pulse as insets in Fig. 9(b) at $y = 0$ and $t = 8.446 T_L$ without the extra filters, which shows that the FWHM of a_z^2 is about 8 as and the attosecond pulse has a half-cycle profile.

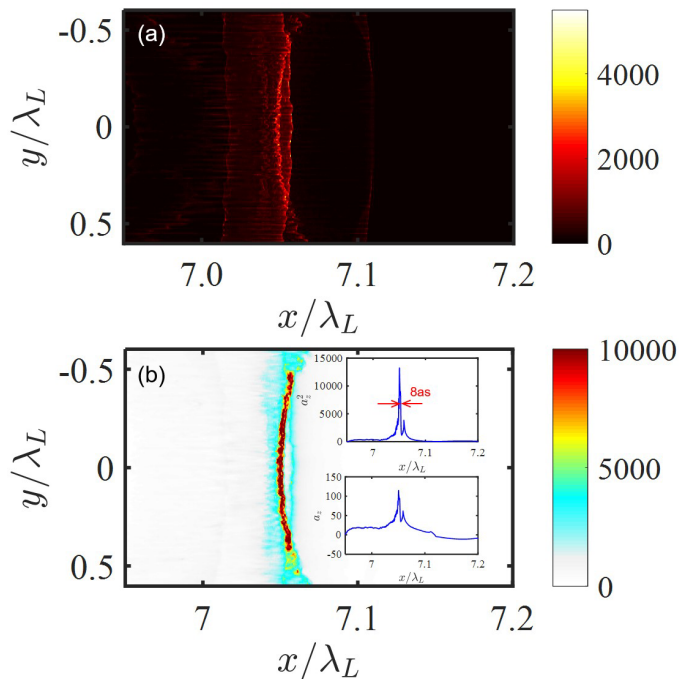


FIG. 9: Two-dimensional(2D) simulation on the transmitted attosecond pulse generation. (a) gives the density of electron sheet n_e standing for the generation of forward attosecond pulse. (b) illustrates the generated isolated attosecond pulse in transmitted direction without the need for extra filters, where the two insets show that the FWHM of a_z^2 is about 8 as and the attosecond pulse has a half-cycle profile.

In the above simulations, the ions are assumed to be immobile as the duration of the interaction between laser and target is very short. We have also performed simulation with mobile ions in order to check the rationality of this assumption, which was illustrated in Fig. 10. The parameters of the laser pulses and plasma are the same as that in Fig. 1. As can be seen from Fig. 10, the ions were only slightly pushed from their equilibrium position towards the transmitted direction by the ponderomotive force of the laser pulses, which shows that there is no oscillation for ions. From the comparison between Fig. 1(a) and Fig. 10(a), the spatiotemporal evolution of the electron number density is almost identical to the case that the ions are assumed to be immobile. The extremely dense electron sheet are also produced in the laser-plasma

interaction and responsible for the attosecond pulse in CSE regime. Accordingly, the forward attosecond pulses are also generated in the case of mobile ions, which also has a half-cycle profile (shown in Fig. 10(b)) and the FWHM of a_z^2 is about 13 as (shown in Fig. 10(c)) and is slightly larger than the case (7 as) in Fig. 1.

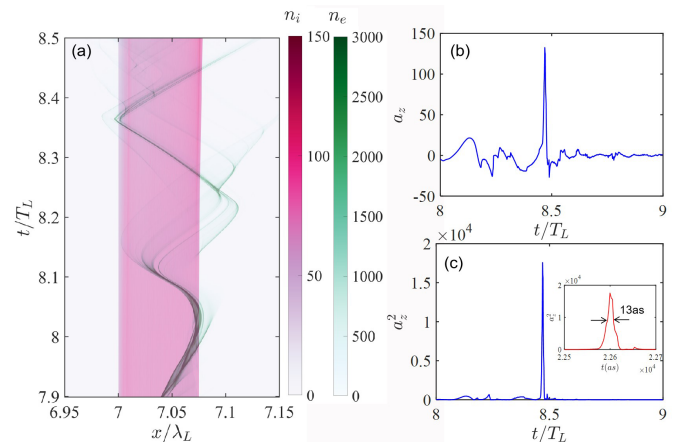


FIG. 10: (a) The spatiotemporal evolution of electron number density n_e/n_c and ion number density from time $t = 7.9 T_L$ to $t = 8.5 T_L$. (b) The half-cycle profile of the forward attosecond pulse a_z . (c) The profile of the intensity of the attosecond pulse a_z^2 and an inset with a close-up of the attosecond pulse having a FWHM of about 13 as.

VI. DISCUSSION AND CONCLUSION

In this paper, a unique ultra-thin foil regime of forward isolated unipolar attosecond pulse emission is proposed by using a three-color laser pulse impinging an ultra-thin foil, of which the thickness is approximately equal to maximum depletion length. As a result, the secondary electron sheet will not be formed. Generally, the secondary electron sheet contributes to coherent synchrotron radiation besides the primary electron sheet [3]. As the primary and secondary electron sheets will both contribute to the transmitted radiation, the attosecond pulses have two distinct subpulses. The distinct subpulse structure in the forward attosecond pulse will no longer appear in our ultra-thin foil regime.

The robustness of the attosecond pulse generation scheme is checked with different laser and foil plasma conditions and the multi-dimensional effects. The density, thickness, the length of linear density ramp at the front of the target, the phase ϕ_1 of the fundamental frequency laser pulse, and the durations of the second harmonics and third harmonics were varied. 2D simulation shows that the dynamics of the electron sheet and the emission process of the attosecond pulse is similar to one dimensional case. The robustness of the regime is also verified with the help of similarity theory, where the isolated unipolar attosecond pulses are emitted with approximate-

ly equal similarity parameters $S \sim 3.51$ for different the laser amplitude a_0 and electron density n_e in all cases. For the case of $n_e = 800n_c$, the attosecond pulse has an intensity of up to $I \sim 8.2 \times 10^{22}$ W/cm² and a FWHM of about 3.5 as.

For reflected radiation in CSE regime, the intensity of the emitted attosecond pulse can be compare or much larger than that of incident pulse. While for transmitted radiation in CSE regime, the intensity of attosecond is usually smaller than that of incident pulse [3, 21, 22]. In this paper, the intensity of forward attosecond pulse generated from ultra-thin foil regime is comparable to that of the incident pulse. Instead of electrons oscillating only at skin depth in the front side of the plasma, in ultrathin foil regime all electrons in the plasma target oscillate twice under the action of the strongest cycle of three-color laser pulses. After that, a dense electron sheet with maximum density $> 11000n_c$ is formed and accelerated to ultra-relativistic velocity, while transversely perturbed by the weak half-cycle of the laser pulse. As a result, an isolated unipolar attosecond pulses is emitted. Since the

extremely dense electron sheet mainly comes from the rear surface of the thin foil, the forward HHG does not need to penetrate from the front of the target to the rear side of the target and the spectrum of the forward HHG has no low-frequency cutoff.

Acknowledgments

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