A COMPARISON OF STRESS ANALYSIS AND LIMIT ANALYSIS APPROACHES FOR SINGLE AND MULTIPLE NOZZLE COMBINATIONS IN CYLINDRICAL PRESSURE VESSELS

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ABSTRACT

The finite element method is the most commonly used approach when solving complex practical cylinder-cylinder junctions in pressure vessels. This is used when the geometrical arrangement is out with the permitted scope of the design-by-rule approaches or when detailed stress information is required as in a fatigue assessment. High-stress concentrations occur on the crotch corner for cylinder-cylinder joints, and it is possible to reach solutions for this problem by using both theoretical and numerical solutions. However, those approaches do not fully overlap nor have the same underlying assumptions. As such, an innovative high-fidelity finite element model has been developed to provide a holistic unified approach which can tackle a wide range of problems. In this study, various detailed nozzle design challenges were investigated including single and multiple nozzle combinations, nozzle-cylinder systems with different size ratios, fillet weld applications, limit loads and external loading cases were analyzed. The results obtained are compared with well-respected calculation methods such as WRC537, and a new approach is presented for the analysis of cylinder-cylinder combinations.

Keywords: Pressure vessel; Nozzle; FEM, Limit Load, External Load

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Mean diameter of nozzle;</td>
</tr>
<tr>
<td>D</td>
<td>Mean diameter of shell;</td>
</tr>
<tr>
<td>r_i</td>
<td>Inner radius of cylinder;</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis;</td>
</tr>
<tr>
<td>M_L</td>
<td>Longitudinal moment;</td>
</tr>
<tr>
<td>P_L</td>
<td>Pitch between nozzles in longitudinal direction;</td>
</tr>
<tr>
<td>d_i</td>
<td>Inner diameter of nozzle;</td>
</tr>
<tr>
<td>D_i</td>
<td>Inner diameter of shell;</td>
</tr>
<tr>
<td>r_o</td>
<td>Outer radius of cylinder;</td>
</tr>
<tr>
<td>M_C</td>
<td>Circumferential moment;</td>
</tr>
<tr>
<td>P</td>
<td>Tensile force;</td>
</tr>
<tr>
<td>P_L</td>
<td>Limit load;</td>
</tr>
<tr>
<td>P_int</td>
<td>Internal pressure;</td>
</tr>
<tr>
<td>T</td>
<td>Thickness of the shell;</td>
</tr>
<tr>
<td>V_L</td>
<td>Longitudinal force;</td>
</tr>
<tr>
<td>t</td>
<td>Thickness of the nozzle</td>
</tr>
<tr>
<td>V_C</td>
<td>Circumferential force;</td>
</tr>
<tr>
<td>\sigma_y</td>
<td>Yield stress;</td>
</tr>
<tr>
<td>P_T</td>
<td>Pitch between nozzles in axial direction;</td>
</tr>
</tbody>
</table>

1 INTRODUCTION

In pressure vessel applications, the system can be exposed to many actions including internal pressure and external loads especially where applied to nozzles in a local load configuration. The detection of these loads on the system is important for the safe operation of the system. If the necessary precautions are not considered, serious damage may occur to the structure. Welding Research Council Bulletins WRC107 [1] and WRC537 [2] provide detailed calculations for nozzles subject to the external loads mentioned. Since these calculations are based on thin shell theory, they do not fully describe the behavior of the stress system especially when more than one is present. More detailed assessment for the combination of two cylindrical nozzles in pressure vessels, which takes account of the opening in the shell, is included in WRC297 [3]. The stresses that occur on the nozzle and at the junction areas can be calculated therein. In addition, PD5500 Annex G [4] also contains similar stress calculations. Although close results are obtained with all these methods, there are some small differences due to the assumptions in the approaches.

It is indispensable to create openings for the insertion of nozzles in pressure vessels and this leads to potential weaknesses due to the high stresses in the intersection region, the effect of internal pressure and the external forces applied.
From Attwater’s study [5], the nozzles attached to the pressure vessels in the radial direction and the intersections under the internal pressure were investigated. The problem details were analyzed and studied by a three-dimensional finite element approach. The key pressure vessel parameters for these problems were investigated and compared with British Standards (BS) at the time. The code values used to obtain the stress intensity factors different from the standards were applied not only to the shell but also to the nozzle. The results show that, when thinner nozzles are used, the stress intensity is higher than the maximum stress values given in BS5500, or in other words, the stress values specified in BS5500 for thinner nozzles were ignored.

Moini et al. [6] attempted to obtain more efficient results from existing studies by using the specified boundary displacement method in a cylindrical pressure vessel with a single nozzle connection. This study was based on Bryson's study [7] and researchers compared the results obtained with this Bryson's work. They have stated that the study caused serious improvements especially in the values of internal surface tension.

In addition, Chandiramani et al. [8] made a comparison study using the calculations in WRC 297 and WRC 537 with the results obtained with the FEA nozzle-shell results. Additionally, Andrade et al. [9], studied nozzle-cylinder connections and stresses due to external loads were examined for a nozzle connection without reinforcement plate. Similarly, the results obtained were compared with WRC107 and WRC297.

In the present study, a new finite element approach has been introduced to consider the high-stress values occurring in nozzle-cylinder connections exposed to internal pressure and external loading conditions. This model can respond to external loading and multiple nozzle connections and also establish the limit load for each case. The ultimate goal is to develop a universal model that can be applied to all geometries and load cases for stress analysis and limit load evaluation.

2 STATEMENT of the PROBLEM

In the design and analysis of pressure vessel problems, the highest stresses usually occur on the nozzle intersection areas with under high internal pressure effect or at the weld depending on the direction and magnitude of the external forces applied. While the maximum stress concentration is expected to occur in the crotch corner only in internal pressure problems, this location can also shift to the welding area depending on the direction and size of the external loading. Such that, possible cracks and even explosion issues in the system mostly start from the crotch corner or welding area.

For this reason, the correct calculations of local stress values in nozzle connection problem are of great importance in the design and analysis phase in order to determine the fatigue life and prevent plastic deformation. The finite element method, stress analysis and limit load analysis are the most widely used method for the evaluation of maximum allowable loadings that may occur in the crotch corners and welding which are potential cracking places in the pressure vessel - nozzle junctions. The purpose of this study is to carry out a wide-ranging study in pressure vessels nozzles, especially on the nozzle intersection zones, and to include a range of common problem variations. In addition, it is a key challenge to create an efficient, high-quality finite element model which can provide physically representative solutions to these variations including the presence of pads and welds. The scope of this model encompasses the solutions obtained for critical regions depending on internal pressure, external loading, and limit loading conditions.

In a general framework is drawn, the study begins with verifying the finite element modeling for internal pressure and external loading conditions. For this, some studies in the literature and WRC537 calculations are used. Then, the appropriate welding and inner fillet size are determined for various d/D and t/T values. Thereafter, the effect of inner fillet and outer fillet (welding) values on the maximum stress values is examined. Thereafter, external load combinations affecting the system are examined. Moreover, research on radial and longitudinal joints for multi-nozzle combinations are investigated. Finally, the study concludes by examining limit loading conditions under various parameters.

3 FINITE ELEMENT MODEL DESCRIPTION

3.1. General

Bozkurt et al., in previous work, conducted studies on internal pressure and local loads observed in single nozzle configurations [10]. The finite element model and meshing techniques used are employed in the present work to ensure consistent and reliable results are obtained. The analysis examines the effects of internal pressure and external local loads on the system. Verification calculation are presented for internal pressure and external loading conditions in the elastic zone. Then, limit load verification was performed with elastic-perfectly plastic analysis and calculations. After these
verification studies, stress analysis of elastic zone will be performed under various parameters for single and multiple nozzle connections. Finally, parametric studies related to limit loadings (elastic-perfectly plastic analysis) are addressed.

3.2. Material Properties and Element Type
In this study, since the stress values are examined at the nozzle-cylinder intersection zones, the model is restrained at the saddle base. As such, the material allocation is reported for the nozzle and shell and the details are given in the Table 1. For the saddle, a 210GPa Young's Modulus value is adequate. In addition, since the weld material was assumed to be solid, it has the same properties as the nozzle material. However, the tensile properties of welds are normally overmatched to the parent material, as welds serve as an intermediate material between the nozzle and the shell. The yield strength values shown in the table are the minimum values to be considered. It is noted, however, that higher yield strength values can be obtained in practical applications. The values shown in the table are used directly in the analysis.

<table>
<thead>
<tr>
<th>Material Name</th>
<th>Young’s Modulus</th>
<th>Poisson’s ratio</th>
<th>Yield Strength</th>
<th>Ultimate Tensile Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell - SA-516 Grade 70 Carbon steel pressure vessel plates [16]</td>
<td>200GPa</td>
<td>0.3</td>
<td>260MPa</td>
<td>485-620MPa</td>
</tr>
<tr>
<td>Nozzle - ASTM A266 Grade 2 Carbon Steel [17]</td>
<td>190GPa</td>
<td>0.29</td>
<td>290MPa</td>
<td>570MPa</td>
</tr>
</tbody>
</table>

The choice of element type when undertaking finite element analysis is important as this is determined the ability of the model to represent the stress distribution experience in the physical world. ANSYS SOLID186, a higher-order 3-D solid element with 20-nodes that exhibits quadratic displacement behavior is employed. The element is defined by 20 nodes having three degrees of freedom per node and supports plasticity, and other phenomenon and is suitable for the present application.

3.3. Geometry and Loading Conditions
An open-end model is used during the analysis meaning there is no physical pipework is attached, but the imposed pressure load is included by means of the end cap pressure. Since the parameters are variable in the analysis, the diameter (d/D) and thickness ratios (t/T) for the nozzle and shell can vary in each case. In other words, the parametric analysis to be made is based on the nozzle diameter, the main cylinder diameter and their wall thickness. For this reason, d/D and t/T combinations for each vessel size will be the main variables for each analysis. These variables (d/D and t/T) are specified in the tables or in the relevant sections in detail in each analysis to be undertaken. Only the cylinder-cylinder connection
was used in the model and the model was placed on saddles and constrained at the base remote from the high stress regions.

The external loads are applied on nozzle centripetal with the application of link elements. All nodes in the nozzle upper area are connected to the nozzle center with link elements so that these external loads can be applied centripetally as shown in Figure 2. Since the requirement is not to progress into the plastic region, the internal pressure, local load and moment values to be applied, the limit loads are calculated first. Analyses related to determining to the limit loads in case of changes in internal pressure and external forces are examined in detail in the Various Limit Load Analysis States section. After these limit loads are found, forces are applied so that the stress values do not exceed the limit load.

4 FINITE ELEMENT MODEL VERIFICATION

4.1. General
Verification work has been completed for three different situations in nozzle-cylinder intersections. Firstly, the finite element model under internal pressure loading only is examined. Then the model is verified for three different external load cases. Finally, the model is validated for various limit load cases.

4.2. First Verification – Internal Pressure Only
Firstly, the finite element model used is validated based on Chandiramani’s [8] study. In summary, in models without a reinforcement plate, allowable stress values on the system for nozzle-shell junction and stress across the shell were obtained. During these processes, a cylindrical vessel with an inner diameter of 1000mm and a wall thickness of 25mm was used to calculate the MAWP (Maximum allowable working pressure) value. The inner diameter of the nozzle was taken as half the inner diameter of the vessel. The allowable working pressure value calculated by Chandiramani with rules of Clause 4.3.3 of Cod [15] was also recalculated and confirmed and this value obtained was applied to all interior walls of the system. Then, using the computed MAWP value, the maximum allowable stress values were calculated with the hand calculation, and the elastic stress values along the shell were found using ANSYS. The results obtained are very close to Chandiramani’s results, with the difference being about 1% and with further details reported by Bozkurt et al. [10].

4.3. Second Verification - External Pipework Loading
In this study, verification is made when the model is exposed to external loading independently from each other in X, Y, and Z directions. Loading conditions are represented in Figure 3.
In the analysis, a constant internal pressure of 0.5MPa was applied to the model. Three different external loading analyses were carried out as tensile force, longitudinal and circumferential moments. These were carried out into three distinct stages, and the load was doubled in each stage. The FEA results obtained were compared to WRC537. Details of the benchmark results can be seen in the Figure 4-6.
The results are similar in comparison made only for the model under tensile stress. In cases where the longitudinal moment and circumferential moment were applied, the results obtained from analysis and WRC537 calculations differ up to 5%. WRC 537 offers the necessary formulas and diagrams for spherical and cylindrical pressure vessels under external loads. This approach is based on shell theory and does not take into account of the opening formed by the nozzle intersection. The stress values obtained with the WRC537 calculations are higher than the 3D finite element analysis. The results obtained for the analyses are thus deemed quite satisfactory.

5. PARAMETRIC STUDY

5.1. General

It is necessary to optimize the model to be used due to make the most accurate design and analysis in cylinder-nozzle intersection problems. The main factors for optimization are the determination of the most suitable weld and inner fillet size according to the nozzle and shell wall thickness, the determination of the allowable forces or pressures acting on the system, and the positioning of the nozzles radially or axially etc. In all these analyses, set-in nozzle connections will be used and a representative image for the nozzle, weld and shell is given in the Figure 7. These mentioned parameters are discussed in detail in the following sections.

5.2. Inner Fillet and Outer Fillet (Welding) Applications Effects

The main purpose of this study is to minimize critical stress values that are likely to occur along the intersection area in the nozzle-shell connection zones (crotch corner) in pressure vessels. Internal fillet and welding applications are the primary application for these methods. When a welded geometry is considered, it is assumed to be continuous and integral to the nozzle and the weld and the nozzle parent material properties are considered identical. A representative image is given in Figure 8 for these applications, with the outer fillet profile referring to the weld, and the inner fillet pertaining to the rounding in the crotch corner region.
The aim herein is to identify the most suitable weld size and fillet size whilst minimizing the maximum stress. In order to determine the outer fillet size, a 5mm fillet is applied on the crotch corner and stress analysis is performed at four different t/T values. Welding radii in the analyses are 15mm, 30mm, and 45mm, respectively. The maximum Von-Mises stresses obtained as a result of these values are given in the Table 2.

<table>
<thead>
<tr>
<th>t/T</th>
<th>15mm</th>
<th>30mm</th>
<th>45mm</th>
<th>Max. Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,12</td>
<td>305</td>
<td>304</td>
<td>303</td>
<td>0,89%</td>
</tr>
<tr>
<td>0,56</td>
<td>413</td>
<td>410</td>
<td>407</td>
<td>1,52%</td>
</tr>
<tr>
<td>0,74</td>
<td>466</td>
<td>463</td>
<td>460</td>
<td>1,32%</td>
</tr>
<tr>
<td>1,08</td>
<td>570</td>
<td>567</td>
<td>564</td>
<td>0,98%</td>
</tr>
</tbody>
</table>

As can be seen in the Table 2, the weld size was increased in 15 mm steps. In all cases, the stress values at the crotch corner were very similar and the difference between these values was less than 1.5%. For this reason, the same process was applied to the inner fillet formed around the crotch corner.

Changing the outer fillet (welding) dimensions does not make a significant difference on the maximum stresses in the crotch corner. For this reason, it is useful to examine the response to changes in the inner fillet dimensions. Four analyses were undertaken for a vessel with a wall thickness of 25 mm, and the inner fillet radius was increased by 5 mm in each successive analysis. The outer weld fillet radius was 15mm in each analysis. The results of the analyzes performed for four different t/T ratios are shown in the Table 3.

<table>
<thead>
<tr>
<th>t/T</th>
<th>5mm</th>
<th>10mm</th>
<th>15mm</th>
<th>20mm</th>
<th>Max. Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,19</td>
<td>470</td>
<td>449</td>
<td>433</td>
<td>426</td>
<td>10,36%</td>
</tr>
<tr>
<td>0,40</td>
<td>490</td>
<td>469</td>
<td>452</td>
<td>438</td>
<td>11,74%</td>
</tr>
<tr>
<td>0,73</td>
<td>516</td>
<td>502</td>
<td>484</td>
<td>471</td>
<td>9,47%</td>
</tr>
<tr>
<td>0,99</td>
<td>549</td>
<td>542</td>
<td>523</td>
<td>502</td>
<td>9,33%</td>
</tr>
</tbody>
</table>

As can be seen from the Table 3, the increase of the inner fillet radius significantly reduces the Von-Mises stresses. At the maximum point, this difference in stress values reaches an average of 10%. As such, the correct determination of the inner fillet dimension has more effect than changing the weld size in order to reduce the maximum stress value.

The next comparison comprises models that do not have welding or external fillet connection, rather only internal fillet, and fillet plus welding applications. For this study, 7 analyses for vessels with 500mm inside diameter, 7 analyses for vessels with 1000mm inside diameter and 9 analyses for vessels with 2000mm inside diameter were completed. The ratio
of the nozzle to vessel thickness (t/T) is the main variable in this comparison. Six elements were applied throughout the nozzle thickness. In addition, the external weld fillet radius is fixed at 15mm and the crotch corner fillet radius is 5mm in each analysis. This ensures t/T is the only variable parameter as the effect of weld and inner fillet size variation were examined previously. Thereafter, the obtained results are compared to vessels with a sharp corner and having outer (weld) and inner fillets. The obtained maximum stress values and comparisons are given in Figure 9-11.
Pressure vessels with a diameter of 500mm, 1000mm and 2000mm were used in the analyses performed. When the stress values occurring in the vessel with the smallest diameter value (Di = 500mm) were examined, an average of 4% reduction in maximum stress values was observed for each nozzle shell thickness ratio (t/T) in external fillet applications (welding effect). On the other hand, when the vessel size increases, the stress values were less than 1.5% for vessels with an inner diameter of 1000 mm and less than average 1% for vessels with an internal diameter 2000 mm.

Considering the cases where the inner fillet and outer fillet were applied at the same time, the maximum stress values were reduced by about 10% for a 500mm diameter container and a 5% reduction for the others. In view of the results obtained, it is clear that the effect of the internal fillet application on the stress values is more effective than the external fillet (welding) effect. In the analysis, the outer fillet and internal radius dimensions have always remained constant. Therefore, the change in outer and inner fillet dimensions is explored further in subsequent studies. In this way, the ideal parameters can be established.

In addition to all these, the maximum stress region was observed on the crotch corner in each case. Since the location of the maximum stress does not change on each analysis, the only variable is the therefore magnitude of the maximum stress.

5.3. Multiple External Loads Applications
Multiple external loads for a cylindrical vessel configuration with single nozzle joints are considered to establish any interaction effects. For this study, the nozzle and cylinder radii ratio (di/Di) are 0.5 and the wall thicknesses ratio (t/T) is 0.24. The loading conditions to be applied simultaneously with the nozzle centre takes place in 5 steps and the details are given in Table 4 below.

<table>
<thead>
<tr>
<th>Case</th>
<th>Loading Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Case</td>
<td>Pint + P</td>
</tr>
<tr>
<td>2nd Case</td>
<td>Pint + MC</td>
</tr>
<tr>
<td>3rd Case</td>
<td>Pint + ML</td>
</tr>
<tr>
<td>4th Case</td>
<td>Pint + MC + ML</td>
</tr>
<tr>
<td>5th Case</td>
<td>Pint + P + MC + ML</td>
</tr>
</tbody>
</table>

The internal pressure in each case is 5MPa in magnitude and constant. Tensile force, Longitudinal Moment and Circumferential Moment are variables in the analysis. Some six analyses are performed for each case, and the loading magnitudes increased in each analysis whilst the pressure remains constant. In these analyses, the crotch corner fillet radius is 5mm and the welding fillet radius is 15mm. The local loads to be applied are represented in Figure 12.
The analysis results for 5 cases mentioned Table 4 before is graphically detailed as follows. The graph is given in Figure 13.

![Figure 13 Maximum Stress Values for Cases Mentioned Table 2](image)

5.4. Pressure Vessel with Two Nozzle Intersections

5.4.1. General

In some practical applications, it is required to have nozzles closely located in combination. If the nozzles are excessively close to each other, with a small ligament, this arrangement can cause the stress to increase significantly. The ligament distance between the nozzles should not exceed a certain ratio to prevent large stresses that may occur. If the nozzles need to be closer to each other than the calculated pitch, it may be necessary to apply additional reinforcement plates to the vessel. If two adjacent nozzles with the same diameter are to be considered, the minimum pitch P between them is determined by the formula below.
\[ P = d + A\sqrt{D T} \]  \hspace{1cm} (Eq.1)

Also, pitch is determined as \( P_L \) and \( P_T \) for longitudinal and axial directions respectively between nozzles. These positions are shown in Figure 14.

\[ \text{Figure 14 Dimensions of the longitudinal and axial distance [13]} \]

Here, the stress distribution for two nozzles with longitudinal, radial and both longitudinal and radial distances without reinforcement plates are examined. The results are obtained and interpreted for the stresses occurring in the crotch corner within the two nozzles arrangement. In this case, only internal pressure is considered. Also, crotch corner fillet radius and welding fillet radius are not changed to be consistent with previous analyses, and they are 5mm and 15mm, respectively.

### 5.4.2. Investigation of the Cylindrical Vessel with 2 Longitudinal Located Nozzles

Von-Mises stress results have been obtained for 3 different distances magnitude for 2 nozzles placed in longitudinal direction on the cylindrical shell. In these analyses, the nozzles are positioned symmetrically to the midpoint of the cylindrical vessel and the spacing of the nozzle centres are defined 200mm, 300mm, and 400mm, respectively. A representative image showing the distance between the nozzles is shown in Figure 15.

\[ \text{Figure 15 Longitudinal placed nozzles} \]

In these analyses, similarly to the single nozzle model, only the inner pressure was applied to the shell and the shell was constrained to move from one side and was exposed to thrust from the other side. Furthermore, the nozzles were also under the influence of thrust with internal pressure. To distinguish the nozzles from each other, the first nozzle is closer to the side that the thrust force affects and the further away nozzle from the axial thrust side is called the 2nd nozzle. The results of the analysis for each of the 2 nozzles are given in the following Figures 16-17.

As can be seen from the two graphs above, the maximum stress point is at the junction area of the first nozzle. Especially when the distance between the nozzles reaches 300mm and 400mm, the results are very close to each other and the
differences are around 1%. On the other hand, the difference between the stress values reached up to 12% in the case of 200 mm pitch. The reason for this is that the stress increases considerably when the nozzles are very close to each other due to the influence of the die-out of the first on the second. In fact, in these 3 cases, additional reinforcement plate is required when the minimum pitches are calculated. However, it is obvious that the stress values are very high for the nozzles with a distance of 200mm.

![Graph showing stress values](image16.png)

*Figure 16 Maximum stress values on 1st nozzle crotch corner*

![Graph showing stress values](image17.png)

*Figure 17 Maximum stress values on 2nd nozzle crotch corner*

5.4.3. *Varying the distance from nozzle centre to the end of the vessel*

Previously, the focus for the analysis was only on the distance between nozzles. On the other hand, as the distance between the nozzles increases, the distance between the nozzle centre and the end of the container decreases. As such, the distance between the nozzles was increased, and therefore the distance between the nozzle centre and the end of the vessel was decreased. Thus, the distance between the nozzle centre and the end of the vessel should be kept constant by adding the material to the end of the vessel and the new cases should be re-examined under these conditions as shown in Figure 18.
For the three cases shown in part b of Figure 18 above, the analysis for 6 different thickness ratios was performed. Results for 1st and 2nd nozzles separately shown in Figures 19-20 below.
When the values in the graphs are analysed, those for the 1st nozzle, closer to the side the thrust force affects have almost the same magnitude, the differences between the values being less than 1%. That is, the addition of material to the end of the vessel has not contributed physically to the stress values on the nozzles. Considering the 2nd nozzle, which is the nozzle close to the side where the boundary conditions are applied, it is observed that the difference between the stress values reaches up to 5% when the distance between the nozzles reaches 400 mm. As noted, the pressure vessel was restricted to the movement from one side and the thrust was applied from the other side. In the light of this, along with the last change, the 2nd nozzle has been located 400 mm further away from the thrust applied. In other words, the positioning of the nozzles far away from the applied force can be said to be an approach which reduces the stress values – which is common with industry practices and the use of a die-out distance.

5.4.4. Investigation of the Cylindrical Vessel with 2 Radial Located Nozzles

This study examines four different cases for pairs of radially located nozzles. Firstly, the Von-Mises stresses are examined for two nozzles positioned in the same plane but offset at 90 degrees. Subsequent studies are carried out with the offset angle between the nozzles at 60 degrees. In the final two cases, a length of 250 mm longitudinal offset is included on the nozzles positioned at 90 and 60 degrees. These situations are compared with each other separately. The representative image for positioning of the nozzles is given in Figure 21. \( P_L \) represents the longitudinal distance and \( P_T \) represents the radial distance between nozzles.

The nozzles placed at 90 degrees are examined in two configurations. The distance between the nozzles in the first arrangement, \( P_L \), is 0. In the second case, there is a 90-degree radial gap between the nozzles and an offset distance of
250 mm. In addition, the models are loaded by internal pressure and axial thrust driven by internal pressure. The results are presented in the following Figures 2

When the results are examined, it was observed that the stress values were lower in both radial and longitudinal nozzles. It is obvious from this that, as the area of ligament material between the nozzles increases, the stress values are observed to have decreased. In addition, in all studies, it is seen that the stress values decrease when the radial distance is reduced by 30 degrees. Also, in the model has P_r is 90 degrees, the maximum tensile stress was observed at the 1st nozzle near the corner positioned at the top of the shell. Conversely, when P_r was 60 degrees, the maximum stress was observed at the 2nd offset nozzle. The internal pressure effect is much more pronounced when the 2nd nozzle deflection angle decreases from 90 degrees to 60 degrees. A summary of the total stress values can be seen in Figure 23 below.

5.5. Limit load Analysis
Maximum limiting loading conditions to be applied in design and analysis approaches to be applied for pressure vessel problems are of great importance. The traditional design methods determine bending stresses as well as membrane stresses and a limit load are determined. A suitable safety factor for the system is then determined. In this study, external loading on the system will be examined together with the effect of internal pressure and limit load analysis is carried out. In this,
the model is free to the movement in all directions. The boundary conditions are applied only under saddle and the saddle base is fully fixed. The load action variables are internal pressure, external moment, and tensile force. It is important to determine the maximum internal pressure and external load values in cylinder-cylinder connection problems to prevent plastic collapse of the system. It is also important to determine whether the limit load values make a significant difference in the nozzle-cylinder intersection region (crotch corner) to reveal the necessity of this study.

Considering Perfect Plasticity and Tresca criteria for thick - walled open cylinders without any nozzle connection, first yield and limit load values can be calculated using the following equations. For vessels with nozzle connection, the results will be calculated with elastic-perfectly plastic analysis.

\[ F_{yield} = \frac{\sigma_y}{2} \left( 1 - \left( \frac{r_i}{r_o} \right)^2 \right) \]  

(Eq.2)

\[ F_{limit} = \sigma_y \ln \left( \frac{r_i}{r_o} \right) \]  

(Eq.3)

In the calculations made, the limit loads of only a perfect cylinder (Di = 500mm) subjected to internal pressure and container with a nozzle (Di = 500mm, di = 125mm, t = 15mm) were taken into consideration. Analyses were repeated for 4 different shell thicknesses (T). The results obtained are shown in the Table 5. As it can be understood from the results in the Table 5, as the wall thickness increases, the resistance of the container against the internal pressure increases, so the limit pressure values increase in both cases. Although the nozzle size remains constant, when T = 10mm, the difference between the two cases is 2.3%, while at T = 25mm, this difference has increased to 6.2%.

<table>
<thead>
<tr>
<th>T(mm)</th>
<th>Limit Pressure (MPa)</th>
<th>Limit Pressure (MPa)</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10,2</td>
<td>9,9</td>
<td>2,4%</td>
</tr>
<tr>
<td>15</td>
<td>15,1</td>
<td>14,4</td>
<td>5,2%</td>
</tr>
<tr>
<td>20</td>
<td>20,3</td>
<td>19,2</td>
<td>5,9%</td>
</tr>
<tr>
<td>25</td>
<td>24,8</td>
<td>23,3</td>
<td>6,2%</td>
</tr>
</tbody>
</table>

In another comparison analysis, the effect of the nozzle opening is examined on the limit pressure values. In these analyses, Di = 500mm, T = 15mm and t = 15mm. Only the nozzle inner diameter will be changed. Limit analysis results for 3 different nozzle inner diameters are given in the Table 6.

<table>
<thead>
<tr>
<th>di(mm)</th>
<th>Limit Pressure (MPa)</th>
<th>Limit Pressure (MPa)</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>15,1</td>
<td>15,1</td>
<td>0,6%</td>
</tr>
<tr>
<td>100</td>
<td>15,1</td>
<td>15,0</td>
<td>1,0%</td>
</tr>
<tr>
<td>125</td>
<td>15,1</td>
<td>14,4</td>
<td>4,9%</td>
</tr>
</tbody>
</table>

In such analysis, it is expected that as the nozzle opening increases, the stress values on the crotch corner increase further and therefore the limit loads are also reduced. It is observed that the difference between the two cases becomes more pronounced as the nozzle opening increases. It is noted that the limit load values make a significant difference in the nozzle-cylinder crotch corner. For this reason, the effects of nozzle diameter variation, nozzle and shell thickness changes and external force magnitude on limit loads are examined in detail.

5.6. Various Limit Load Analysis States

5.6.1. Changing Nozzle Diameter

Under this section, the variation of the limit pressure values according to the nozzle size will be discussed. In the analyses to be realized, the main shell inner diameter (Di) is 500mm, the main shell outer diameter (Do) is 530mm and the nozzle
thickness (t) is fixed as 15mm. The only variable is the nozzle inner diameter (di), and analyses are performed for 6 different nozzle sizes. The limit pressure values obtained for t/T = 1.0 are shown in the Figure 24.

As can be seen in Figure 24, as the nozzle diameter increases, the maximum internal pressure that the system can bear decreases. The reason why the limit pressure magnitude decreases as the nozzle diameter increases, is that the opening in the main shell increases and reduces the available material to support the load.

5.6.2. Changing Nozzle and Shell Thicknesses

Here, the nozzle and shell inner diameters are kept constant. The thicknesses start from 5mm and is increased in 5mm increments and the analyses repeated. The load acting on the system is internal pressure only and limit internal pressure is established as seen in Figure 25.

In both cases, as the wall thickness increases, the vessel becomes stiffer in supporting the internal pressure and larger loads are needed to develop plasticity in the system. While a 20mm thickness increase in the nozzle increases the limiting internal pressure by about 14%, this rate is close to 80% greater for the shell. These results show that the effect of thickness change on the limiting internal pressure is about 5 times higher for the nozzle than the shell – that is the nozzle carries the load more effectively than the nozzle in this case.

5.6.3. Changing External Loads
Here, the limit external loads are investigated whilst maintaining constant internal pressure values. The external loads investigated comprise the axial, longitudinal and circumferential forces. Internal pressure magnitudes are increased by 5MPa in each analysis. The results obtained from the analysis are shown in detail in Figure 26.

It is clear the tensile force (P) applied in the Z direction according to the graph is much more dominant than the other loads. The increase in pressure of 25MPa in the system leads to a 30% decrease in limit tensile force. In addition, the change in limit longitudinal and circumferential forces for the same rate of pressure increase is less than 2%. This is an indication that the limit load change is not affected by the pressure change when moment loads are present.

![Figure 26 Limit loads for changing external loads.](image)

6. RESULT AND DISCUSSIONS

A wide range of studies have been presented based on the high-quality model developed by the authors and a comprehensive parametric study undertaken. The accuracy of the model is supported by verification checks, based on some studies in the literature, for a variety of loads condition. After verification, various parametric studies related to limit loading conditions were made with single nozzle connections and multiple nozzle connections. The main purpose of the study was to reduce the maximum stress values that occur in the nozzle junction and to determine the maximum load values on the system under a variety of conditions. The following outcomes have been discerned from the various analyses.

Firstly, the stress conditions occurring in the nozzle-cylinder combinations were examined with sharp corner, inner and outer fillet applications for pressure loading only. The position of the maximum stress was found to be consistent with previous work [1], [2] and in line with WRC537 and other industry procedures. However, the present work addresses the adoption of inner and outer fillet radii, which is not taken into account in current codes and standards. The use of internal and external fillets is prevalent in industry and hence the current work has directly relevance to address the effect of those features.

The effect of outer fillet size on the stress values was found to have a minor influence of 1.5% for the cases considered, whilst the effect of the inner fillet size was around 10% on average. Hence these results show that one of the most effective ways to reduce the maximum stress is the provision of appropriate inner fillet geometry. Even with a small inner fillet, the stress is significantly reduced which minimised the potential for yielding or crack initiation. As such, a stress reducing factor of 0.9 could be introduced into codes and standards to take account of the present of an inner fillet at the crotch corner.
When considering the effect of combinations of sharp corner, inner fillet only and inner + outer fillet (welding), these variations were compared to the baseline case of the sharp corner inner fillet case as found in conventional design codes. It was noted that there was an improvement in reducing stress values by up to 10%, while this ratio decreases to 2% in the outer fillet. This is consistent with the earlier work and no significant additional benefit is found to have resulted as part of the combined effects. It is also worth noting that the only action on the system was that of the internal pressure and the nozzle was placed in the centre of the cylinder, and as such, the stress distribution was symmetrical, and the maximum stress was formed on the crotch corner. This is again as expected Figure 27 shows the maximum stress area on the crotch corner of the model under internal pressure only in the cutaway plot below.

From this, an investigation of the flexibility of the system and these analyses were repeated by changing the base vessel size. In this, the nozzle and cylinder diameter ratio was kept constant for each analysis (di / Di = 0.5) and analyses were completed for 3 different Di values (Di=500mm, 1000mm and 2000mm). From these studies (Figure 9-11), a reduction in maximum stress is always present, but the difference between the effectiveness of the internal fillet and external fillet decreases considerably. With the smaller vessel a 10% reduction is achievable whilst for larger diameter vessels, this reduces to 1.5%. The reason for this is that the fillet radius remained constant while the vessel size increased and this highlights the influence of flexibility in the system.

Thereafter, the model was exposed to various external load actions and examined for five different situations. In this study, the nozzle and cylinder radii ratio (di/Di) was fixed at 0.5, Di was 500mm and the wall thicknesses ratio (t/T) was 0.24. As the loading direction changed, changes in the stress location were observed as expected. In the case of external loading applications, the maximum stress moves to the welding zone location on the exterior and not on the crotch corner which was the case when under internal pressure. A representation of the stress distribution for combined loading is given in Figure 28.

In addition, the values obtained in external loading situations were also compared with the calculations in WRC537, with the results shown in the Figures 4-6. As can be seen in the graphs, the FEA and WRC537 results are nearly same, especially when the model is only under tensile loading. In models with the longitudinal and circumferential moment applied, the differences between the results of FEA and WRC537 reached up to 5% as the value of the applied moment increased. It
is noted that WRC537 calculations are always higher and hence more conservative. The FEA approach is therefore more accurate which can be helpful in cyclic loading and fatigue cases.

When considering multiple nozzle combinations, it can be seen the stress values decrease when the radial distance is reduced by 30 degrees. In addition, the stress values obtained increase when the longitudinal nozzles are positioned close to each other. Therefore, if the nozzles get too close to each other radially or longitudinally, there is the potential for plastic collapse to be present in the system. Therefore, maintaining the die-out distance as shown by the code equation remains important even in limit load analysis. It is also noted that the maximum stresses in the multiple nozzle analyses occurred on the crotch corner in all cases, when only the internal pressure is applied. Changes in the maximum stress magnitudes were observed when their relative positions changed. However, the stress magnitudes in the 1st nozzles, which are close to the thrust force, are greater in both longitudinal and axially positioned situations rather than at the 2nd nozzle position – hence showing the result of the combined interaction. These situations therefore need to be assessed on a case by case basis.

Finally, when considering limit load analysis for combined load cases, the variation of the load parameters shows the potential for high load carrying capability when combinations of load act together with internal pressure. This therefore can lead to the development of an interaction equation in future studies.

7. CONCLUSIONS AND FUTURE STUDY
The validated high-fidelity model developed by Bozkurt et al. has been further developed and made more comprehensive. A wide-ranging parameter study has been undertaken to examine the effect of various geometrical configurations for single and multiple load combinations. These arrangements include consideration of the effects of inner and outer nozzle fillet radii, the presence of weldments and nozzles in single and paired configurations. In addition, both stress analysis and limit load analyses have been considered for a range of d/D ratios for internal pressure acting alone and also in addition to combinations of axial thrust and moment loads. The result of these studies has also been compared to standard industrial practices as found in WRC Bulletin 537 where appropriate.

Overall, the models developed give excellent results compared with the literature. The key load variables have been examined and their influences quantified for single and multiple load case application. The use of radii at the junctions in nozzle arrangements has a real benefit, but only up to certain sizes. Externally profiled radii have more influence than internally profiled contouring. This can have up to 10% reduction in maximum stress values for certain cases. The models have also been extended to include for two nozzle combinations and also for single and combined load case analysis. The results show the benefit of maintaining the die-out distance and minimum ligament between nozzles especially when moment loadings are present. The results also show that WRC 537 is conservative which is important when designing against fatigue loading is a primary consideration.

Finally, based on the successful usage of this model, the effects of pad reinforcement, fracture applications and shakedown analyses are planned in further work. The ultimate goal is to develop a unified approach and an interaction equation that can be used to treat all cylinder-cylinder configurations.

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REFERENCES


