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On the similarities of the sPTT and FENE-P models for polymeric fluids

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ABSTRACT

For many commonly used single mode viscoelastic constitutive equations of differential type, it is well known that they share many features. For example, in certain parameter limits the models due to Giesekus, Phan-Thien Tanner and FENE-type models approach the Oldroyd-B model. In this talk, I'll compare the response of the linear form of the simplified Phan-Thien Tanner model [due to Phan-Thien and Tanner, Journal of Non-Newtonian Fluid Mechanics, 2 (1977) 353–365] (the “sPTT”) and the Finitely Extensible Nonlinear Elastic model that follows the Peterlin approximation [due to Bird et al., Journal of Non-Newtonian Fluid Mechanics, 7 (1980) 213–235] (the “FENE-P"). I'll show that for steady homogeneous flows such as steady simple shear flow or pure extension the response of both models is identical under certain conditions.

For more general flows we show analytically that the results from the two models only formally approach each other when both the polymer concentration and Weissenberg number is small. We then use a numerical approach to investigate the response of the two models when the flow is “complex” under two different definitions: firstly, when the applied deformation field is homogenous in space but transient in time (so called “start-up" shear and planar extensional flow) and then for “complex” flows (through a range of geometries) which, although Eulerian steady, are unsteady in a Lagrangian sense. Under the limit that the flows remain Eulerian steady (so the Weissenberg number is typically small), we see once again a very close agreement between the FENE-P and sPTT models.

Videos to this article can be found online at https://doi.org/10.1016/j.sctalk.2022.100015.

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Figures and tables

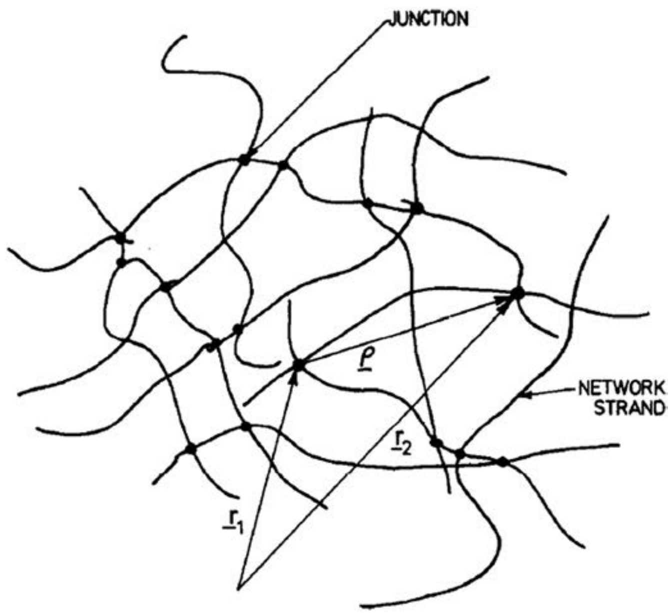

Fig. 1. Typical network of polymer concentrated solutions and melts. Figure taken from [1] with permission (Elsevier).

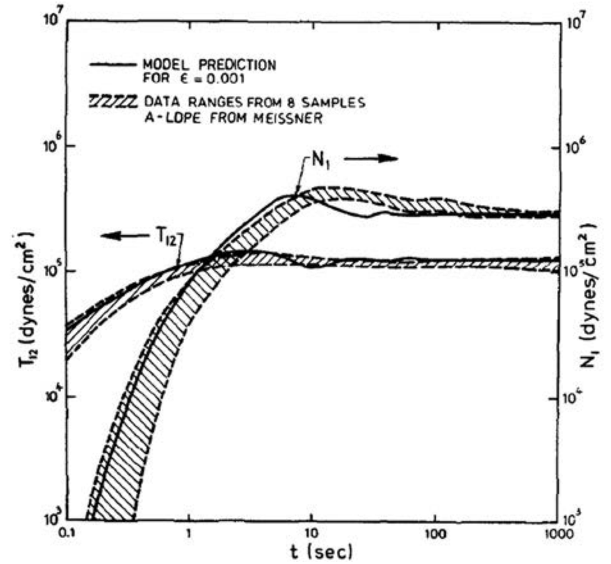

Fig. 3. Starting up of a shear flow. $\gamma = 1\ s^{-1}$ predictions for PTT model including comparison to experimental LDPE data. Figure taken from [1] with permission (Elsevier).

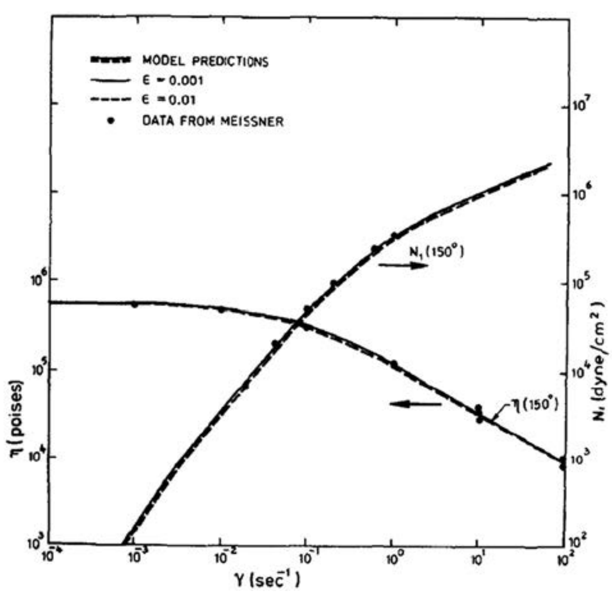

Fig. 2. Viscosity and first normal stress difference predictions for PTT model including comparison to experimental LDPE data. Figure taken from [1] with permission (Elsevier).

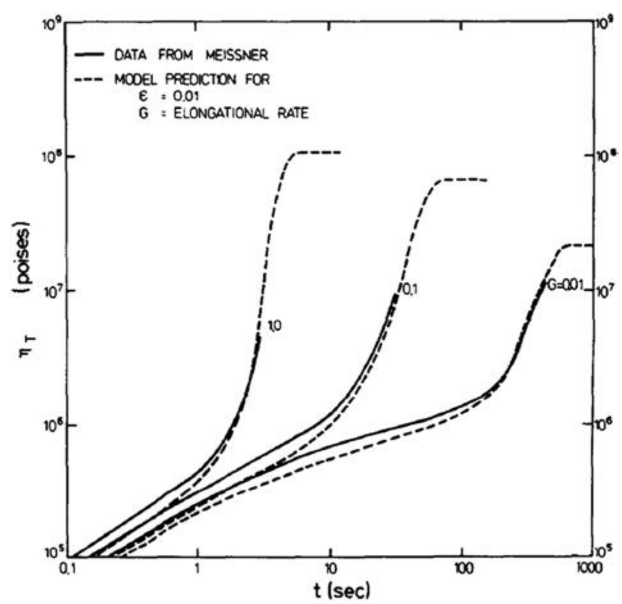

Fig. 4. Tensile viscosity predictions for PTT model. Figure taken from [1] with permission (Elsevier).

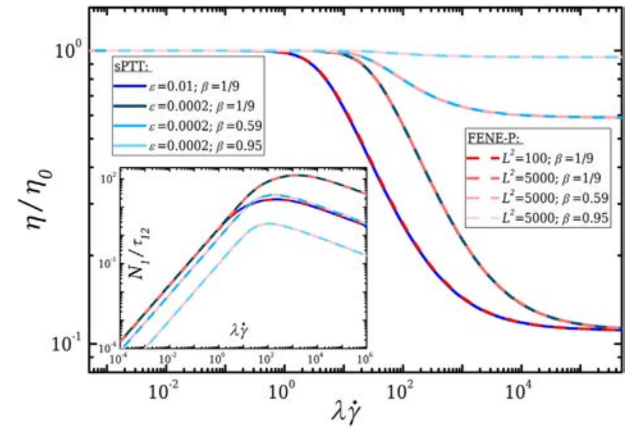

Fig. 5. Steady simple shearing predictions for sPTT and FENE-P model for both viscosity and (inset) first normal-stress difference versus non-dimensional shear rate.

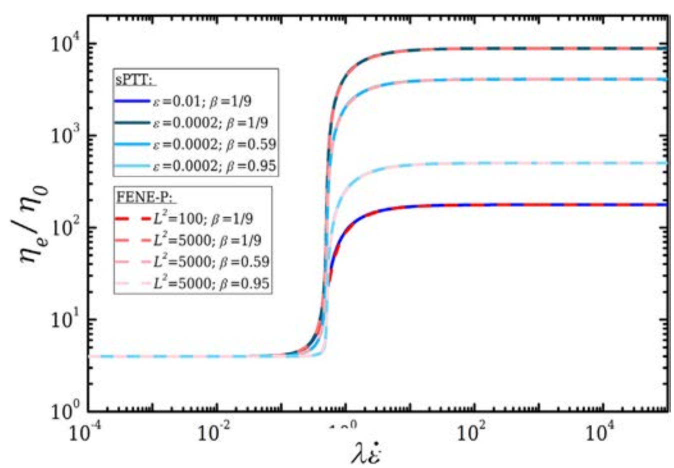

Fig. 6. Steady planar extensional viscosity predictions for sPTT and FENE-P models versus non-dimensional shear rate.

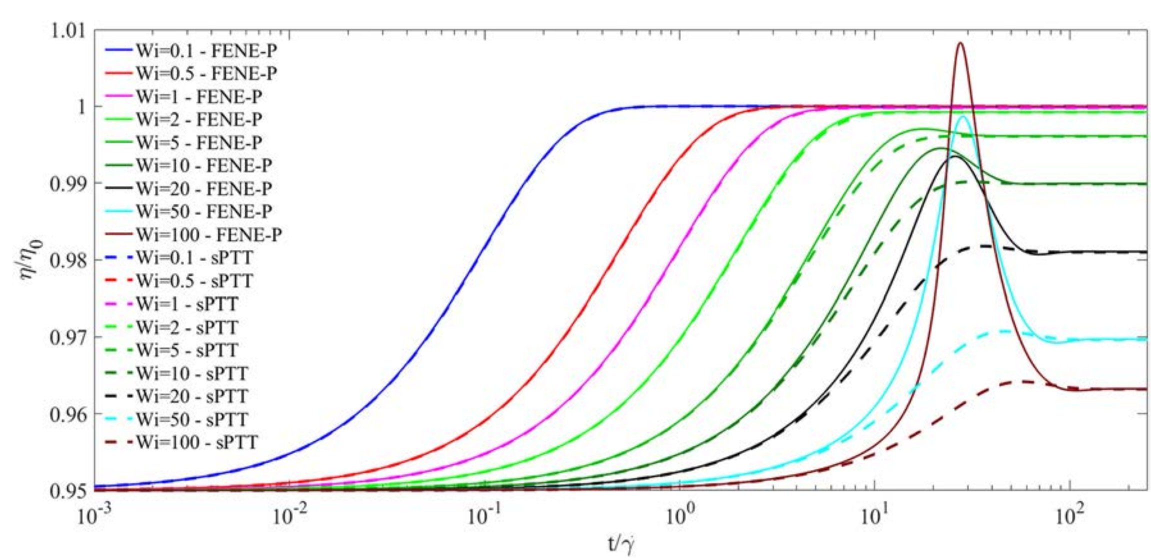

Fig. 7. Start-up of simple shear predictions of viscosity for sPTT and FENE-P models.

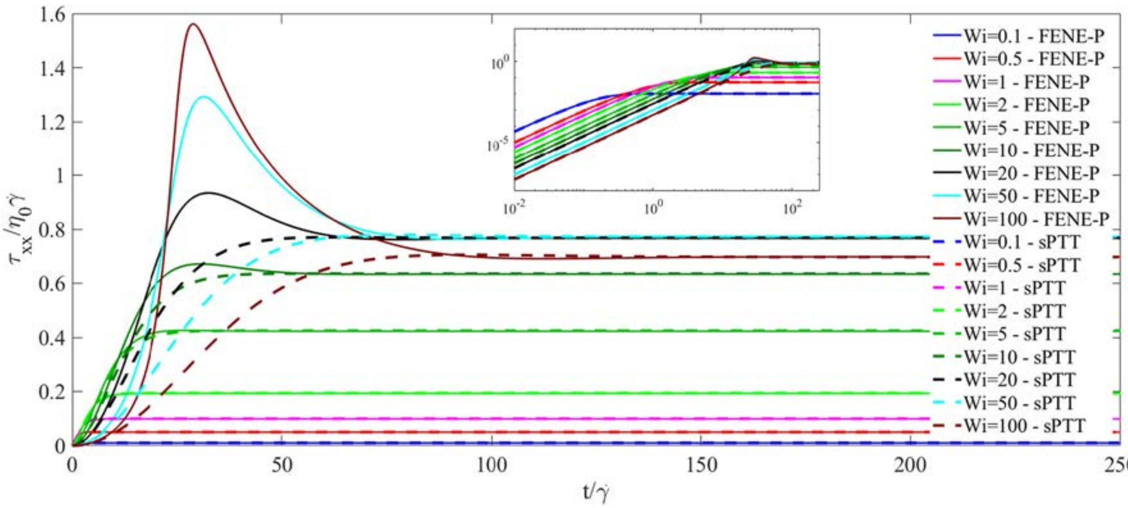

Fig. 8. Start-up of simple shear predictions of axial normal-stress for sPTT and FENE-P models.

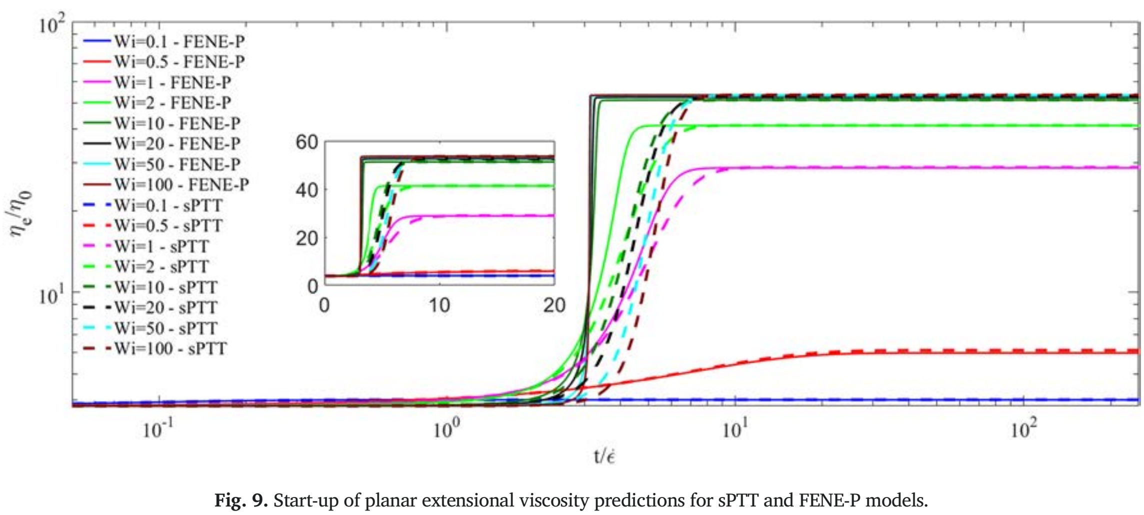

Fig. 9. Start-up of planar extensional viscosity predictions for sPTT and FENE-P models.

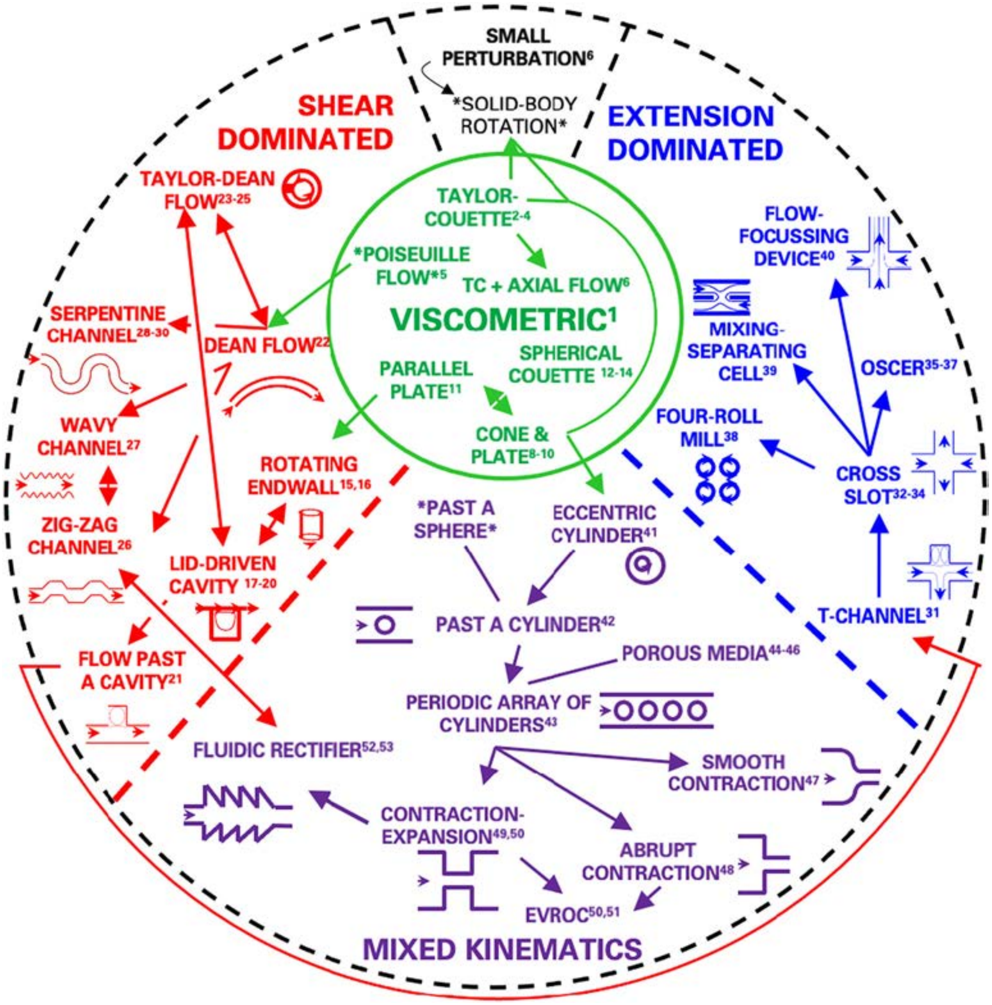

Fig. 10. The Purely-Elastic Instability Flow Map ("PEFIM") adapted from Poole [2]. For further information and list of references cited herein see Datta, et al. [4].

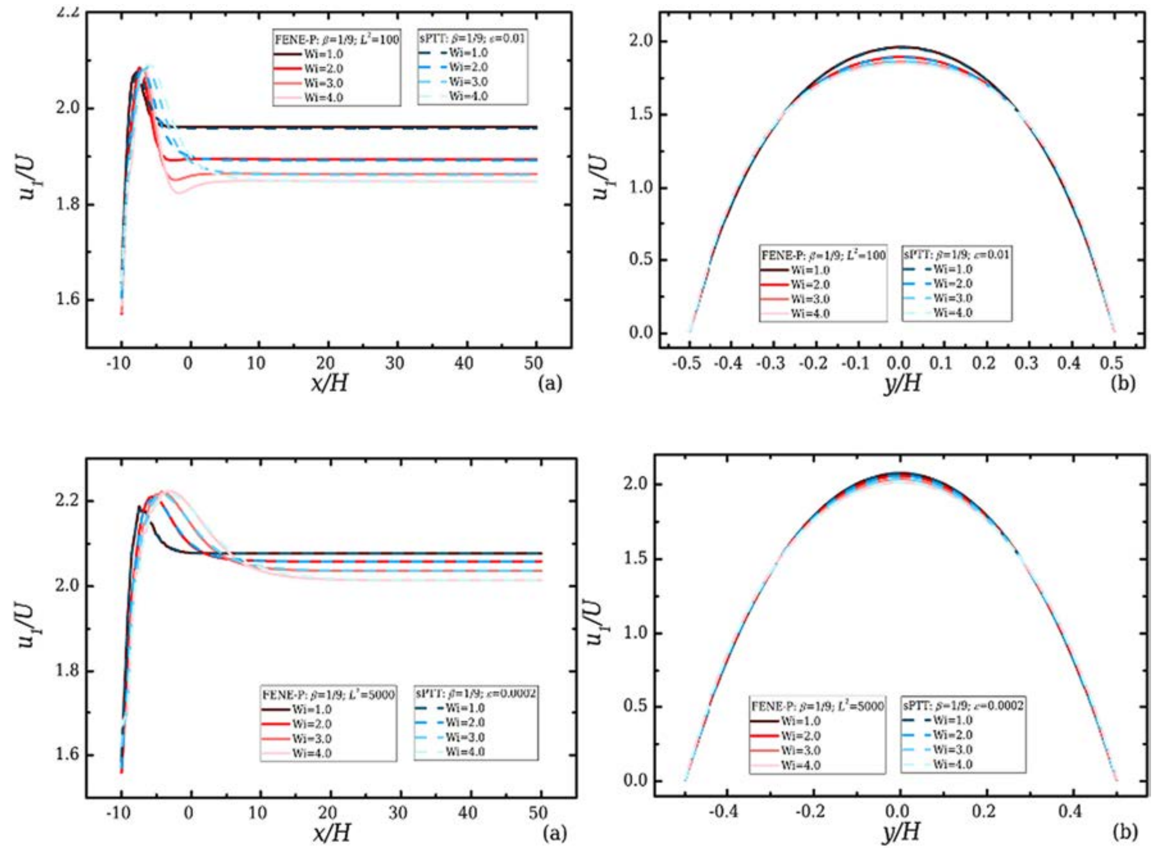

Fig. 11. Comparison of sPTT and FENE-P models in developing flow within a square duct. Streamwise (x) variation shown along duct centeline in left hand side panels and transverse (y) variation shown in fully-developed region far downstream in right hand side panels.

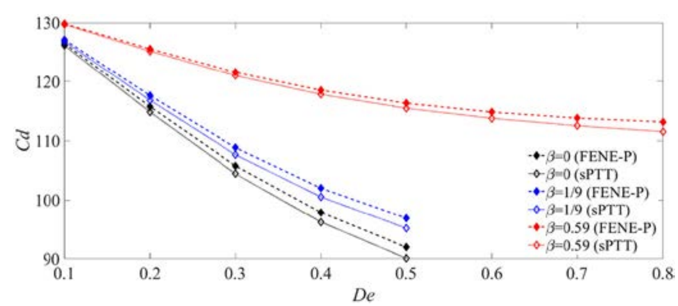

Fig. 12. Comparison of sPTT and FENE-P model results ($L^2 = 100$ $\varepsilon = 0.01$) for drag force on flow past a confined cylinder (50% blockage) in a two-dimensional channel. β is the solvent-to-total viscosity ratio.

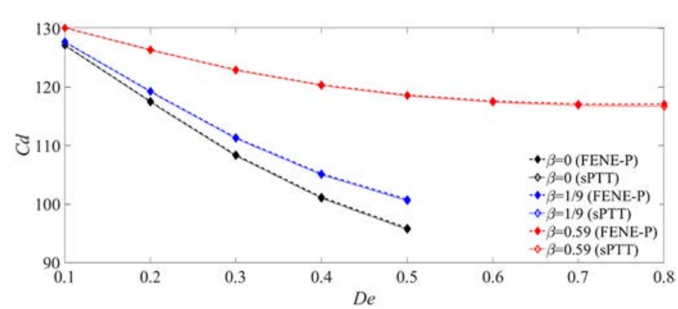

Fig. 13. Comparison of sPTT and FENE-P model results ($L^2 = 1000$ $\varepsilon = 0.001$) for drag force on flow past a confined cylinder (50% blockage) in a two-dimensional channel. β is the solvent-to-total viscosity ratio.

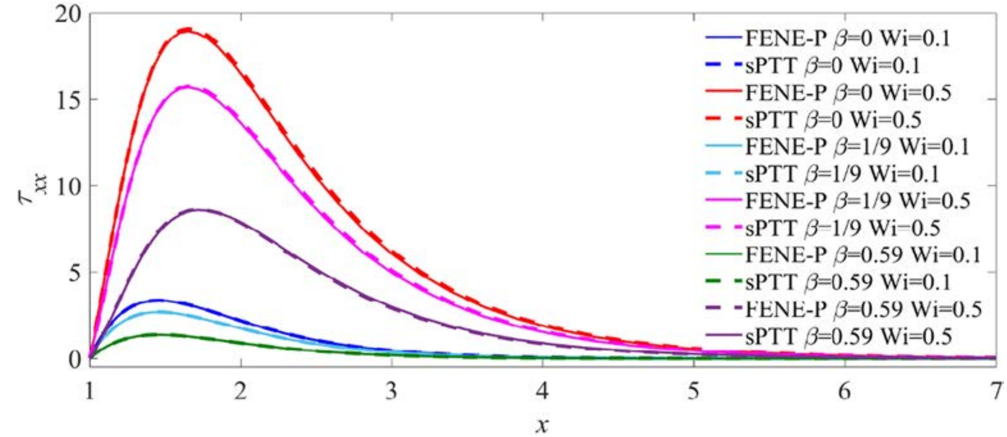

Fig. 14. Comparison of sPTT and FENE-P model results ($L^2 = 1000$ $\varepsilon = 0.001$) for streamwise normal-stress along centreline downstream of a confined cylinder (50% blockage) in a two-dimensional channel. β is the solvent-to-total viscosity ratio.

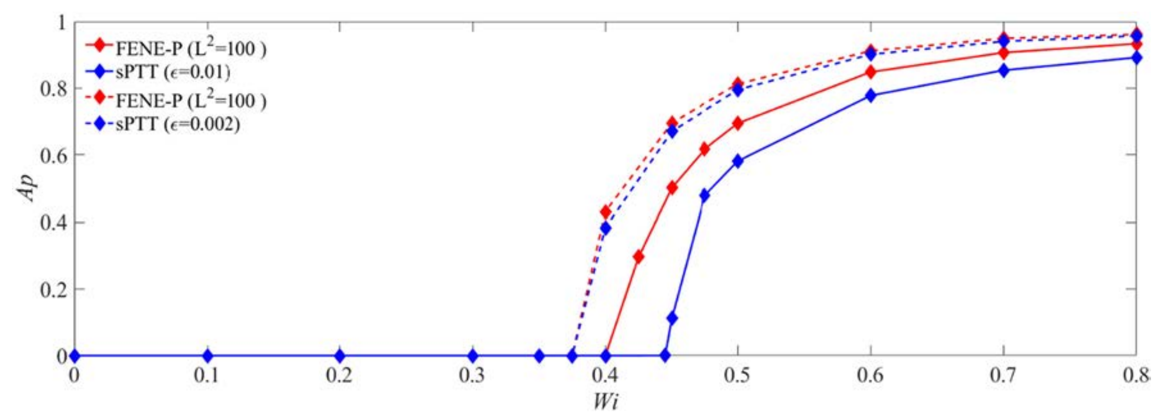

Fig. 15. Variation of cross-slot asymmetry parameter with Weissenberg number for sPTT and FENE-P models. Parameters are: $L^2 = 500$ $\varepsilon = 0.002$ *and* $L^2 = 100$ $\varepsilon = 0.01$ $\beta = 1/9$. For definition of asymmetry parameter see Davoodi et al. [5].

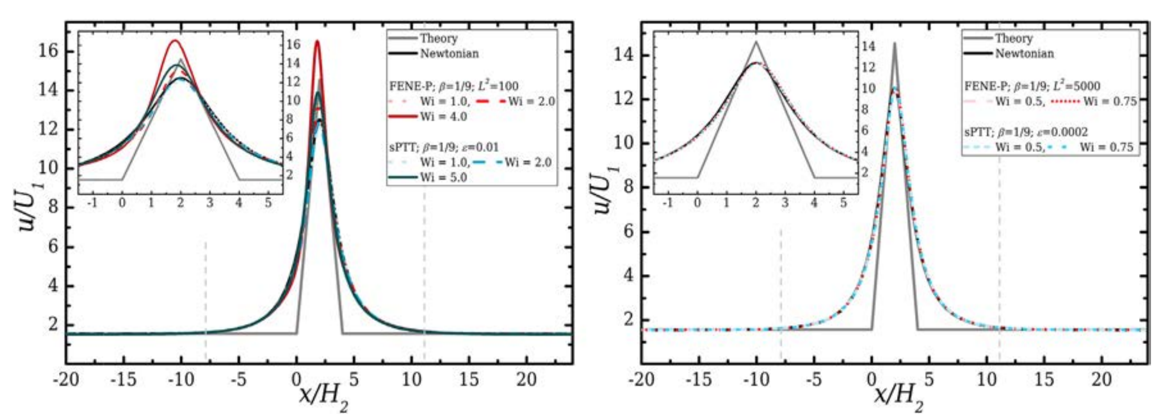

Fig. 16. Variation of centreline velocity for sPTT and FENE-P models in “EVROC” geometry (see Ober et al. [6] and Zografos et al. [7]).

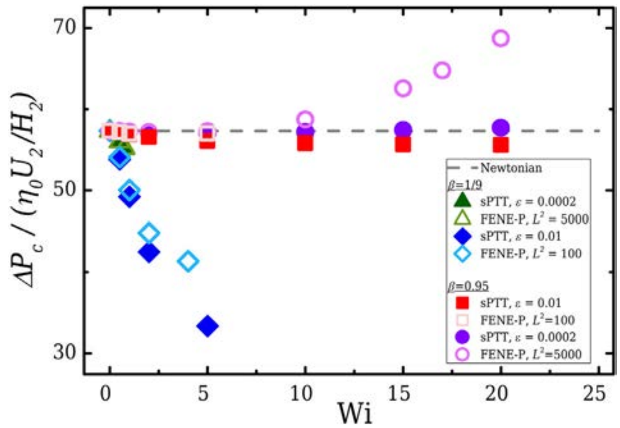

Fig. 17. Comparison of pressure-drop in EVROC geometry for sPTT and FENE-P models.

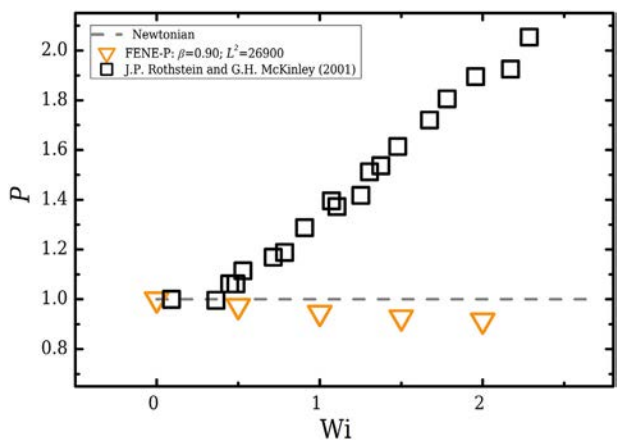

Fig. 18. Comparison of pressure-drop in 4:1:4 axisymmetric contraction-expansion of Rothstein and McKinley [3] with predictions from the FENE-P model with model parameters suggested in Ref. 3.

CRediT authorship contribution statement

Mahdi Davoodi: Data curation, Investigation, Methodology, Software, Validation, Visualization. **Konstantinos Zografos:** Data curation, Investigation, Methodology, Software, Validation, Visualization. **Robert J. Poole:** Conceptualization, Methodology, Supervision, Writing – original draft, Writing – review & editing, Visualization, Project administration, Funding acquisition.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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