Three-phase OPF Based Local Flexibility Market for Mitigating Unbalanced Voltage in Distribution Systems

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***Abstract*—This paper presents a three-phase optimal power flow (TOFP) based method for mitigating the voltage unbalance factor (VUF) with the help of demand-side flexibility. It can be expected that issues with the unbalanced voltage can increase in the future distribution system with significant levels of single-phase Distributed Energy Resources (DERs). Unhealthy voltage unbalance status reduces the efficiency of the distribution systems, leading to energy loss and heat problems in electrical devices. This paper has proposed a three-phase OPF (TOPF) based local flexibility market (LFM) framework, where Distribution System Operators (DSOs) can utilize the upwards or downwards flexibility to manage the voltage unbalance. A modified IEEE-34 bus distribution network is used to illustrate the implementation of the proposed methodology, with the simulation results showing that the VUF value varies with the unbalanced load and connected photovoltaic (PV) levels, while the VUF drops below the limiting threshold when the flexible phase-by-phase flexibility services are introduced. This means that the proposed approach could inform modification of flexible service product in the future local flexibility market framework to help DSOs mitigate the unbalanced voltage.**

Keywords—Distributed energy resource (DER), local flexibility market, three-phase optimal power flow (TOPF), demand-side flexibility, voltage unbalance factor (VUF).

I. INTRODUCTION

Most power system analysis and tools used for enabling the provision of flexibility by Distributed Energy Resources (DERs) assume that three-phase distribution networks operate under balanced conditions. In practice, however, this may not be the case. In addition to existing problems due to unbalance of load connections, integration of Distributed Generators (DGs) may exuberate the problem and lead to exceeding three-phase balance limitations defined by the security standard [1]. This possible negative impact of, particularly, small-scale renewable resources is due to the nature of their, often single-phase, connections. Similarly, Electric Vehicle (EV) charging points at home and demand-side response are or expect, to be single-phase connections. This leads to additional system management challenges, requiring a cost-efficient method for meanwhile, the increasing number of small-scale renewable energy units lead to system management challenges, requiring a cost-efficient method for Distribution System Operators (DSOs) to operate distribution system safely.

The local flexibility market (LFM) approach has started to emerge as one of the choices for DSOs to address issues due to DER integration and manage distribution network operation. Such markets can provide an efficient solution to allocate the available flexibility resources, yet, so far, the focus has been on a market design, congestion management, voltage management, and power balance for balanced systems [2]. There has been less attention towards using LFM to address the voltage unbalance, which is also an important factor in safe system operation [3]. For example, there are ever-increasing connections of single-phase rooftop photovoltaic (PV) panels integrated into the low voltage network.

In the past, solutions to deal with a voltage unbalance in the three-phase network were based on some costly devices such as phase switches, on-load tap changers (OLTC), and compensators [4]. Thus, there is an increasing need to consider a more cost-efficient approach to enable DSOs to monitor and manage the operation status of each phase, which can include phase balancing by utilizing flexible resources connected at each of the phases. This requires the three-phase network representation, as well as the three-phase optimal power flow (TOPF) to model operation and clearing of flexibility services and LFM. Bruno et al. [5] presented TOPF formulation and simulation software based on OpendDSS tool, however, it does not include management of the voltage unbalance, while Araujo [6] put forward a primal-dual interior-point TOPF to reduce the overall voltage unbalance status of the whole system using the reactive power from DGs. Furthermore, Fanjun [7] discussed both power balance and current injection methods for TOPF formulation, however, the objective function included only costs associated with energy production, which would not be suitable for clearing of flexibility markets.

In this paper, a TOPF based flexibility market has been presented, where the DSOs can access the upwards and downwards flexibility from the demand side to manage the unbalanced voltage in each three-phase busbars. A modified IEEE 34-bus network is used to illustrate the proposed approach under different penetration levels of unbalanced load and PV.

The main contribution of this paper includes:

1. Propose a TOPF based local flexibility market framework, including both upwards and downwards flexibility market.
2. Consideration of voltage unbalances limitation as inequality constraint which is convent for DSOs to manage the voltage unbalance in each of three-phase busbars.

II. LOCAL FLEXIBILITY MARKET

The organization and design of LFMs have been introduced for managing network capacity issues [8]. Similar to other formulations, this paper assumes the active load would participate into the flexibility market, often through an aggregator. Typically, the demand side flexibility has been classified as: (i) upwards and (ii) downwards, which are separately aggregators on the behavior of a group of active loads and submit the bids, available upwards and downwards flexibility power, and flexibility connection position into LFM, while on the other hand, the DSOs need to provide their requirement such as flexibility amount and voltage unbalance value into the market operators, and after the market-clearing process, the corresponding single will be sent back to aggregators and system operators.

The available upwards and downwards flexibility power provided from an aggregator is shown in Fig. 1 and Fig. 2. The demand side flexibility provider is treated as a negative generator, and, in this case, the available upwards flexibility amount is the gap between power consumption baseline and low limitation, while the available downwards flexibility is the distance from baseline power to up limitation.

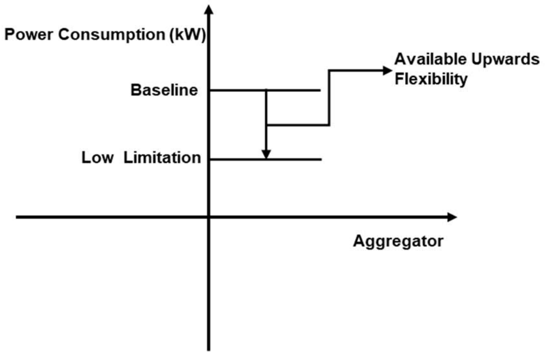

Fig. 1. Upwards flexibility of an aggregator

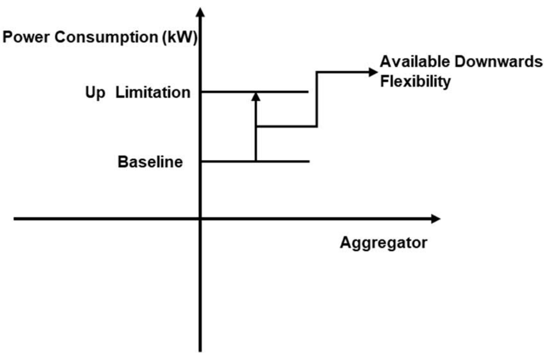

Fig. 2. Downwards flexibility of an aggregator

The upwards flexibility in this paper means the flexibility provider is willing to reduce its power consumption for compensation, with the cost function of using upwards flexibility defined as:

$$= \quad \times . \quad - \bar{A} \quad) \times \Delta \qquad (1)$$

where is the upwards flexibility cost coefficient, and are the actual power consumption value and pre-scheduled demand-side consumption plan, respectively, while ΔĀis the duration of a flexibility service.

Similarly, the downwards flexibility in this paper means the flexibility provider willing to adjust their power consumption plan to a higher level, also for receiving a reward, with the cost function of using downwards flexibility defined as:

$$= \bar{A} \quad * (\quad - \bar{A} \quad) * \Delta \qquad (2)$$

where is the downwards flexibility cost coefficient, and are the actual power consumption value and initial demand-side consumption data. It should be noted here the cost function of upwards and downwards flexibility are always positive in this paper.

III. PROBLEM FORMULATION

To address the voltage unbalance problem in future distribution systems, the unbalanced network model and the three-phase OPF need to be considered in the LFM framework.

A. Unbalanced Feeder Model

A three-phase overhead line, underground cable, or other conductors in a distribution system are modeled as -model, shown in Fig. 3.

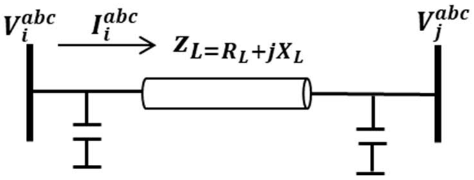

Fig. 3. -model of unbalanced three-phase feeder

Furthermore, the relationship between injection current from bus *i* to bus *j* and phase voltage at each of the buses *i* and *j* are defined as,

$$\begin{bmatrix} \bar{A},\bar{A} \\ \bar{A},\bar{A} \\ \bar{A}, \\ \bar{A},\bar{A} \\ \bar{A},\bar{A} \\ \bar{A}, \end{bmatrix} = \bar{A} \quad \begin{bmatrix} \bar{A}, \\ \bar{A}, \\ \bar{A}, \\ \bar{A}, \\ \bar{A}, \\ \bar{A}, \end{bmatrix} \qquad (3)$$

where is the three-phase admittance matrix of the distribution lines. The value of the can be estimated by the well-known Carson's equation [9]. Thus the structure of is a 6×6 matrix, where the diagonal elements correspond to self-admittance, while off-diagonal elements are mutual admittance elements. It should be mentioned that the phase voltage, etc., depends on the considered and adjacent phase as the mutual effect between each phase in the distribution system. The typical distribution system includes single and two-phase feeders, and therefore in this paper, any missing elements of the feeder matrix will be set to zero. For simplicity, transformers, capacitor banks, and voltage regulators have been omitted in this paper.

Noted that the admittance matrix is complex and usually written as $= \bar{A} +$, where and are the conductance and electrical susceptance matrix, respectively.

B. Distribution System Model

Assume a distribution system contains N bus in the set $= \{1, \dots, \bar{A}\}$, and $\emptyset = \{\bar{A}, \bar{A}, \ \}$ represent different phase of each bus, the set of line connected between buses represented by $= \{1, \dots, \}$. Let $3 \times N$-dimensional complex vectors $= [\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{C}^{\times}$ represent the voltage $_{,Āh}$ at bus $\in$ in each phase $h \in \emptyset$, with the voltage represented in a rectangular coordinate, so that,

$$= \quad + \tag{4}$$

where, $= [\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$ and $= [\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$ are the real and image component of **v**. If any bus of the modeling system contains only one or two phases, then zero elements are used to replace the corresponding elements in vectors and . According to the system topology, the vector $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{C}^{\times}$ and $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{C}^{\times}$ stand for the *from* end and *to* end voltage of the distribution feeder.

The load can be divided into non-flexible and flexible. With the $_{,Āh}$ and $_{,Āh}$ denoting the non-flexible load at bus $\in$ in each phase $h \in \emptyset$, and then define the $3 \times N$-dimensional vectors $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$ and $= [\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$.

For the flexible load, $_{,Āh}$ and $_{,Āh}$ are, respectively, the upwards and downwards flexibility power from aggregators at bus $\in$ in each phase $h \in \emptyset$. We can define $3 \times N$-dimensional vectors $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$ and $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$. If there is not upwards or downwards flexibility connected at bus $\in$, in one of the phases $h \in \emptyset$, then the element at the corresponding position in the above vectors is zero.

For modeling PV generators, $_{,Āh}$ and $_{,Āh}$ terms represent the real and reactive power output from PV units at bus $\in$ in each phase $h \in \emptyset$, defining the $3 \times N$-dimensional vectors $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$ and $=$ $[\ _{1,}\ ,\ _{1,}\ ,\ _{1,}\ , \dots ,\ _{,}\ ,\ _{,}\ ,\ _{,}\ ] \in \mathbb{R}^{\times}$. Again, if no PV is connected in each phase $h \in \emptyset$ at bus $\in$, then $_{,Āh} = \ _{,Āh} = 0$.

C. Three Phase OPF

TOPF can help DSOs to manage the unbalanced network in high penetration level of single-phase DERs scenarios. DSOs need to know the phase information and, with the help of TOPF, address the problems of how much, when, and where to utilize flexible services to ensure safe and cost-effective operation of the system. This section describes in detail the different components of the TOPF model.

1) Objective function

The objective function in this paper is minimum of the total upwards and downwards flexibility cost defined as,

$$= \Sigma \quad + \Sigma \tag{5}$$

where $\sum$ and $\sum$ are the total upwards and downwards flexibility cost, respectively.

2) Phase power balance constraint

In this paper, a rectangular coordinate system is used to represent the phase power balance equality constraint,

$$= \mathrm{diag}(\ )(\quad - \quad) + \mathrm{diag}(\ )(\quad + \quad) \tag{6}$$

$$= \mathrm{diag}(\ )(\quad - \quad) - \mathrm{diag}(\ )(\quad + \quad) \tag{7}$$

$$= \quad - \quad - \quad - \tag{8}$$

$$= \quad - \tag{9}$$

The net power injection of active and reactive power is first defined by (6) and (7), with diag( ) and diag( ) returning square matrix with and as the diagonal. These net injections and are equal to active and reactive power injections from connected PVs, load, and flexible services, as defined in (8) and (9).

3) Distribution feeder congestion constraint

With the high penetration level of DERs, a bidirectional power flow scenario should be considered, which, for distribution feeders, are included via (10) and (11), within bidirectional real power flow limits defined by (12) and (13), where the $\Re(\cdot)$ and $(\cdot)^*$ operations denote the real parts of a complex number and complex conjugate, respectively, so that,

$$= \mathrm{diag}(\quad)(\qquad)^* \tag{10}$$

$$= \mathrm{diag}(\quad)(\qquad)^* \tag{11}$$

$$\Re(s\ ) \leq \tag{12}$$

$$\Re(s\ ) \leq \tag{13}$$

In the above equations $\in \mathbb{C}^{(3\times\ )\times(3\times\ )}$ and $\in \mathbb{C}^{(3\times\ )\times(3\times\ )}$ are the *from* end and *to* end branch matrix of the distribution system, while and are the real power flow limit of *from* end and *to* end of distribution feeders, respectively.

4) Voltage and voltage unbalance constraint

The voltage magnitude constraint at bus *n* in each phase $\emptyset$ is defined as,

$$\leq \left|\ _{,Āh}\right| \leq \qquad \forall \quad \in \quad, \forall \quad h \in \emptyset \tag{14}$$

where, and are the lower and upper limit of voltage, and $|\cdot|$ denote the absolute value of a corresponding complex voltage. To maintain the voltage angle shift of 120° for each phase, and at each phase, the secondary side of the distribution transformer is considered as the reference bus where voltage angle shift of 120° is enforced, as defined below,

$$_{1,} = \ _{1,} * \tan(0^{\circ})$$

$$_{1,} = \ _{1,} * \tan(-120^{\circ})$$

$$_{1,} = \ _{1,} * \tan(120^{\circ}) \tag{15}$$

A true definition of voltage unbalance called voltage unbalanced factor (VUF) is adopted in this paper. It is equal to the ratio of the negative sequence voltage component to the positive voltage sequence component [10], that is,

$$= \frac{\quad}{\quad} * 100\% \tag{16}$$

Furthermore, the negative and positive sequence voltage at bus $\in$ is defined as,

$$= \frac{1}{3}(\ _{,} + \ ^{2}\ _{,} + \ _{,}) \quad \forall \quad \in \tag{17}$$

$$=\frac{1}{3}\left({}_{\bar{A},\bar{A}} + \bar{A}\ {}_{\bar{A},} \ + \bar{A}^{2}\ {}_{\bar{A},} \right) \quad \forall\, \bar{A} \in \bar{A} \quad (18)$$

where $= \frac{-\bar{A}2}{3}$. Then, which represent the voltage unbalance degree at bus $\bar{A} \in$, is defined as,

$$\bar{A}\bar{A}\bar{A} = \left| \overline{\qquad\qquad} \right| \qquad \forall\, \bar{A} \in \bar{A} \qquad (19)$$

Let represent the voltage unbalance limit at bus $\bar{A} \in \bar{A}$, and then define an *N*-dimensional vector $= [\bar{A}\bar{A}\bar{A}_1 \quad , \ldots , \qquad] \in \mathbb{R}$. Before launching flexibility services, DSO can submit a predefined safety value of VUF to the market operator, and, in general, the value of is equal to 2% according to the suggestion of the Engineering Recommendation P29 [11]. The voltage unbalance security constraint used in TOPF is defined as,

$$\bar{A}\bar{A}\bar{A} \le \qquad\qquad \forall\, \bar{A} \in \qquad (20)$$

It should be noticed here, for the non-three-phase bus of the distribution system, a high default value needs to be set at the corresponding position in the vector so that the constraint is not activated during the TOPF process. The proposed approach is flexible enough for DSOs to manage the voltage unbalance status at each three-phase busbar.

5) *Flexibility constraint*

The flexibility limitation of upwards and downwards is defined as,

$$\bar{A}_{\bar{A},\bar{A}h}^{\ \_\bar{A}\bar{A}\bar{A}} \le \bar{A}_{\bar{A},\bar{A}h} \le \bar{A}_{\bar{A},\bar{A}h}^{\bar{A}\bar{A}\_\bar{A}\bar{A}\bar{A}\bar{A}} \qquad \forall\, \bar{A} \in \ , \bar{A}h \in \emptyset \quad (21)$$

$$\bar{A}_{\bar{A},\bar{A}h}^{\bar{A}\bar{A}\bar{A}\_} \qquad \le \bar{A}\ {}_{,\bar{A}h} \quad \le \bar{A}_{\bar{A},\bar{A}h}^{\bar{A}\bar{A}\bar{A}\_} \qquad \forall\, \bar{A} \in \bar{A},\, \bar{A}h \in \emptyset \ (22)$$

where ${}_{\bar{A},\bar{A}h}^{\ \_\bar{A}\bar{A}\bar{A}\bar{A}}$ and ${}_{\bar{A},\bar{A}h}^{\bar{A}\bar{A}\bar{A}\_}$ are the pre-scheduled power consumption plan submitted by aggregators. ${}_{\bar{A},\bar{A}h}^{\ \_\bar{A}\bar{A}\bar{A}}$ and ${}_{\bar{A},\bar{A}h}^{\bar{A}\bar{A}\bar{A}\_}$ are the minimum and maximum limitation of upwards and downwards flexibility.

IV. CASE STUDY

A modified IEEE-34 bus network was used to illustrate the proposed method, in which the base demand includes unbalanced three-phase load, single-phase load, and also balanced three-phase load. The active load in Fig. 4 represents the flexibility providers, and the flexibility location and capacity are shown in Table I.

In this section, two case studies will be analyzed, the first one to study the impact of different penetration levels of PVs in three-phase networks on VUF, while the second seeks to investigate the impact of different levels of unbalanced loads on VUF, as well as how the voltage unbalance restriction conditions should be considered to ensure secure system operation.

The proposed methodology is implemented in MATLAB 2020b and this non-linear optimization problem is solved by the Interior Point algorithm through Optimization Toolbox.

Assuming the bus voltage is in the range of 0.94 ~ 1.06 p.u., the VUF limit is set to 2% in this paper. The distribution transformers and shunt capacitors have been neglect in this study, with an additional assumption that the power flowing through the distribution line does not reach the maximum rating of each line. Therefore, the congestion scenario is not considered here.

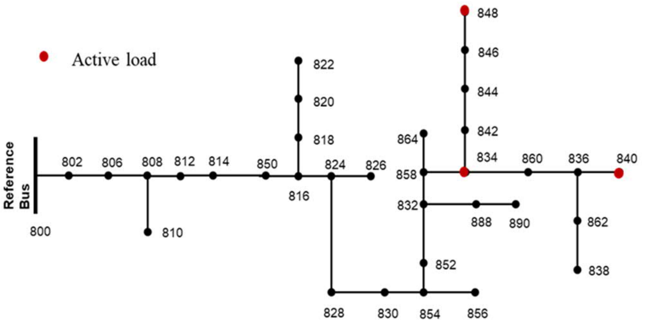

Fig. 4. Modified IEEE-34 nodes test network

TABLE I. FLEXIBILITY CAPACITY AND POSITION

Bus	**Upward (kW)**			**Downwards (kW)**		
	Ph- A	***Ph-B***	***Ph-C***	***Ph- A***	***Ph-B***	***Ph-C***
834	12	n/a	n/a	20	n/a	n/a
840	10	7	n/a	10	9	n/a
848	8	13	6	8	8	7

To study the overall unbalanced voltage trend of different scenarios in the distribution system, the average VUF index has to be introduced in (23).

$$= \frac{\Sigma\bar{A}\bar{A}\bar{A}}{\bar{A}h\bar{A}\bar{A}\bar{A}-\bar{A}h\bar{A}\bar{A}\bar{A}} * 100\% \qquad (23)$$

A. *Case study 1: PV integration*

The power output from PVs will make the net power injection deviated from the predicted state and therefore affect the VUF value. It is assumed that the maximum penetration level of PVs in each phase is 276kW, 56kW, and 356kW, respectively. In this case, for a benchmark test, the VUF values of the system when the distribution system is not connected to renewable generators are used.

This simulation tested the influence of phase-by-phase flexibility service on the penetration level of PVs from 0% to the maximum value in steps of 25%. The results recorded the corresponding bus VUF values for different scenarios, as presented in Fig. 5. It can be observed that the VUF increases as the percentage of PV generation rise, with the VUF value approaching the set maximum when the penetration rate reaches 50%. If the unbalance voltage constraint is not considered in the process of three-phase optimal power flow, the VUF value of the system will not meet the safe operation standards when the PV continues to increase above 75%, as indicated by the black horizontal dotted line in Fig. 5. To address this problem of high VUF values, it is necessary to include VUF limitations, which, for the high PV injections start becoming active at the renewable power penetration rate of 75% and above, as can be seen from Fig.5. Due to the utilization of phase-by-phase flexibility services, which are managed by TOPF, the system returns to a safe operating state and within the VUF constraint.

The average VUF value considers without and with voltage unbalance limitation is shown in Table II. It can be seen that the proposed method in this paper can reduce the overall voltage unbalance of the distribution system and thus improve the hosting capacity of renewable energy.

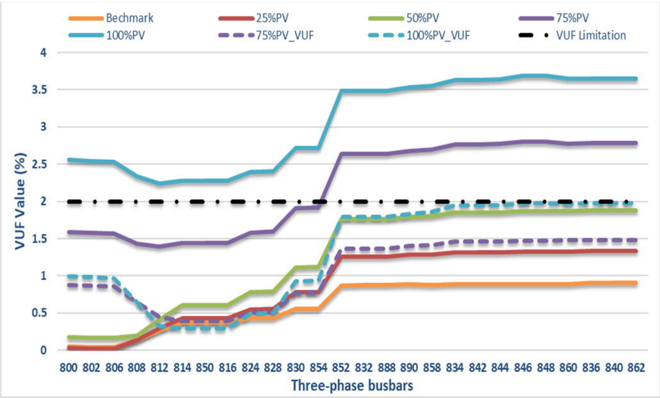

Fig. 5. VUF results comparison between with and without voltage unbalanced constraint in different penetration level of PV

TABLE II. AVERAGE VUF VALUE IN DIFFERENT PV LEVEL

(%)	The different penetration level of PV				
	0%	25%	50%	75%	100%
Without limit	0.616	0.873	1.247	2.202	3.065
With limit	0.342	0.480	0.474	1.061	1.322

The total demand-side flexibility cost is shown in Fig. 6. When the PV penetration level is low, whether or not the voltage unbalance constraint is included in the demand-side response has no impact on the overall flexibility cost, as the voltage unbalance constraint is not active, but when the system unbalance voltage begins to exceed the safe range as PV continues to increase, operators need to reallocate demand-side flexibility to invoke more downwards flexibility resources to bring the system back within the safe state and therefore incur additional costs in the process.

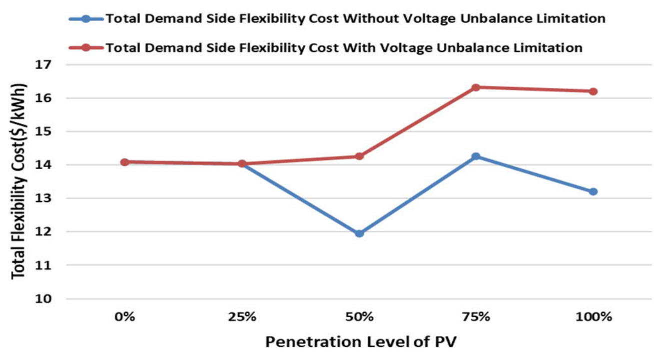

Fig. 6. Overall demand-side flexibility cost

B. *Case study 2: Unbalanced load integration*

In addition, unbalanced load distribution is an important factor, which may cause voltage unbalance, and this simulation assumes that the baseline load of the system is increased proportionally to 160% of the initial state in steps of 20%. It can be seen from Fig. 7, that the unbalance voltage of the system gradually increases with the increase of load demand, which is because the unbalance current gradually rises with the unbalance of the load, thus causing the deviation of the system voltage to increase, and when the unbalance load increased to 140%, the VUF of the system is already not within the allowed safe operating range. As can be seen from Fig. 7, adding unbalance voltage limitation constraints for 140% and 160% of the baseline load condition can reduce the unbalanced voltage level of the system to the allowable range.

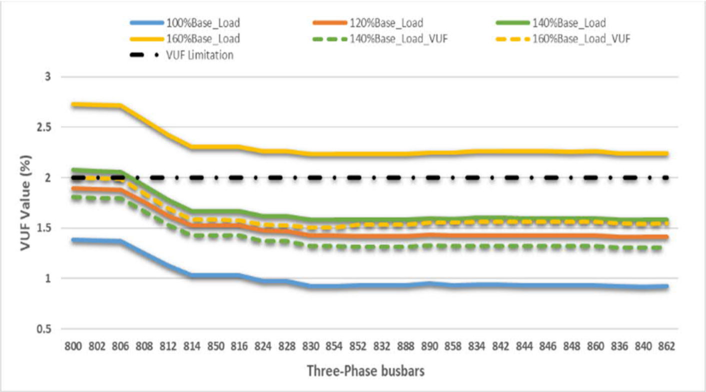

Fig. 7. VUF results comparison between with and without voltage unbalanced constraint in different level of unbalanced load

V. CONCLUSIONS

Presented case studies show the benefits of allowing the provision of flexible services for restoring voltage balance in a three-phase distribution network. In the case of PVs, it allowed additional DG connections. Besides, we found that if the unbalanced loads level continued to increase, or PVs level becomes very high, it might be hard to remedy the voltage unbalance problem if there is insufficient phase-by-phase flexibility provision. This issue, however, is not within the scope of this paper.

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