Debt structure and debt overhang [*]

Liu Gan[†] Xin Xia[‡] Hai Zhang[§]

April 20, 2022

Abstract

We study the impact of heterogeneous debt structures on corporate financing and investment decisions in a dynamic trade-off model. The issuance of bank debt along with market debt accelerates investment and mitigates the ex-post debt overhang relative to exclusive market debt structures. A growth firm optimally increases its reliance on bank debt and decreases its usage of market debt when it has fewer valuable growth opportunities, its asset volatility is higher, its bankruptcy cost is lower, or it faces a low tax rate environment. We identify the non-monotonic effects of the cyclicality of growth opportunities on firms' optimal debt composition.

JEL Classification: G13; G32; G33

Keywords: Debt Structure; Corporate Investment; Financing; Debt Overhang

---

[*] We sincerely thank the editor Heitor Almeida and the anonymous referee for helpful comments. Any remaining errors are ours. Research presented in this paper was supported by the National Natural Science Foundation of China (Project No. 72161016), the Fundamental Research Funds for the Central Universities, Zhongnan University of Economics and Law (Project No. 2722021BZ055), the Natural Science Foundation of Jiangxi Province (Project No. 20202BABL201002), and the Innovation and Talent Base for Digital Technology and Finance (Project No. B21038).

[†] Newhuadu Business School, Minjiang University, Fuzhou, China. Email: ganliu2016@163.com

[‡] Corresponding author. School of Finance, Zhongnan University of Economics and Law, Wuhan, China. Email:doublexinxiao@yeah.net

[§] Department of Accounting and Finance, Strathclyde Business School, UK. Email:hai.zhang@strath.ac.uk

1 Introduction

In corporate finance theory, most existing capital structure models assume that: (i) firms are optimally financed by a single type of debt that can force the firm into immediate liquidation in default (i.e., public or market debt) due to the dispersion of its creditors (Leland, 1994, 1998; Hackbarth and Mauer, 2012; Diamond and He, 2014; Sundaresan, Wang, and Yang, 2015; Bolton, Wang, and Yang, 2020; Hackbarth, Rivera, and Wong, 2021), which ignores the stylized fact that firms often simultaneously use different types of debt in practice; and (ii) the capital structure choice for a firm with heterogeneous (mixed) debt financing is independent of its real investment decisions (Bolton and Freixas, 2000; Park, 2000; DeMarzo and Fishman, 2007; Hackbarth, Hennessy, and Leland, 2007), which overlooks the most important ex-post debt overhang problem first analyzed by Myers (1977). Empirically, extensive studies have recognized heterogeneity in the sources of debt financing (Rauh and Sufi, 2010; Colla, Ippolito, and Li, 2013; Becker and Ivashina, 2014; Chen, Maslar, and Serfling, 2020) and the negative impact of debt-in-place on corporate investment (Hennessy, 2004; Alanis, Chava, and Kumar, 2018).

Notably, unlike market debt, bank (private or negotiable) debt can generally be renegotiated with a single creditor, delaying inefficient and costly bankruptcy when a firm is in financial distress (see Lummer and McConnell, 1989; Gilson, John, and Lang, 1990; Fan and Sundaresan, 2000; Sundaresan and Wang, 2007). However, a large body of theoretical studies has treated these two debt instruments as uniform within a firm, providing no insights into the precise roles of heterogeneous debt structures in determining corporate policies and the substantial capital structure variation documented in practice.[1] Remarkably, Morellec, Valta, and Zhdanov (2015) and Shibata and Nishihara (2015b) incorporate both investment decisions and various debt structures into trade-off capital structure models to investigate the intertwined effects between a firm's marginal financing decisions (market versus bank debt) and its investment decisions. Nevertheless, neither considers the simultaneous usage of both market and bank debt within a firm, thus overlooking the possible interactions of these two debt instruments.[2]

Motivated by corporate finance theory and empirical evidence, this paper develops a theoretical link between heterogeneous debt structures and corporate financing and investment policies. In particular, we adopt a setting that resembles that of Hackbarth, Hennessy, and Leland (2007) and extend their discussions by incorporating a series of flexible investment opportunities, in the spirit of Diamond and He (2014), DeMarzo and He (2021), and Wong and Yu (2022). Specifically, we consider a firm that has both assets-in-place and flexible future investment opportunities. The firm is initially financed with an optimal mixture of equity, senior bank debt and junior market debt, maximizing

---

[1] Theoretical studies have often treated corporate debt as uniform because they want to build more tractable models (Rauh and Sufi, 2010).

[2] Both Morellec, Valta, and Zhdanov (2015) and Shibata and Nishihara (2015b) assume that a firm makes discrete financing choices between market and bank debt in a real options framework.

the ex ante value of equity (Hackbarth, Hennessy, and Leland, 2007). A default on bank debt leads to a renegotiation with a permanent reduction of coupon payments and a corresponding proportion of the equity stake being paid to compensate bank lenders, i.e., an irreversible debt-for-equity swap to retire some existing bank debt in the spirit of Morellec et al. (2015). Similar to Leland (1994), the maturity of private and public debt contracts is perpetual. At any future time, shareholders can make an all-or-nothing decision regarding the growth of assets-in-place, and they have the timing option to default on debt obligations, leading to either renegotiation or formal bankruptcy. Asset growth is financed by internal funds (or equity) (Diamond and He, 2014; DeMarzo and He, 2021; Wong and Yu, 2022), facilitating the analysis of debt overhang, i.e., the pure negative effect of debt in place on corporate investment.

Shareholders generally prefer debt renegotiation to formal bankruptcy, implying that bank debt might always be superior to market debt. However, the opposite might occur when our model considers shareholders' bargaining power in default (Fan and Sundaresan, 2000; Sundaresan and Wang, 2007; Davydenko and Strebulaev, 2007; Shibata and Nishihara, 2015a,b; Wong and Yu, 2022) and renegotiation frictions (Davydenko and Strebulaev, 2007; Favara, Schroth, and Valta, 2012; Morellec, Valta, and Zhdanov, 2015). Private bank debt delays inefficient and costly bankruptcies during financial distress should renegotiation be successful. Nevertheless, bank debt does not always dominate market debt, especially when shareholders have stronger bargaining power or the renegotiation friction is larger, further justifying demandable market debt issuance. Thus, our model allows us to explore optimal debt composition within a firm, significantly differing from existing studies focusing on the choice of bank debt versus bank debt (Morellec, Valta, and Zhdanov, 2015; Shibata and Nishihara, 2015b).

In our model setting, a firm's optimal debt structure is driven by the trade-off among tax shield benefits, bankruptcy costs, and debt overhang costs. To further emphasize the main implications of the interaction between debt heterogeneity and future growth opportunities, on the one hand, we construct two extreme benchmark models, each considering either an exclusive bank debt structure or an exclusive market debt structure. On the other hand, we provide another benchmark model in which the firm never invests, and thus its asset growth rate features an exogenous constant. We then focus on the following questions that existing trade-off models leave unresolved. First, how do growth firms financed with a combination of bank and market debt choose their ex-post corporate policies in the presence of debt overhang problems analyzed by Myers (1977)? What factors drive such choices? Second, how will firms' optimal debt composition be determined in the presence of a series of future growth opportunities? Finally, how do firm characteristics affect the interaction between investment dynamics and debt structure decisions?

Our research emphasizes the negative effect of debt overhang on the equity incentives to undertake investments under a heterogeneous debt structure. Many theoretical studies with homogeneous market debt structures, such as Mauer and Ott (2000), Hennessy (2004), Hackbarth and Mauer

(2012), Diamond and He (2014), Sundaresan, Wang, and Yang (2015), and Chen and Manso (2017), have elaborated on debt overhang in corporate finance from the perspective of different market frictions. Our main contribution to this line of research is to introduce bank debt in addition to market debt and to provide new predictions concerning the role of bank debt in the corporate capital structure.[3]

We show that compared with the widely used exclusive market debt structure, mixed bank and market debt financing accelerates firm investment and thus mitigates the ex-post debt overhang problem. The intuition underlying the mechanism of this result differs from the direct wealth transfer effect on investment timing investigated in Mauer and Ott (2000), Hackbarth and Mauer (2012), Sundaresan, Wang, and Yang (2015), and Chen and Manso (2017) with a single irreversible investment option in the spirit of McDonald and Siegel (1986); our model follows Diamond and He (2014) and captures the indirect effects of debt overhang by which the inclusion of bank debt delays bankruptcy and preserves more future investment opportunities, leading to a larger marginal impact of the firm's assets-in-place on the equity value, thus strengthening shareholders' current investment incentives (i.e., setting a lower investment threshold).[4]

After demonstrating the effects of heterogeneous debt financing on shareholders' investment incentives, we then address how the firm's ex-post growth opportunities affect its ex ante optimal choice of debt composition. In general, firms with growth opportunities tend to optimally increase their usage of market debt and reduce their reliance on bank debt in total debt relative to an otherwise identical firms with no growth opportunities. This result reflects the trade-off between the tax shield benefits of market debt and the cost reduction of bankruptcy and debt overhang from bank debt.[5] Furthermore, this change in debt composition becomes more pronounced when growth firms have more valuable growth opportunities. Additionally, we provide several insights into the determinants of the optimal debt composition across other firm characteristics, the changing of which affects the attractiveness of senior bank debt and junior market debt. Notably, we find that

---

[3] Many theoretical studies have particularly emphasized the role of bank debt in a firm's capital structure because strong covenant restrictions in bank debt contracts can help firms to achieve a less severe moral hazard problem (see, e.g., Park, 2000; DeMarzo and Fishman, 2007).

[4] We thank the anonymous referee for noting that the debt overhang effect in our model, with a series of reversible investment opportunities in the spirit of Diamond and He (2014), is significantly different from that investigated by the model with a single irreversible investment in the spirit of McDonald and Siegel (1986). In our model, "activating the investment does not lead to the wealth transfer from shareholders to debtholders in the same way a lumpy irreversible investment does as default occurs at the same level of the cash flow process before and after investment. Therefore, investment timing is solely related to the curvature of the shareholders' value function and to the moment its slope is sufficiently steep to trigger the flow of investment. In general, renegotiable debt, which allows for deferring liquidation for longer, would generally result in a higher slope of the shareholders' value function for a given cash flow level, hence fostering efficient investment."

[5] Bank debt also has the shield benefits, but they are smaller than those of market debt due to the debt-for-equity swap during renegotiation.

higher (lower) asset volatilities, lower (higher) bankruptcy costs and lower (higher) corporate tax rates enhance a firm's reliance on senior bank debt (subordinated market debt) in total debt. These model predictions reconcile several empirical findings (Johnson, 1997; Rauh and Sufi, 2010) and yield a set of novel empirical tests of the joint determinants of a firm's debt types.

Our model also sheds light on two additional theoretical implications and thus generates novel empirical tests. First, we find a U-shaped relationship between shareholders' investment incentives and debt renegotiation frictions for growth firms with mixed debt financing. The non-monotonic relationship is driven by the variation in the debt renegotiation friction regarding shareholders' trade-offs between the positive effect of the reduction in total debt and the negative effect of the change in debt composition (i.e., decreased reliance on bank debt and increased usage of market debt) when making an ex-post investment decision. To the best of our knowledge, the linkage between debt renegotiation frictions and a growth firm's choice of investment has rarely been discussed in the literature. As a result, this finding offers a testable hypothesis on the empirical prediction regarding the relationship between debt renegotiation frictions and shareholders' investment incentives based on the role of debt heterogeneity in corporate capital structure decisions.[6]

Second, we extend our baseline model by assuming that the cost of investment is time varying, representing that the firm faces time-varying growth opportunities. In such an environment, we provide a testable prediction of how the cyclicality of a firm's growth opportunities affects its optimal debt composition. That is, our extended model predicts that firms with stronger cyclicality of growth opportunities tend to first decrease (increase) and then increase (decrease) their reliance on bank (market) debt in total debt, exhibiting a (an inverted) U-shaped pattern. Theoretically, our extended model also complements the discussions in Chen and Manso (2017), who employ an irreversible real options model of the type described by McDonald and Siegel (1986) and investigate the impact of time-varying growth opportunities on a firm's capital structure decisions with homogeneous market debt structures.

Related literature To emphasize the contributions of our article, following Hackbarth, Hennessy, and Leland (2007), we summarize the features of related structural trade-off models in Table 1. Specifically, this table collects the two main strands of the recent literature on debt financing (columns (1-2)) and dynamic trade-off models with endogenous investment (column (5)).

Our paper contributes to the growing literature examining the interaction between corporate financing and investment decisions associated with debt overhang in dynamic trade-off models with exclusive market debt financing described by Leland (1994). Hackbarth and Mauer (2012) develop a dynamic model to study debt structure and its impact on corporate investment and find that jointly optimal capital and priority structures can virtually eliminate the debt overhang that results from

---

[6] Favara, Schroth, and Valta (2012) empirically explore the relationship between equity beta and debt renegotiation frictions across countries.

agency conflicts. Diamond and He (2014) provide a thorough analysis of the effects of debt maturity on the debt overhang problem. Sundaresan, Wang, and Yang (2015) develop an analytically tractable model in which a firm has multiple rounds of investment with financing flexibility, and they emphasize that debt structures have quantitatively significant implications for debt overhang. Furthermore, Chen and Manso (2017) offer several new predictions on how the presence of macroeconomic risk affects a firm's investment and capital structure decisions. Our work complements these studies by incorporating heterogeneous debt financing into the analysis of corporate financing and investment decisions.

Table 1: Related structural trade-off models

	Negotiable debt (1)	Market debt (2)	Bargaining power (3)	Closed-form solution (4)	Investment flexibility (5)
Leland (1994)	N	Y	N	Y	N
Fan and Sundaresan (2000)	Y	N	Y	Y	N
Hackbarth et al. (2007)	Y	Y	Y	Y	N
Sundaresan and Wang (2007)	Y	N	Y	Y	N
Pawlina (2010)	Y	N	Y	Y	N
Hackbarth and Mauer (2012)	N	Y	N	Y	N
Diamond and He (2014)	N	Y	N	Y	Y
Morellec, Valta, and Zhdanov (2015)	Y	Y	Y	Y	N
Shibata and Nishihara (2015a)	Y	N	Y	Y	N
Shibata and Nishihara (2015b)	Y	Y	Y	Y	N
Sundaresan, Wang, and Yang (2015)	N	Y	N	Y	N
Chen and Manso (2017)	N	Y	N	Y	N
Wong and Yu (2022)	Y	N	Y	Y	Y
This Paper	Y	Y	Y	Y	Y

In addition, our paper ties into a strand of the literature that explores the role of exclusive negotiable debt financing in firm decision making. Fan and Sundaresan (2000) adopt a Nash bargaining game for debt renegotiation to investigate the impact of varying bargaining power on renegotiation and payout policies. Sundaresan and Wang (2007) further employ a real-options framework and explore how the possibility of debt renegotiation during financial distress impacts ex ante growth-option-exercising decisions, debt capacity and firm value. Pawlina (2010) shows that the presence of negotiable debt exacerbates a firm's underinvestment problem. By incorporating debt financing constraints into the model framework of Sundaresan and Wang (2007), Shibata and Nishihara (2015a) examine the effects of negotiable debt on a firm's investment decisions. However, these studies describe such investment as either an irreversible and one-shot decision or an exogenous policy. Recently, based on the model framework of Diamond and He (2014), Wong and Yu (2022) investigate the real effect of credit default swap (CDS) trading on investment dynamics and debt financing in a dynamic trade-off model with negotiable debt and flexible investment opportunities.

Our article is closely related to Hackbarth, Hennessy, and Leland (2007), who employ the classical Leland-type capital structure framework and explore the optimal mixture of bank and market debt, in the following two respects. First, inspired by Diamond and He (2014) and Wong and Yu (2022), we incorporate flexible investment decisions into Hackbarth, Hennessy, and Leland's model. Since the debt structure and investment decisions interact with one another, the optimal investment timing varies under different debt structures. Thus, it is necessary to incorporate the investment decision into a dynamic trade-off model. Second, we consider an irreversible debt-for-equity swap with renegotiation frictions to reflect the situation that we are seeking and capture the feature of bank debt contracts in the model, which is different from Hackbarth, Hennessy, and Leland's model that uses strategic debt service with costless renegotiation. This setting allows us to yield a rich set of novel insights and empirical predictions on heterogeneous debt financing.

Finally, the present work is also closely related to a set of papers that study the effects of various debt structures on corporate policy choices in a unified framework. Typically, Morellec, Valta, and Zhdanov (2015) investigate the choice between bank and market debt in a firm's marginal financing decision and its effects on investment decisions. Shibata and Nishihara (2015b) develop a model to study the impact of bank and market debt structure under issuance limit constraints on corporate financing and investment decisions. However, neither Morellec, Valta, and Zhdanov nor Shibata and Nishihara consider mixed debt financing since the debt choice in their models is assumed to be mutually exclusive. To the best of our knowledge, this paper is the first attempt to investigate the interaction between debt heterogeneity and investment dynamics in a structural trade-off model in which firms simultaneously use bank and market debt. In this manner, we can jointly determine the optimal composition of bank and market debt in our model and shed light on how optimal mixed debt financing affects corporate investment and valuation.

The remainder of this article is organized as follows. Section 2 develops a dynamic trade-off model that incorporates both flexible investment opportunities and heterogeneous debt financing. Section 3 solves the model, and Section 4 provides key model implications. Section 5 explores the impact of time-varying investment opportunities on firms' mixed debt financing choices, followed by a summary of novel empirical predictions in Section 6. Finally, Section 7 concludes the study. Technical information is gathered in Appendix A.

2 Model setup

Following Hackbarth, Hennessy, and Leland (2007) and Diamond and He (2014), we extend the classical Leland-type capital structure framework to a dynamic structural model that simultaneously incorporates both dynamic investment opportunities and heterogeneous debt financing. Throughout the paper, we assume that assets can be continuously traded in a complete and arbitrage-free market.

2.1 Firm cash flows and investment opportunities

Consider a firm with assets-in-place that is endowed with dynamic investment opportunities to expand its assets at any time t. The assets-in-place generate an operating cash flow, which is given by $\{X_t : t \geq 0\}$ and evolves according to[7]

$$dX_t = (\mu + \bar{i}(X_t))X_t dt + \sigma X_t dZ_t, \tag{1}$$

where constants μ and σ are the baseline growth rate and volatility of the cash flow, respectively, and $\{Z_t : t \geq 0\}$ is standard Brownian motion defined for the risk-neutral probability space, $(\Omega, \mathcal{F}, \mathbb{P})$. At each point in time, the firm's asset growth rate is subject to investment rate $\bar{i}(X)$, which is dependent on the current cash flow level X. Notably, we follow Diamond and He (2014) and Wong and Yu (2022) and assume that the investment policy is controlled by shareholders and takes a binary constant value, in that

$$\bar{i}\,(X) = \begin{cases} i, & if \quad X \geq X_i; \\ 0, & if \quad X < X_i. \end{cases} \tag{2}$$

Here, X_i denotes the investment threshold, which indicates that a firm invests (cuts investment) when its cash flow level exceeds (is less than) X_i. The random first passage time, T_i, that the firm's cash flow level reaches the threshold, X_i, satisfies

$$T_i = \inf\{t \geq 0, X_t = X_i\}.$$

In addition, investment incurs a cost, which is given by $\phi\bar{i}(X)X$ and is financed by internal funds (retained earnings)(Diamond and He, 2014).

Let the constant $r > 0$ denote the risk-free interest rate. Following Wong and Yu (2022) and Diamond and He (2014), the after-tax present value of an all-equity-financed firm that always invests satisfies

$$\mathbb{E}_t\left[\int_t^{\infty} e^{-r(u-t)}((1-\tau)X_u - \phi i X_u)du\right] = U_i X_t, \tag{3}$$

where $U_i \equiv \frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)}$ and $\tilde{\phi} \equiv \frac{\phi}{(1-\tau)}$; $\tau \in (0,1)$ denotes the corporate tax rate. For convergence, we assume that the interest rate is higher than the maximal (risk-neutral) cash flow growth rate in that $r > \mu + i$. Conversely, when a firm never invests, its present value of assets is given by $U_0 X_t$, where $U_0 \equiv \frac{1-\tau}{r-\mu}$. As shown in Wong and Yu (2022), we impose the parametric restriction to ensure that $\Pi \equiv U_i/U_0 > 1$, which captures the positive marginal benefit of investment and thus measures the profitability of the firm's future investment opportunities.[8]

---

[7] This cash flow specification is similar to that used in the dynamic investment models of Diamond and He (2014) and Wong and Yu (2022).

[8] This ratio $\Pi \equiv U_i/U_0$ can also be interpreted as the all-equity-financed firm's Tobin's Q (see Wong and Yu, 2022).

2.2 Debt structure and default

By featuring debt heterogeneity, our model assumes that a firm simultaneously issues two classes of perpetual debt, namely bank and market debt, at the initial time.[9] Bank debt and market debt have promised coupon payments b and c, respectively. A firm that falls into financial distress (i.e., experiences a credit event) in the future may default on its existing debt obligations. Because of limited liability, suppose that shareholders first attempt to initiate a private workout instead of declaring bankruptcy to renegotiate the coupon payment of bank debt upon default. Let T_n be the first passage time of the firm's cash flow, X_t, through the endogenous renegotiation threshold, X_n, from above, which is given by

$$T_n = \inf\{t \geq 0, X_t = X_n\}.$$

Debt restructuring allows a firm to deleverage by removing a fraction $m \in [0, 1)$ of the coupon payment b at the expense of paying a corresponding fraction ρ of the total equity to bank lenders (i.e., debt-for-equity swaps).[10] The reduced coupon payment for bank lenders after debt restructuring is denoted by $b_n = (1 - m)b$.[11] We further assume that the debt renegotiation process is formalized as Nash bargaining, consistent with Fan and Sundaresan (2000), Sundaresan and Wang (2007), and Wong and Yu (2022). Because of bargaining, both shareholders and bank lenders can obtain a proportion of the total surplus according to their bargaining powers η and $1 - \eta$, respectively. In contrast, the coupon payment to market lenders cannot be modified outside the formal bankruptcy process; this feature is presented by Leland (1994) and supported by the empirical evidence in Gilson et al. (1990) and Asquith et al. (1994).[12]

The restructuring of bank debt entails a cost and thus might fail.[13] Our model captures such frictions by defining the probability of failure, $q \in [0, 1]$, of debt renegotiation, as discussed in Davydenko and Strebulaev (2007), Favara, Schroth, and Valta (2012), and Morellec, Valta, and

---

[9] In the model, we consider a one shot capital structure decision at the initial time, which is a standard assumption in the capital structure literature. See, for example, Leland (1994), Hackbarth, Hennessy, and Leland (2007), and Morellec, Valta, and Zhdanov (2015).

[10] Because debt structure and investment dynamics are central to our analysis, we do not consider the optimal terms of the bank debt contract in our model, and we calibrate the parameter value of m based on theoretical estimation and empirical evidence to the extent that it is available.

[11] Morellec, Valta, and Zhdanov (2015) also introduce a debt-for-equity swap to capture the feature of the bank debt contract. In their model, unlike ours, all existing bank debt is removed with an equity stake when firms are in a period of financial distress. However, some empirical evidence has documented that bank lenders typically receive a combination of permanent debt reduction and an equity stake in exchange for their debt (see, e.g., Franks and Torous, 1994; Altman and Karlin, 2009).

[12] Due to the dispersion of debt holders, market debt cannot be renegotiated when a firm falls into financial distress, which is the only difference between market debt and bank debt in our model (see, e.g., Morellec, Valta, and Zhdanov, 2015; Shibata and Nishihara, 2015b).

[13] Restructuring costs include various costs during both out-of-court renegotiation and formal bankruptcy, such as legal costs, loss of reputation, accounting expenses, and so forth (see, e.g., Favara, Schroth, and Valta, 2012).

Zhdanov (2015). A higher q implies a more costly renegotiation process, costs that are borne by shareholders. Once renegotiation fails, the shareholders declare bankruptcy, and the firm is liquidated. In such a scenario, the absolute priority rule (APR) is enforced, and the outside option value for shareholders is zero. Furthermore, we assume that the new owner will not re-level the restructured firm and lose any future investment opportunities when a liquidation event occurs (see, for example, Leland, 1994, 1998). Let $\alpha \in (0, 1)$ denote the bankruptcy loss rate. At bankruptcy, both market debt holders and bank debt holders then jointly share liquidation value $L(X_t)$, which is equal to

$$L(X_t) = (1 - \alpha)U_0 X_t. \tag{4}$$

In this model, we assume that bank debt is senior to market debt upon bankruptcy, consistent with the theoretical prediction in Hackbarth, Hennessy, and Leland (2007) and the empirical evidence reported by Rauh and Sufi (2010). That is, lenders of junior market debt will not receive payoffs until senior bank lenders are fully paid off at bankruptcy. Thus, senior bank debt holders receive reservation value $R_b(X_t)$, which is given by

$$R_b(X_t) = \min\left\{\frac{b}{r}, L(X_t)\right\}, \tag{5}$$

while junior market debt holders collect the remaining value $R_m(X_t)$, which satisfies

$$R_m(X_t) = \max\left\{L(X_t) - \frac{b}{r}, 0\right\}. \tag{6}$$

For simplicity, we assume that shareholders and bank lenders renegotiate only once. We could generalize the model to a case with multiple (finite) costly renegotiations (see, e.g., Moraux and Silaghi, 2014). However, in addition to some quantitative (but not qualitative) differences, we believe that our main results also hold in such a setting. Then, provided that debt renegotiation is successful, the firm continues to operate with the restructured total coupon payment $(b_n + c)$. However, the firm could still default on its restructured debt obligations if its fundamental condition deteriorates. The first passage time of the firm's cash flow, X_t, through the endogenous bankruptcy threshold, X_d from above, is defined as

$$T_d = \inf\{t \geq 0, X_t = X_d\}.$$

At this time, the firm is formally liquidated, and outside debt holders receive the corresponding liquidation value.

3 Model solution

The model is solved by backward induction. At time 0, the firm's assets-in-place generate a cash flow X_0, and it issues both senior bank debt and junior market debt. Thereafter, shareholders can make decisions regarding renegotiation, future investment and bankruptcy. Note that the renegotiation threshold X_n splits firm's operations into two stages: before debt restructuring and after debt restructuring. The investment threshold X_i also divides a firm's cash flow into the two regions of an investment region for $X_t > X_i$ with $\bar{i}(X) = i$ and a non-investment region for $X_t < X_i$ with $\bar{i}(X) = 0$. We follow Wong and Yu (2022) and assume that the ordering of the firm's optimal thresholds is described as follows:

$$X_d < X_i < X_n < X_0.$$

Although this ordering is endogenous, it can be easily satisfied with our calibrated model parameters. Intuitively, shareholders would cut investment at any future time if the firm falls into financial distress. However, as discussed in Wong and Yu (2022), they would prefer to negotiate down the debt burden first and keep the investment alive in this situation because investment creates persistent value in this modeling environment.

Based on the firm's decision-making process, Figure 1 plots five possible paths that a firm might take during its life cycle. At $t = 0, X_0$, there are two scenarios regarding the firm's performance. The first path shows the best outcome, as the firm grows at rate $\mu + i$ and does not experience any financial distress. The second path reflects financial distress if the firm's cash flow continues declining, where debt restructuring occurs at $t = 30, X_n$. After debt restructuring, we assume that no further debt renegotiation occurs, and there are three scenarios. First, the decreased debt burden may allow the firm to escape financial distress; thus, the firm's cash flow continues rising along the third path. Second, the fourth path shows that debt restructuring could lead to a more severe cash-flow reduction, and the firm might choose to cut investment at $t = 50, X_i$. Subsequently, if the firm's cash flow level can increase to meet the threshold X_i, shareholders would optimally exercise growth opportunities again to increase their asset growth rate, expecting that the cash flow will continue rising along the fifth path. However, if the firm's fundamental condition continues deteriorating along the fourth path, it ultimately chooses to file for bankruptcy and loses any future investment opportunities to increase the asset growth rate at $t = 70, X_d$.

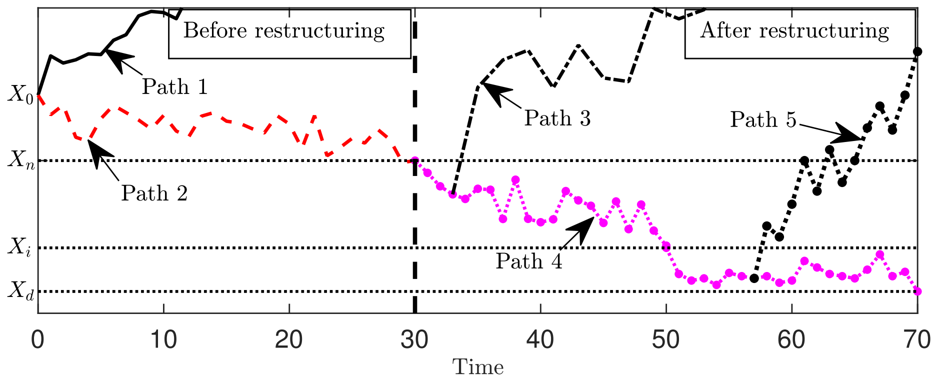

Figure 1: This figure shows five possible paths that firms might take during their life circle.

Let $E_n(X;b_n,c)$, $D_n(X;b_n,c)$, and $M_n(X;b_n,c)$ denote the new values of equity, bank and market debt after restructuring, respectively. Then, let us denote by $E(X;b,c)$, $D(X;b,c)$, and $M(X;b,c)$ the values of equity, bank and market debt before restructuring, respectively. The total firm value is therefore defined by the sum of the values of equity, bank and market debt.

3.1 After the debt restructuring stage

3.1.1 Equity value and equity holders' decisions

After debt restructuring ($T_d > T > T_n$), the total equity value function $E_n(X)$ captures the ex-post optimal investment and bankruptcy decisions selected by shareholders. The after-tax cash flow net of the investment cost (i.e., dividend) to shareholders is $(1-\tau)(X_t-c-b_n)-\phi\bar{i}(X_t)X_t$ at each time t. Moreover, the capital gains over each time interval $(t, t+dt)$ are given by $\mathbb{E}[dE_n]$, where $\mathbb{E}[\bullet]$ is an expectation operator. By applying the dynamic programming method and It$\hat{o}$'s lemma, we have that the market value of equity $E_n(X)$ satisfies the following ordinary differential equation (ODE):

$$rE_n(X)=\max_{\bar{i}(X)\in\{0,i\}}\left\{(1-\tau)(X-c-b_n)-\phi\bar{i}(X)X+(\mu+\bar{i}(X))XE_n'(X)+\frac{\sigma^2}{2}X^2E_n''(X)\right\}. \tag{7}$$

In Equation (7), the no-arbitrage rule implies that, at any time interval, the required return for holding a firm's equity (the left-hand side) should be equal to the sum of the after-tax cash flow net of the investment cost (the first two terms on the right-hand side) and the expected change in the equity value (the last two terms). The first-order condition (FOC) for investment $\bar{i}(X)$ generates the optimal investment threshold that satisfies the following:

$$E_n'(X_i)=\phi. \tag{8}$$

Equation (7) is simultaneously solved by using the following four boundary conditions:

$$\begin{cases} lim_{X\to\infty}\frac{E_n(X)}{X}<+\infty, & \text{the standard no-bubble condition;}\\ lim_{X\downarrow X_i}E_n\left(X\right)=lim_{X\uparrow X_i}E_n\left(X\right), & \text{the value-matching condition at } X_i;\\ lim_{X\downarrow X_i}E_n'\left(X\right)=lim_{X\uparrow X_i}E_n'\left(X\right), & \text{the non-arbitrage condition at } X_i;\\ lim_{X\downarrow X_d}E_n\left(X\right)=0, & \text{the value-matching condition at } X_d. \end{cases} \tag{9}$$

In Equation (9), because the possibility of bankruptcy in the normal region is negligible when X is sufficiently large, we impose the no-bubble condition for equity to exclude speculative bubbles. Value matching at the investment threshold X_i is a continuity condition. The non-arbitrage condition at the investment threshold X_i requires that the first derivative of the equity value on the left-hand side at X_i be equal to the first derivative of the equity value on the right-hand side at X_i.[14] The

---

[14] Please see the detailed discussion of the non-arbitrage condition in Dumas (1991).

value-matching condition at the default threshold X_d demonstrates the equality between the equity value after debt restructuring and the recovery value at final bankruptcy.

For the bankruptcy threshold, the smooth pasting of $E_n(X)$ at X_d delivers the following endogenous bankruptcy threshold (see, e.g., Leland, 1994):

$$E_n'(X_d) = 0. \tag{10}$$

The endogenous default threshold X_d is uniquely determined by a nonlinear equation that is solved numerically. Then, the equity value is summarized as follows.

Proposition 1 *Given the shareholders' optimal investment threshold X_i and optimal default threshold X_d, after debt restructuring, the equity value $E_n(X)$ is given by*

$$E_n\left(X\right) = \begin{cases} U_iX - (1-\tau)\frac{c+b_n}{r} + A_1X^{\gamma_1}, & \text{if} \quad X_i \le X, \\ U_0X - (1-\tau)\frac{c+b_n}{r} + A_2X^{\beta_1} + A_3X^{\beta_2}, & \text{if} \quad X_d < X < X_i. \end{cases} \tag{11}$$

Constants γ_1, β_1, β_2, A_1, A_2, and A_3 are defined in Appendix A.1.

The expression for the equity value is intuitive. The firm in the after debt restructuring stage ($T > T_n$) faces two different regions depending on whether shareholders stop investing. First, in the interval $[X_i, +\infty)$, the equity value consists of three components. The first two terms denote the expected present value of the unlevered firm that always invests net of the expected present value of the perpetual after-tax total coupon payment when there is no default risk. The last term captures the value adjustments when the firm enters the non-investment region in the future. Second, in the interval $[X_d, X_i)$, the firm is in the non-investment region, and the market value of equity consists of four components. The first two terms are the expected present value of the unlevered firm that never invests net of the expected present value of the perpetual after-tax total coupon payment when there is no default risk. The last two terms capture the changes in value by considering both possible future defaults and re-entering the investment region again.

3.1.2 Values of market debt and bank debt

We derive the market debt value in the after debt restructuring stage ($T_d > T > T_n$) as follows. Market lenders receive coupon payment c each time. Moreover, they also receive capital gains $\mathbb{E}[dM_n]$ over each time interval $(t, t + dt)$. On the basis of the standard variation principle, the market debt value $M_n(X)$ is solved by the following ODE:

$$rM_n(X) = c + (\mu + \bar{i}(X))XM_n'(X) + \frac{\sigma^2}{2}X^2M_n''(X), \tag{12}$$

which is dependent on the shareholders' optimal bankruptcy threshold X_d, optimal investment policy $\bar{i}(X)$ and the corresponding investment threshold X_i. In Equation (12), the left-hand side represents the return required by market debt holders, while the right-hand side is the sum of debt service (the first term) and the expected change in the value of market debt at any time interval (the last two terms).

Equation (12) is solved subject to the following four boundary conditions:

$$\begin{cases} lim_{X\to\infty} M_n(X) = \frac{c}{r}, & \text{the standard no-bubble condition;} \\ lim_{X\downarrow X_i} M_n\left(X\right) = lim_{X\uparrow X_i} M_n\left(X\right), & \text{the value-matching condition at } X_i; \\ lim_{X\downarrow X_i} M_n'\left(X\right) = lim_{X\uparrow X_i} M_n'\left(X\right), & \text{the non-arbitrage condition at } X_i; \\ lim_{X\downarrow X_d} M_n\left(X\right) = R_m(X_d), & \text{the value-matching condition at } X_d. \end{cases} \tag{13}$$

The standard no-bubble condition shows that market debt becomes risk-free when a firm's cash flow level becomes sufficiently large; thus, its value equals the expected present value of perpetual debt service c. The interpretation of the other boundary conditions is similar to those discussed in Equation (9). Then, using Equations (12)–(13), we can summarize the solutions for the value of market debt as the following proposition.

Proposition 2 *Given a shareholder's optimal investment threshold X_i and optimal bankruptcy threshold X_d, after debt restructuring, the value of market debt is given by*

$$M_n\left(X\right) = \begin{cases} \frac{c}{r} + C_1X^{\beta_1}, & if \quad X_i \leq X, \\ \frac{c}{r} + C_2X^{\beta_1} + C_2X^{\beta_2}, & if \quad X_d < X < X_i. \end{cases} \tag{14}$$

Constants C_1, C_2, and C_3 are defined in Appendix A.1.

Proposition 2 shows that in the interval $[X_i, +\infty)$, the value of market debt consists of two components. The first term is the default-free debt value c/r. The last term captures the value adjustments when the firm enters the non-investment region in the future. In the interval $[X_d, X_i)$, the value of market debt consists of three components. The default-free debt value c/r is given in the first term. The last two terms capture the changes in value for possible future bankruptcy and re-entering the investment region.

The value function $D_n(X)$ for bank debt with the reduced coupon payment b_n satisfies an ODE similar to Equation (12). The boundary conditions are:

$$\begin{cases} lim_{X\to\infty} D_n(X) = \frac{b_n}{r}, & \text{the standard no-bubble condition;} \\ lim_{X\downarrow X_i} D_n\left(X\right) = lim_{X\uparrow X_i} D_n\left(X\right), & \text{the value-matching condition at } X_i; \\ lim_{X\downarrow X_i} D_n'\left(X\right) = lim_{X\uparrow X_i} D_n'\left(X\right), & \text{the non-arbitrage condition at } X_i; \\ lim_{X\downarrow X_d} D_n\left(X\right) = R_b(X_d), & \text{the value-matching condition at } X_d. \end{cases} \tag{15}$$

We then summarize the solutions for the value of bank debt as the following proposition.

Proposition 3 *Given a shareholders' optimal investment threshold* X_i *and optimal bankruptcy threshold* X_d*, after debt restructuring, the value of bank debt is given by*

$$D_n(X) = \begin{cases} \frac{b_n}{r} + F_1 X^{\gamma_1}, & if \quad X_i \leq X, \\ \frac{b_n}{r} + F_2 X^{\beta_1} + F_3 X^{\beta_2}, & if \quad X_d < X < X_i. \end{cases} \tag{16}$$

Constants F_1*,* F_2*, and* F_3 *are defined in Appendix A.1.*

3.2 Before the debt restructuring stage

3.2.1 Bank debt value and Nash bargaining

To obtain the bank debt value before the debt restructuring stage, we first solve the Nash bargaining game between the original shareholders and bank lenders (Fan and Sundaresan, 2000; Sundaresan and Wang, 2007; Morellec, Valta, and Zhdanov, 2015) in debt renegotiation. The outcome of this Nash bargaining process depends on the outside options (reservation value) of original shareholders and bank lenders, i.e., the payoffs that both parties will obtain when the firm is liquidated. The outside option for shareholders is zero, and the outside option for bank lenders is $R_b(X)$. In the event that shareholders initiate renegotiation, the threat of liquidation should be credible to induce acceptance by the bank. That is, the debt-for-equity swap must be chosen to ensure that the liquidation value obtained by the bank at bankruptcy is weakly less than the value that it acquires by accepting the renegotiation offer. Intuitively, once the threat of bankruptcy by shareholders is not credible, we can easily verify that simultaneous default on both types of debt is optimal since the bankruptcy threshold chosen by shareholders is identical to that derived by issuing only market debt with coupon $c^{new} = b + c$.[15] Thus, to focus on mixed debt policies that entail renegotiation, we follow previous studies, such as Hackbarth, Hennessy, and Leland (2007), and make the following assumption about parameter choices:[16]

Assumption 1. We restrict our attention to model parameter choices that ensure that $\rho(X)E_n(X) + D_n(X) > R_b(X)$ to induce acceptance of the renegotiation offer by the bank.

Under the above assumption, when a successful renegotiation between the two parties is realized at renegotiation time T_n, the net gain for the original shareholders is $(1-\rho(X_n))E_n(X_n)$, and the net gain for bank lenders is $\rho(X_n)E_n(X_n) + D_n(X_n) - R_b(X_n)$. Following Fan and Sundaresan (2000),

---

[15] For some extreme parameter values, the renegotiation between shareholders and bank lenders in our model would not occur. For example, based on our calibrated model, when shareholders' bargaining power or renegotiation friction is sufficiently high, the equity dilution, ρ, would be greater than 100%, indicating that shareholders must give more than they have in the company. As a result, there will be no debt restructuring in such cases since the threat of shareholder default is not credible.

[16] This assumption can be satisfied when we calibrate the model based on the existing empirical evidence and theoretical predictions.

the optimal swap rate $\rho^*(X_n)$ is a result of the following optimization problem:

$$\begin{aligned}\rho^*(X_n) &= \underset{\rho(X_n)}{\arg max}\left\{[(1-\rho(X_n))E_n(X_n)]^{\eta}\,[\rho(X_n)E_n(X_n)+D_n(X_n)-R_b(X_n)]^{1-\eta}\right\}\\ &= 1-\frac{\eta(E_n(X_n)+D_n(X_n)-R_b(X_n))}{E_n(X_n)}.\end{aligned} \tag{17}$$

Before the debt structuring stage ($T < T_n$), bank lenders receive coupon payment b at each time. By anticipating a shareholders' realized investment policy $\bar{i}(X) = i$, the value function of bank debt $D(X)$ satisfies the following ODE:

$$rD(X) = b + (\mu + i)XD'(X) + \frac{\sigma^2}{2}X^2D''(X), \tag{18}$$

given the bank lender's rational expectation of the shareholders' realized investment policy $\bar{i}(X) = i$. The market value of bank debt is solved subject to the following value-matching and no-bubble conditions:

$$\begin{cases} lim_{X\downarrow X_n} D(X) = lim_{X\uparrow X_n}\{(1-q)[D_n(X) + \rho^*(X)E_n(X)] + qR_b(X)\},\\ lim_{X\to\infty} D(X) = \frac{b}{r}.\end{cases} \tag{19}$$

The value-matching condition shows that, at renegotiation, bank lenders receive the expected value, which is equal to the weighted average of the total surplus received by bank lenders after restructuring and the reservation value. The standard no-bubble condition shows that bank debt becomes risk free when a firm's cash flow level becomes sufficiently large; thus, its value equals the expected present value of perpetual debt service b. Using standard calculations as in Leland (1994), we can summarize the solutions for the value of bank debt as the following proposition.

Proposition 4 *Given a shareholders' optimal investment threshold X_i, optimal renegotiation threshold X_n and optimal default threshold X_d, in the before debt restructuring stage ($T < T_n$), bank lenders bearing a coupon flow of b are worth the following:*

$$D(X) = \frac{b}{r} + \left[(1-q)(D_n(X_n) + \rho^*(X_n)E_n(X_n)) + qR_b(X_n) - \frac{b}{r}\right]\left(\frac{X}{X_n}\right)^{\gamma_1}. \tag{20}$$

Proposition 4 shows that $D(X)$ is given by the sum of the expected value of the perpetual coupon flow absent default risk (b/r), the changes in value for possible future financial distress and the temporary halt in investment.

3.2.2 Values of equity and market debt

Prior to debt restructuring, the after-tax cash flow net of the investment cost (i.e., dividend) to shareholders is $(1-\tau)(X_t - b - c) - \phi iX_t$ at each time t. Thus, the equity value function $E(X)$ before debt restructuring must satisfy the following ODE:

$$rE(X) = (1-\tau)(X - b - c) - \phi iX + (\mu + i)XE'(X) + \frac{\sigma^2}{2}X^2E''(X). \tag{21}$$

Solving the value function $E(X)$ is subject to the following value-matching and no-bubble conditions:

$$\begin{cases} lim_{X\downarrow X_n} E(X) = lim_{X\uparrow X_n}(1-q)(1-\rho^*(X))E_n\left(X\right), \\ lim_{X\to\infty}\frac{E(X)}{X} < +\infty. \end{cases} \tag{22}$$

Then, we can present the following proposition.

Proposition 5 *Given shareholders' optimal investment threshold X_i, optimal renegotiation threshold X_n and optimal default threshold X_d, the market value of equity $E(X)$ before the debt restructuring stage ($T < T_n$) satisfies*

$$E(X) = \left(U_i X - (1-\tau)\frac{b+c}{r}\right) + \left[(1-q)(1-\rho^*(X_n))E_n(X_n) - U_i X_n + (1-\tau)\frac{b+c}{r}\right]\left(\frac{X}{X_n}\right)^{\gamma_1}. \tag{23}$$

Equation (23) indicates that the equity value function in the before debt restructuring stage ($T < T_n$) is given by the sum of the expected present value of a perpetual flow of dividends and the adjustment in the equity value caused by potential future debt renegotiation, the halting of investment and bankruptcy. Shareholders select the renegotiation threshold X_n to maximize the equity value, consistent with, for instance, Fan and Sundaresan (2000) and Sundaresan and Wang (2007). Thus, the following smooth-pasting condition at threshold level X_n should hold:

$$E'(X_n) = (1-q)[E_n'(X_n) - (\rho^*(X_n)E_n(X_n))']. \tag{24}$$

Again, the above condition translates into nonlinear equations that can be solved numerically because of the absence of closed-form solutions.

We derive the market debt value as follows. Market lenders receive the coupon payment c at each time point, and the value of market debt $M(X)$ satisfies

$$rM(X) = c + (\mu + i)XM'(X) + \frac{\sigma^2}{2}X^2M''(X). \tag{25}$$

Equation (25) is solved by using the following value-matching and no-bubble conditions:

$$\begin{cases} lim_{X\downarrow X_n} M\left(X\right) = lim_{X\uparrow X_n}\{(1-q)M_n\left(X\right) + qR_m(X)\}, \\ lim_{X\to\infty} M(X) = \frac{c}{r}. \end{cases} \tag{26}$$

Using Equations (25)-(26), we can summarize the solutions for the value of market debt as the following proposition.

Proposition 6 *Given a shareholders' optimal investment threshold X_i, optimal renegotiation threshold X_n and optimal default threshold X_d, the value of market debt $M(X)$ before the debt restructuring stage ($T < T_n$) satisfies*

$$M\left(X\right) = \frac{c}{r} + \left[(1-q)M_n\left(X_n\right) + qR_m(X_n) - \frac{c}{r}\right]\left(\frac{X}{X_n}\right)^{\gamma_1}, \quad if \quad X \geq X_n. \tag{27}$$

3.3 Optimal debt structure

We now turn to the discussion of a firm's optimal debt structure. At the initial financing time, shareholders select the optimal mixture of bank and market debt to maximize the ex ante equity value, which is equal to the initial firm value $V(X_0; b, c)$ (Leland, 1994). That is, the firm's optimal debt structure is solved by the following optimization problem:

$$\{b^*(X_0), c^*(X_0)\} = \arg\max_{b,c} \{V(X_0; b, c)\}, \tag{28}$$

given the shareholders' ex-post optimal investment, debt renegotiation and default policies. The optimal coupon payment on bank and market debt cannot be obtained analytically. The optimal initial market leverage ratio of the firm, LR, is measured by the ratio of the market value of total debt to the market value of the firm, which is given by

$$LR(X_0; b^*(X_0), c^*(X_0)) = \frac{M(X_0; b^*(X_0), c^*(X_0)) + D(X_0; b^*(X_0), c^*(X_0))}{V(X_0; b^*(X_0), c^*(X_0))}. \tag{29}$$

To fully understand the endogenous distribution of the composition of debt finance, we decompose total debt into two components, namely the ratio of bank debt to total firm debt defined by $LR_b = D/(M + D)$ and the ratio of market debt to total firm debt defined by $LR_m = M/(M + D)$.

4 Model implications

In this section, we conduct numerical analyses to better illustrate our model implications, which are novel in the literature (see, e.g., Hackbarth, Hennessy, and Leland, 2007; Morellec, Valta, and Zhdanov, 2015).

4.1 Parameter calibration and choice

Following the capital structure and corporate investment literature, we use the following annualized parameter values for our numerical analysis. The initial level of productivity is normalized to $X_0 = 1$. Similar to Diamond and He (2014) and Wong and Yu (2022), the risk-free interest rate is $r = 5\%$, the baseline growth rate is $\mu = 0.5\%$, the asset volatility is $\sigma = 25\%$, and the effective tax rate is $\tau = 20\%$. We take the empirical estimate in Glover (2016) and set the bankruptcy cost rate to $\alpha = 45\%$.

Shareholders' bargaining power, the probability of renegotiation failure, and the coupon concession are key parameters that drive the differences between bank debt and market debt. Shareholders' bargaining power is taken to be $\eta = 40\%$, consistent with the average value in the sample analyzed by Favara, Schroth, and Valta (2012), who use insiders' share of equity as a proxy variable to measure shareholders' bargaining power. The probability of renegotiation failure is taken to be $q = 54\%$, which is also in line with the average value in the sample reported by Favara, Schroth, and Valta

(2012). We consider a coupon concession parameter of $m = 50\%$, which falls into the range of the theoretical predictions in Moraux and Silaghi (2014).[17]

For the real investment parameters, we set the incremental productivity growth rate to be $i = 2\%$, which is close to the calibration reported by Wong and Yu (2022). We fit the firm's Tobin's Q to approximate the average Tobin's Q (i.e., 1.89) in the sample of large firms in Compustat from 1993 to 2003 used by Eberly and Rebelo (2009) by setting the investment cost parameter to $\phi = 5$.[18] This parameter choice is also consistent with the calibration in Wong and Yu (2022).

4.2 Heterogeneous debt structures and corporate policies

In this section, we conduct quantitative analysis based on our baseline model to evaluate the effects of debt heterogeneity on corporate investment dynamics, bankruptcy policy, the capital structure decision, and firm value. Although a large number of firms simultaneously use more than one type of debt for debt financing (see Rauh and Sufi, 2010), some firms issue only one type of debt in debt financing. Thus, it is worthwhile to compare the results revealed by our model with those derived from two other models using alternative debt structures, i.e., the exclusive market debt structure and the exclusive bank debt structure. These two special cases are derived in Appendix A.2. Moreover, we focus on the impact of shareholders' bargaining power η and renegotiation frictions q on corporate policies since both parameters drive the different features between bank and market debt.[19] Such an analysis allows us to illustrate a novel result that firms with mixed debt financing can mitigate ex-post debt overhang relative to the benchmark firms with an exclusive market debt structure.[20]

4.2.1 Optimal leverage and ex ante firm value

Debt composition varies across the distribution of shareholders' bargaining power and renegotiation frictions, as shown in Figure 2. Panel A of Figure 2 illustrates that shareholders with sufficiently low bargaining power rely exclusively on bank debt, consistent with the discussions in Hackbarth, Hennessy, and Leland (2007). In contrast, when shareholders' bargaining power is sufficiently large (i.e., $\eta \in [\eta^*, 1]$), the value of bank debt would fall to less than the reservation value, and rene-

---

[17] Moraux and Silaghi (2014) predict that the coupon is reduced at least to 67% and at most to 27% of the initial coupon payment in renegotiation. Additionally, Silaghi (2018) predicts that the coupon reduction obtained in renegotiation is substantial.

[18] Tobin's Q is defined as the ratio of the total market value of the firm to the value of assets-in-place, where the value of assets-in-place is given by $U_0 X_t$.

[19] In the extreme case of $q = 1$, bank debt is non-renegotiable, which is equivalent to market debt. However, to highlight our main results, we do not consider this special case in the following numerical analysis.

[20] In the literature on corporate finance theory, a number of studies have been devoted to discussing Myers's debt overhang problem under an exclusive market debt structure. See, for example, Mauer and Ott (2000), Hennessy (2004), Hackbarth and Mauer (2012), Diamond and He (2014), Sundaresan et al. (2015), and Wong and Yu (2022).

gotiation would not occur. In such a situation, bank and market debt are equivalent. Thus, as η increases, the total leverage with mixed debt financing exhibits a downward jump at $\eta^* = 0.50$. We find a similar pattern of optimal leverage with mixed debt financing regarding renegotiation frictions q in Panel C of Figure 2. The downward jump reflects that the optimal leverage is identical to the leverage achieved by issuing market debt only with coupon $c^{new} = c + b$. In the following, we confine our attention to the situations in which the threat of liquidation is credible.

Debt structures have significant implications for optimal leverage and firm value, as indicated in Figure 2. First, in all simulated scenarios, a comparison between the two constructed benchmarks reveals that the exclusive bank debt structure dominates the exclusive market debt case because it entails a lower implied bankruptcy cost. Therefore, firms issuing bank debt only would take on higher leverage and achieve higher firm value. Second, as indicated in Figure 2, a firm achieves the highest market leverage when issuing a mixture of bank and market debt. The reason for this finding is that firms with mixed debt financing can enjoy greater tax benefits of debt relative to the case of financing with bank debt only, and they bear lower bankruptcy costs relative to the case of financing with market debt only. This evidence implies that using both market and bank debt might mitigate the financial constraints for financially constrained firms. Similar to market leverage, we show that firms with mixed debt financing outperform other firms with only bank debt or only market debt. In our base case, our model indicates that heterogeneous debt financing could enhance firm value by 0.25% (0.67%) relative to the exclusively bank (market) debt case. These results are consistent with the observation that many corporations choose mixed debt financing (see Rauh and Sufi, 2010).

4.2.2 Ex-post corporate policies: investment and bankruptcy

To understand how optimal dynamic corporate investment and bankruptcy decisions vary with different debt structures, we perform a comparative static analysis regarding ex-post corporate policies. Figure 3 shows that compared with the exclusive market debt financing, the exclusive bank debt structure significantly delays bankruptcy thanks to its renegotiation feature. Additionally, the postponing of default caused by bank debt financing preserves more future investment opportunities and leads to a larger marginal impact of the firm's assets-in-place on the equity value (i.e., a steeper $E_n'(X)$ curve). This positive effect feeds back to the present and increases shareholders' current investment incentives. Therefore, a firm issuing bank debt exclusively invests earlier and has a higher asset growth rate than a firm financed exclusively with market debt, mitigating underinvestment distortions due to debt overhang. This result is in sharp contrast to the model prediction of Pawlina (2010) that firms financed with negotiable (bank) debt are more hesitant to invest. Pawlina assumes that a firm's expansion investment is financed exclusively by negotiable debt and that such investment features an irreversible and one-shot decision (of the type described by McDonald and Siegel, 1986), and he finds that shareholders set a higher investment threshold with exclusively negotiated debt than those of a firm financed with market debt only. Our result regarding the impact of nego-

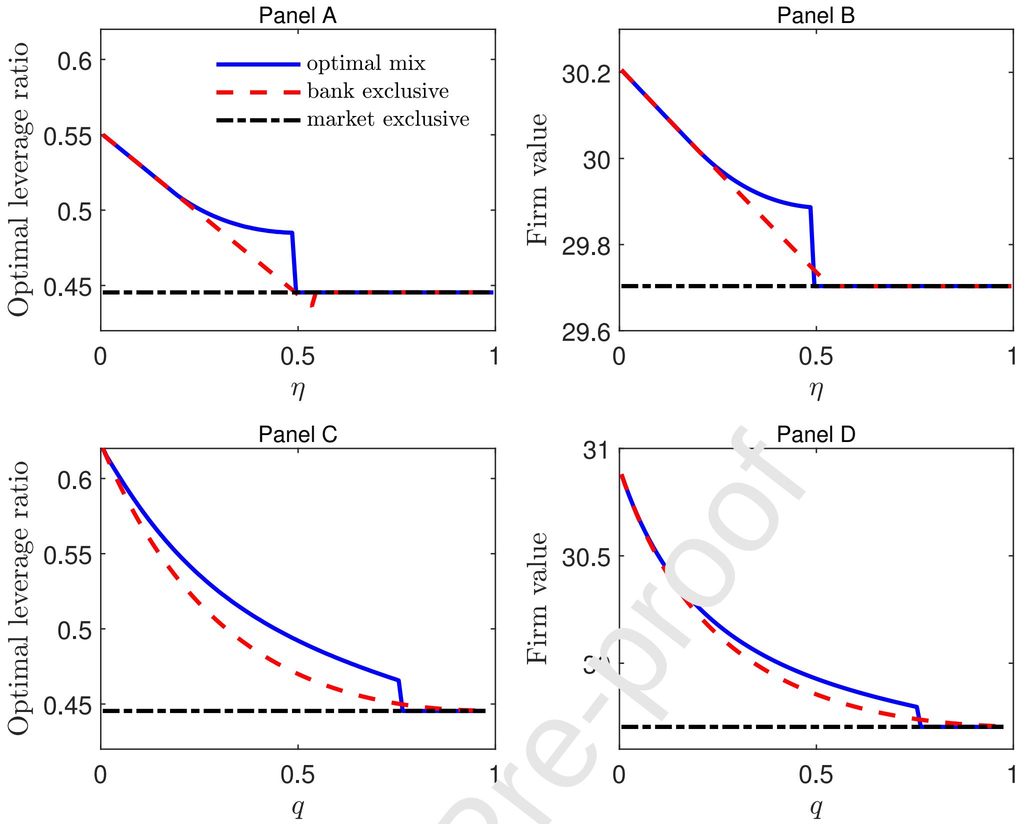

Figure 2: The comparative statics of the effects of shareholders' bargaining power, η, and renegotiation frictions, q, on optimal leverage and firm value under three different types of debt structure: i.e., a mix of bank and market debt, bank debt only, and market debt only.

tiable debt on the investment incentive problem in a setting with multiple investment opportunities extends and complements the discussion in Pawlina (2010).

Intuitively, the bankruptcy threshold and investment threshold for a firm that simultaneously issues bank and market debt remains between the two benchmark cases. This statement is confirmed in Figure 3. Panels A and B of Figure 3 further show the non-monotonic effects of shareholders' bargaining power η on bankruptcy and investment thresholds for firms with heterogeneous debt financing. There are two opposing effects on the ex-post bankruptcy decision. First, all else being equal, stronger shareholders (with higher bargaining power) can extract more renegotiation surplus in a private workout; thus, they delay bankruptcy at a lower threshold (see Panel A of Figure 3).[21]

Second, as noted in Hackbarth, Hennessy, and Leland (2007), bank (market) debt represents a smaller (larger) proportion of the total capital structure when shareholders have stronger bargaining power because rational bank lenders react to a decreasing proportion of the renegotiation surplus by reducing the firm's bank debt capacity, further leading to a higher bankruptcy threshold. Considered together, the effect of the change in debt composition dominates (is dominated by) the effect of

---

[21] This effect of shareholders' bargaining power on bankruptcy policy is also discussed in Wong and Yu (2022).

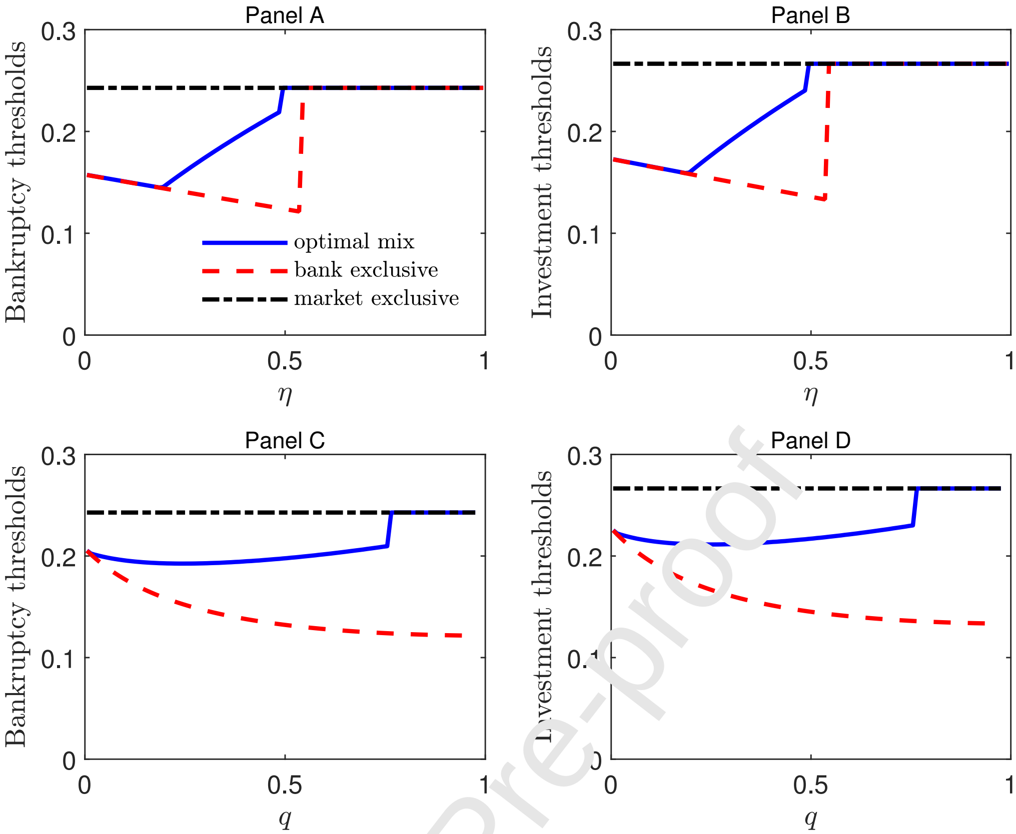

Figure 3: The comparative statics of shareholders' bargaining power, η, and renegotiation frictions, q, regarding a firm's optimal default and investment policies under three different types of debt structure, i.e., a mix of bank and market debt, bank debt only, and market debt only.

surplus extraction for stronger (weaker) shareholders, thus implying a non-monotonic relationship between bargaining power and the bankruptcy threshold. In addition, a longer (shorter) distance to bankruptcy preserves more future investment opportunities, strengthening (mitigating) shareholders' incentive to invest. Thus, the corporate investment policy presents the same pattern as the default policy with respect to shareholders' bargaining power.

We also observe a non-monotonicity of bankruptcy and investment thresholds for firms with heterogeneous debt financing in the renegotiation friction q, as shown in Panels C and D of Figure 3. An increase in q indicates two indirect effects on the final bankruptcy threshold. First, bank debt loses its advantage because shareholders would bear higher renegotiation costs; thus, they would tend to use more market debt and less bank debt in total debt, inducing a higher bankruptcy threshold. Second, since shareholders anticipate that high leverage might trigger ex-post renegotiation earlier, they would choose to optimally issue less total debt ex ante, inducing a lower bankruptcy threshold. At low values of q, the former effect is dominated by the the latter effect, and shareholders delay formal bankruptcy as q increases. In contrast, when q is sufficiently large, the former effect dominates, and thus, shareholders accelerate formal bankruptcy as q increases. An earlier (later) bankruptcy caused by more renegotiation frictions destroys (preserves) more future investment opportunities; thus, shareholders' current incentive to invest is weaker (stronger).[22]

4.3 Flexible growth opportunities and corporate policies

Hackbarth, Hennessy, and Leland (2007) provide a thorough analysis of the optimal mix of bank and market debt and its determinants in a traditional trade-off model. However, much less is known about how optimal corporate policies will be determined in the presence of a series of future flexible growth opportunities and heterogeneous debt structures. The answer to this question is important given that a firm's ability to adjust its size through investing influences its financial policy (see, e.g., Hennessy, 2004; Alanis, Chava, and Kumar, 2018). A novel feature of our model is the interaction between debt heterogeneity and flexible growth opportunities, as suggested by Diamond and He (2014). This setting allows us to analyze the implications of future flexible growth opportunities for the optimal mix of bank and market debt. To facilitate discussion, we provide a benchmark model in which the firm never invests, and thus its asset growth rate features an exogenous constant μ. The model solution of this special case is given in Appendix A.3. Then, the difference between the solutions of these two models exclusively captures the impact of flexible growth opportunities, which is shown in Figure 4.

In this model, shareholders make a trade-off between the tax benefit of debt and the costs induced by higher market leverage that they bear by themselves when deciding how much debt to issue (see Leland, 1994). Panel A of Figure 4 conveys the following insights. First, we note that firms with flexible growth opportunities tend to use lower leverage to finance the initial firm size than an otherwise identical firm with no growth opportunities. This point partly reflects that growth firms are concerned about the future debt-overhang costs arising from shareholder-creditor conflicts over investment policy when the leverage level is too high. Second, a firm with more profitable growth opportunities rationally uses lower leverage to mitigate the future debt-overhang burdens than a firm with less profitable growth opportunities. In a related analysis based on a real-options model, Sundaresan, Wang, and Yang (2015) discussed a similar issue involving the effects of multiple growth opportunities on a firm's initial leverage usage. However, they focused only on a homogeneous market debt structure. Our study complements the discussions in Sundaresan, Wang, and Yang (2015) by considering the simultaneous issuance of bank and market debt in a standard trade-off capital structure model with dynamic investment, in the spirit of Diamond and He (2014) and Wong and Yu (2022).

As a complement to Panel A in Figure 4, Panel B plots the optimal composition of bank and

---

[22] Firms delay investment (i.e., a higher threshold X_i) when there is sufficiently higher renegotiation friction in private workouts, consistent with the empirical findings of Favara et al. (2017) that "debt renegotiation frictions that strengthen the enforcement of debt contracts induce underinvestment distortion."

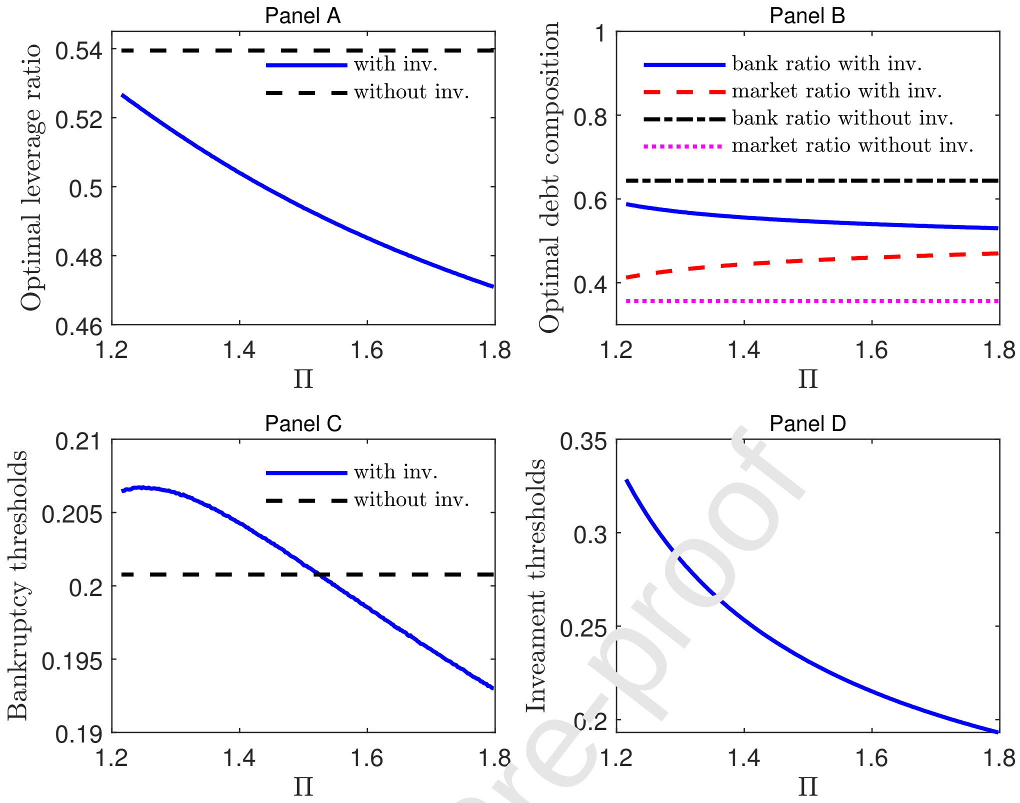

Figure 4: The comparative statistics of the effect of the profitability of growth opportunities, Π, on a firm's optimal leverage (Panel A), default (Panel C) and investment policies (Panel D). Panel B decomposes the optimal mixed debt financing in Panel A and shows the composition of bank and market debt.

market debt as a function of the profitability of growth opportunities. Shareholders optimally issue a greater share of bank debt out of total debt across two types of firms (with or without investment options), mainly driven by bank debt mitigating the potential downside of bankruptcy risk. However, the presence of flexible future growth opportunities encourages shareholders to optimally reduce the usage of bank debt and issue more market debt than no-growth firms. For example, for the base case, the ratio of bank debt to total debt is 15.86% lower for growth firms than for no-growth firms. Accordingly, the ratio of market debt to total debt is 28.64% higher for growth firms than for the corresponding no-growth firms. Furthermore, a growth firm tends to significantly reduce its reliance on bank debt and increase the usage of market debt when future growth opportunities are more valuable. This evidence suggests that, although bank debt can render firms less exposed to debt overhang and expensive bankruptcy costs, the tax shield benefit of market debt dominates for high-growth firms.

In Panels C and D in Figure 4, we examine the effects of future investment profitability on corporate bankruptcy and investment policies. Future investment opportunities have two opposing effects on bankruptcy. First, firms with more profitable investment opportunities are less likely to default (a lower bankruptcy threshold) because of good future prospects. In addition, firms tend to use more market debt and less bank debt when they have more profitable investment opportunities (illustrated in Panel B), leading to a higher bankruptcy threshold. Our model predicts that high-growth firms trigger later bankruptcy (a lower threshold X_d) since the former effect dominates the latter. Thanks to the good prospects of future investment opportunities, the lower probability of bankruptcy for high-growth firms strengthens shareholders' investment incentives and increases the asset growth rate (a lower threshold X_i).

4.4 Other comparative static results

Following the literature on classical capital structure trade-off models (see, e.g., Leland, 1994), we provide comparative statics of corporate policies for three additional parameters, namely asset volatility (σ), bankruptcy costs (α), and the corporate tax rate (τ). These comparative static results might depend on the value of growth opportunities. To ensure the robustness of our results, we also consider the change in the value of growth opportunities Π by choosing two different investment cost parameter values $\phi \in \{0.5, 12.5\}$. As we see in the following discussion, confirming our previous key results, firms with more profitable investment opportunities tend to use more market debt and less bank debt than firms with less profitable investment opportunities, regardless of the values of σ, α, and τ.

4.4.1 Asset volatility (σ)

Figure 5 presents corporate policies when asset volatility varies from $\sigma = 0.2$ to $\sigma = 0.6$ for a firm with $\Pi = 1.24$ (low-growth) and $\Pi = 1.78$ (high-growth). As shown in Panel A of Figure 5, a firm with mixed debt financing would take on lower market leverage when its cash flow is riskier in both high- and low-growth firms. The intuition comes from the value effect. First, an increase in asset volatility increases the equity value because of the standard real-options effect, as discussed in Leland (1994) and Sundaresan, Wang, and Yang (2015). Second, the total debt value is greatly increased under significant uncertainty because of the endogenous coupon effect. Overall, the equity value has greater sensitivity to asset volatility than the total debt value due to its convexity feature. Thus, the market leverage decreases with asset volatility. Regarding the optimal debt structure, we identify a change in debt composition whereby firms tend to increase (decrease) their proportion of bank (market) debt out of total debt when asset growth is subject to significant uncertainty (Panel B of Figure 5). This result is straightforward since firms optimally issue more bank debt to absorb the downside risk when asset risk is higher.

Panel C of Figure 5 illustrates the decreasing monotonic effect of asset volatility on the bankruptcy threshold, driven by the real option value of waiting (Sundaresan, Wang, and Yang, 2015; Wong and

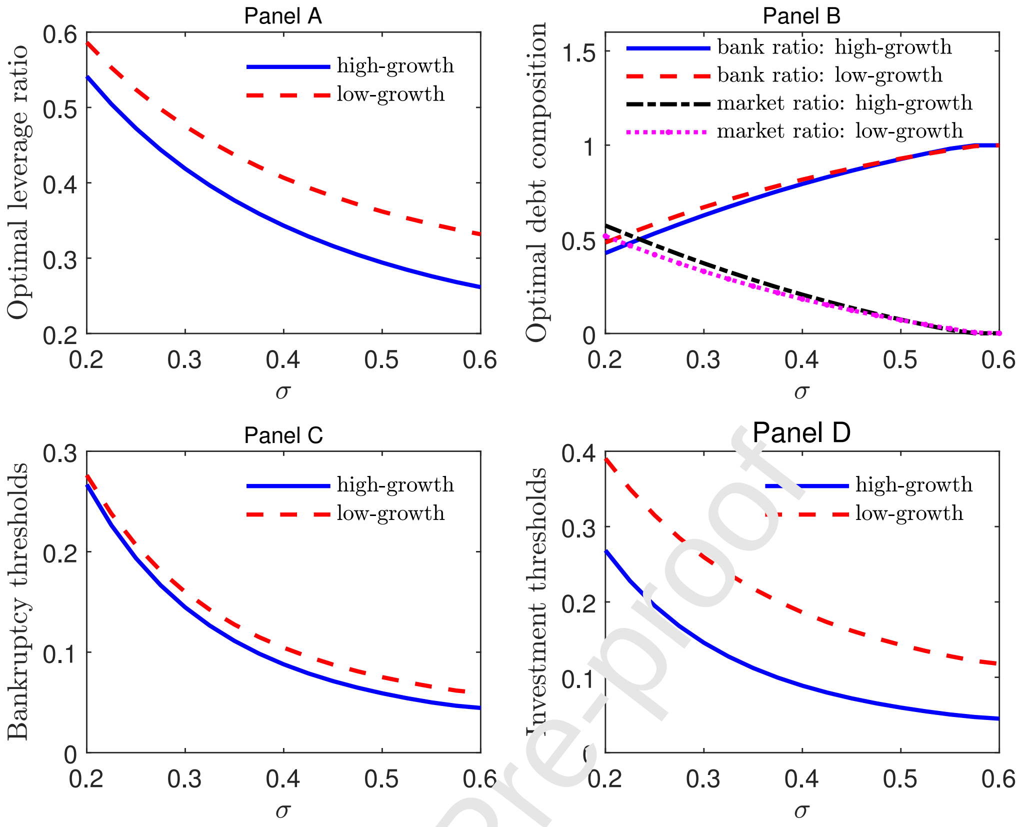

Figure 5: The comparative statics of the effect of asset volatility σ on corporate policies for two types of firms, i.e., high-growth firms ($\Pi = 1.78$) and low-growth firms ($\Pi = 1.24$).

Yu, 2022). Additionally, when uncertainty is higher, the issuance of a larger (smaller) share of bank (market) debt also delays the bankruptcy trigger. As a result, the delay of bankruptcy preserves more future growth opportunities and thus strengthens shareholders' current investment incentives and induces a higher asset growth rate (i.e., a lower investment threshold), as shown in Panel D of Figure 5.

4.4.2 Bankruptcy costs (α)

Figure 6 depicts the impact of bankruptcy costs (α) on corporate policies for two given values of Π (1.24 and 1.78). Overall, as shown in Panel A of Figure 6, the optimal leverage decreases with the bankruptcy cost, the effect of which is more significant for low-growth firms. In addition, the value of outside options for bank lenders decreases with bankruptcy costs (i.e., a smaller strategic benefit), causing the proportion of bank debt in total debt to decline significantly and firms with high bankruptcy costs to rely on more market debt, regardless of the value of growth opportunities (Panel B of Figure 6).

An increase in bankruptcy costs has two effects on shareholders' bankruptcy policies. First, with an increase in bankruptcy costs, firms will optimally issue less total debt ex ante to delay

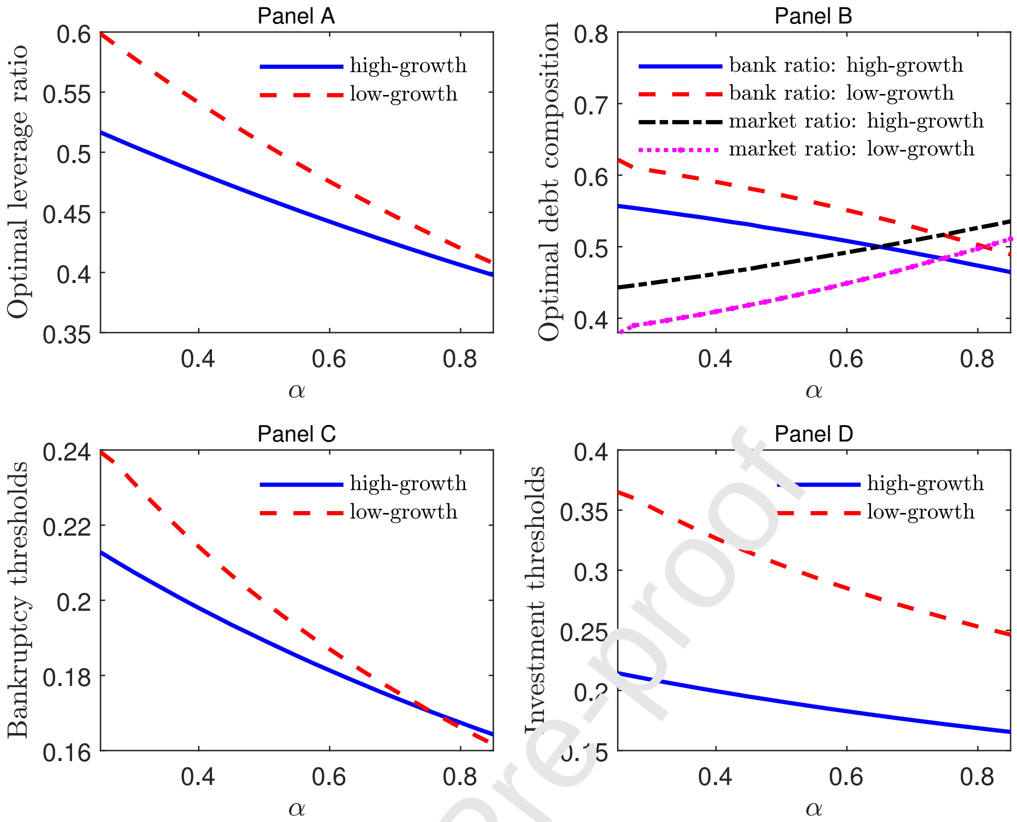

Figure 6: The comparative statics of the effect of bankruptcy costs α on corporate policies for two types of firms, i.e., high-growth firms ($\Pi = 1.78$) and low-growth firms ($\Pi = 1.24$).

ex-post bankruptcy. Second, an increase in bankruptcy costs reduces the total renegotiation surplus and results in the issuance of a larger (smaller) share of market (bank) debt, leading to a higher bankruptcy threshold. The former effect always dominates the latter, leading to a decreasing bankruptcy threshold with bankruptcy costs for both high- and low-growth firms (see Panel C of Figure 6). However, low-growth firms reduce their bankruptcy thresholds at a faster rate. Similarly, postponing bankruptcy preserves more future investment opportunities; thus, shareholders have a stronger incentive to invest (a lower investment threshold and a higher asset growth rate) when α increases, as shown in Panel D of Figure 6.

4.4.3 Corporate tax rate (τ)

Although it is well known in the capital structure literature that the tax rate τ is highly correlated with a firm's market leverage ratio, few studies, whether theoretical or empirical, link the tax effect to a firm's choice of debt type. Figure 7 presents corporate policies across τ for a firm with $\Pi = 1.24$ (low-growth) and $\Pi = 1.78$ (high-growth). Generally, shareholders have a stronger incentive to issue debt ex ante, when the tax shield benefit dominates the bankruptcy and debt overhang costs. Therefore, firm leverage increases with tax rate τ for both high- and low-growth firms (Panel A of

Figure 7)). By further exploring the optimal debt composition in a higher tax rate environment, growth firms tend to decrease their usage of bank debt and increase their reliance on market debt due to the dominance of the tax shield benefits, as indicated in Panel B of Figure 7. Empirically, there have been few studies that have linked the corporate tax rate and the choice of debt type.[23] This finding thus motivates a hypothesis to empirically test the relationship between firm debt structure choice and the corporate tax rate environment. Panel C of Figure 7 illustrates that an increase in the corporate tax rate increases a firm's debt capacity, leading to a higher bankruptcy threshold. In such cases, shareholders have a lower incentive to invest for both high- and low-growth firms due to the severe debt overhang (i.e., a higher investment threshold and a lower asset growth rate, as shown in Panel D of Figure 7). Moreover, low-growth firms reduce their asset growth rate at a faster rate as τ increases.

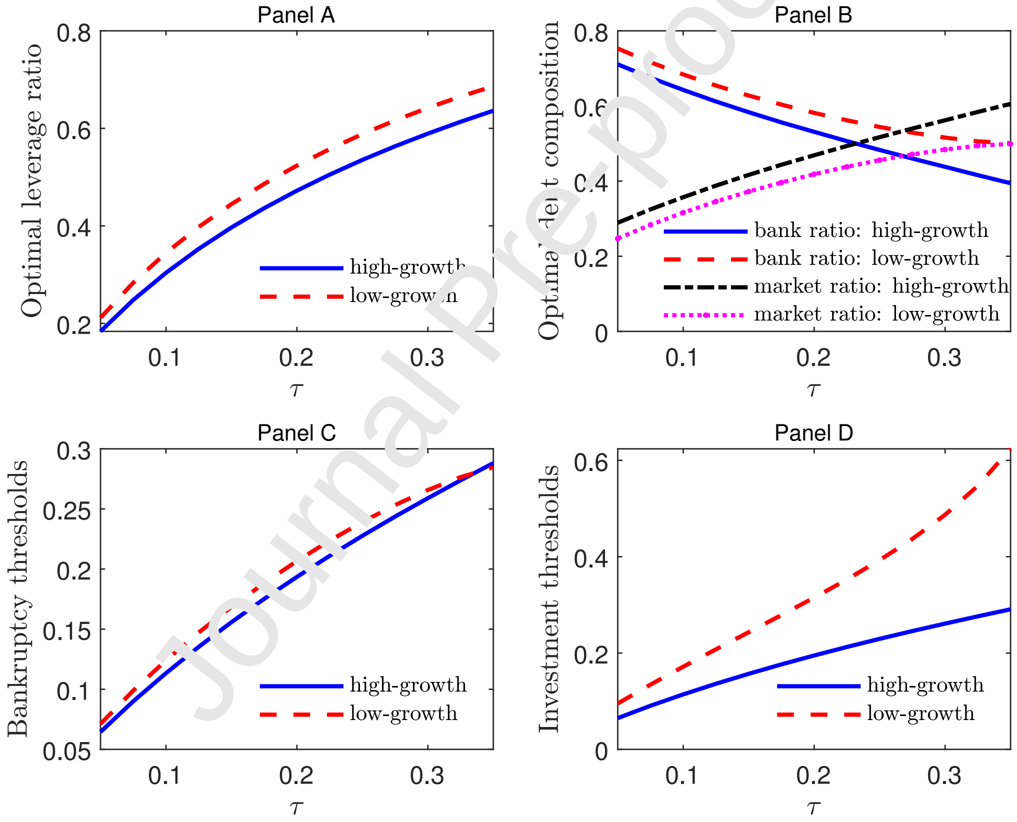

Figure 7: The comparative statics of the effect of the tax rate τ on corporate policies for two types of firms, i.e., high-growth firms ($\Pi = 1.78$) and low-growth firms ($\Pi = 1.24$).

5 Time-varying investment opportunities

---

[23] Sharpe and Nguyen (1995) and Graham, Lemmon, and Schallheim (1998) examine the relevant issues regarding tax effects.

In our previous setting, we assume that a firm faces only constant investment opportunities throughout its life. Chen and Manso (2017) demonstrate that in an exclusive market debt structure, stochastic growth opportunities have a significant impact on firms' capital structure choices. A natural question is then what would happen to heterogeneous debt financing in our model when we introduce time-varying investment opportunities. Inspired by Chen and Manso (2017), we extend our baseline model by assuming that the economy has two observable aggregate states, denoted by $s_t \in \{G, B\}$, with G representing a good (boom) state and B representing a bad (recession) state. Let $\lambda(G)$ (or $\lambda(B)$) be the risk-adjusted transition intensity out of state G (or B) to B (or G). Thus, the probability of the state of the world switching from one state to the other within an infinitesimal time interval $(t, t + dt)$ is $\lambda(s_t)dt$.

To capture the time-varying investment opportunities, we assume that a firm in each state faces different investment costs ϕ. The investment cost is countercyclical (i.e., $\phi(G) < \phi(B)$), representing that the profitability of the firm's growth opportunities is cyclical. This assumption is consistent with the existing literature on investment over the business cycle (Chen and Manso, 2017).[24] Following the standard approach, we solve our extended model in Appendix A.4. Such an extension facilitates the investigation of how a firm optimally adjusts its leverage and debt composition when anticipating time-varying investment conditions. We provide a brief numerical analysis as follows.

Based on the calibration in Bolton, Chen, and Wang (2013), the risk-adjusted transition intensities for the aggregate state are considered to be $(\lambda(G), \lambda(B)) = (0.2, 0.25)$. We set the investment cost in the bad state B to $\phi(B) = 8$, implying that the profitability of a growth opportunity in state B is 1.4. We then define the profitability difference between state G and state B as $\Pi(G)/\Pi(B)$. In Figure 8, we vary the investment cost $\phi(G) \in [0.5, 8]$ to obtain $\Pi(G)/\Pi(B)$. A higher $\Pi(G)/\Pi(B)$ implies that the cyclicality of growth opportunities is stronger. Figure 8 reports the optimal market leverage and debt composition in both states when the profitability difference changes.[25]

Panel A of Figure 8 shows that the firm takes on more market leverage in bad state B than in good state G because of the value effect. That is, when the firm's growth opportunity switches from the good state to the bad state, shareholders lose more value than debt holders, leading to higher market leverage.[26] Moreover, firms optimally decrease their total market leverage in the good state as the profitability difference between two states increases, as they anticipate higher debt overhang costs in the future. Similarly, in bad state, shareholders are still reluctant to issue more debt since they expect that the current bad state could change to the good state, and they might face a higher debt overhang cost.

---

[24] To emphasize the impact of time-varying investment opportunities on corporate financing policies, all other parameters are constant over time and are not affected by macroeconomic conditions.

[25] Upon a necessary relabeling process, we can obtain the corresponding results for the case in which we vary the investment cost $\phi(B)$.

[26] This result is similar to the discussion in Hackbarth, Miao, and Morellec (2006), who demonstrate that leverage with homogeneous market debt structures is countercyclical.

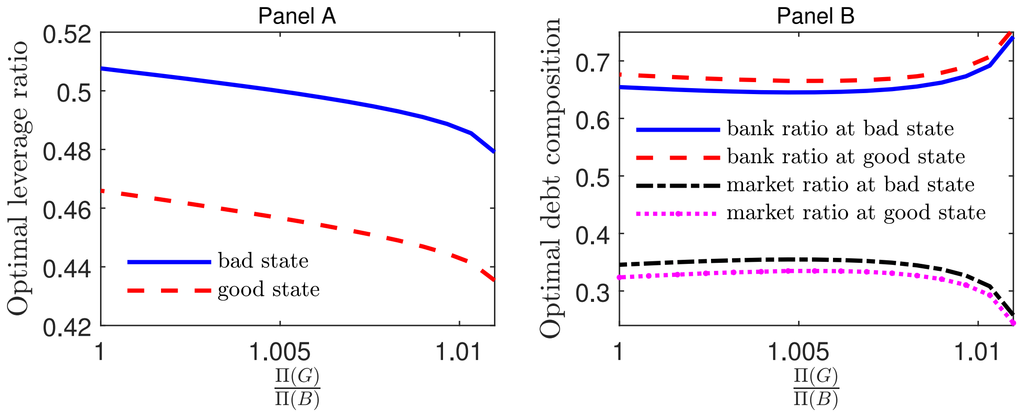

Figure 8: The effects of the future investment profitability difference, $\frac{\Pi(G)}{\Pi(B)}$, on a firm's optimal leverage and its decomposition of bank and market debt between good and bad states.

Panel B of Figure 8 further analyzes the endogenous distribution of debt composition in two aggregate states. In particular, we show that firms optimally choose to increase the usage of market (bank) debt and decrease their reliance on bank (market) debt in total debt when future growth opportunities shift from a good (bad) prospect state to a bad (good) prospect state. This result reflects the variation in the trade-off between the tax shield benefits of market debt and the reduction of bankruptcy and the debt overhang costs of bank debt. Moreover, our results reconcile with the stylized fact and theoretical prediction in De Fiore and Uhlig (2015) that "nonfinancial corporations started shifting the composition of their debt from bank loans toward debt securities early in 2008."

Another implication from our dynamic model is that Panel B of Figure 8 illustrates the opposite effect of the profitability difference on bank and market debt issuance. With an increase in the profitability difference, the firm first decreases (increases) and then increases (decreases) the optimal bank (market) debt proportion in total debt, exhibiting a (an inverted) U-shaped relation in both states. This finding reflects that when growth opportunities are less cyclical, the firm's default risk becomes lower; thus, it tends to use more market debt and less bank debt, as the tax shield benefit of market debt dominates the reduction in bankruptcy and debt overhang costs from bank debt. In contrast, when the profitability difference is sufficiently high (i.e., stronger cyclical of growth opportunities), the opposite holds true, as the firm's incentive to lower bankruptcy costs and debt overhang costs outweighs the tax benefits.

6 Heterogeneous debt structure and its empirical predictions

We have shown in our numerical analyses that our theoretical results reconcile several empirical findings. In this section, we summarize some novel empirical tests regarding how firm characteristics affect the optimal debt composition and corporate policies.

Observation 1. *The issuance of bank debt along with market debt mitigates the ex-post debt* *overhang effect relative to the benchmark firms with an exclusive market debt structure.*

Our model generates a testable prediction for debt overhang and debt renegotiability. Pawlina (2010) employed a real-options framework and found that firms financed exclusively with renegotiated debt face a higher magnitude of Myers's debt overhang problem than the benchmark firms financed exclusively with market debt. In contrast, in Subsection 4.2.2, we develop a dynamic investment model and find that the combination of negotiated debt and market debt financing delays bankruptcy and accelerates current investment and therefore mitigates the ex-post debt overhang cost relative to the exclusive market debt case. As a consequence, in a regression model of the investment rate run jointly for both types of firms, we expect the coefficient of the interaction term of the debt overhang proxy (see Hennessy, 2004) and debt renegotiability to be positive.

Observation 2. *Higher friction in the renegotiation of debt restructuring first increases and then decreases shareholders' investment incentives for growth firms with mixed debt financing.*

Our model generates an empirical prediction regarding shareholders' investment incentives for growth firms with senior bank debt and subordinated market debt financing and debt renegotiation frictions. In Subsection 4.2.2, we find that, for growth firms with lower debt renegotiation frictions, the shareholder's investment incentive increases with q (i.e., a lower investment threshold). In contrast, for growth firms with higher debt renegotiation frictions, the shareholder's investment incentive decreases with q (i.e., a higher investment threshold). Empirically, there are few empirical predictions about the effects of debt renegotiation frictions on corporate investment policy within a growth firm with mixed debt financing. As a result, in a regression model of the investment rate run for growth firms with mixed debt financing, we expect that the coefficient of the term of debt renegotiation frictions (the explanatory variable) has a positive or negative sign, depending on the severity of debt renegotiation frictions faced by growth firms.

Observation 3. *Firms with more profitable growth opportunities are expected to reduce their reliance on bank debt and increase their usage of market debt relative to firms with less profitable growth opportunities.*

Some empirical studies have investigated the impact of the presence of growth opportunities on the choice between an exclusive market debt structure and an exclusive bank debt structure (see, e.g., Morellec, Valta, and Zhdanov, 2015). However, these studies have overlooked the complex debt structures within a firm. In Subsection 4.3, our model predicts that the presence of growth opportunities is expected to reduce the firm's reliance on bank debt and increase the usage of market debt. This effect is much more pronounced among growth firms, in which the profitability of growth opportunities is larger. Thus, to investigate the impact of the presence of growth opportunities on the choice of bank debt and market debt within a firm, we can estimate simultaneous equation models with bank debt and market debt as endogenous variables. Furthermore, we can test how firms' debt compositions vary when growth opportunities are more valuable.

Observation 4. *Firms with stronger cyclicality of growth opportunities tend to first decrease* *(increase) and then increase (decrease) their reliance on bank (market) debt as a share of total debt.*

Time-varying investment opportunities play an important role in simultaneously determining the optimal composition of bank and market debt in the corporate debt structure. In our extended model discussed in Section 5, we document a novel theoretical result that a firm with stronger cyclicality of growth opportunities first decreases (increases) and then increases (decreases) the usage of bank (market) debt as a share of total debt. To the best of our knowledge, there have been few empirical predictions about how the cyclicality of growth opportunities affects a growth firm's joint choice of debt types. As a result, our model's prediction could motivate hypotheses to empirically test this relationship.

7 Conclusion

Most of the extant theoretical literature on corporate financing and investment, such as Hackbarth and Mauer (2012), Morellec, Valta, and Zhdanov (2015), Sundaresan, Wang, and Yang (2015), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b), has ignored the stylized fact that firms simultaneously use bank and market debt (see Rauh and Sufi, 2010; Hackbarth, Hennessy, and Leland, 2007). In this paper, we integrate two strands of the literature, namely endogenous investment and heterogeneous debt structure, and develop a dynamic trade-off capital structure model that captures both flexible investment opportunities and heterogeneous debt financing. In our model, private bank debt is renegotiable during financial distress; thus, it delays an inefficient and costly bankruptcy should renegotiation be successful. This setting allows us to shed light on the interactions between mixed debt financing and investment decisions when shareholders optimally choose growth option exercises, renegotiation and bankruptcy policies and when the growth firm's optimal debt structure is driven by the trade-off among tax shield benefits, bankruptcy costs and debt overhang costs.

Our model generates the following novel and important results. First, our analysis uncovers a positive effect of bank debt on the debt overhang problem stemming from its ability to postpone inefficient firm bankruptcy in private workouts. This delay of costly bankruptcy preserves more future growth opportunities, leading to a larger marginal impact of the firm's assets-in-place on the equity value, strengthening shareholders' current investment incentives. As a result, we show that the issuance of bank debt along with market debt mitigates the ex-post debt overhang effect relative to the benchmark case of firms with an exclusive market debt structure. Second, our model demonstrates that a growth firm tends to optimally increase (decrease) the usage of bank debt and decrease (increase) its reliance on market debt as a share of total debt when it has fewer (more) valuable growth opportunities, its asset volatility is higher (lower), its bankruptcy cost is lower (higher), or there is a low (high) tax rate environment. These results complement and extend the discussions in Morellec, Valta, and Zhdanov (2015) and Shibata and Nishihara (2015b) who explored how firm characteristics affect the choice between an exclusive bank debt structure and an exclusive market debt structure. Finally, we extend our baseline model by incorporating time-varying investment opportunities and investigate how a firm optimally adjusts its debt composition in such circumstances. The model predicts that firms with more cyclical growth opportunities should first decrease (increase) and then increase (decrease) the usage of bank (market) debt as a share of total debt.

Although our model generates some novel insights into corporate financing and investment decisions by recognizing that debt heterogeneity is an important feature of public firms, we consider only secured bank debt and subordinated market debt. However, in the real world, firms often simultaneously use various other types and priorities of debt, e.g., convertible debt (Lyandres and Zhdanov, 2014; Giambona, Golec, and Lopez-de Silanes, 2021) and mortgage debt. Thus, the natural step to extend our model is to incorporate these types of debt with different control provisions and re-examine the relevant issues.

Appendix A

A.1 Appendix for Section 3

To derive the value of corporate securities in our model, we first provide the following constants:

$$\gamma_1 = -\frac{1}{\sigma^2}\left[\left(\mu + i - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu + i - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] < 0,$$

$$\gamma_2 = -\frac{1}{\sigma^2}\left[\left(\mu + i - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu + i - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 1,$$

$$\beta_1 = -\frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] < 0,$$

$$\beta_2 = -\frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 1.$$

As shown in Section 3, the value of corporate securities after debt restructuring is solved in two regions:

$$\mathbb{N}_1 = [X_i, +\infty), \quad \mathbb{N}_2 = [X_d, X_i),$$

where $\mathbb{N}_1$ denotes the investment region, while $\mathbb{N}_2$ denotes the non-investment region. Similar to Wong and Yu (2022) and Diamond and He (2014), the total equity value satisfies ODE (7). Then its general solution can be expressed by (11). The constant coefficients A_1, A_2, and A_3 are obtained from the boundary conditions given in Equation (9), which yields

$$A_1 = (U_0 - U_i)X_i + A_2 X_i^{\beta_1} + A_3 X_i^{\beta_2},$$

$$A_2 = \frac{(\beta_2 - \gamma_1)X_i^{\beta_2}[(1-\tau)(c+b_n)/r - U_0 X_d] - X_d^{\beta_2}(U_0 - U_i)(\gamma_1 - 1)X_i}{(\beta_2 - \gamma_1)X_i^{\beta_2}X_d^{\beta_1} - (\beta_1 - \gamma_1)X_i^{\beta_1}X_d^{\beta_2}},$$

$$A_3 = \frac{X_d^{\beta_1}(U_0 - U_i)(\gamma_1 - 1)X_i - (\beta_1 - \gamma_1)X_i^{\beta_1}[(1-\tau)(c+b_n)/r - U_0 X_d]}{(\beta_2 - \gamma_1)X_i^{\beta_2}X_d^{\beta_1} - (\beta_1 - \gamma_1)X_i^{\beta_1}X_d^{\beta_2}}.$$

For the value of market debt, the general solution is given by (14) and subject to the boundary conditions given in Equation (13). Then, the constant coefficients C_1, C_2, and C_3 are given as follows:

$$C_1 = c/r + C_2 X_i^{\beta_1} + C_3 X_i^{\beta_2},$$

$$C_2 = \frac{(\beta_2 - \gamma_1)X_i^{\beta_2}[R_m(X_d) - c/r] - X_d^{\beta_2}(\gamma_1 - 1)c/r}{(\beta_2 - \gamma_1)X_i^{\beta_2}X_d^{\beta_1} - (\beta_1 - \gamma_1)X_i^{\beta_1}X_d^{\beta_2}},$$

$$C_3 = \frac{X_d^{\beta_1}(\gamma_1 - 1)c/r - (\beta_1 - \gamma_1)X_i^{\beta_1}[R_m(X_d) - c/r]}{(\beta_2 - \gamma_1)X_i^{\beta_2}X_d^{\beta_1} - (\beta_1 - \gamma_1)X_i^{\beta_1}X_d^{\beta_2}}.$$

The general solution for the value of bank debt is given by (16). The three constant coefficients F_1, F_2, and F_3 are simultaneously determined by the boundary conditions given in Equation (15). We then obtain

$$
\begin{aligned}
F_1 &= b_n/r + F_2 X_i^{\beta_1} + F_3 X_i^{\beta_2},\\
F_2 &= \frac{(\beta_2-\gamma_1)X_i^{\beta_2}[R_b(X_d)-b_n/r] - X_d^{\beta_2}(\gamma_1-1)b_n/r}{(\beta_2-\gamma_1)X_i^{\beta_2}X_d^{\beta_1} - (\beta_1-\gamma_1)X_i^{\beta_1}X_d^{\beta_2}},\\
F_3 &= \frac{X_d^{\beta_1}(\gamma_1-1)b_n/r - (\beta_1-\gamma_1)X_i^{\beta_1}[R_b(X_d)-b_n/r]}{(\beta_2-\gamma_1)X_i^{\beta_2}X_d^{\beta_1} - (\beta_1-\gamma_1)X_i^{\beta_1}X_d^{\beta_2}}.
\end{aligned}
$$

Before debt restructuring, a similar derivation process for the bank debt value, the equity value and the market debt value is shown as before; thus, we can obtain Propositions 4, 5, and 6, respectively.

A.2 Appendix for Section 4.2

A.2.1 Exclusive market debt structure

First, we consider the model with firms financed exclusively with equity and market debt. Let $V_c(X)$, $E_c(X)$, and $M_c(X)$ denote the total values of the firm, equity and market debt, respectively. The corresponding default and investment thresholds are denoted by X_d^c and X_i^c, respectively. In the absence of bank debt, the value of corporate securities is only solved in two regions: the investment region and non-investment region. We thus take $b=0$ and remove the debt restructuring opportunity from our baseline model. Using a similar derivation process to that shown in Section A.1, we can summarize the main results as the following proposition.[27]

Proposition 7 *For firms financed exclusively with equity and market debt, the market value of equity $E_c(X)$ satisfies*

$$
E_c(X) = \begin{cases} \left(U_i X - (1-\tau)\frac{c}{r}\right) + W_1 X^{\gamma_1}, & if \quad X_i^c \le X,\\ \left(U_0 X - (1-\tau)\frac{c}{r}\right) + W_2 X^{\beta_1} + W_3 X^{\beta_2}, & if \quad X_d^c < X < X_i^c, \end{cases} \tag{A.1}
$$

where

$$
\begin{aligned}
W_1 &= (U_0-U_i)(X_i^c)^{1-\gamma_1} + W_2 (X_i^c)^{\beta_1-\gamma_1} + W_3 (X_i^c)^{\beta_2-\gamma_1},\\
W_2 &= \frac{(1-\gamma_1)(U_0-U_i)X_i^c(X_d^c)^{\beta_2} - \left(U_0 X_d^c - (1-\tau)c/r\right)(\beta_2-\gamma_1)(X_i^c)^{\beta_2}}{(\beta_2-\gamma_1)(X_i^c)^{\beta_2}(X_d^c)^{\beta_1} - (\beta_1-\gamma_1)(X_i^c)^{\beta_1}(X_d^c)^{\beta_2}},\\
W_3 &= \frac{\left(U_0 X_d^c - (1-\tau)c/r\right)(\beta_1-\gamma_1)(X_i^c)^{\beta_1} - (1-\gamma_1)(U_0-U_i)X_i^c(X_d^c)^{\beta_1}}{(\beta_2-\gamma_1)(X_i^c)^{\beta_2}(X_d^c)^{\beta_1} - (\beta_1-\gamma_1)(X_i^c)^{\beta_1}(X_d^c)^{\beta_2}}.
\end{aligned}
$$

The optimal investment threshold X_i^c is also determined by the condition $E_c'(X_i^c) = \phi$. The optimal bankruptcy threshold, which is selected by shareholders, is determined by the smooth-pasting condition

---

[27] Similar expressions are derived by Diamond and He (2014).

$lim_{X\downarrow X_d^c} E_c'(X) = 0$. *By anticipating shareholders' investment and default policies, the value of market debt* $M_c(X)$ *satisfies*

$$M_c(X) = \begin{cases} \frac{c}{r} + H_1 X^{\gamma_1}, & if \quad X_i^c \le X, \\ \frac{c}{r} + H_2 X^{\beta_1} + H_3 X^{\beta_2}, & if \quad X_d^c < X < X_i^c. \end{cases} \tag{A.2}$$

Here, the coefficients H_1, H_2, *and* H_3 *are given by*

$$\begin{aligned} H_1 &= H_2 (X_i^c)^{\beta_1 - \gamma_1} + H_3 (X_i^c)^{\beta_2 - \gamma_1}, \\ H_2 &= \frac{(\beta_2 - \gamma_1)(X_i^c)^{\beta_2} \left(L(X_d^c) - c/r\right)}{(\beta_2 - \gamma_1)(X_i^c)^{\beta_2}(X_d^c)^{\beta_1} - (\beta_1 - \gamma_1)(X_i^c)^{\beta_1}(X_d^c)^{\beta_2}}, \\ H_3 &= \frac{(\gamma_1 - \beta_1)(X_i^c)^{\beta_1} \left(L(X_d^c) - c/r\right)}{(\beta_2 - \gamma_1)(X_i^c)^{\beta_2}(X_d^c)^{\beta_1} - (\beta_1 - \gamma_1)(X_i^c)^{\beta_1}(X_d^c)^{\beta_2}}. \end{aligned}$$

The expressions given in Equations (A.1) and (A.2) for the values of equity and market debt, respectively, are similar to the expressions obtained previously. We thus obtain a similar interpretation. Finally, the optimal capital structure decisions, which are chosen to maximize the ex ante equity value (initial firm value), are determined by $c_c^*(X_0) = arg \max_c V_c(X_0; c)$.

A.2.2 Exclusive bank debt structure

We now consider firms financed exclusively with equity and bank debt. To facilitate exposition, let us denote by $E_b(X)$, $D_b(X)$, and $V_b(X)$ the equity value, bank debt value, and firm value, respectively, in the before debt restructuring stage. Moreover, let $E_{nb}(X)$ and $D_{nb}(X)$ denote the equity value and bank debt value, respectively, in the after debt restructuring stage. The corresponding investment, bankruptcy, and renegotiation thresholds are denoted by X_i^b, X_d^b, and X_n^b, respectively.

Under the exclusive bank debt structure, the value functions of equity after and before debt restructuring resemble Equations (11) and (23), respectively, with $c = 0$. Here, the optimal investment, renegotiation and bankruptcy thresholds are given by the condition $E_{nb}'(X_i^b) = \phi$, smooth-pasting condition $E_b'(X_n^b) = (1-q)[(E_{nb}'(X_n^b) - (\rho(X_n^b) E_{nb}(X_n^b))')]$ and smooth-pasting condition $E_{nb}'(X_d^b) = 0$, respectively. Similarly, the value functions of bank debt after and before debt restructuring resemble Equations (16) and (20), respectively, but with $c = 0$. Finally, shareholders choose the optimal coupon payment c_b^* to maximize the ex ante equity value (initial firm value), in that $c_b^*(X_0) = arg \max_b V_b(X_0; b)$.

A.3 Appendix for Section 4.3

The setting in the absence of flexible growth opportunities serves as a benchmark to better understand the determinants of the optimal mixture of bank and market debt. In such situations, we need only consider two possible events triggered by the evolution of the cash flow X: renegotiation and final bankruptcy. Using the same derivation procedure as that in our baseline model, we can obtain the valuation of corporate securities in the benchmark model. We summarize the main results as follows.

Proposition 8 *In the after debt restructuring stage ($T > T_n$), the value of the firm's total equity ($E_n(X)$) is given by*

$$E_n(X) = U_0X - \frac{(1-\tau)(c+b_n)}{r} - \left[U_0X_d - \frac{(1-\tau)(c+b_n)}{r}\right]\left(\frac{X}{X_d}\right)^{\gamma_1}, \tag{A.3}$$

where the optimal bankruptcy threshold is given by $X_d = \gamma_1(1-\tau)(c+b_n)/[(\gamma_1-1)U_0r]$. The values of bank debt ($D_n(X)$) and market debt ($M_n(X)$) satisfy

$$D_n(X) = \frac{b_n}{r} + \left[R_b(X_d) - \frac{b_n}{r}\right]\left(\frac{X}{X_d}\right)^{\gamma_1}, \tag{A.4}$$

$$M_n(X) = \frac{c}{r} + \left[R_m(X_d) - \frac{c}{r}\right]\left(\frac{X}{X_d}\right)^{\gamma_1}, \tag{A.5}$$

respectively. At the time of renegotiation, the Nash bargaining process between shareholders and bank lenders remains the same as that in the baseline model. In the before debt restructuring stage ($T < T_n$), the value of the firm's original equity ($E(X)$) is given by

$$E(X) = \left(U_0X - (1-\tau)\frac{b+c}{r}\right) + \left[(1-q)(1-\rho^*(X_n))E_n(X_n) - U_0X_n + (1-\tau)\frac{b+c}{r}\right]\left(\frac{X}{X_n}\right)^{\gamma_1}, \tag{A.6}$$

where the optimal renegotiation threshold is also determined by the condition (24). The expressions for the value of bank debt ($D(X)$) and market debt ($M(X)$) are the same as Equations (20)) and (27)), respectively, given the specified bank debt value $D_n(X)$, market debt value $M_n(X)$ and equity value $E_n(X)$ in the absence of flexible growth opportunities.

A.4 Technical details for Section 5

We first define the following two functions to solve the model with time-varying investment opportunities:

$$g_{s,L}(y) \equiv r + \lambda_s - \mu y - \frac{\sigma^2}{2}y(y-1),$$

$$g_{s,H}(y) \equiv r + \lambda_s - (\mu+i)y - \frac{\sigma^2}{2}y(y-1),$$

where y is a variable and $s \in \{G, B\}$.

Let $\gamma_{1,L}$ and $\gamma_{2,L}$ be the two negative roots and $\gamma_{3,L}$ and $\gamma_{4,L}$ be the two positive roots of the quadratic equation $g_{B,L}(y) \times g_{G,L}(y) = \lambda_G\lambda_B$.

Let $\gamma_{1,M}$ and $\gamma_{2,M}$ be the two negative roots and $\gamma_{3,M}$ and $\gamma_{4,M}$ be the two positive roots of the quadratic equation $g_{B,L}(y) \times g_{G,H}(y) = \lambda_G \lambda_B$.

Let $\gamma_{1,H}$ and $\gamma_{2,H}$ be the two negative roots and $\gamma_{3,H}$ and $\gamma_{4,H}$ be the two positive roots of the quadratic equation $g_{B,H}(y) \times g_{G,H}(y) = \lambda_G \lambda_B$.

Let $\beta_{1,v}$ and $\beta_{2,v}$ be the two roots of the quadratic equation $g_{G,v}(y) = 0$ for $v \in \{H, L\}$, respectively.

Next, we derive the solution for the equity value. The presentation of the value function for equity is dependent on the ordering of the renegotiation, investment and bankruptcy boundaries in both states, which are endogenously determined. We first conjecture and then verify the ordering of these decisions, as discussed in the existing literature on investment and capital structure decisions under business cycles (see, e.g., Chen and Manso, 2017). Specifically, first, the firm takes $i = 0$ only in the after debt restructuring stage in both states, as noted previously. Second, it is straightforward to show that shareholders exercise the growth option earlier and exercise the bankruptcy option later in the good state than in the bad state after debt restructuring, i.e., $X_i(G) < X_i(B)$ and $X_d(G) < X_d(B)$. We then assume that the following ordering after debt restructuring holds:[28]

$$X_d(G) < X_d(B) < X_i(G) < X_i(B).$$

This ordering yields four relevant intervals for cash flow X_t when we solve the value of equity: $\mathbb{L}_1 = (X_d(G), X_d(B)]$, $\mathbb{L}_2 = (X_d(B), X_i(G)]$, $\mathbb{L}_3 = (X_i(G), X_i(B)]$ and $\mathbb{L}_4 = (X_i(B), +\infty)$. Before debt restructuring, it is intuitive that

$$X_n(G) < X_n(B).$$

That is, shareholders exercise the renegotiation option later in the good state than in the bad state. Thus, the equity value can be solved in two intervals: $\mathbb{L}_5 = (X_n(G), X_n(B)]$ and $\mathbb{L}_6 = [X_n(B), +\infty)$.

After debt restructuring. After the restructuring of bank debt, for $X \in \mathbb{L}_1$, the firm is solvent but does not exercise investment in the good state, while it goes bankrupt in the bad state. Thus, the equity value $E_n(X, B) = 0$, while $E_n(X, G)$ satisfies the following ODE:

$$(r + \lambda_G) E_n(X, G) = (1 - \tau)(X - c - b_n) + \mu X E_n^{'}(X, G) + \frac{\sigma^2 X^2}{2} E_n^{''}(X, G). \tag{A.7}$$

For $X \in \mathbb{L}_2$, the firm is solvent but does not exercise investment in either state. Thus, equity values $E_n(X, B)$ and $E_n(X, G)$ satisfy the following system of ODEs:

$$\begin{cases} (r + \lambda_B) E_n(X, B) = (1 - \tau)(X - c - b_n) + \mu X E_n^{'}(X, B) + \frac{\sigma^2 X^2}{2} E_n^{''}(X, B) + \lambda_B E_n(X, G), \\ (r + \lambda_G) E_n(X, G) = (1 - \tau)(X - c - b_n) + \mu X E_n^{'}(X, G) + \frac{\sigma^2 X^2}{2} E_n^{''}(X, G) + \lambda_G E_n(X, B). \end{cases} \tag{A.8}$$

---

[28] This order can easily be satisfied with our calibrated model parameters.

For $X \in \mathbb{L}_3$, the firm has already exercised investment in state G but has not made investment in state B. $E_n(X,B)$ and $E_n(X,G)$ thus satisfy the following system of ODEs:

$$\begin{cases} (r+\lambda_B)E_n(X,B) = (1-\tau)(X-c-b_n) + \mu X E_n^{'}(X,B) + \frac{\sigma^2 X^2}{2} E_n^{''}(X,B) + \lambda_B E_n(X,G), \\ (r+\lambda_G)E_n(X,G) = (1-\tau)(X-c-b_n) - \phi_G i X + (\mu+i) X E_n^{'}(X,G) + \frac{\sigma^2 X^2}{2} E_n^{''}(X,G) + \lambda_G E_n(X,B). \end{cases} \tag{A.9}$$

For $X \in \mathbb{L}_4$, the firm has always made the investment in either state, and $E_n(X,B)$ and $E_n(X,G)$ satisfy the following system of ODEs:

$$\begin{cases} (r+\lambda_B)E_n(X,B) = (1-\tau)(X-c-b_n) - \phi_B i X + (\mu+i) X E_n^{'}(X,B) + \frac{\sigma^2 X^2}{2} E_n^{''}(X,B) + \lambda_B E_n(X,G), \\ (r+\lambda_G)E_n(X,G) = (1-\tau)(X-c-b_n) - \phi_G i X + (\mu+i) X E_n^{'}(X,G) + \frac{\sigma^2 X^2}{2} E_n^{''}(X,G) + \lambda_G E_n(X,B). \end{cases} \tag{A.10}$$

The solutions of the above ODE system can be calculated as

$$E_n(X,B) = \begin{cases} 0, & X \in (0, X_d(B)]; \\ \sum_{j=1}^{4} A_j^E X^{\gamma_{j,L}} + (1-\tau)[w_L X - k(c+b_n)], & X \in (X_d(B), X_i(G)]; \\ \sum_{j=1}^{4} H_j^E X^{\gamma_{j,M}} + (1-\tau)[w_{B,M} X - k(c+b_n)], & X \in (X_i(G), X_i(B)]; \\ \sum_{j=1}^{4} J_j^E X^{\gamma_{j,H}} + (1-\tau)[w_{B,H} X - k(c+b_n)], & X \in (X_i(B), \infty); \end{cases} \tag{A.11}$$

and

$$E_n(X,G) = \begin{cases} 0, & X \in (0, X_d(G)]; \\ \sum_{j=1}^{2} N_j^E X^{\beta_{j,L}} + \frac{(1-\tau)X}{r+\lambda_G-\mu} - \frac{(1-\tau)(c+b_n)}{r+\lambda_G}, & X \in (X_d(G), X_d(B)]; \\ \sum_{j=1}^{4} B_j^E X^{\gamma_{j,L}} + (1-\tau)[w_L X - k(c+b_n)], & X \in (X_d(B), X_i(G)]; \\ \sum_{j=1}^{4} I_j^E X^{\gamma_{j,M}} + (1-\tau)[w_{G,M} X - k(c+b_n)], & X \in (X_i(G), X_i(B)]; \\ \sum_{j=1}^{4} K_j^E X^{\gamma_{j,H}} + (1-\tau)[w_{G,H} X - k(c+b_n)], & X \in (X_i(B), +\infty); \end{cases} \tag{A.12}$$

where

$$\begin{aligned} B_j^E &= \frac{\lambda_G}{g_{G,H}(\gamma_{j,L})} A_j^E, \quad j \in \{1,2,3,4\}, \\ I_j^E &= \frac{\lambda_G}{g_{G,H}(\gamma_{j,M})} H_j^E, \quad j \in \{1,2,3,4\}, \\ K_j^E &= \frac{\lambda_G}{g_{G,H}(\gamma_{j,H})} J_j^E, \quad j \in \{1,2,3,4\}, \\ w_{G,M} &= \frac{(r+\lambda_B-\mu)[1-\phi_G i/(1-\tau)] + \lambda_G}{(r+\lambda_B-\mu)(r+\lambda_G-\mu-i) - \lambda_B\lambda_G}, \\ w_{B,M} &= \frac{r+\lambda_G+\lambda_B[1-\phi_G i/(1-\tau)] - \mu - i}{(r+\lambda_B-\mu)(r+\lambda_G-\mu-i) - \lambda_B\lambda_G}, \\ w_{G,H} &= \frac{[r+\lambda_B-\mu-i][1-\phi_G i/(1-\tau)] + \lambda_G[1-\phi_B i/(1-\tau)]}{(r+\lambda_G-\mu-i)(r+\lambda_B-\mu-i) - \lambda_G\lambda_B}, \\ w_{B,H} &= \frac{[r+\lambda_G-\mu-i][1-\phi_B i/(1-\tau)] + \lambda_B[1-\phi_G i/(1-\tau)]}{(r+\lambda_G-\mu-i)(r+\lambda_B-\mu-i) - \lambda_G\lambda_B}, \end{aligned}$$

$$w_L = \frac{r+\lambda_G+\lambda_B-\mu}{(r+\lambda_G-\mu)(r+\lambda_B-\mu)-\lambda_G\lambda_B},$$
$$k = \frac{\lambda_G+\lambda_B+r}{(r+\lambda_G)(r+\lambda_B)-\lambda_G\lambda_B}.$$

The coefficients A_1^E, A_2^E, A_3^E, A_4^E, H_1^E, H_2^E, H_3^E, H_4^E, N_1^E, N_2^E, J_1^E, J_2^E, J_3^E, and J_4^E are determined by the following boundary conditions. First, we have the standard no-bubble conditions:

$$\lim_{X\uparrow\infty}\frac{E_n(X,G)}{X} < \infty,$$
$$\lim_{X\uparrow\infty}\frac{E_n(X,B)}{X} < \infty.$$

Second, $E_n(X,G)$ and $E_n(X,B)$ satisfy the value-matching conditions at the boundary of regions $\mathbb{L}_1$, $\mathbb{L}_2$, $\mathbb{L}_3$, and $\mathbb{L}_4$, in that

$$\begin{aligned}
&\lim\nolimits_{X\downarrow X_d(B)} E_n(X,B) = 0, \quad \lim\nolimits_{X\downarrow X_d(G)} E_n(X,G) = 0,\\
&\lim\nolimits_{X\uparrow X_i(G)} E_n(X,B) = \lim\nolimits_{X\downarrow X_i(G)} E_n(X,B), \quad \lim\nolimits_{X\uparrow X_i(B)} E_n(X,B) = \lim\nolimits_{X\downarrow X_i(B)} E_n(X,B),\\
&\lim\nolimits_{X\uparrow X_d(B)} E_n(X,G) = \lim\nolimits_{X\downarrow X_d(B)} E_n(X,G), \quad \lim\nolimits_{X\uparrow X_i(G)} E_n(X,G) = \lim\nolimits_{X\downarrow X_i(G)} E_n(X,G),\\
&\lim\nolimits_{X\uparrow X_i(B)} E_n(X,G) = \lim\nolimits_{X\downarrow X_i(B)} E_n(X,G).
\end{aligned}$$

Finally, $E_n(X,G)$ and $E_n(X,B)$ should satisfy the non-arbitrage conditions at the boundary of regions $\mathbb{L}_1$, $\mathbb{L}_2$, $\mathbb{L}_3$, and $\mathbb{L}_4$, in that

$$\begin{aligned}
&\lim\nolimits_{X\uparrow X_i(G)} E_n'(X,B) = \lim\nolimits_{X\downarrow X_i(G)} E_n'(X,B), \quad \lim\nolimits_{X\uparrow X_i(B)} E_n'(X,B) = \lim\nolimits_{X\downarrow X_i(B)} E_n'(X,B),\\
&\lim\nolimits_{X\uparrow X_d(B)} E_n'(X,G) = \lim\nolimits_{X\downarrow X_d(B)} E_n'(X,G), \quad \lim\nolimits_{X\uparrow X_i(G)} E_n'(X,G) = \lim\nolimits_{X\downarrow X_i(G)} E_n'(X,G),\\
&\lim\nolimits_{X\uparrow X_i(B)} E_n'(X,G) = \lim\nolimits_{X\downarrow X_i(B)} E_n'(X,G).
\end{aligned}$$

The optimal bankruptcy and investment decisions $\{X_d(B), X_d(B), X_i(B), X_i(B)\}$ after debt restructuring satisfy the following four smooth-pasting conditions:

$$\begin{aligned}
&E_n'(X_d(B), B) = 0, \qquad E_n'(X_d(G), G) = 0,\\
&E_n'(X_i(B), B) = \phi(B), \quad E_n'(X_i(G), G) = \phi(G).
\end{aligned}$$

Before debt restructuring. For $X \in \mathbb{L}_5$, the firm has already performed the debt restructuring in state B but has not performed the debt restructuring in state B. Thus, $E_n(X,B) = (1-q)(1-\rho(B))E_n(X,B)$ and $E(X,G)$ satisfy the following ODE:

$$(r+\lambda_G)E(X,G) = (1-\tau)(X-c-b) - \phi_G iX + (\mu+i)XE^{'}(X,G) + \frac{\sigma^2X^2}{2}E^{''}(X,G) + \lambda_G E(X,B). \tag{A.13}$$

For $X \in \mathbb{L}_6$, the firm has not yet performed the debt restructuring in either state; therefore, $E(X,G)$ and $E(X,B)$ satisfy the following system of ODEs:

$$\begin{cases}
(r+\lambda_B)E(X,B) = (1-\tau)(X-c-b) - \phi_B iX + (\mu+i)XE^{'}(X,B) + \frac{\sigma^2X^2}{2}E^{''}(X,B) + \lambda_B E(X,G),\\
(r+\lambda_G)E(X,G) = (1-\tau)(X-c-b) - \phi_G iX + (\mu+i)XE^{'}(X,G) + \frac{\sigma^2X^2}{2}E^{''}(X,G) + \lambda_G E(X,B).
\end{cases} \tag{A.14}$$

Given the optimal swap rate $\{\rho^*(X,G), \rho^*(X,B)\}$ in the bank debt restructuring process in each state, the solutions of the above ODE system are given by

$$E(X,B) = \begin{cases} (1-q)(1-\rho^*(X,B))E_n(X,B), & X \in (0, X_n(B)]; \\ \sum_{j=1}^{4} a_j^E X^{\gamma_{j,H}} + (1-\tau)[w_{B,H}X - k(b+c)], & X \in (X_n(B), +\infty); \end{cases} \tag{A.15}$$

and

$$E(X,G) = \begin{cases} (1-q)(1-\rho^*(X,G))E_n(X,G), & X \in (0, X_n(G)]; \\ \sum_{j=1}^{2} n_j^E X^{\beta_{j,H}} + (1-q)\eta \sum_{j=1}^{2} \frac{\lambda_G J_j^E}{g_{G,H}(\gamma_{j,H})} X^{\gamma_{j,H}} + & \\ \frac{(1-\tau)[1+(1-q)\eta\lambda_G(w_{B,H}-(1-\alpha)w_L)]}{r+\lambda_G-\mu-i_G} X - \frac{(1-\tau)[b+c+(1-q)\eta\lambda_G k(b_n+c-b_n/(1-\tau))]}{r+\lambda_G}, & X \in (X_n(G), X_n(B)]; \\ \sum_{j=1}^{4} b_j^E X^{\gamma_{j,H}} + (1-\tau)[w_{G,H}X - k(b+c)], & X \in (X_n(B), +\infty). \end{cases} \tag{A.16}$$

Here,

$$b_j^E = \frac{\lambda_G}{g_{G,H}(\gamma_{j,H})} a_j^E, \quad j \in \{1, 2, 3, 4\}.$$

Again, the coefficients a_1^E, a_2^E, a_3^E, a_4^E, n_1^E, and n_2^E are determined by the following boundary conditions. First, to exclude bubbles, we have

$$\lim_{X\uparrow\infty} \frac{E_n(X,G)}{X} < \infty,$$

$$\lim_{X\uparrow\infty} \frac{E_n(X,B)}{X} < \infty.$$

Next, at the restructuring threshold of bank debt, the value-matching conditions imply

$$\begin{aligned} &\lim_{X\uparrow X_n(B)} E(X,B) = \lim_{X\downarrow X_n(B)} (1-q)(1-\rho^*(X,B))E_n(X,B), \\ &\lim_{X\uparrow X_n(G)} E(X,G) = \lim_{X\downarrow X_n(G)} (1-q)(1-\rho^*(X,G))E_n(X,G), \\ &\lim_{X\uparrow X_n(B)} E(X,G) = \lim_{X\downarrow X_n(B)} E(X,G), \\ &\lim_{X\uparrow X_n(B)} E'(X,G) = \lim_{X\downarrow X_n(B)} E'(X,G). \end{aligned}$$

The optimal restructuring boundary $\{X_n(B), X_n(G)\}$ of bank debt satisfies two smooth-pasting conditions:

$$E'(X_n(G),G) = (1-q)[E_n'(X_n(G),G) - (\rho^*(X_n(G),G)E_n(X_n(G),G))'],$$

$$E'(X_n(B),B) = (1-q)[E_n'(X_n(B),B) - (\rho^*(X_n(B),B)E_n(X_n(B),B))'].$$

In each state s_t, we can also obtain the solutions for bank debt $(D(X_t,s_t)$ and $D_n(X_t,s_t))$ and market debt $(M(X_t,s_t)$ and $M_n(X_t,s_t))$ in the same way as we do for equity. We omit their descriptions for brevity reasons. Finally, the optimal debt structure is determined by

$$\{b^*(X_0,s_0), c^*(X_0,s_0)\} = \arg\max_{b,c} \{E(X_0,s_0;b,c) + D(X_0,s_0;b,c) + M(X_0,s_0;b,c)\}, \quad s_0 \in \{G,B\}. \tag{A.17}$$

References

Alanis, E., Chava, S., Kumar, P., 2018. Shareholder bargaining power, debt overhang, and investment. Review of Corporate Finance Studies 7 (2), 276–318.

Altman, E., Karlin, B., 2009. The re-emergence of distressed exchanges in corporate restructurings. NYU Working Paper.

Asquith, P., Gertner, R., Scharfstein, D., 1994. Anatomy of financial distress: An examination of junk-bond issuers. The Quarterly Journal of Economics 109 (3), 625–658.

Becker, B., Ivashina, V., 2014. Cyclicality of credit supply: Firm level evidence. Journal of Monetary Economics 62, 76–93.

Bolton, P., Chen, H., Wang, N., 2013. Market timing, investment, and risk management. Journal of Financial Economics 109 (1), 40–62.

Bolton, P., Freixas, X., 2000. Equity, bonds, and bank debt: Capital structure and financial market equilibrium under asymmetric information. Journal of Political Economy 108 (2), 324–351.

Bolton, P., Wang, N., Yang, J., 2020. Leverage dynamics and financial flexibility. Working Paper. National Bureau of Economic Research.

Chen, H., Manso, G., 2017. Macroeconomic risk and debt overhang. Review of Corporate Finance Studies 6 (1), 1–38.

Chen, H., Maslar, D. A., Serfling, M., 2020. Asset redeployability and the choice between bank debt and public debt. Journal of Corporate Finance 64, 101678.

Colla, P., Ippolito, F., Li, K., 2013. Debt specialization. The Journal of Finance 68 (5), 2117–2141.

Davydenko, S. A., Strebulaev, I. A., 2007. Strategic actions and credit spreads: An empirical investigation. The Journal of Finance 62 (6), 2633–2671.

De Fiore, F., Uhlig, H., 2015. Corporate debt structure and the financial crisis. Journal of Money, Credit and Banking 47 (8), 1571–1598.

DeMarzo, P., He, Z., 2021. Leverage dynamics without commitment. The Journal of Finance 76 (3), 1195–1250.

DeMarzo, P. M., Fishman, M. J., 2007. Optimal long-term financial contracting. The Review of Financial Studies 20 (6), 2079–2128.

Diamond, D. W., He, Z., 2014. A theory of debt maturity: The long and short of debt overhang. The Journal of Finance 69 (2), 719–762.

Dumas, B., 1991. Super contact and related optimality conditions. Journal of Economic Dynamics and Control 15 (4), 675–685.

Eberly, J. C., Rebelo, S., 2009. Investment and value: a neoclassical benchmark. Working Paper. Northwestern University and HEC Montréal.

Fan, H., Sundaresan, S. M., 2000. Debt valuation, renegotiation, and optimal dividend policy. The Review of Financial Studies 13 (4), 1057–1099.

Favara, G., Morellec, E., Schroth, E., Valta, P., 2017. Debt enforcement, investment, and risk taking across countries. Journal of Financial Economics 123 (1), 22–41.

Favara, G., Schroth, E., Valta, P., 2012. Strategic default and equity risk across countries. The Journal of Finance 67 (6), 2051–2095.

Franks, J. R., Torous, W. N., 1994. A comparison of financial recontracting in distressed exchanges and chapter 11 reorganizations. Journal of Financial Economics 35 (3), 349–370.

Giambona, E., Golec, J., Lopez-de Silanes, F., 2021. Do firms purposefully change capital structure? evidence from an investment-opportunity shock to drug firms. Journal of Financial and Quantitative Analysis 56 (3), 915–944.

Gilson, S. C., John, K., Lang, L. H., 1990. Troubled debt restructurings: An empirical study of private reorganization of firms in default. Journal of Financial Economics 27 (2), 315–353.

Glover, B., 2016. The expected cost of default. Journal of Financial Economics 119 (2), 284–299.

Graham, J. R., Lemmon, M. L., Schallheim, J. S., 1998. Debt, leases, taxes, and the endogeneity of corporate tax status. The Journal of Finance 53 (1), 131–162.

Hackbarth, D., Hennessy, C. A., Leland, H. E., 2007. Can the trade-off theory explain debt structure? The Review of Financial Studies 20 (5), 1389–1428.

Hackbarth, D., Mauer, D. C., 2012. Optimal priority structure, capital structure, and investment. The Review of Financial Studies 25 (3), 747–796.

Hackbarth, D., Miao, J., Morellec, E., 2006. Capital structure, credit risk, and macroeconomic conditions. Journal of Financial Economics 82 (3), 519–550.

Hackbarth, D., Rivera, A., Wong, T.-Y., 2021. Optimal short-termism. Management Science, forthcoming.

Hennessy, C. A., 2004. Tobin's Q, debt overhang, and investment. The Journal of Finance 59 (4), 1717–1742.

Johnson, S. A., 1997. An empirical analysis of the determinants of corporate debt ownership structure. Journal of Financial and Quantitative analysis 32 (1), 47–69.

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. The Journal of Finance 49 (4), 1213–1252.

Leland, H. E., 1998. Agency costs, risk management, and capital structure. The Journal of Finance 53 (4), 1213–1243.

Lummer, S. L., McConnell, J. J., 1989. Further evidence on the bank lending process and the capital-market response to bank loan agreements. Journal of Financial Economics 25 (1), 99–122.

Lyandres, E., Zhdanov, A., 2014. Convertible debt and investment timing. Journal of Corporate Finance 24, 21–37.

Mauer, D. C., Ott, S. H., 2000. Agency costs, underinvestment, and optimal capital structure. Perfect Flexibility, Agency, and Competition, New Developments in the Theory and Application of Real Options, Oxford University Press, New York.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. The Quarterly Journal of Economics 101 (4), 707–727.

Moraux, F., Silaghi, F., 2014. Inside debt renegotiation: Optimal debt reduction, timing, and the number of rounds. Journal of Corporate Finance 27, 269–295.

Morellec, E., Valta, P., Zhdanov, A., 2015. Financing investment: The choice between bonds and bank loans. Management Science 61 (11), 2580–2602.

Myers, S. C., 1977. Determinants of corporate borrowing. Journal of Financial Economics 5 (2), 147–175.

Park, C., 2000. Monitoring and structure of debt contracts. The Journal of Finance 55 (5), 2157–2195.

Pawlina, G., 2010. Underinvestment, capital structure and strategic debt restructuring. Journal of Corporate Finance 16 (5), 679–702.

Rauh, J. D., Sufi, A., 2010. Capital structure and debt structure. The Review of Financial Studies 23 (12), 4242–4280.

Sharpe, S. A., Nguyen, H. H., 1995. Capital market imperfections and the incentive to lease. Journal of Financial Economics 39 (2-3), 271–294.

Shibata, T., Nishihara, M., 2015a. Investment-based financing constraints and debt renegotiation. Journal of Banking and Finance 51, 79–92.

Shibata, T., Nishihara, M., 2015b. Investment timing, debt structure, and financing constraints. European Journal of Operational Research 241 (2), 513 – 526.

Silaghi, F., 2018. The use of equity financing in debt renegotiation. Journal of Economic Dynamics and Control 86, 123–143.

Sundaresan, S., Wang, N., 2007. Investment under uncertainty with strategic debt service. American Economic Review 97 (2), 256–261.

Sundaresan, S., Wang, N., Yang, J., 2015. Dynamic investment, capital structure, and debt overhang. The Review of Corporate Finance Studies 4 (1), 1–42.

Wong, T.-Y., Yu, J., 2022. Credit default swaps and debt overhang. Management Science 68 (3), 2069–2097.

Highlights

Debt heterogeneity has a significant impact on corporate financing and investment.

The inclusion of bank debt financing can mitigate the ex-post debt overhang problem.

Firms with more growth opportunities tend to use more market debt and less bank debt.

Non-monotonic effects of the cyclicality of growth opportunities on debt composition.