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1:1 RESONANCE CAPTURE OF A LOW-THRUST SPACECRAFT AROUND VESTA

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Vesta is the second largest celestial object of the main asteroid belt and it was visited and studied by the DAWN mission in 2011. The spacecraft used solar-electric propulsion that generates continuous low-thrust.As the spacecraft slowly descends from high altitude mission orbit (HAMO) to low altitude mission orbit (LAMO), it crosses the 1:1 resonance, putting the spacecraft at risk of being permanently trapped at this altitude. The objective of this paper is to analyze the probability that the spacecraft has to be captured into the 1:1 resonance with Vesta. Firstly, we model the dynamics considering the irregular gravitational field up to the fourth order and degree and the thrust constant in magnitude and opposite to the velocity direction of the spacecraft. Then, we calculate the probability of capture for orbits with different combinations of semi-major axis and true anomaly. In addition, we simplify the dynamical model by considering the harmonic terms related to the 1:1 resonance and the second order degree harmonics, respectively. It is found that the simplified models are not capable of estimating this probability promisingly. Therefore, through pure numerical simulations of the complete model, we investigate the sensitivity of this capture to different orbital geometries and physical properties of the spacecraft. The results show that the probability of capture is more dependent on the value of the mass of the spacecraft and the magnitude of the thrust, and is less dependent on the value of the specific impulse. In addition, it is found that the spacecraft is more prone to be captured into the 2:3 resonance with Vesta if the descent starts from a non-polar orbit.

INTRODUCTION

The DAWN mission was one of the first missions to use electric propulsion during the cruise phase and the approach to the asteroid. It demonstrates the possibility of relying on electric propulsion for the majority of the mission duration. As the spacecraft slowly approaches the asteroid, there is a possibility that it is captured by the 1:1 resonance and being permanently trapped at this altitude. Since the application of the electric propulsion is the future tendency, the study of the probability of capture into resonance of a spacecraft around a celestial body needs to be investigated. Resonance orbit is defined as the trajectory for which the ratio of the revolution period of the spacecraft around the asteroid and the rotation period of the asteroid around itself is equal to an integer number , e.g., 1:1 resonance orbit for which the spacecraft does one orbit revolution with the same period with which the asteroid rotates around itself. The spacecraft at each revolution will encounter the same gravitational configuration,the effect of which will sum up over the revolutions and will change noticeably the orbit eccentricity and inclination.

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Tricarico and Sykes[1] considered this scenario and identified the possibility for the spacecraft to be captured into the 1:1 resonance that was found to be between the orbital radius of 500km and 600km. The authors simulated the descent starting from 1000km and considered 12 different initial conditions shifted by 30°of true anomaly. Among the 12 simulations, the capture occurred only once, estimating the probability of capture $1/12 \sim 8.3\%$ with a limited set of simulations. Delsante[2] simulated the scenario with a greater number of initial conditions and different values of thrust, improving the results by obtaining a probability of capture equal to 8.26%. It is found that the dependence of the probability of capture on the magnitude of the thrust is related to the initial semi-major axis.

With this work, we extend the above research and advance the knowledge about this phenomenon by considering a larger and broader set of initial conditions. This paper is structured as follows. In Section2, we define the equations of motion (EoM) describing the dynamics of the spacecraft's motion around Vesta. Section 3 performs sensitivity analysis of the capture probability to different initial orbital geometries and spacecraft parameters such as the specific impulse, the mass and the thrust. Section 4 analyzes the performance of the simplified models on estimating the probability of capture, Section 5 concludes this study.

DYNAMICAL MODELING

In this section we define the EoM of a spacecraft around a rotating asteroid. The model considered is the two-body problem with perturbations from the irregular gravitational field of the asteroid and the low-thrust to which the spacecraft is subject to. The gravitational field is represented by the spherical harmonics model and is truncated to the fourth order and degree, the low-thrust is constant in magnitude and it always directs to the opposite direction of the spacecraft's velocity.

Following Kaula[3] , the potential of the gravitational field of an asteroid V in spherical coordinates can be expressed as the sum of the keplerian component and a spherical harmonic expansion up to the degree n and order m

$$V_{r\lambda\phi} = \frac{\mu}{r} + \sum_{n=2}^{\infty}\sum_{m=0}^{n} \frac{\mu}{r}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\phi)(C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda) \tag{1}$$

where μ is the gravitational constant of Vesta, R_e is the reference radius of the asteroid, $P_{n,m}(\sin\phi)$ are the associated legendre polynomials, r is the radial distance, ϕ is the geocentric latitude, λ is the longitude, $C_{n,m}$ and $S_{n,m}$ are the spherical harmonics coefficients and n, m are integers.

By transforming in the potential in cartesian coordinate and taking the gradient of the potential and adding the low thrust component to the acceleration, we can define the EoM which describe the absolute satellite motion in the asteroid centered inertial frame as Eq.2 where V_{xyz} represents the potential expanded in spherical harmonics as a function of the cartesian coordinates (x, y, z), T is the thrust, m is the spacecraft's mass and $\hat{v}$ is the spacecraft's velocity unit vector. To that, we add the differential equation describing the rate of change of the spacecraft's mass over time as Eq.3

where I_{sp} and g_0 represent the specific impulse and Earth's gravitational acceleration respectively.

$$\ddot{\vec{x}} = \nabla V_{xyz} - \frac{T}{m(t)}\hat{v} \quad (2)$$

$$\dot{m} = -\frac{T}{I_{sp}g_0} \quad (3)$$

NUMERICAL SIMULARIONS

Validation

Firstly , we validate our codes reproducing the results obtained by Tricarico and Sykes[1] and Delsante.[2] Next, we proceed to consider how the results vary by simplifying the dynamic model and by considering different initial conditions. The goal is to identify critical situations in which the spacecraft could find itself and at the same time identify the conditions that minimizes the probability of capture for a safer mission.

The initial conditions in which DAWN was when it reached Vesta are given in Table 1.

Table 1: DAWN spacecraft initial conditions at its arrival at Vesta.

Mass	1000kg
Thrust	20mN
Specific Impulse	3000s
SMA	1000km
eccentricity	0
Inclination	90°
Longitude of the ascending node	0°
Argument of periapsis	0°

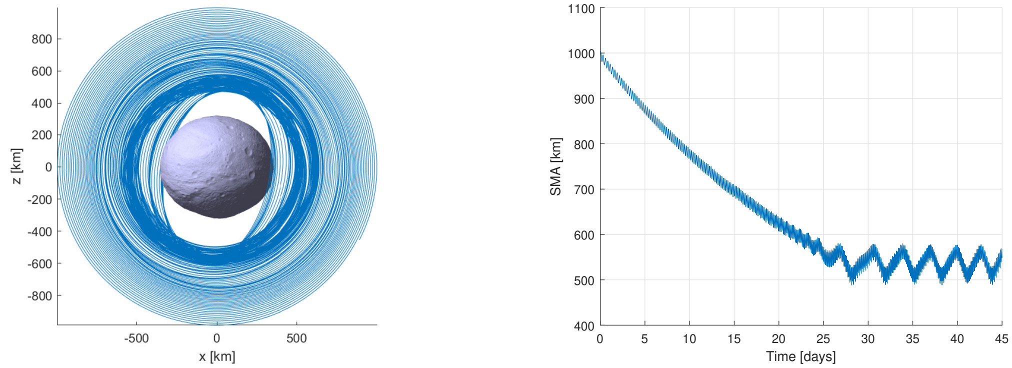
For an initially circular polar orbit the 1:1 resonance locates around 540km considering the fourth order and degree gravity field of Vesta as shown in Figure 1. In Figure 1(a), we give a realistic representation of the spacecraft descent from HAMO until the resonance capture happens; while, in Figure 1(b), we illustrate the evolution of the semi-major axis (SMA) over 45 days, from which we can locate the $1 : 1$ resonance around 540km.

Model reduction analysis

We extend the study of Tricarico varying the initial SMA and true anomaly. In this paper, we simulate each 10km of orbital radius from 600km to 1000km and each 9°of true anomaly from 0°to 360°for 45 days. The probability of capture at different SMA is given in Figure 2. The probability of capture remains confined below 15% and the mean value is 7.3171%. The orbital radii that are more sensible to the capture probability are 690km, 750km and near the radius of 1000km.

Then we simplify the dynamical model by considering only the harmonic terms of the potential related to the 1:1 resonance as in Eq.4 where V_0 is the keplerian gravitational term, V_{20} and V_{40} are the zonal harmonics of second and fourth order respectively, V_{22} and V_{44} are the tesseral harmonics of of second and fourth order respectively, V_{32} and V_{42} are the remaining sectorial harmonics of the gravitational expansion.

$$V_{1:1}^{4th} = V_0 + V_{20} + V_{22} + V_{32} + V_{40} + V_{42} + V_{44} \quad (4)$$



((a)) DAWN descent from HAMO to LAMO

((b)) SMA evolution during descent

Figure 1

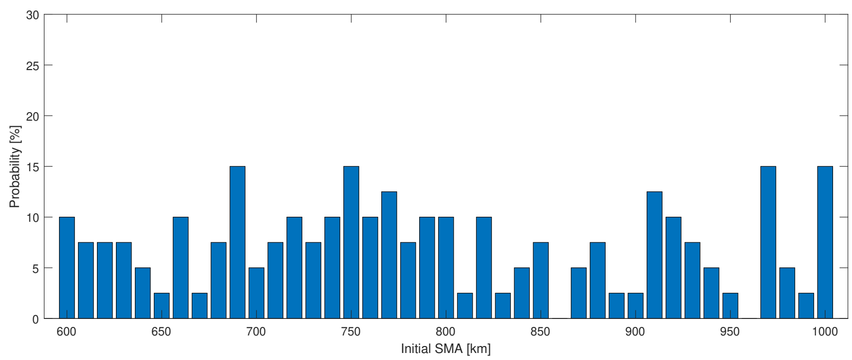


Figure 2: Probability of capture into 1 : 1 resonance for different initial SMA.

By performing the same simulations of varying SMA and true anomaly we obtain the probability of capture at different SMA in Figure 3. .

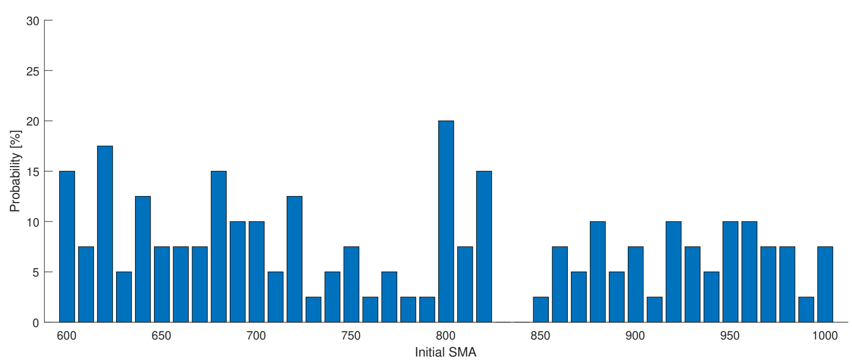


Figure 3: Probability of capture into 1 : 1 resonance for different initial SMA with a simplified dynamical model up to the fourth order harmonics

The simplified dynamics up to the fourth order shows a similar but noticeably different probability for each SMA. There are many spikes in the probability distribution, but there are also a reasonable amount of probabilities which remain unchanged from the complete model. For this reason the reduced fourth order model could be used for specific cases which should be defined beforehand through a Monte Carlo approach. Nevertheless, the mean probability of this distribution stays close to the value computed for the complete fourth order model with a value of 8.2658%.

Further, we simplify the model by considering the gravitational potential up to the second order and degree and this potential is given as

$$V_{1:1}^{2nd} = V_0 + V_{20} + V_{22} \tag{5}$$

For the same simulations, the probability of capture is given as in Figure 4.

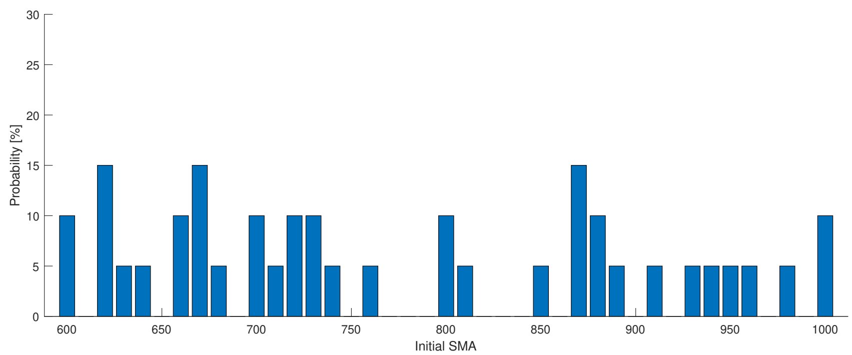


Figure 4: Probability of capture into $1:1$ resonance for different initial semi-major axis with a reduced dynamics up to the second order.

This result of the simplification demonstrates obvious difference from the previous two, which is reflected on the mean probability of capture. In fact, the mean of the probability distribution is estimated to be 5.5715%. This result shows that this model cannot be reliable for further analysis and this section's results are summarized in Table 2.

Table 2: Summary of the mean probabilities of capture for the complete and reduced fourth order model. $I_{sp} = 3000$s, $m = 1000$kg, $T = 20$mN, $\Delta a = 10$km, $\Delta\theta = 9°$and $\Delta t = 45$days.

Model	Probability
4th order Complete	7.3171%
4th order Simplified	8.2658%
2nd order	5.5715%

Effects of $1:1$ **resonance**

In this section, we evaluate how the 1:1 resonance affects the spacecraft's descent, in particular we focus on the eccentricity and inclination. It is worth noting that, other than the 1:1 resonance, the descent is influenced also by the 1:2 resonance and 2:3 resonance which are located at 871km and 717km respectively, as show in Figure 5. Considering the case in Figure 1, the evolution of the eccentricity and inclination is illustrated in Figure 6.

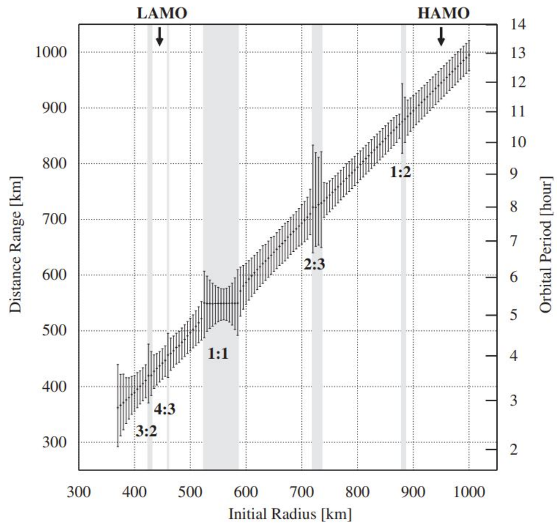


Figure 5: Location of the main resonances around Vesta. Source: Tricarico and Sykes[1]

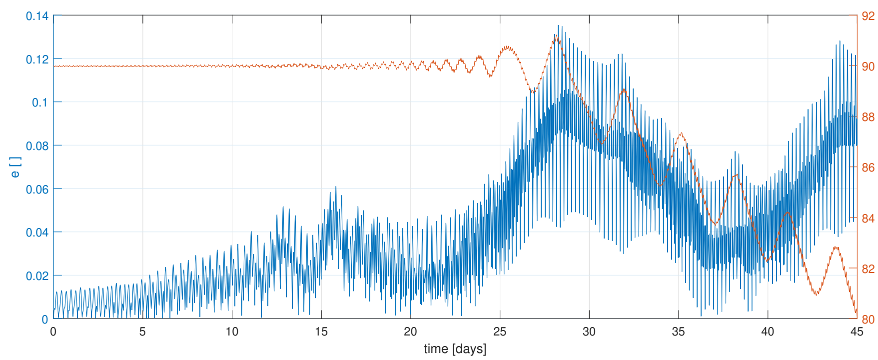


Figure 6: Eccentricity and inclination evolution during DAWN's descent.

Eccentricity The initial eccentricity has a value equal to 0 and it starts increasing around day 5. At this moment the spacecraft is located at below 900km from the asteroid and at this distance the dynamics is influenced by the 1:2 resonance which slightly increases the eccentricity. A major pump of the eccentricity is noticeable between 10 and 15 days, in which the spacecraft crosses the 2:3 resonance and it is highly influenced by it. As the spacecraft reaches the 1:1 resonance altitude in day 25, the eccentricity starts to change increasing its value the more the spacecraft stays trapped.

Inclination Following the same checkpoints, it is possible to notice that: the effect of the 1:2 resonance on the inclination is null as the spacecraft inclination remains unchanged; the 2:3 resonance causes the value of the inclination to start oscillating, without any major change; as the spacecraft crosses the 1:1 resonance the value of the inclination oscillates considerably while almost linearly decreasing its value.

Sensitivity of the capture probability on spacecraft properties

In this section, we analyze the sensitivity of the capture probability on spacecraft properties such as the specific impulse, its mass and the low-thrust. The typical value of specific impulse of electric thrust id 3000s for several different missions. The magnitude of the thrust varies over a range, e.g. 20mN for the DAWN spacecraft and 28mN for the recent Hayabusa 2 spacecraft. The spacecraft mass does not have a typical value since it depends on the mission and the payloads on board. We take the mass of DAWN as a reference and consider other two cases in which the mass has a smaller value.

The specific impulse The specific impulse for space mission using electric thrust typically ranges from 2000s to 3000s. For the sensitivity analysis, we use values of 2000s, 2500s and 3000s respectively. The values of the spacecraft mass and magnitude of the thrust values are fixed to 1000kg and 20mN respectively.

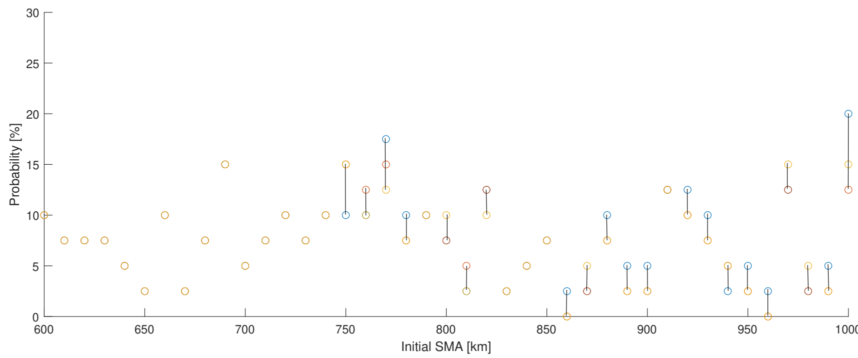


Figure 7: Probability of capture into 1:1 resonance for different initial semi-major axis considering different values of specific impulses. Blue: 2000s, red: 2500s and yellow: 3000s

By repeating the simulations in Section 2, we obtain the capture probability for three different specific impulse as Figure 7. It is seen that change in the specific impulse does change the probability of capture and a change tendency can be identified. For all the three cases, we locate the peak probabilities at 690km, 770km and 1000km. For initial SMA smaller than 750km, the probability of capture generally remains unchanged. Then the probabilities start to differentiate among the three cases, their difference generally do not exceed 2.5%. For the difference that goes up to 7.5%, the descent starts near the SMA at 850km.

The spacecraft's mass For this section, we take the mass of the DAWN spacecraft as reference, which was 1000kg at its arrival at Vesta. The other two mass values that we are using are 750kg and 875kg for analysis. The specific impulse and the magnitude of the thrust are fixed at 3000s and 20mN respectively.

We notice a great sensibility of the dynamics with respect to the mass change. In fact, the probability distribution changes at each altitude and the variation can differ up to 15% in some cases. The mean probability of capture of the three cases are 7.2561% for the 750kg case, 10.6098% for the 750kg case and 7.3171% for the 1000kg case. This result does not show any direct relation between mass and probability of capture.

The magnitude of the thrust The last property under our analysis focuses on the thrust sensibility. As in Figure 9, the change in the thrust value impacts noticeably the probability of capture for different orbital radii. In our simulations, we considered the following values for the thrust: 20mN, 22.5mN and 25mN. In this case the error spans from 0% to 17%, making the value of the thrust also crucial to analyze the probability of capture. But the error does not stay consistent, making sudden

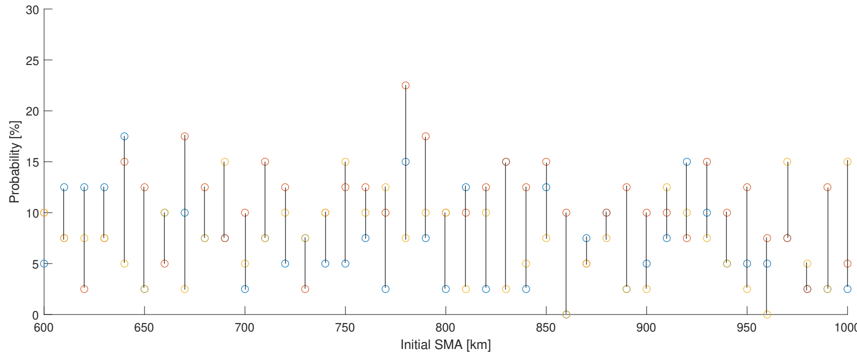


Figure 8: Probability of capture into 1:1 resonance for different initial semi-major axis considering different values of the spacecraft mass. Blue: 750kg, red: 875kg and yellow: 1000kg

changes along the different initial SMA.

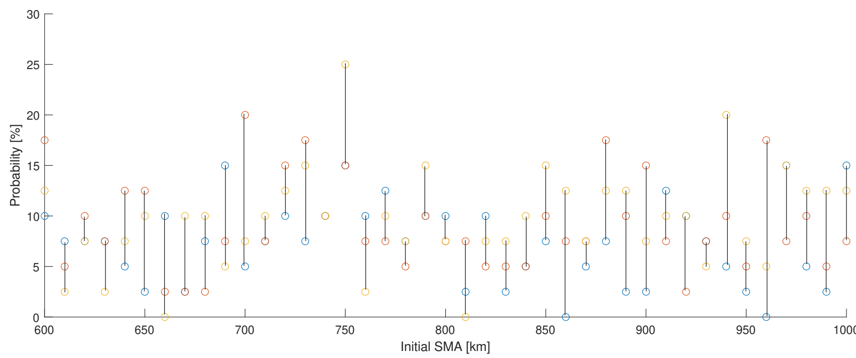


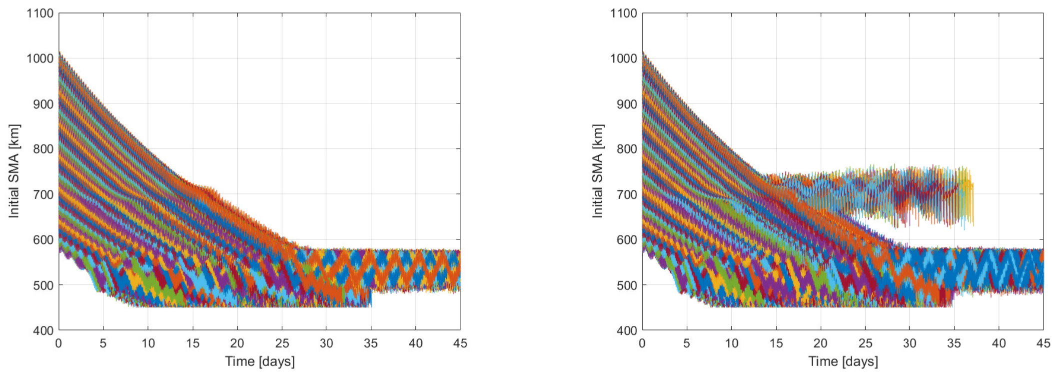
Figure 9: Probability of capture into 1:1 resonance for different initial semi-major axis considering different values of thrust. Blue: 20mN, red: 22.5mN and yellow: 25mN

For the different value of thrust the mean probability of capture results to be 7.3171% for the 20mN case (as previously calculated), 9.0854% for the 22.5mN case and, lastly, 9.5732% for the 25mN case. These simulation show a direct relationship between thrust thrust and mean probability of capture. However, more simulation with a better resolution should be made to better understand this relationship.

Sensitivity with respect to orbital geometries

Sensibility with respect to different orbit inclinations The previous research on the resonance capture around Vesta mainly focused on two cases: circular polar orbit and circular equatorial orbit. This section analyzes the resonance capture for orbits at a different inclination. We simulate orbits at different with the inclination of 85°that are slightly deviating from the polar orbit. Figure 10 shows the SMA evolution of all orbits with the same combination of SMA and true anomaly as Section 2, but for the inclination at 90°and 85°respectively.

It is noticed that for the initial inclination at 85°, the spacecraft is more at risk of being captured into the 2:3 resonance around 700km. The analysis of the probability of capture into the 2:3 reso-



((a)) SMA evolution with initial inclination $i = 90°$ ((b)) SMA evolution with initial inclination $i = 85°$

Figure 10

nance is out of the scope of this paper. As for the 1:1 resonance capture, the mean of the probability of capture is 6.6463%, while for the circular polar case the probability was estimated to be 7.3171%. Even if the probability is lower than the polar case, this analysis does not inform on which option is safer, as the spacecraft is more at risk of being trapped into the 2:3 resonance. In Figure 11 we illustrate the probability of capture for different SMA at 85°.

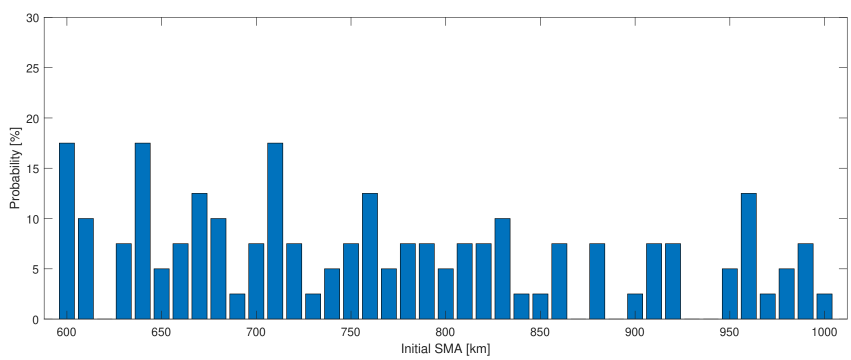


Figure 11: Probability of capture into $1:1$ resonance for different initial semi-major axis ($i = 85°$).

CONCLUSION

In this work, we expanded the work of the author who previously dealt with this problem. We analyzed if a model reduction was possible maintaining a good degree of accuracy. We considered the effects of the resonance on the eccentricity and inclination. Then, we showed how the results that were obtained in the previous sections are sensible to changes in the spacecraft properties and orbit geometry, mainly concentrating on the inclination. The simulations showed that: the model simplification up to the fourth order introduced anomalies in the probability distribution, while was accurate on many other points. This suggests that the simplified model up to the fourth order can be used for specific cases. Instead the simplification up to the second order completely underestimates the probability distribution and it is not suitable for any analysis. Also, we showed that the spacecraft's motion is influenced by all the resonance that crosses during its descent. While he 2:3 resonance strongly influenced the spacecraft's descent, the 1:1 resonance is the main cause of change in the orbit eccentricity and inclination. The spacecraft descent is slightly sensible when considering different values of specific impulses. Instead, it shows a strong sensibility on the change of spacecraft mass and the value of the thrust. Lastly, we showed that for non-polar orbits the spacecraft is at risk of being captured into the 2:3 resonance around Vesta.

ACKNOWLEDGEMENTS

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APPENDIX

The gravitational coefficient for Vesta are reported in Table 3.

Table 3: Gravitational coefficient for Vesta[4]

Coefficients	Vesta
C00	1.0
C20	-6.872555 $\times 10^{-2}$
C21	0.0
S21	0.0
C22	3.079667 $\times 10^{-3}$
S22	0.0
C30	6.286305$\times 10^{-3}$
C31	-7.982112 $\times 10^{-4}$
S31	1.825409 $\times 10^{-4}$
C32	-3.162892 $\times 10^{-4}$
S32	5.943231 $\times 10^{-5}$
C33	2.565757$\times 10^{-5}$
S33	7.264998$\times 10^{-5}$
C40	9.6 $\times 10^{-3}$
C41	6.394125$\times 10^{-4}$
S41	-1.347130$\times 10^{-4}$
C42	-3.152856 $\times 10^{-5}$
S42	6.551679 $\times 10^{-5}$
C43	-3.113571$\times 10^{-5}$
S43	-2.689264$\times 10^{-6}$
C44	3.190457 $\times 10^{-6}$
S44	5.514632 $\times 10^{-6}$

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