Structuring Light to Rotate Optical Turing Patterns and Solitons

Alison M. Yao
Department of Physics
University of Strathclyde
Glasgow, Scotland.
alison.yao@strath.ac.uk

Christopher J. Gibson
Department of Physics
University of Strathclyde
Glasgow, Scotland.
christopher.gibson.100@strath.ac.uk

Gian-Luca Oppo
Department of Physics
University of Strathclyde
Glasgow, Scotland.
g.l.oppo@strath.ac.uk

***Abstract*—The interplay of diffraction and self-focussing nonlinearity in an optical cavity results in the formation of spatial Turing patterns. Using helically-phased pumps, the patterns form on rings and rotate. We show how fully-structured light can be used to gain precise control over the rotation speed.**

Keywords—orbital angular momentum, Turing patterns, cavity solitons

I. INTRODUCTION

The interplay of diffraction and self-focussing (Kerr) nonlinearity in an optical cavity driven by a plane wave is well-known to result in the formation of stationary spatial patterns in the transverse profile of the transmitted beam. This is well described by the Lugiato-Lefever equation (LLE) in two transverse dimensions [1]:

$$\partial_t E = P - (1 + i\theta)E + i\beta|E|^2 E + i\nabla^2 E \qquad (1)$$

where E is the intracavity field, P is the amplitude of the input pump, θ is the detuning between the input pump and the closest cavity resonance, β is proportional to the Kerr coefficient of the nonlinear material, and the term with the transverse Laplacian ∇^2 describes diffraction and can be written in either Cartesian or polar coordinates.

II. STRUCTURED LIGHT PUMPS

When the pump is structured to have a helical phase profile, $P = P_m e^{im\varphi}$, and therefore carry orbital angular momentum (OAM) of quantum number m, the resultant Turing patterns form on rings that rotate [2] about the on-axis optical vortex. We show analytically that the speed of rotation is $\omega = 2m/R^2$, demonstrating that it can be controlled both by the magnitude of the OAM, m, and by the radius of the ring, R.

A. Cylindrical Vector Beams

Cylindrical vector beam [3] pumps consist of orthogonally polarized Laguerre-Gaussian (LG) eigenmodes with equal and opposite OAM:

$$\boldsymbol{E} = \cos\gamma\ \mathrm{LG}_m e^{i\alpha}\,\boldsymbol{e}_L + \sin\gamma\ \mathrm{LG}_{(-m)}\boldsymbol{e}_R, \qquad (2)$$

where $\boldsymbol{e}_L$, $\boldsymbol{e}_R$ correspond to left and right-handed circular polarization, respectively, m is the OAM, and γ and α control the weighting of the modes and the phase difference between them, respectively. These have a cylindrically-symmetric spatially inhomogeneous polarization distribution.

We show that by varying γ from 0 to $\pi/2$ to change the relative weightings of the eigenmodes, we can obtain full control over the angular velocity of the pattern in the range $-2m/R^2 \leq \omega \leq 2m/R^2$.

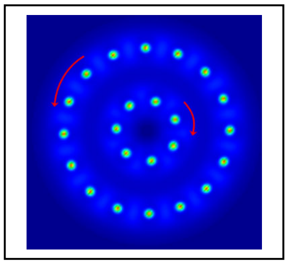

Figure 1: "Optical peppermill" formed by vector pump with eigenmodes with OAM of m = 8 and m = −1.

B. Poincaré Beams

Using Poincaré beams [4] that consist of orthogonally polarized eigenmodes with different magnitudes of OAM, m_L, m_R, we show that the resultant angular velocity of the optical patterns is $\omega = (m_L + m_R)/R^2$ if there is good overlap between the eigenmodes. However, if there is no, or very little, overlap between the modes then concentric Turing pattern rings, each with angular velocity $\omega = 2\, m_{L,R}/R^2$ will result. This can lead to, for example, concentric, counter-rotating Turing patterns creating an optical peppermill-type structure, as shown in the figure above.

Full control over the speeds of multiple rings has potential applications in particle manipulation and stretching, atom trapping, and circular transport of cold atoms and BEC wavepackets.

C. Cavity Solitons

The LLE admits homogeneous stationary transverse solutions according to

$$|P|^2 = I_s[1 + (I_s - \theta)^2], \qquad (3)$$

where I_s is the stationary intensity. For detunings $\theta > \sqrt{3}$ the steady-state curve is S-shaped, allowing the formation of bright, stable, non-diffracting spots of light known as cavity solitons (CS) [5]. We demonstrate that the effect of pumping with light carrying OAM is to increase the effective detuning by an amount m^2/R^2, allowing the cavity solitons to form at lower values of the detuning. The CS are shown to rotate at angular velocities determined by the net OAM and distance from the beam centre, as before. Moreover, by including a radial phase profile, the CS can be made to travel in unusual spiral paths.

III. CONCLUSION

We have demonstrated formation and rotation of spatio-temporal Turing patterns in self-focussing (Kerr) nonlinear optical cavities pumped by beams carrying orbital angular momentum, m. For scalar pumps we see the formation of patterns on a ring, or concentric rings, around an optical vortex that rotate at angular velocity $\omega = 2m/R^2$, where R is the radius of the ring. Using cylindrical vector beams, that have eigenmodes with equal and opposite OAM m, the biasing of the relative weightings of the eigenmodes allows the angular velocity to be controlled precisely in the range of

$-2m/R^2 \leq \omega \leq 2m/R^2$. Using Poincaré beams, that have overlapping eigenmodes with different magnitudes of OAM m_L, m_R, the resultant angular velocity is $\omega = (m_L + m_R)/R^2$. Finally, by increasing the cavity detuning and adding a radial phase, we demonstrate the existence of spiralling cavity solitons. These results are of interest for particle manipulation and control as well as for inducing the circular transport of cold atoms and BEC wavepackets using opto-mechanic nonlinearities instead of Kerr media [6].

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